# SELF-BALANCING BICYCLE USING HYBRID OF REACTION WHEEL AND STEERING CONTROL

by

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Master of Engineering in Mechatronics

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# AUTHOR'S DECLARATION

I, Pornchanok Vanich declare that the research work carried out for this thesis was under the regulations of the Asian Institute of Technology. The work presented in it is my own and has been generated by me as the result of my original and has not been submitted to any other institution to obtain another degree or qualification. This is a true copy thesis, including final revisions.

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Lastly, I would like to express my gratitude to my family for having my back.

# ABSTRACT

According to the study, the self-balancing bike has a new design and is now available in adult sizes. Additionally, the self-balancing bike must be in the upright position without the rider. To combine the two balancers together can get the better performance. By combining a 250-Watt DC speed motor to rotate the reaction for producing torque and a 100-Watt DC motor to control the steering handle, a system can be balanced. IMU sensor MPU 6050 will be used to gauge the bicycle's tilt angle. The microcontroller of the self-balancing bicycle is Arduino Due and Arduino Mega 2560. Since the lithium-ion battery's discharge rate is higher than that of a lead-acid battery, it is used in the system. Additionally, the battery's current does not drop, and it can function as the system's power source.

Linear quadratic regulation (LQR) is used as a controller in the self-balancing bicycle that uses a hybrid reaction wheel and steering control. The LQR is then used in conjunction with state space to determine the gain G. The dynamic model's calculated gain G will be used to stabilize the system. Additionally, this controller may choose the Q and R matrices to match the system. Finally, the system can reach zero on its own.

Keywords: Self-balancing Bicycle, LQR, Reaction Wheel, Steering Control

# CONTENTS

# LIST OF FIGURES

**Page**

# CHAPTER 1

# INTRODUCTION

## 1.1 Background of Study

The first bicycle was built in 1817 by german. The bicycle that we know has been designed since the $19^{th}$ century. In the 1880's and 1890's the electric bike was built in France and United States. For now, it has a self-balancing bicycle or riderless bicycle.

For the normal bicycle, the rider must control the balance of it by themselves. They make it move by using pedals, crank, handle, and wheel. Thus, it is not easy for disabled people for riding a bicycle by themselves. Moreover, for the young children who still cannot balance a bicycle, it may make them fall or injure. Thus, the electric bike or Ebike has an important role in our daily life.

The electric bicycle uses a motor to make it move forward. But the children cannot ride it because most electric bicycle is for adults. It has a higher speed than a normal bicycle. The electric bicycle has been developing for the future with cleaner technology. In the next several years, we may have the opportunity to use the electric bike for making our life easier. Because the bicycle is a friendly environment vehicle for transportation or exercise.

On the other hand, not all humans can ride a bicycle by themselves for example children. Therefore. The self-balancing bicycle was built for people who cannot ride a bicycle and make it safer for the rider. The self-balancing bicycle has the ability for stabilizing itself without falling. The normal bicycle cannot stand without a rider, but the self-balancing bicycle can balance with or without a rider. In the past, self-balancing was balanced by many controllers and solutions.

Thus, this self-balancing bicycle is using concept hybrid of a reaction wheel and steering for making the bicycle stand. When the bicycle moves reaction wheel will spin, and the steering rotates in the opposite direction of the leaning angle for balancing a bicycle. The reaction wheel and steering are creating the torque and force to control the bicycle. Then, the bicycle will be stable.

## 1.2 Statement of the Problem

In the past, a self-balancing bicycle robot was controlled by several solutions. Another self-balancing bicycle robot does not work well. However, the self-balancing bicycle robot needs to stabilize and another balancing algorithm. The self-balancing bicycle needs to be further investigated. Thus, this self-balancing bicycle uses the hybrid of reaction wheel and steering to control the stability of a bicycle.

## 1.3 Objectives

This research is aiming to develop a self-balancing bicycle using hybrid of reaction wheel and control steering to balance a bicycle. The objectives are as follows.

1. To balance a bicycle using hybrid of reaction wheel and control steering.
2. To design the controller to balance

## 1.4 Limitation and Scope

1. The maximum speed of self-balancing bicycle is 2m/s.
2. The width and height are 60*100 centimeter.
3. The self-balancing bicycle robot do not has a rider.

# CHAPTER2

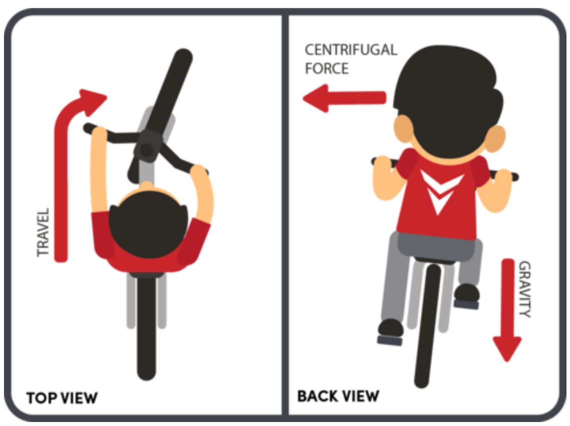
# LITERATURE REVIEW

## 2.1 Bicycle Balancing by the Rider

The rider can balance a bicycle by using the steering handle. When the bicycle leans to the ground, the rider should move the steering handle in the opposite direction to balance the bicycle. The steering generates centrifugal force in a different way from the way gravity pulls the bicycle over. For balancing a bicycle, the rider should lean their body in the opposite direction.

**Figure 2.1**

*Centrifugal Force*



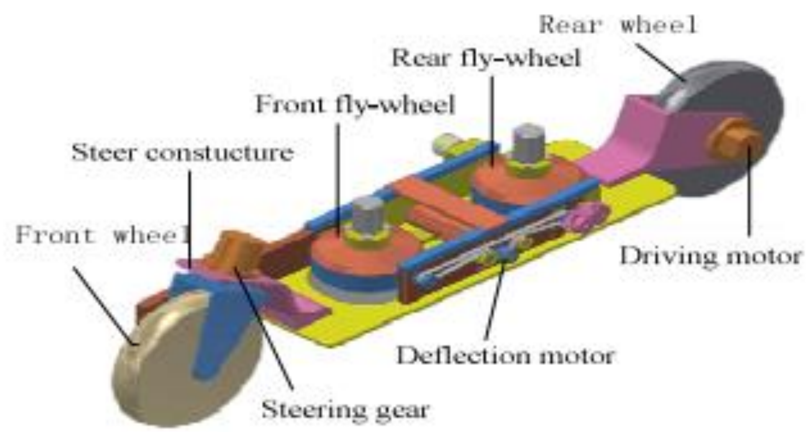

## 2.2 The Previous Design of the Self-balancing Bicycle

The previous version of the self-balancing bicycle uses many concepts to balance the bicycle. The system combines the steering handle and balancer, the reaction wheel, or only the steering handle. However, only the steering handle uses for high-speed systems, sometimes it cannot balance during low speed. In addition, the bicycle is a nonlinear system. Thus, it is challenging to find the best balancing system for the bicycle.

Gyroscope effect theory and sliding mode control (SMC) are used by the two-wheel self-balancing robot to stabilize it. The robot can be balanced by using sliding mode control from the initial roll angle up to ±45°. The front wheel's steering can stabilize around ±10°from the equilibrium. The two flywheels make the robot balance. Thus, the robot should has steering control, because it has the effect to stabilize the robot.

**Figure 2.2**

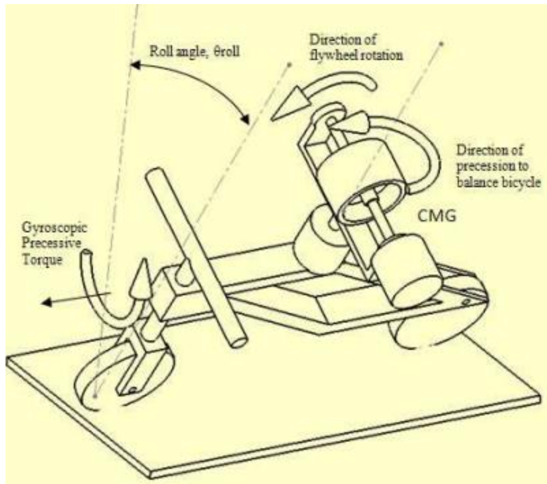
*The Two-wheel Self-balancing Robot Model*




Control Moment Gyroscope (CMG) has a spinning rotor and gimbals to stabilize the bicycle. The spinning rotor generates the torque for applying to the gimble. Then, the bicycle on the leaning angle is back to the equilibrium point. The control moment gyroscope can apply to a delicate system with a small angle and responds quickly to balance the robot.

**Figure 2.3**

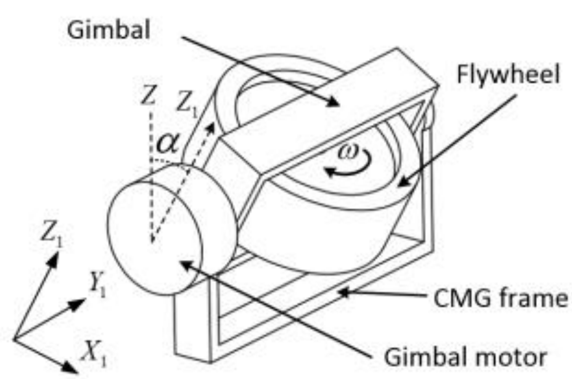
*Gyroscopic Balancing with Kid-size Bicycle*




The control moment gyroscope (CMG) has the flywheel inside. When the sensor measures the tile angle of the robot. Then, the flywheel will start to move at a high speed to generate torque. Because when the center of gravity of the robot is superior to the ground, it can fall. Thus, the control moment gyro uses to make the system stable. The robot uses a control moment gyroscope (CMS) and always uses the concept of an inverted pendulum.

**Figure 2.4**

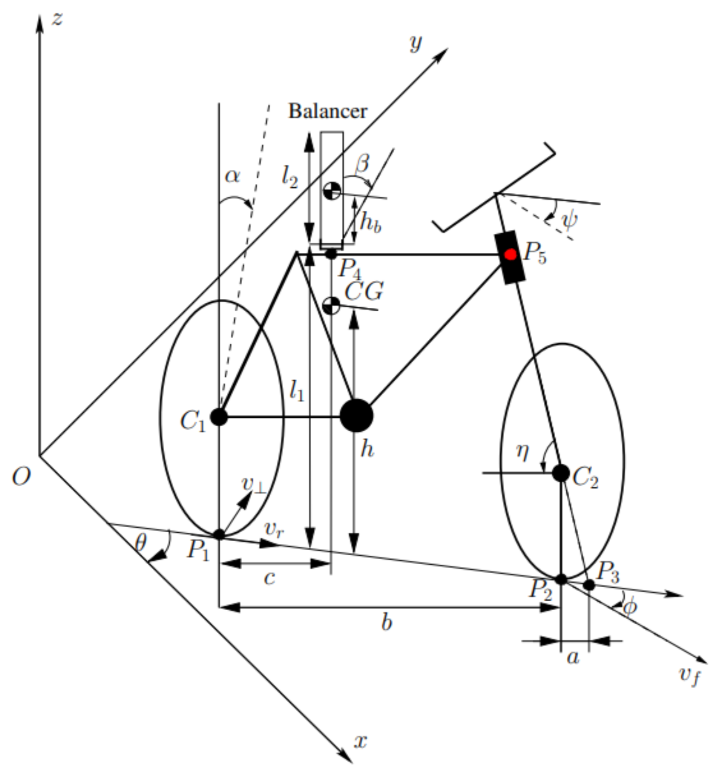
*Control Moment Gyroscope Model*



The self-balancing bicycle was balanced in 2009 by an inverted pendulum balancer and a steering handle. The outcome shows that it can balance quite effectively at zero forward velocity. The idea of an output-zeroing controller was applied to steady this self-balancing bicycle. This self-balancing bicycle can only balance with a tilt angle of no greater than 7°.

**Figure 2.5**

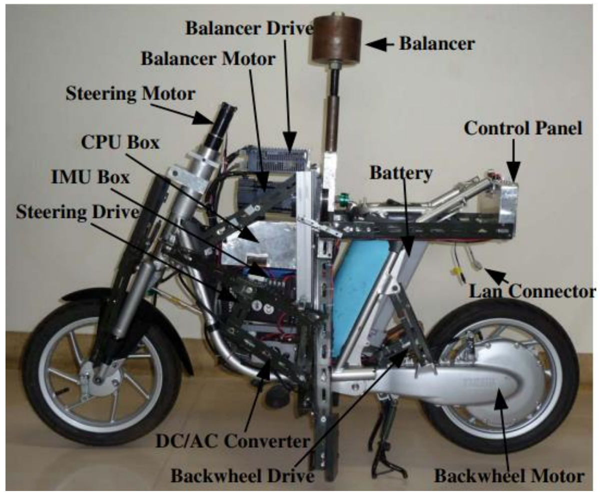
*The Self-balancing Bicycle With Inverted Pendulum Balancer*




The steering handle and balancer make the bicycle steady. This technique was applied for the first time in 2009. The bicycle uses a balancer together with a steering handle, it can balance easier. When the bicycle leans, the balancer and steering handle will move in the opposite direction of the leaning angle. Then, the bicycle will be stabilized by itself.

**Figure 2.6**

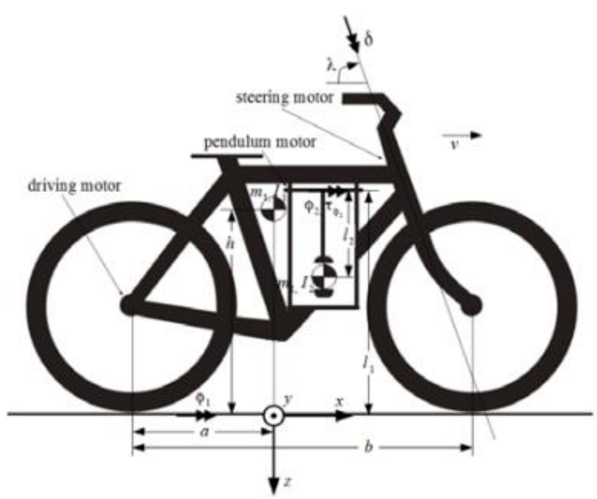
*The Bicycle With a Balancer*




Moreover, a steering handle and mass-moving are also combined for stability on the self-balancing bicycle. The way to use the steering handle and another balancer, makes the bicycle can be balance easier. Because only a steering handle cannot balance at low speed or zero speed, it should balance at high speed. Thus, mass-moving is helping to stabilize the bicycle. When a bicycle is leaning, the steering handle and mass-moving will change the angle for making the system steady. This balance system can use with low or zero speed, but mass-moving is cause of unstable.

**Figure 2.7**

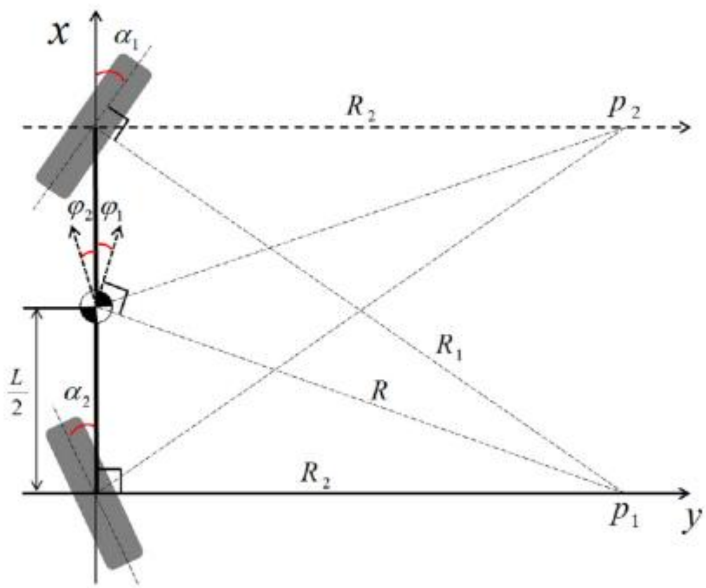
*The Steering and Mass-moving Self-balancing Bicycle Model*



The self-balancing motorcycle uses the concept of steering without a handlebar was presented in 2016. It is like Segway when it moves at high or low speed. The front and rear wheels are swaying to keep the system in an upright position. Only the difference between this concept and Segway is that Segway's wheels move forward and backward to maintain balance. The self-balancing motorcycle has steering angle less than 90°.

**Figure 2.8**

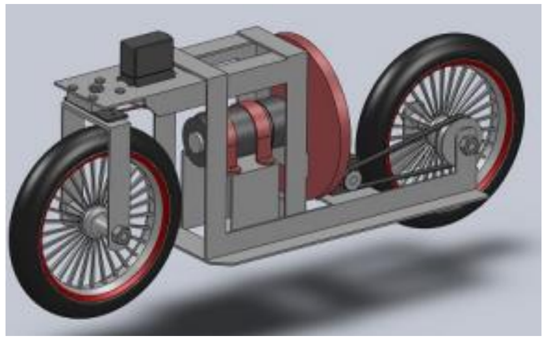
*Top View of Self-balancing Motorcycle*




In addition, the inverted pendulum theory is applied to a self-balancing bicycle. to stabilize the system. This self-balancing bicycle uses a reaction wheel to make bicycle at an upright position. In 2005, the reaction wheel is applying for balancing self-balancing bicycle named Murata boy. The reaction wheel will generate the torque to steady the system in parallel with the bicycle's fame. However, this concept uses more energy, but it is low cost and simple.

**Figure 2.9**

*A Design of a Reaction Wheel with Self-balancing Bicycle*



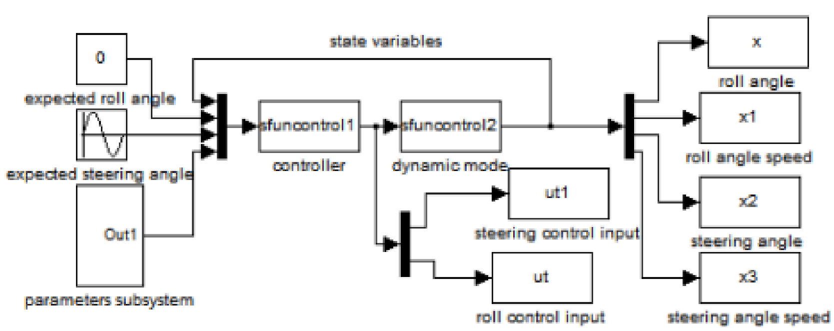
## 2.3 Control System of Self-balancing Bicycle

The bicycle with self-balancing technology's earlier design uses the controller as a balancer such as PID, LQR, LQG, and SMC. Some of those bicycles compare controllers to determine which is the best controller. Most of the time a PID controller is all that is used for balancing. Nevertheless, for this self-balancing choose LQR as a controller.

One sort of nonlinear control is sliding mode control (SMC). It makes a system steady by sliding the surface. Some self-balancing bicycle uses sliding mode control to balance a bicycle. When the robot starts learning and it makes the robot return to the stable position. This technique involves moving along system’s cross section. The advantages of sliding mode control (SMC) are good performance, taking short time response, and easiness.

**Figure 2.10**

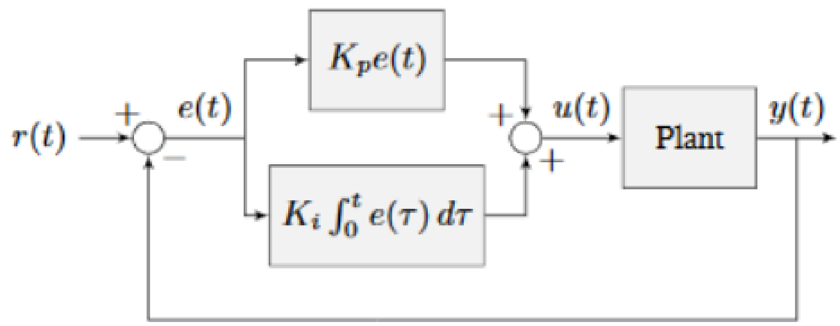
*A Controller Framework of a Two-wheel Self-balancing Robot*



A PID controller, also known as a controller for proportional integral derivatives is the most well-known technique for balancing a self-balancing bicycle. P-proportional, I-inertial, and D-derivative control are all combined in this controller. Thus, the PID controller can be separated to PI or PD controller. However, PI controller is slower 50% just P-controller. As a result, it is always used in conjunction with a derivative controller.

**Figure 2.11**

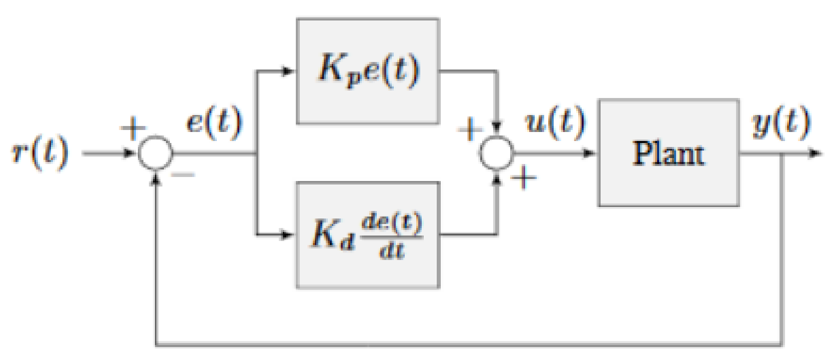
*PI-controller Block Diagram*




the PD controller is a combination between the proportional controller and derivative controller. The error signal is changing with the time response. It takes a short time respond. Additionally, the derivative controller and proportional controller combined have higher productivity than individuals use a derivative controller.

**Figure 2.12**

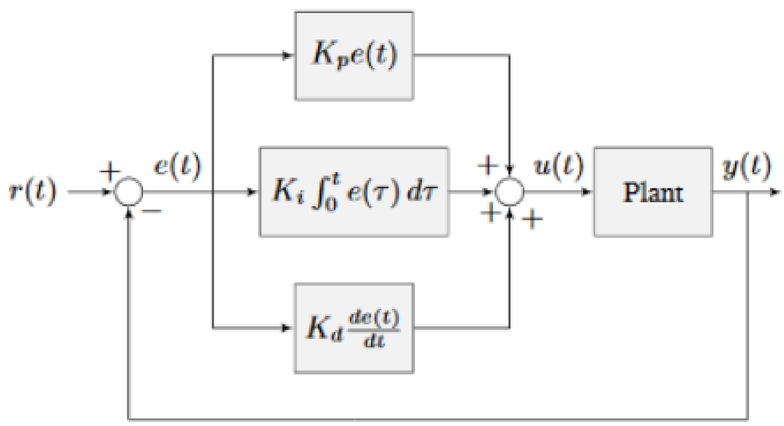
*PD-controller Block Diagram*



The PID controller is famous because of its simplicity and easy for steady the bicycle. However, this controller is not good enough to deal with the high-complexity system for example the multiple input and multiple output system. This system evaluates the feedback variable using a fixed point to produce an error signal. These types of problems often involve two main control techniques, there are robust control and optimal control.

**Figure 2.13**

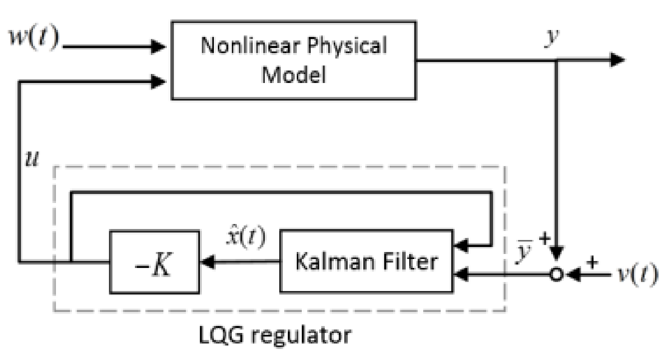
*PID-controller Block Diagram*




Linear Quadratic Gaussian is a kind of optimal control. Modern state-space techniques can be used to create controllers for servos and dynamic regulators with inherent action or setpoint trackers. When this controller applies with control moment gyroscope (CMG) is better performance than PID. The LQG will recover the gimbal in short time response. In addition, the LQG can apply to the error with a small leaning angle.

**Figure 2.14**

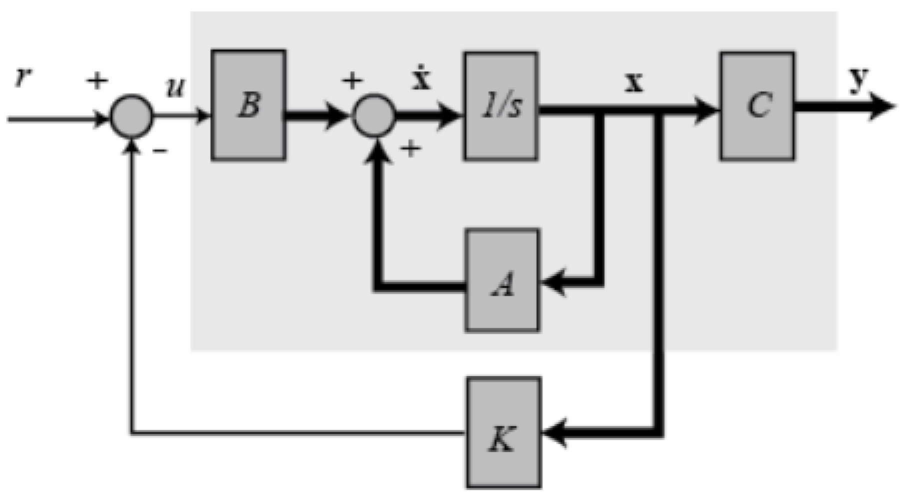
*Linear Quadratic Gaussian (LQG) Feedback Diagram*



Furthermore, the most general optimal controller to apply with a nonlinear system like a self-balancing bicycle is Linear Quadratic Regulation controller (LQR). The Q and R matrixes are a powerful approach used by the LQR controller. The  Q and R matrices can be chosen by the designer to stabilize the system close to equilibrium. The combination of Q and R element have significant effects on a system's performance. Sometimes the system has disturbed by the environment. It cannot make a system responds differently than it should. However, the controller can deal with these disturbances and other sensitivity of the system.

**Figure 2.15**

*Linear Quadratic Regulator (LQR) Block Diagram*



# CHAPTER 3

# METHODOLOGY

## 3.1 Mechanical Design

The self-balancing bicycle has its own design. It has 3 motors for steady control. The first motor is steering handle control. The second motor controls the reaction wheel's rate of rotation. And the third motor of the rear wheel is running speed. The speed control of a self-balancing bicycle is controlled by an electric scooter speed throttle grip.

**Figure 3.1**

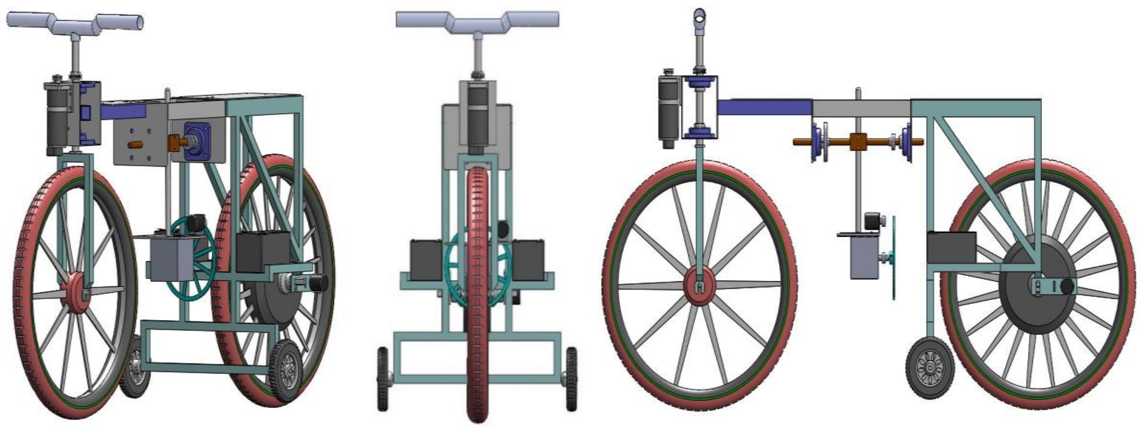
*Self-balancing Bicycle Using a Hybrid of Reaction Wheel and Steering Control*

### 3.1.1 Cad Model and Final Design of the System

Solid Work 2019 software was used to create the CAD model.

**Figure 3.2**

*Cad Model of Self-balancing Bicycle*



After the self-balancing bicycle design. The next figure is the actual model of the self-balancing bicycle.

**Figure 3.3**

*Self-balancing Bicycle Model*

### 3.1.2 The Composition of Self-balancing Bicycle

#### ***3.1.2.1*** **Steering Handle**

The steering handle was controlled by the satellite dc motor. Furthermore, the steering angle needs to be adjusted since a large angle might cause the bicycle to lose control. As a result, the steering angle is measured by using an encoder. The self-balancing bicycle can be managed once it has the steering angle. The angle should be not more than ±10 degree.

**Figure 3.4**

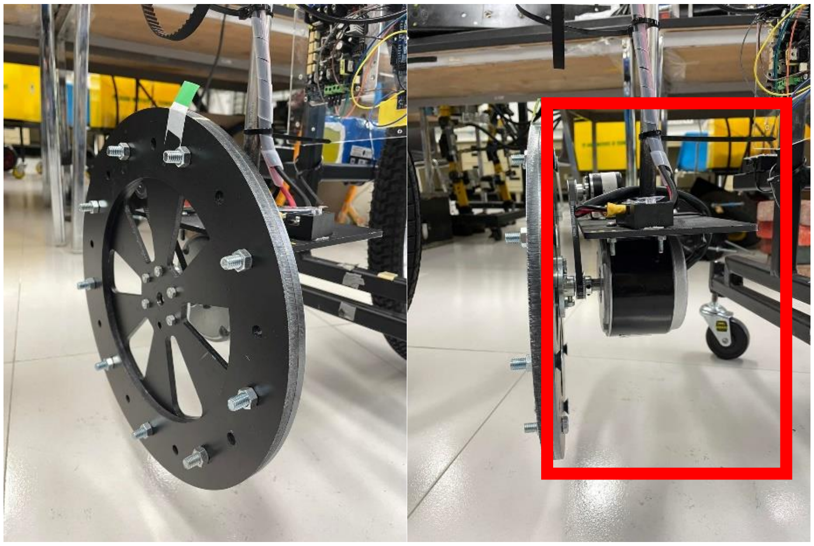
*The Control of the Steering Handle Angle*


#### ***3.1.2.2*** **Reaction Wheel Composition**

The reaction wheel is helping to balance a bicycle. It will spin in a parallel direction with a bicycle. The lean angle affects the reaction wheel's speed of rotation. However, it should not move too much because it can cause of an unstable situation. Thus, the speed of the motor should not be more than 100 rpm. The motor's speed is managed by the reaction wheel.

**Figure 3.5**

*Reaction Wheel Composition*



#### *3.1.2.3* Rear Wheel

As the bicycle moves, the rear wheel controls speed. The motor is brushless. Thus, the bicycle's back wheel is what propels it forward. An electric scooter speed throttle grip is also used to manage the motor's speed. The speed of a self-balancing bicycle is limited. If it runs too fast, it can lose balancing control.

**Figure 3.6**

*The Rear Wheel of a Self-balancing Bicycle*

**Figure 3.7**

*An Electric Scooter Speed Throttle Grip*

#### *3.1.2.4* The Safety Wheel

Moreover, the self-balancing bicycle must have safety wheels. The self-balancing bicycle is protected by safety wheels. Without a rider, a bicycle can tumble at any time while in moving. However, the safety wheel will be protected the system from falling. Furthermore, the self-balancing bicycle cannot lean more than ±5 degrees.

**Figure 3.8**

*The Safety Wheels of a Self-balancing Bicycle*

### 3.2 Electrical Design

This section is separated into 2 parts.

1. System power distribution
2. System inputs and outputs
3. Components

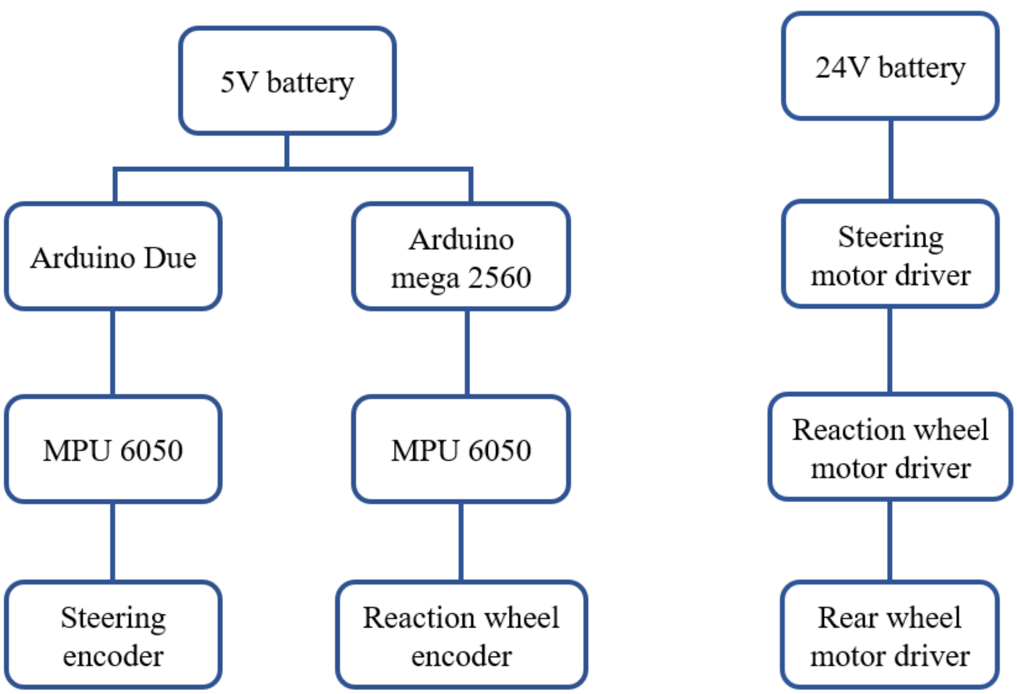
#### 3.2.1 System Power Distribution

The microcontroller is connected to the 5V battery by a micro-USB port. The power supply supports IMU and encoder that directly connects with Arduino 5V power pin.

The motor drivers need 24V for the system. The lithium-ion battery is chosen for all the motor divers because of the discharge larger than the lead-acid battery. Moreover, the current of lithium-ion is not dropped and can use for a long period.

**Figure 3.9**

*System Power Distribution*



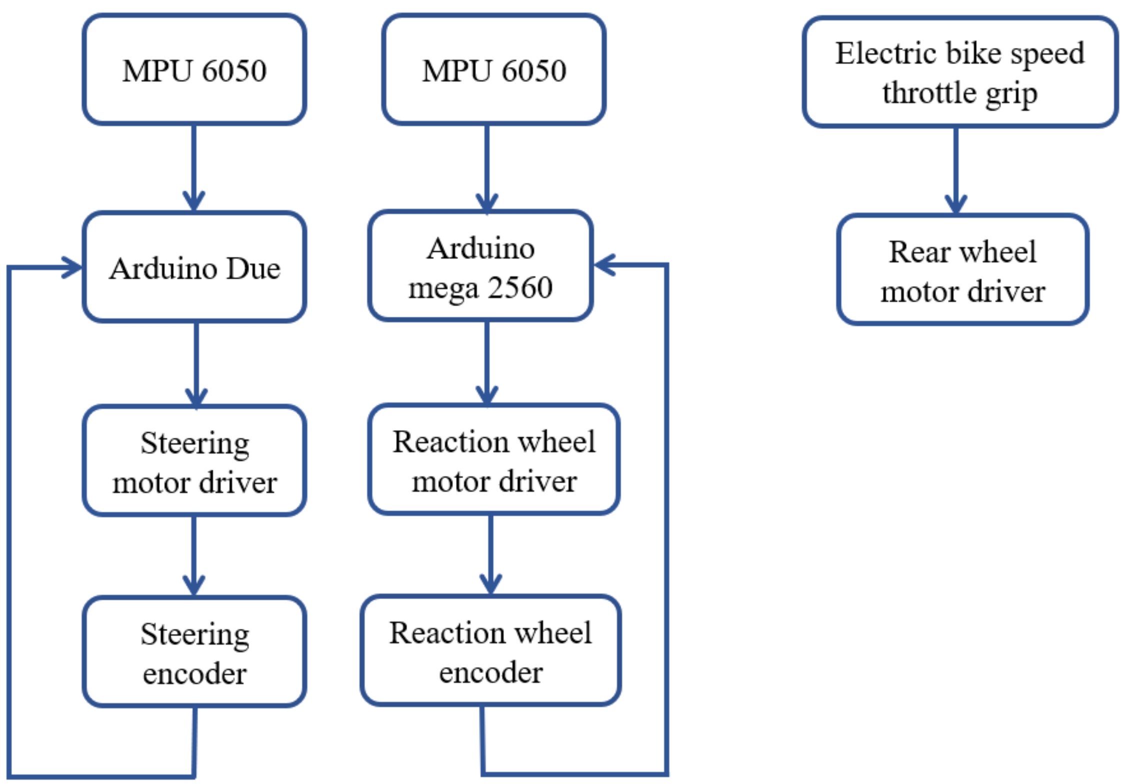
### 3.2.2 System Inputs and Outputs

Each Arduino in the system has two inputs from the sensors. The one output that goes to the driver. The steering handle encoder, reaction wheel encoder, and the IMU sensor are the two inputs of each Arduino. The IMU sensor is connected to the system by using I2C, SDA, and SCL. The rear wheel motor of a self-balancing bicycle is directly controlled by the speed throttle grip. Each encoder is connected to the digital interrupt pins of Arduino.

For the output, the system has two motor drivers. The first motor is required to connect with digital pins. The first pin is for direction control. The next is for PWM control and the last pin is enabling the motor. Two digital pins are required by another motor driver to control the motor's direction.

**Figure 3.10**

*System Inputs and Outputs*



### 3.2.3 The Component of a Self-balancing Bicycle

**Microcontroller**

The control signal is required for making the system steady. Thus, the system of a self-balancing bicycle is separate for two microcontrollers, Arduino Due and Arduino Mega 2560. Arduino Due is a high-performance microcontroller with several input and output ports. Arduino Due comes with 54 digital input or output pins. Moreover, it has up to 12 pins that can be used for PWM control. For another Arduino is for the reaction wheel system. It has 16 MHz crystal oscillator and up to 15 pins for PWM control.

**Figure 3.11**

*Arduino Due*

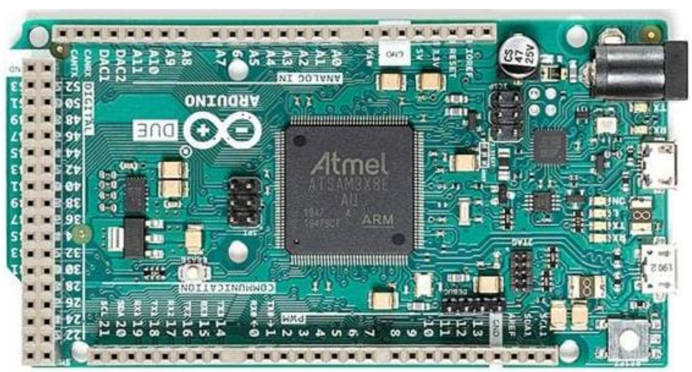
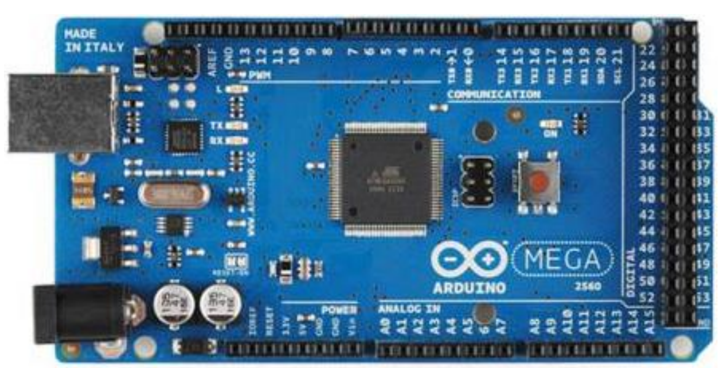



**Figure 3.12**

*Arduino Mega 2560*

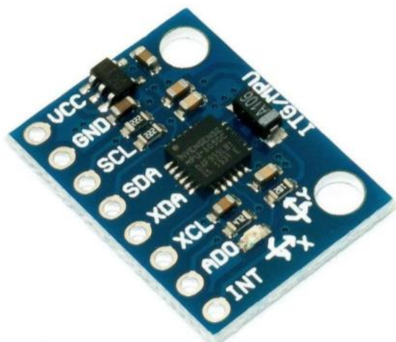


**Inertia Measurement Unit (IMU)**

The important part of the balance system is the measuring sensor. The tilt angle causes an unstable system. Thus, the measuring sensor of the self-balancing bicycle is MPU 6050 for measuring angle and angular velocity. MPU 6050 is a 6-axis accelerometer gyroscope sensor. It connects with the microcontroller by I2C.

**Figure 3.13**

*MPU 6050*




**Electric Bike Speed Throttle Grip**

The part makes the bicycle move forward with a 24V electric bike speed throttle grip. The rear wheel can control the speed. The speed control is sensitive because the bicycle can lose control. Moreover, the electric bike speed throttle grip can limit the speed control of a self-balancing bicycle.

**Figure 3.14**

*Electric Bike Speed Throttle Grip*

**Rotary Encoder**

The encoder works well in this component to determine velocity and system position. This self-balancing bicycle has 2 encoders to measure the system The rotation of the steering wheel and reaction wheel is measured by the two encoders. The first encoder is a 700 P/R rotary encoder and another one is a 200 P/R encoder.

**Figure 3.15**

*Rotary Encoder*

**Motor Driver**

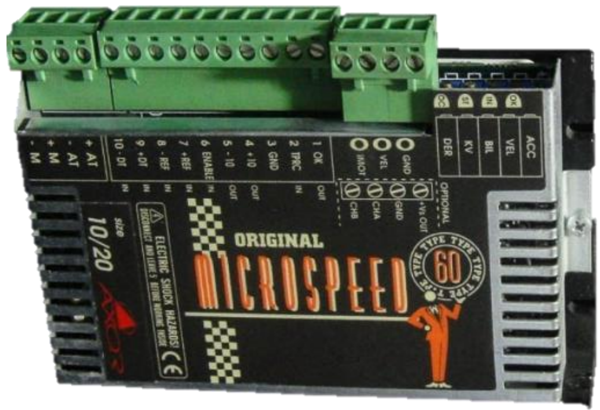
Normally a self-balancing bicycle requires a motor driver that can control both of direction and speed of the motor To maintain the bicycle-like leaning angle, the steering handle pivots in the yaw direction. The motor should also use a lot of currents when changing directions. As a result, the steering motor driver used for this self-balancing bicycle is MICROSPEED. Additionally, the direction of the reaction wheel's change in motion is selected by the DC-motor H-bridge diver.

**Figure 3.16**

*DC-motor H-bridge Diver for Reaction Wheel Motor*

**Figure 3.17**

*The Steering Handle Motor Driver*




### Actuator

A self-balancing bicycle weighs around 45 kilograms and has dimensions of 165 * 75 centimeters. In addition, the motor that applies for the self-balancing bicycle should have enough torque and speed to stabilize a system. Thus, the researcher should 250W DC motor for the reaction wheel and a 100W satellite DC motor to control the speed and direction of the steering handle.

**Figure 3.18**

*Satellite DC motor*

**Figure 3.19**

*DC Motor*


### 3.3 Dynamics Model

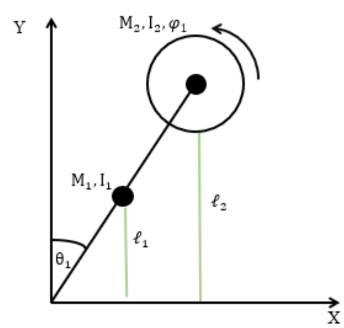
This is one of the most important parts of this thesis. The gain from the dynamic model will be applied to balance the self-balancing bicycle. This part will separate into five sections. There are

1. Dynamic model of bicycle
2. Dynamic of DC motor
3. Dynamic model of a steering handle
4. System State-space Representation
5. Linear Quadratic Regulator

#### 3.3.1 Dynamic Model of Bicycle

**Figure 3.20**

*Self-balancing Bicycle Free Body Diagram*



The y-axis represents the bicycle's direction of motion in forward motion, while the x-axis represents its right side.

Parameter of the dynamics model of bicycle defined as below.

$M_1$ : Bicycle mass

$M_2$ : Reaction wheel mass

$I_1$ : the inertia of bicycle along the x-axis

$I_2$ : the inertia of the reaction wheel along a rotation axis

$\ell_1$ : the length from ground to center of gravity of a bicycle

$\ell_2$ : the length from ground to center of gravity of reaction wheel

$\theta_1$ : the bicycle tile angle

$\varphi_1$ : the reaction wheel rotation angle

Using the Euler-Lagrange equation of motion to find gain, the first step is finding kinetic and potential energy of rigid bodies as generalized coordinates. The equation is show below

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}_1}\right) - \frac{\partial L}{\partial \theta_1} = 0 \tag{1}$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\varphi}_1}\right) - \frac{\partial L}{\partial \varphi_1} = \tau_1 \tag{2}$$

Where

- *L* is the Lagrange function, describe as *L = K.E. – P.E.*
- *K.E.* is total kinetic energy of reaction wheel and bicycle, shown in Eq. (3)
- *P.E.* is total potential energy of bicycle and reaction wheel, shown in Eq. (4)
- $\tau_1$ is driving torque of reaction wheel

#### *3.3.1.1* Kinetic Energy of Bicycle and Reaction Wheel

$$K.E = \left[\frac{1}{2}M_1(\dot{x}_1 + \dot{y}_1)^2 + \frac{1}{2}I_1\dot{\theta}_1^2\right] + \left[\frac{1}{2}M_2(\dot{x}_2 + \dot{y}_2)^2 + \frac{1}{2}I_2(\dot{\theta}_1 + \dot{\varphi}_1)^2\right]$$

$$K.E. = \left[\frac{1}{2}M_1\left(\ell_1^2\dot{\theta}_1^2(cos^2\theta_1 + sin^2\theta_1)\right) + \frac{1}{2}I_1\dot{\theta}_1^2\right]$$

$$+\left[\frac{1}{2}M_2\left(\ell_2^2\dot{\theta}_2^2(cos^2\theta_1 + sin^2\theta_1)\right) + \frac{1}{2}I_1\dot{\theta}_1^2 + I_2\dot{\theta}_1\dot{\varphi}_1 + \frac{1}{2}I_2\dot{\varphi}_1^2\right]$$

$$\boldsymbol{K.E. = \left[\frac{1}{2}M_1\ell_1^2\dot{\theta}_1^2 + \frac{1}{2}I_1\dot{\theta}_1^2\right] + \left[\frac{1}{2}M_2\ell_2^2\dot{\theta}_1^2 + \frac{1}{2}I_1\dot{\theta}_1^2 + I_2\dot{\theta}_1\dot{\varphi}_1 + \frac{1}{2}I_2\dot{\varphi}_1^2\right]} \qquad (3)$$

#### *3.3.1.2* Potential Energy of Bicycle and Reaction Wheel

$$P.E. = mgh$$

$$P.E. = M_1g\ell_1cos\theta_1 + M_2g\ell_2cos\theta_1$$

$$\boldsymbol{P.E. = (M_1\ell_1 + M_2\ell_2)gcos\theta_1} \qquad (4)$$

Then substituting (3) and (4) into Lagrange function

$$L = K.E - P.E.$$

$$L = \left[\frac{1}{2}M_1\ell_1^2\dot{\theta}_1^2 + \frac{1}{2}I_1\dot{\theta}_1^2\right] + \left[\frac{1}{2}M_2\ell_2^2\dot{\theta}_1^2 + \frac{1}{2}I_1\dot{\theta}_1^2 + I_2\dot{\theta}_1\dot{\varphi}_1 + \frac{1}{2}I_2\dot{\varphi}_1^2\right] - (M_1\ell_1 + M_2\ell_2)gcos\theta_1$$

$$L = \left[\frac{1}{2}(M_1\ell_1^2 + M_2\ell_2^2 + I_1 + I_2)\dot{\theta}_1^2 + I_2\dot{\theta}_1\dot{\varphi}_1 + \frac{1}{2}I_2\dot{\varphi}_1^2\right] - (M_1\ell_1 + M_2\ell_2)gcos\theta_1 \qquad (5)$$

Substituting (5) into (1) of Euler-Lagrange equation of emotion

$$\frac{d}{dt}\left(\frac{\partial L}{\partial\dot{\theta}_1}\right) - \frac{\partial L}{\partial\theta_1} = 0$$

$$\frac{\partial L}{\partial\dot{\theta}_1} = \frac{d}{dt}\left[\frac{1}{2}(M_1\ell_1^2 + M_2\ell_2^2 + I_1 + I_2)\dot{\theta}_1^2 + I_2\dot{\theta}_1\dot{\varphi}_1\right]$$

$$= (M_1\ell_1^2 + M_2\ell_2^2 + I_1 + I_2)\ddot{\theta}_1 + I_2\ddot{\varphi}_1$$

$$\frac{\partial L}{\partial\theta_1} = -(M_1\ell_1 + M_2\ell_2)gcos\theta_1$$

$$= (M_1\ell_1 + M_2\ell_2)gsin\theta_1$$

Thus,

$$(M_1\ell_1^2 + M_2\ell_2^2 + I_1 + I_2)\ddot{\theta}_1 + I_2\ddot{\varphi}_1 - (M_1\ell_1 + M_2\ell_2)gsin\theta_1 = 0$$

Linearization by

$$sin\theta \approx \theta, cos\theta \approx 1, \dot{\theta}^2 and\ \dot{\theta}\theta \approx 0$$

$$(M_1\ell_1^2 + M_2\ell_2^2 + I_1 + I_2)\ddot{\theta}_1 + I_2\ddot{\varphi}_1 - (M_1\ell_1 + M_2\ell_2)g\theta_1 = 0$$

$C_1$ represent for $(M_1\ell_1^2 + M_2\ell_2^2 + I_1 + I_2)$

$C_2$ represent for $(M_1\ell_1 + M_2\ell_2)g$

Thus

$$C_1\ddot{\theta}_1 + I_2\ddot{\varphi}_1 - C_2\theta_1 = 0 \tag{6}$$

Then, substituting (5) into (2) of Euler-Lagrange equation of emotion

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\varphi}_1}\right) - \frac{\partial L}{\partial \varphi_1} = \tau_1$$

$$\frac{\partial L}{\partial \dot{\varphi}} = \frac{d}{dt}[I_2\dot{\theta}_1\dot{\varphi}_1 + \frac{1}{2}I_2\dot{\varphi}_1^2]$$

$$= I_2\ddot{\theta}_1 + I_2\ddot{\varphi}_1$$

$$\frac{\partial L}{\partial \varphi} = 0$$

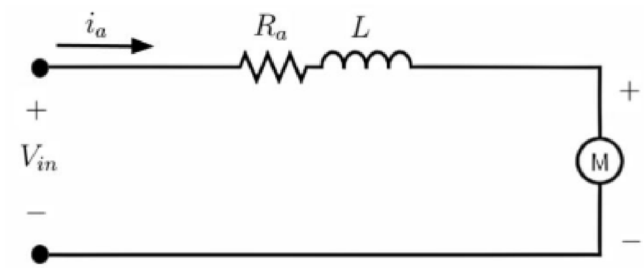
Thus,

$$I_2\ddot{\theta}_1 + I_2\ddot{\varphi}_1 = \tau_1 \tag{7}$$

### 3.3.2 Dynamic Model of DC Motor

**Figure 3.21**

*DC Motor Circuit*




| | |
|---|---|
| $V_b$ | is back Electromotive force (EMF) |
| $\tau_1$ | is motor torque |
| $R_a$ | is armature resistant |
| $i_a$ | is armature current |
| $K_b$ | is constant of back EMF |
| $K_T$ | is motor torque constant |
| $\omega_1$ | is motor angular velocity |
| $V_{in}$ | is input voltage |
| $L$ | is inductance |

$$V_b \text{ proportional to } \omega : \ V_b = K_b \omega_1 \quad (8)$$

$$\tau \text{ proportional to } i_a = K_t i_a \quad (9)$$

Relative to Kirchhoff voltage law (KVL)

$$i_a R_a + L_a \frac{d_{ia}}{d_t} + V_b = V_a \quad (10)$$

Assume L by $R_a$ because L is very small

$$i_a R_a + V_b = V_a \tag{11}$$

Substituting (8) and (9) into (11)

$$\frac{\tau_1}{K_t}(R_a) + K_b\omega_1 = V_a$$

$$\frac{\tau_1}{K_t}(R_a) = V_a - K_b\omega_1$$

$$\tau_1 = K_t \frac{V_a}{R_a} - K_t \frac{K_b\omega_1}{R_a} \tag{12}$$

Substituting $\omega_1$ by

$$\frac{2\pi r_2}{\dot{\varphi}} = \frac{r_2}{r_1}\frac{2\pi}{\omega_1}$$

$$\omega_1 = \frac{\dot{\varphi}_1}{r_1} \tag{13}$$

*r* is the radius of a pulley

Substitute (13) into (12)

$$\tau_1 = K_t \frac{V_a}{R_a} - K_t \frac{K_b\dot{\varphi}_1}{R_a r_1} \tag{14}$$

Write in short by

$$\tau_1 = C_3 V_a - C_4 \frac{\dot{\varphi}_1}{r_1} \tag{15}$$

$C_3$ represent for $\frac{K_t}{R_a}$

$C_4$ represent for $K_t \frac{K_b}{R_a}$

After that substitute (15) into (7)

$$\ddot{\theta}_1 = \left[\frac{C_3}{I_2}\right] V_a - \left[\frac{C_4}{I_2 r_1}\right] \dot{\varphi}_1 - \ddot{\varphi}_1 \tag{16}$$

$C_5$ represent for $\frac{C_3}{I_2}$

$C_6$ represent for $\frac{C_4}{I_2 r_1}$

Substituting (16) into (6) and get

$$\boldsymbol{\ddot{\varphi}_1 = \frac{-C_1 C_5 V_a}{C_7} + \frac{C_1 C_6 \dot{\varphi}_1}{C_7} + \frac{C_2 \theta}{C_7}} \quad \textbf{(17)}$$

$C_7$represent for $[-C_1 + I_2\ ]$

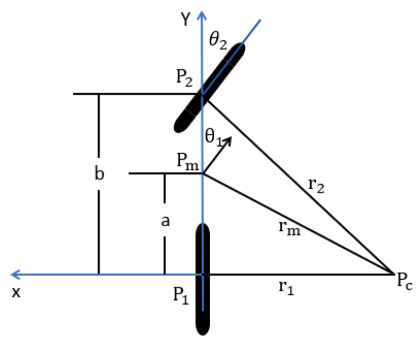
Then substitute (17) into (16)

$$\boldsymbol{\ddot{\theta}_1 = \left[\frac{C_3}{I_2} + \frac{C_1 C_5}{C_7}\right] V_a - \left[\frac{C_4}{I_2 r_1} - \frac{C_1 C_6}{C_7}\right] \dot{\varphi}_1 - \left[\frac{C_2}{C_7}\right] \theta} \quad \textbf{(18)}$$

### 3.3.3 Dynamic Model of the Steering Handle

**Figure 3.22**

*Top View of Self-balancing Bicycle*




$\theta_2$ represent for steering angle turn counterclockwise and $\theta_1$ is the angle of bicycle leaning. The equation below calculates from the geometry of a bicycle.

$$b = r_1 tan\theta_2$$

$$a = r_1 tan\theta_1$$

$$a = r_m sin\theta_1$$

$$r_m = \frac{a}{\sin\left[tan^{-1}\left[\frac{a}{b} \cdot tan\theta_2\right]\right]} \quad (19)$$

Around the equilibrium point, the steering angle is very small. Thus, it can be linearized by $sin\,\theta \approx tan\,\theta \approx \theta, and\ tan^{-1}\theta \approx \theta$

$$\theta_1 \approx \frac{a}{b} \cdot \theta_2 \tag{20}$$

$$r_m \approx \frac{a}{a \cdot \theta_2 / b} = \frac{b}{\theta_2} \tag{21}$$

From relation of steering and motor, $\dot{\theta}_2 = \omega_{steering}, \omega_{motor} = N \cdot \omega_{steering}$. Then, can get the equation below

$$V = \frac{J_{ei}R}{K_t N} \cdot \ddot{\theta} + \left[\frac{b_{ei}R}{K_t N} + K_b N\right] \dot{\theta} \tag{23}$$

Thus, the state space of the steering motor is $x = [\theta_2 \quad \dot{\theta}_2]$

### 3.3.4 System State-space Representation

Then, the linearized of dynamic model will be represent in the state-space model.

$$\dot{x} = Ax + Bu$$

$$y = Cx$$

When

A is state matrix

B is input matrix

C is output matrix

x is state variable

u is input

The state variable of self-balancing bicycle can separate to 6 parameters.

$\theta$ stand for bicycle tilt angle

$\dot{\theta}_1$ stand for angular velocity

$\theta_2$ stand for steering angle

$\dot{\theta}_2$ stand for steering angular velocity

$\varphi$ stand for reaction wheel rotating angle

$\dot{\varphi}$ stand for reaction wheel angular velocity

Then, transform dynamic equation (17), (18), and (23) into a state-space form.

$$\begin{bmatrix} \dot{\theta}_1 \\ \ddot{\theta}_1 \\ \dot{\theta}_2 \\ \ddot{\theta}_2 \\ \dot{\varphi} \\ \ddot{\varphi} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ \dfrac{C_2}{C_7} & 0 & 0 & 0 & 0 & \dfrac{C_4}{I_2 r_1} - \dfrac{C_1 C_6}{C_7} \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -\left[\dfrac{b_{ei}}{J_{ei}} + \dfrac{K_b K_t N^2}{J_{ei} R}\right] & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ \dfrac{C_2}{C_7} & 0 & 0 & 0 & 0 & \dfrac{C_1 C_6}{C_7} \end{bmatrix} \begin{bmatrix} \theta_1 \\ \dot{\theta}_1 \\ \theta_2 \\ \dot{\theta}_2 \\ \varphi \\ \dot{\varphi} \end{bmatrix} + \begin{bmatrix} 0 \\ \dfrac{c_3}{I_2} + \dfrac{C_1 C_5}{C_7} \\ 0 \\ \dfrac{K_t N}{J_{ei} R} \\ 0 \\ \dfrac{-C_1 C_5}{C_7} \end{bmatrix} V_a$$

$$y = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \theta_1 \\ \dot{\theta}_1 \\ \theta_2 \\ \dot{\theta}_2 \\ \varphi \\ \dot{\varphi} \end{bmatrix}$$

When

$C_1$ is $(M_1 \ell_1^2 + M_2 \ell_2^2 + I_1 + I_2)$

$C_2$ is $(M_1 \ell_1 + M_2 \ell_2) g$

$C_3$ is $\frac{K_t}{R_a}$

$C_4$ is $K_t \frac{K_b}{R_a}$

$C_5$ is $\frac{C_3}{I_2}$

$C_6$ is $\frac{C_4}{I_2 r_1}$

$C_7$ is $[-C_1 + I_2\ ]$

### 3.3.5 Linear Quadratic Regulator

The Linear Quadratic Regulator (LQR) controller is finding the optimal gain for steady the system. It can determine by closed-loop poles to balance the importance of the needed is control energy with the acceptable response.

The state space model

$$\dot{x} = Ax + Bu$$

Linear control law

$$u = -Gx \tag{24}$$

The gain G reduces the cost function V

$$V = \int_{\tau}^{T} [x'(t)Q(t)x(t) + u'R(t)u(t)]dt$$

When

> $Q$ and $R$ are the symmetric matrices
>
> $Q$ is the state weighting matrix
>
> $R$ is control weighting matrix

The gain G for control signal (24) can be found from

$$G = R^{-1}B'M \tag{25}$$

Where M is finding from algebraic Riccati equation

$$A'M + MA - MBR^{-1}B'M + Q = 0$$

With chosen values for Q and R, the LQR function in MATLAB can determine the system's optimum gain G. LQR controllers are also used by self-balancing bicycles. The fixing error value of that variable will be the reference of the system. The way to select some constant for that error make the system will not fail. The range of the error will be selected value. Additionally, if the chosen value is not high enough, the device may fail. On the other hand, the system will not recognize because select value is too low. As a result, not only does this strategy not diminish overshoot at startup, but it also drives a system gradually to any position.

# CHAPTER 4

# RESULT AND DISCUSSION

Gain G and simulation will be the main topics of this chapter. Then, this chapter show the result of self-balancing bicycle.

## 4.1 State-space Representation of LQR

**Table 4.1**

*Parameter of State-space*

| Constant parameter | Value | Units |
|---|---|---|
| $\boldsymbol{M_1}$ | 50 | Kg |
| $\boldsymbol{M_2}$ | 8 | Kg |
| $\boldsymbol{I_1}$ | 1 | $\mathrm{kg \cdot m^2}$ |
| $\boldsymbol{I_2}$ | 0.68 | $\mathrm{kg \cdot m^2}$ |
| $\boldsymbol{\ell_1}$ | 0.375 | m |
| $\boldsymbol{\ell_2}$ | 0.18 | m |
| $\boldsymbol{K_b}$ | 0.137 | - |
| $\boldsymbol{K_t}$ | 0.137 | - |
| $\boldsymbol{g}$ | 9.81 | $m/s^2$ |
| $\boldsymbol{R_a}$ | 0.7 | Ω |

Substitute the value into state-space equation

$$\begin{bmatrix} \dot{\theta}_1 \\ \ddot{\theta}_1 \\ \dot{\theta}_2 \\ \ddot{\theta}_2 \\ \dot{\varphi} \\ \ddot{\varphi} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ \frac{C_2}{C_7} & 0 & 0 & 0 & 0 & \frac{C_4}{I_2 r_1} - \frac{C_1 C_6}{C_7} \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -\left[\frac{b_{ei}}{J_{ei}} + \frac{K_b K_t N^2}{J_{ei} R}\right] & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ \frac{C_2}{C_7} & 0 & 0 & 0 & 0 & \frac{C_1 C_6}{C_7} \end{bmatrix} \begin{bmatrix} \theta_1 \\ \dot{\theta}_1 \\ \theta_2 \\ \dot{\theta}_2 \\ \varphi \\ \dot{\varphi} \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{c_3}{I_2} + \frac{C_1 C_5}{C_7} \\ 0 \\ \frac{K_t N}{J_{ei} R} \\ 0 \\ \frac{-C_1 C_5}{C_7} \end{bmatrix} V_a$$

$$y = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \theta_1 \\ \dot{\theta}_1 \\ \theta_2 \\ \dot{\theta}_2 \\ \varphi \\ \dot{\varphi} \end{bmatrix}$$

**Open Loop Pole Location**

Finding system pole location will be used for steady the system. The pole is stable at a left-half plane. On the other hand, the right-half plane is unstable. The way to determine the pole is using MATLAB command.

$$pole = eig(A)$$

**Linear Quadratic Regulator**

LQR controller needs matrix Q and R to complete the system. For the self-balancing bicycle selected Q and R matrix below

$$Q = \begin{bmatrix} 10 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 10 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 10 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} R = \begin{bmatrix} 10 & 0 \\ 0 & 0.001 \end{bmatrix}$$

The calculation of gain G is using MATLAB function **lqr()**

$$[1.85 \quad 0.38 \quad 9.97 \quad 3.19 \quad -0.03 \quad 0.0031]$$

**4.2 Simulation**

The simulation is using Simulink.

**Figure 4.1**

*State-space Representation Without Reference*

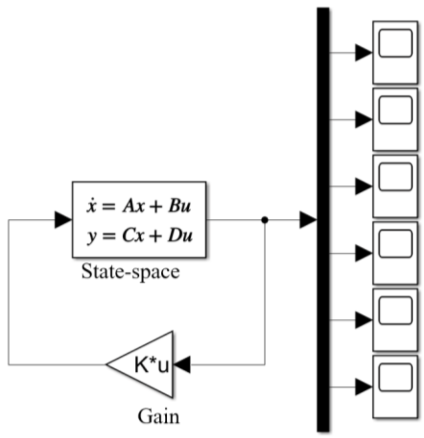
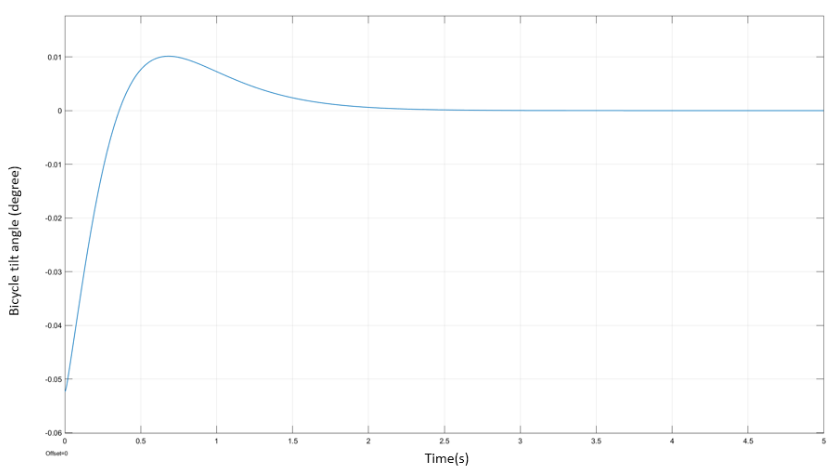



**Figure 4.2**

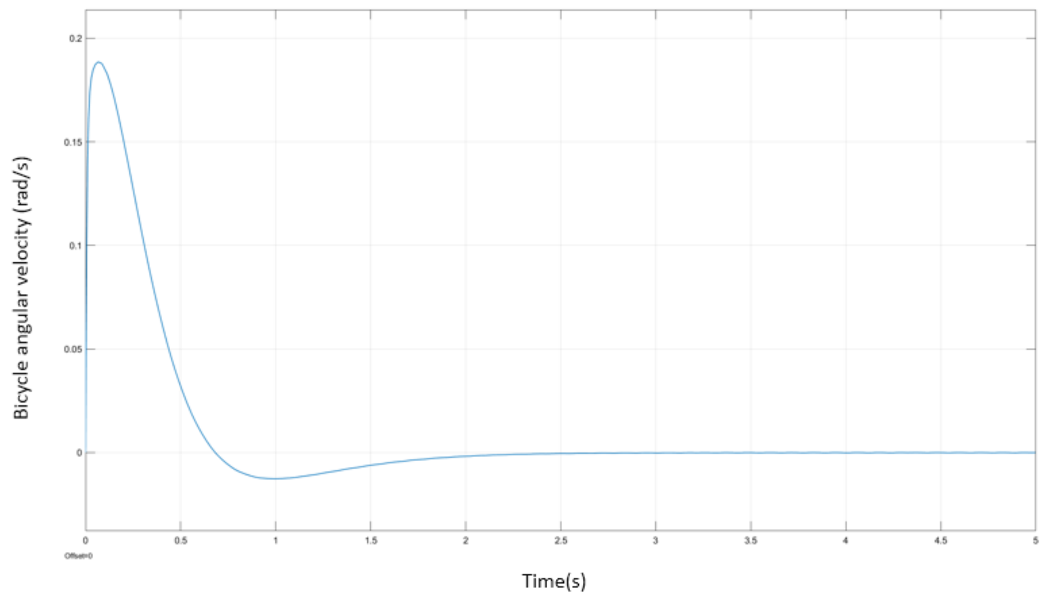
*The Simulation Result of Bicycle Leaning Angle*




Rise time(s) = 0.4, Settling time(s) = 2.1, Steady-state error = 0

**Figure 4.3**

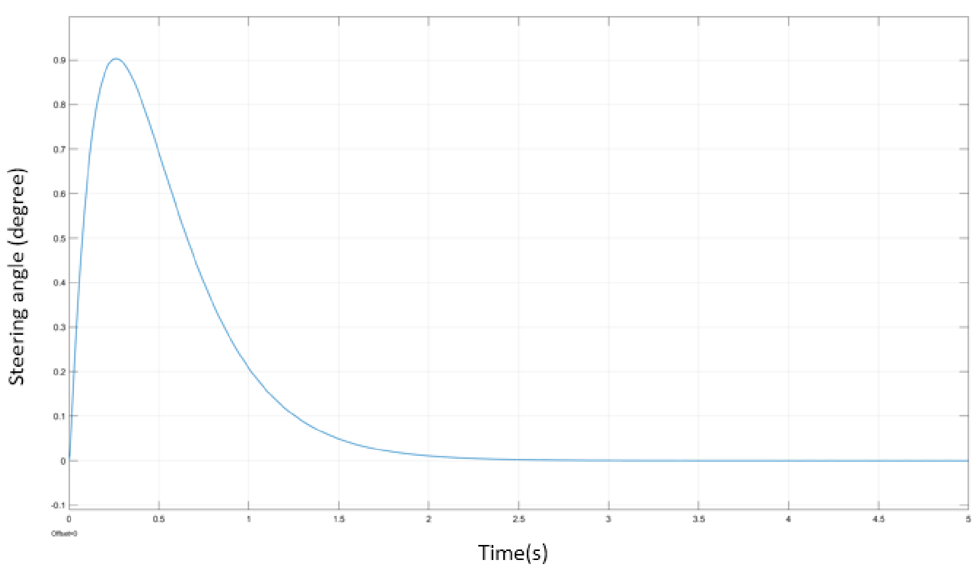
*The Simulation Result of Bicycle Angular Velocity*




Rise time(s) = 0.7, Settling time(s) = 1.8, Steady-state error = 0

**Figure 4.4**

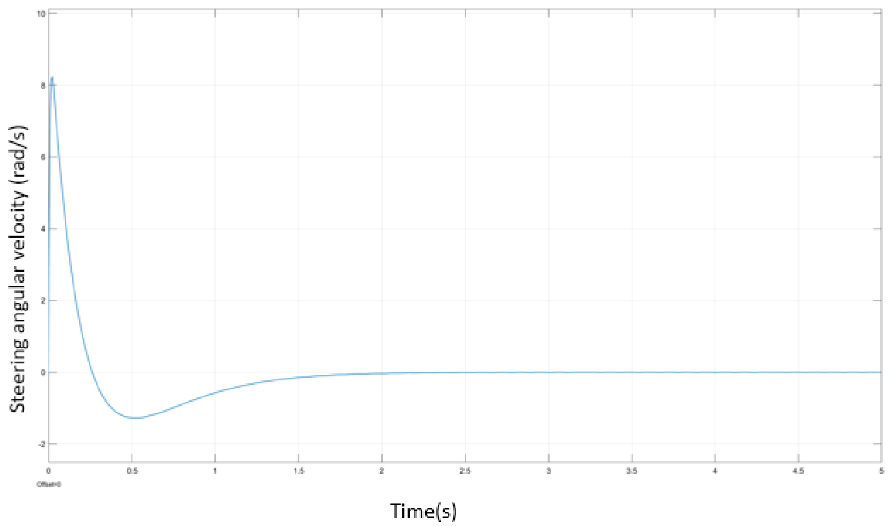
*The Simulation Result of Steering Angle*




Rise time(s) = 1.3, Settling time(s) = 1.9, Steady-state error = 0

**Figure 4.5**

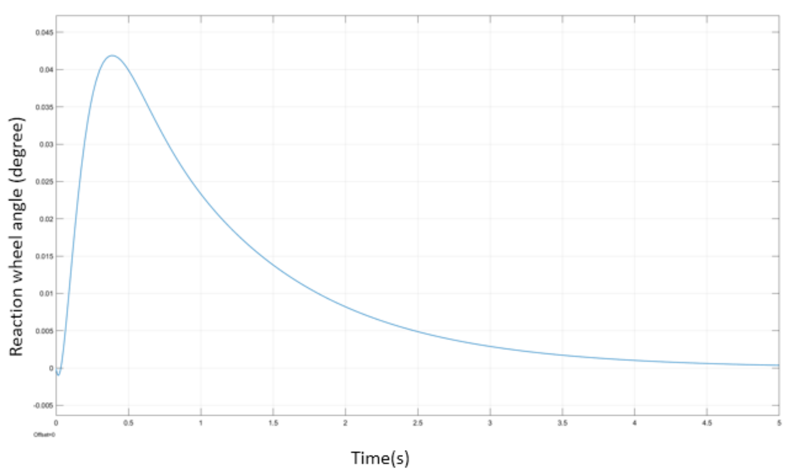
*The Simulation Result of Steering Angular Velocity*




Rise time(s) = 0.6, Settling time(s) = 2, Steady-state error = 0

**Figure 4.6**

*The Simulation Result of Steering Angular Velocity*




Rise time(s) = 0.3, Settling time(s) = 4.5, Steady-state error = 0

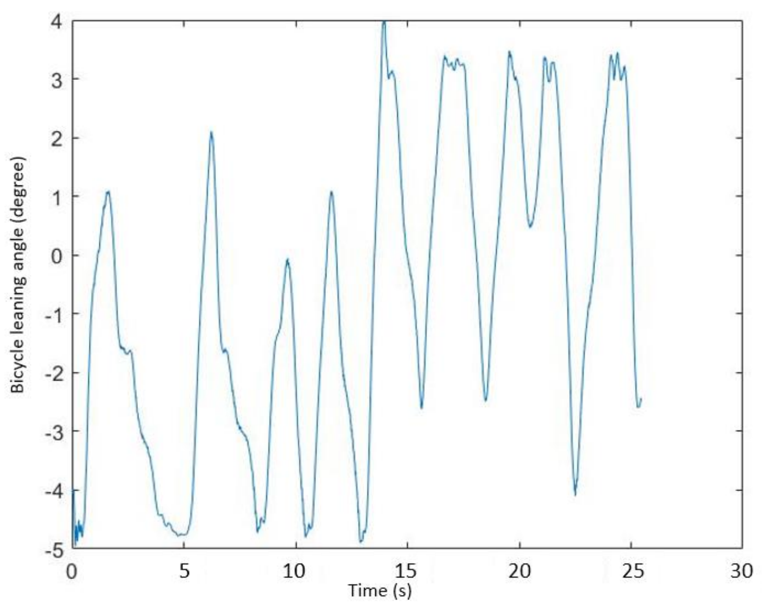
### 4.3 Experiment and Result

The self-balancing cannot stable by itself while the system is being tested. It consistently falls to the left or right. The self-balance is very heavy. Furthermore, the system is unable to regulate balance due to the environment's unpredictability.

In the graph below shown the result while self-balancing bicycle testing the balance system. The result shown that self-balancing bicycle cannot control at the upright position by itself.

**Figure 4.7**

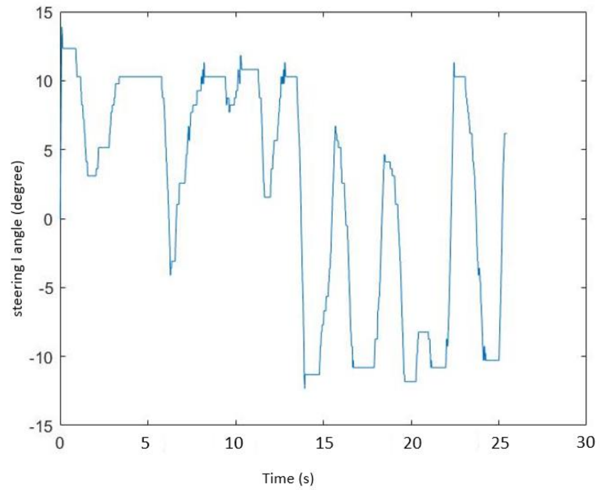
*Bicycle Leaning Angle*




Next, the steering angle during the testing period shown that it try to move to the starting point or zero degree position. However, the self-balancing bicycle cannot stay at the upright position. The steering also cannot back to the zero angle.

**Figure 4.8**

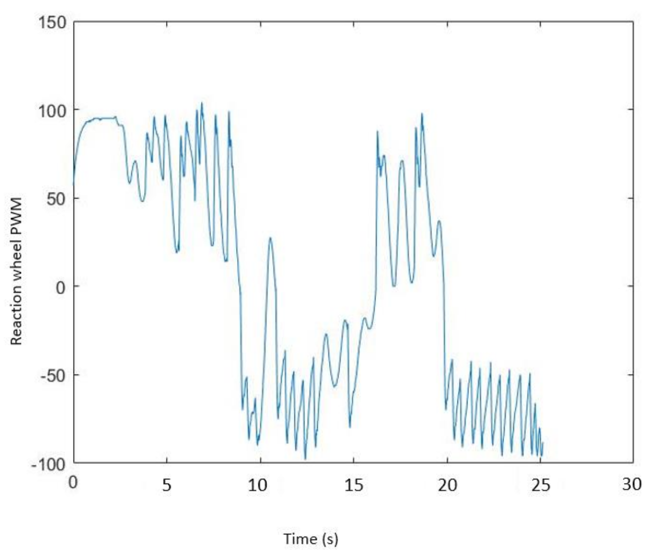
*Steering Handle Angle*




The last graph is the reaction wheel PWM. It shows the direction of reaction wheel that move parallel to the bicycle for generating torque to stabilize the system. In the graph below it has positive and negative side. The positive side is reaction wheel move counterclockwise same as the bicycle that leaning to the right side. On the other hand, if at the negative side is reaction move clockwise parallel with the bicycle leaning left.

**Figure 4.9**

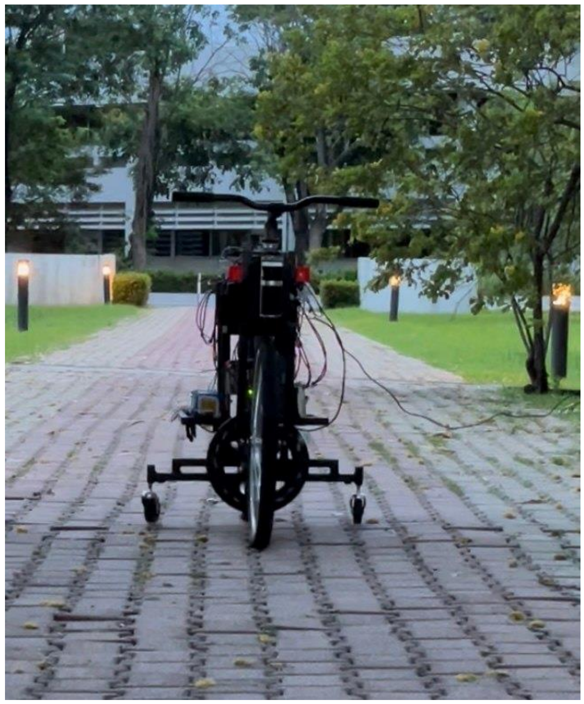
*Reaction Wheel Direction Control*



This picture below shows during the bicycle is testing on the footpath. The footpath is bumping the safety wheels of the self-balancing bicycle. Thus, it can be balanced in a short period of time.

**Figure 4.10**

*Reaction Wheel Direction Control*



# CHAPTER 5

# CONCLUSION AND FUTURE WORK

## 5.1 Conclusion

The self-balancing bicycle has a new design same as adult-size bicycle. The steering control is applying for control steering handle. Moreover, the reaction wheel combines with the steering handle can reduce the torque for reaction wheel. The reaction wheel rotates in the parallel direction with the bicycle to generate the torque for balancing a bicycle. When bicycle is leaning, the steering handle will try to move in the opposite direction of tile angle for making system steady

The dynamic model is the impart of balancing system. Finding dynamic model will apply with the control algorithm. This self-balancing bicycle selects LQR as a controller. For LQR controller can select the matrix Q and R to match with the system. Furthermore, it also can adjust the gain to control signal of the system.

Successful control balance was the outcome of the simulation of a self-balancing bicycle employing a hybrid reaction wheel and steering motor. On the other hand, when the self-balancing bicycle was test in the real environment. Then, this self-balancing bicycle discovered that it was unable to maintain equilibrium on its own. Because this self-balancing bicycle is very heavy, the uncertainty of the environment make bicycle easy to fall.

## 5.2 Future Work

This self-balancing bicycle should reduce the robot size because it is heavy and big. Thus, it should be smaller. Moreover, the motor will replace with the high-power motor for better performance. Additionally, the sensor that detects tilt angle needs to upgrade to a high-performance level since it may detect the noise. It could have an impact on how a bicycle balances.

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