AN OPTIMAL LATERAL TRANSSHIPMENT MODEL BETWEEN TWO RETAILERS UNDER THE CENTRALIZED INVENTORY SYSTEM FOR SLOW-MOVING ITEMS

by

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**AUTHOR’S DECLARATION**

I, Pitchaporn Yochana, declare that the research work carried out for this thesis was in accordance with the regulations of the Asian Institute of Technology. The work presented in it are my own and has been generated by me as the result of my own original research, and if external sources were used, such sources have been cited. It is original and has not been submitted to any other institution to obtain another degree or qualification. This is a true copy of the thesis, including final revisions.

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# ABSTRACT

Supply chain management becomes very significant in highly competitive market in order to maintain a satisfactory customer service level. Utilizing an appropriate inventory strategy is crucial for the archive of any supply chain in cases of supply chain improvement. One strategy is named a lateral transshipment which allows the transportation of inventory between locations at the same stage of a supply chain. In this research, the focus aims to develop the mathematical models for the emergency lateral transshipment model between two retailers where the retailers follow compound Poisson demand under the centralized inventory system, specifically for slow-moving items. The developed mathematical model was applied to determine the optimal retailer’s order quantity of both retailers and the cycle length such that the expected total inventory cost of the whole system is minimized. Numerical experiments and sensitivity analyses are conducted in order to analyze the effect on the optimal solutions when the model’s base input parameters change.

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# LIST OF ABBREVIATIONS

|  |  |
| --- | --- |
| ELT = | Emergency Lateral Transshipment |
| FMI = | Fast-Moving Item |
| PLT = | Preventive Lateral Transshipment |
| SMI = | Slow-Moving Item |
| 3PL = | Third-Party Logistics Provider |

# INTRODUCTION

## Background of the Study

A supply chain management becomes very significant in highly competitive market in recent years in order to maintain a satisfactory customer service level. Supply chain management is an efficient strategy to manage the flow of information, goods, and services in fulfillment of customer requirements. The supply chain members comprise suppliers, manufacturers, distributors, retailers and customers. Multiple efficient supply chain management techniques have been established in academia, organizations, and industry over the last few decades.

One technique is named a lateral transshipment which allows the transportation of inventory between locations at the same stage of a supply chain. There are two categories of lateral transshipment.

• Preventive lateral transshipment or PLT happens at a specific point in the cycle time before the demand is realized or before stockout is observed. PLT mainly aims to decrease the future stockout risk.

• Emergency lateral transshipment or ELT occurs after the realization of stockout. ELT intends to respond to the actual stockouts.

The main advantage of transshipment is that demand and inventory at various locations are better matched. It redistributes available inventory among retail locations by using inventory data and up-to-date sales. It is possible that an item sells well in some locations but not so well in other locations because of socioeconomic and geographic differences among retailer locations.

To promote the rapid movement of goods, this behavior can be implemented by bypassing the central depot. Consequently, the profits are increased while costs are decreased as compared to a system where does not use the transshipment (Tagaras, 1989). Over the selling season, the transshipment will be needed when there is the mismatch somewhere between supply and demand.

Shipping, inventory orders, storage, and selling become increasingly important decisions as companies expand. The demand for more industrial storage grows, allowing goods to enter a broader geographical region while also increasing product availability. During this time of expansion, a decision about whether to ship goods from a centralized location or to have smaller warehouses in different areas is essential.

Based on the organization structure, personal interests, and management styles, the advantages and disadvantages of both centralized and decentralized inventory should be employed.

• Centralized inventory system

 A centralized inventory system is an inventory management system that performs all operations from a central location. Despite the fact that there might be different storage parts for different items, the storage used is often one large warehouse. The transportation of all inventories is done in the same way.

 Advantages of using centralized inventory system include:

 - Increased margins are achieved by cost reduction. - It makes simpler to maintain and foster the company culture. - There is an important reduction in operation costs.

 - Management responds quickly to issues resulting from processes and goods.

Not all inventories are going to be equally significant and a management of inventory should use classification schemes to put different emphasis on different categories of goods. Wintel and Patch (2003) observed that organizations traditionally concentrate only on the improvement of fast-moving products due to their ability to generate sales, ignoring possible benefits from efficiently handling slow-moving items. In general, most businesses with slow-moving items keep much more inventory than is actually needed.

According to the existing literature, most of the research works mention the lateral transshipment in multiple retailers under the centralized and decentralized inventory system. However, the centralized inventory system has high long-term transportation costs, and rush deliveries which be passed on to the consumer. Hence, dealing with the lateral transshipment in centralized inventory system is a very interesting topic.

## Statement of the Problem

The inventory control is a common issue to all organizations of a supply chain. At the beginning of a selling season, the distribution center will be replenishment products to retailers. Over the selling season, the lateral transshipment will be needed when there is the mismatch somewhere between supply and demand.

The lateral transshipment problem is the problem between multiple members at the same stage of a supply chain. When one retailer experiences shortage, that retailer may request for transshipment from other retailers who still have stock on-hand. The retailers who have stocks on-hand might not agree to transship all the stock they have. Instead, they want to keep some minimum stock which is called hold-back level to avoid shortage after transshipment.

The unit transshipment price must be less than the price when ordering from an upstream member but should be less than the unit selling price of the product on the market. Therefore, the retailer who request for transshipment will have some profit. With that, many researchers studied and developed many transshipment models. However, there are remaining issues which are unmanageable because of the complexity of the supply chain.

In fact, the existing researchers mostly study on the transshipment in case of a supplier provides product which is the fast-moving item to multiple retailers under the centralized and decentralized inventory system. There is no research works conducted on slow moving items. Therefore, there still exists a gap in dealing with centralized system for the slow-moving items. In a centralized system, the decision will be made by a central decision maker.

To fulfill this gap, this research considers the emergency lateral transshipment model in case of a centralized system in which a supplier provides product which is a slow-moving item to two retailers.

## Objective of the Study

This research aims to develop an emergency lateral transshipment model such that the expected total inventory cost of the whole system under centralized inventory system is minimized.

## Scopes and Limitations

In this research, the scope is to develop a lateral transshipment model for one supplier and two retailers. The research will be conducted under the following limitations.

1. The demand at both retailers comes following a compound Poisson processes with different rates and the elapsed time between two consecutive demand requests exponential distribution.
2. The transshipment costs are the same at both retailers and when a transshipment occurs, there will exist unit holding costs which are different for the two retailers.
3. The shortage cost which are different for the two retailers will be incurred when the demand at a retailer is not fulfilled.
4. The demand shortages are fully backlogged.
5. The replenishment cycles of the two retailers are the same.
6. This study only deals with the emergency lateral transshipment.

# LITERATURE REVIEW

## Introduction

This research aims to develop a transshipment model such that the expected total cost of the whole system under centralized inventory system is minimized. The lateral transshipment is a transportation of product between retailers at the same stage in a supply chain. Retailers with excess inventories can transship their stocks to retailers that are at risk of stockouts in the future or actual stockouts. Many actions can be applied to the lateral transshipment such as inventory pooling, redistribution of stock, and lateral resupply.

The lateral transshipment problem is the problem between multiple members at the same stage of a supply chain. When one retailer experiences shortage, that retailer may request for transshipment from other retailers who still have stock on-hand. However, there still have some points which have not been addressed in the past research papers. According to the past research paper, researchers mostly study on the transshipment in case of a supplier provides product which is the fast-moving items to multiple retailers under the centralized and decentralized inventory system.

Because all past research works aimed at fast moving items, there is no research works conducted on slow moving items. Therefore, there still exists a gap in which a centralized system in which a supplier provides product which is the slow-moving items to two retailers. To fulfill this gap, this research considers the transshipment model in case of a centralized inventory system in which a supplier provides the slow-moving item to two retailers.

## The Transshipment in Supply Chain

Lateral transshipment is one of the highly discussed topics among researchers in the inventory management field. The transshipment can be expressed as the monitored movement of material between locations at the same echelon in a supply chain. Many research studies on the issue of lateral transshipment have been published in the last decade. These researches attempted to answer the questions that which lateral transshipment method should be used; when should the lateral transshipment happens; and how much each retailer should order from the supplier at the beginning of a replenishment cycle.

Moreover, the research conducted in the lateral transshipment is Feng et al. (2016). They studied preventive lateral transshipment between multiple locations inventory system with dynamic approach. Decisions are made before demands are realized to prevent stockout risk in the future. Their objective is to determine the quantity and timing of preventive lateral transshipment decisions. Their result illustrates that how demand is distributed at the retailer has an effect on how transshipment performed. Finally, the perishability of a commodity has an obvious impact on transshipment decisions.

Yousuk and Luong (2011) discussed preventive lateral transshipment model of two retailers using planned route approach. The study shows how to use the expected route approach to create a mathematical model such that to minimize the total expected cost per cycle in case of inventory system between two retailers where the retailers use base stock periodic review policy. The lateral transshipment model follows Poisson demand. The hold back inventory level regulates the lateral transshipment amount. The advantages of inventory redistribution are shown by a decrease in total cost as a percentage. The results illustrate that cost of parameters are significant effect in transshipment decisions.

Optimal joint transshipment and replenishment policies in a multi-period inventory system with lost sales was studied by Mehrizi et al. (2015). They proposed transshipment as a forceful mechanism to alleviate the mismatch between supply and demand. They determine a multi-period inventory system with a finite horizon, in which two retailers have the option of replenishing their inventory from a supplier or transshipping from the other retailer during each period. They studied a stochastic control problem where the aim of figuring out the optimal policies such that minimizing the expected total inventory cost over the selling season. Finally, they demonstrated that the optimal policy's structure holds true for any known sequence, combination of ordering and transshipment over time.

Naderi et al. (2018) studied a deterministic model for the transshipment problem of a fast fashion retailer under capacity constraints. They presented a unique transshipment challenge for a major apparel company with a large retail network. Transshipments are used to rebalance stocks across the distribution network in order to best align supply and demand.

Simultaneous inventory rivalry and lateral transshipment between retailers were discovered by Zhang et al. (2019) They looked at a supply chain of two competitors and one supplier. They made a significant contribution by researching simultaneous inventory rivalry and retailer transshipment. They first derived the optimal order quantities of retailer under both centralize and decentralize inventory system. Finally, they discovered that the decentralized inventory system can complete coordination by setting an acceptable transshipment cost if the competitive strength, as measured by customer switching rate, is poor.

Lateral transshipment of critical medical products which are slow-moving items was study by Agirbas (2008) The lateral transshipment of essential medical products with low demand was investigated in this study. In a medical system, lateral transshipment between hospitals offers opportunities to decrease the cost of expiration. The decision rule for lateral transshipment in a two-hospital setting was studied in this paper, and the rule was extended to multiple hospital situations. The rule of decision took the most myopic course of action by believing that no potential transshipments could be done. Numerical experiments illustrate the important savings of cost and the rules of decision have a small gap from the upper bound of the total saving.

Goh and Lim (2014) studied a case study which is slow-moving items in centralized inventory system of a retail network. At the moment, the network stores a large number of products which are a slow-moving item, resulting in crucial working capital being locked up in inventory. Results of this research illustrate that a strategy of selective centralization can be designed to be resilient over the wide variety of slow-moving items definitions. Therefore, the slow-moving items which are stocked in centralized inventory system can reduce the fulfillment costs.

In conclusion, the lateral transshipment problem is critical in supply chain management. There are many past research papers studied this problem. Most of them aimed to minimize the expected total cost which consists of many costs such as holding costs, transshipment costs, and shortage costs. This includes different assumptions on demand distributions, opportunities for backordering, shortage cost, holding cost, ordering cost and transshipment cost. However, there exists some gap from those research works. Hence, this research will consider the preventive lateral transshipment between two retailers under centralized inventory system to be more specific on slow moving items.

## Identifying the Gaps for the Research

According to the literature reviews, it can be observed that each study is different objective. The previous study examines the transshipment problem with PLT and ELT. Some of them consider the centralized system. Most of them study on the transshipment between two retailers for the fast-moving item. Also, no research work carried out the emergency lateral transshipment model between two retailers under the centralized inventory system for slow-moving items.

This research will examine the lateral transshipment problem that how much stocks should be ordered to each retailer at the beginning of each cycle and time period of each cycle in system so as the expected total inventory cost of the system under centralized inventory system is minimized.

**Table 2.1**

*The Literature Review Summarization Related to the transshipment*

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Author | Objective | Transshipment | Inventory system | Retailer | Item |
| Agirbas (2008) | Reduction the expiration costs | ELT | - | Two | SMI(Medical item) |
| Mehrizi et al. (2015) | Minimize the total costs | PLT, ELT | - | Two | - |
| Feng et al. (2016) | Figure out the timing ang quantity | PLT | - | Multiple | FMI(Perishable product) |
| Feng et al. (2018) | Deciding which transshipment policy should be preferred | PLT, ELT | Decentralized | Two | FMI |
| Naderi et al. (2018) | Rebalance stocks | PLT | - | Multiple | FMI |
| Zhang et al. (2019) | Optimal quantity order | ELT | Centralized, Decentralized | Two | - |

# MATHEMATICAL MODEL DEVELOPMENT

To develop the lateral transshipment model, a mathematical model of two retailers with emergency lateral transshipment is developed in order to determine the optimal order quantities of both retailers, cycle length so as the expected total inventory cost of the system under centralized inventory system is minimized.

## Assumptions

The research will be conducted under the following assumptions.

1. The demand at both retailers comes following a compound Poisson processes with different rates and the elapsed time between two consecutive demand requests exponential distribution.
2. The transshipment costs are the same at both retailers and when a transshipment occurs, there will exist unit holding costs which are different for the two retailers.
3. The shortage cost which are different for the two retailers will be incurred when the demand at a retailer is not fulfilled.
4. The demand shortages are fully backlogged.
5. The replenishment cycles of the two retailers are the same.
6. This study only deals with the emergency lateral transshipment.

## Notations

In this research, the mathematical model is developed using the notations listed below.

Indices:

: retailers

 : cases, = 1, 2, 3, and 4

General parameters and variables:

 : cycle length (time unit)

 : order quantity of retailer at the beginning of a cycle (unit)

 : total demand at retailer during the period (unit)

 : the average arrival rate of order at retailer (unit)

 : cumulative distribution function of the amount requested in one order and is the fold convolution of

: the emergency lateral transshipment amount from the sending retailer to the

 receiving retailer (unit)

 : the probability of case

 : the holding cost at retailer ($ per unit per time unit)

 : the shortage cost at retailer ($ per unit per time unit)

 : the ordering cost at retailer ($ per order)

 : the emergency lateral transshipment cost ($ per unit)

 : the expected number of demand arrivals of retailer in case (unit)

 : the expected requested amount of retailer in case (unit)

 : the expected excess inventory of retailer in case (unit)

 : the shortage amount at retailer in case (unit)

 : the expected holding cost in case

 : the expected total inventory cost in case

 : the expected total inventory cost of the whole system per time unit

## Development of Model Framework

Case 1: when and , there will be no the lateral transshipment since the demand is lower than the order quantity at both retailers.

Case 2: when and , there is the lateral transshipment from the sending retailer to the receiving retailer with the lateral transshipment amount (.

 (3.1)

Case 3: when and , there is the lateral transshipment from the sending retailer to the receiving retailer with the lateral transshipment amount (.

 (3.2)

Case 4: when and , there will be no the lateral transshipment since the demand is higher than the order quantity at both retailers.

First, the probability distribution of the demand at retailer during time period defined as follows,

 (3.3)

Let the amount requested in one demand arrival at retailer is a random variable which follows normal distribution then can be approximated by a normally distributed random variable with mean and variance, , that can be determined as follows.

 (3.4)

And,

 (3.5)

Similarly, the probability distribution of the demand at retailer during time period defined as follows,

 (3.6)

Also,

 (3.7)

And,

 (3.8)

### Case 1: when and

**Figure 3.1**

*Inventory Level of Retailers when and*

**T**

**Inventory Level**

**Time**

**Retailer**

**T**

**Inventory Level**

**Time**

**Retailer**

For case 1, the demand of both retailers is lower than the order quantity. Therefore, there are no the emergency lateral transshipment in case 1.

The behavior of both retailers is a similar situation. The probability of case 1 is given by,

 (3.9)

Let be the conditional average demand rate at retailer in case 1 then

 (3.10)

We have

 (3.11)

So,

 (approximately) (3.12)

Similarly, let be the conditional average demand rate at retailer in case 1 then

 (3.13)

We have

 (3.14)

So,

 (approximately) (3.15)

The expected holding cost in a cycle can be expressed as,

 (3.16)

In this case, there are no the lateral transshipment cost. Furthermore, there are fixed cost components, i.e., ordering costs to the two retailers at the beginning of the cycle. Thus, the expected total minimum cost in a cycle of case 1, i.e., , can be expressed as,

 (3.17)

### Case 2: when and

**Figure 3.2**

*Inventory Level of Retailers when and*

**T**

**Inventory Level**

**Time**

**Retailer**

**T**

**Inventory Level**

**Time**

**Retailer**

For case 2, the demand of retailer is lower than its initial inventory level and the demand of retailer is higher than its initial inventory level. Therefore, there will be the emergency lateral transshipment from retailer to retailer .

The emergency lateral transshipment amount can be derived as the minimum value between the excess inventory of the sending retailer and the amount requested by the receiving retailer . Hence, it can be expressed as:

 (3.18)

The probability of case 2 is given by,

 (3.19)

Let be the conditional average demand rate at retailer in case 2 then

 (3.20)

We have

 (3.21)

So,

 (approximately) (3.22)

Similarly, let be the conditional average demand rate at retailer in case 2 then

 (3.23)

We have

 (3.24)

So,

 (approximately)

 (3.25)

In case 2, the whole inventory of retailer is fully consumed before the end of the cycle. The expected time to consume at retailer , i.e., is given by,

 (3.26)

The expected holding cost in a cycle can be expressed as,

 (3.27)

It is noted that the emergency lateral transshipment amounts will be placed by retailer in this case. The expected total demand during at retailer is determined as,

 (3.28)

Let be the expected requested amount of retailer during then

 (3.29)

And, the expected excess inventory of retailer during is given by,

 (3.30)

Then, the expected transshipped amount from to is given by,

 (3.31)

Also, the shortage amount at retailer can be determined as,

 (3.32)

In this case, there is the lateral transshipment cost. Furthermore, there are fixed cost components, i.e., ordering costs to the two retailers at the beginning of the cycle. Therefore, the expected total minimum cost in a cycle of case 2, i.e., , can be expressed as,

 (3.33)

### Case 3: when and

**Figure 3.3**

*Inventory Level of Retailers when and*

**T**

**Inventory Level**

**Time**

**Retailer**

**T**

**Inventory Level**

**Time**

**Retailer**

For case 3, the demand of retailer is higher than its initial inventory level and the demand of retailer is lower than its initial inventory level. Therefore, the inventories of retailer are transshipped from retailer to retailer .

The emergency lateral transshipment amount can be expressed as the minimum value between the excess inventory of the sending retailer and the amount requested by the receiving retailer . Hence, it can be expressed as:

 (3.34)

The probability of case 3 is given by,

 (3.35)

Let be the conditional average demand at retailer in case 3 then

 (3.36)

We have

 (3.37)

So,

 (approximately)

 (3.38)

Similarly, let be the conditional average demand at retailer in case 3 then

 (3.39)

We have

 (3.40)

So,

 (approximately) (3.41)

In case 3, the whole inventory of retailer is fully consumed before the end of the cycle. The expected time to consume at retailer , i.e., is given by,

 (3.42)

The expected holding cost in a cycle can be expressed as,

 (3.43)

It is noted that the emergency lateral transshipment amounts will be placed by retailer in this case. The expected total demand during at retailer is determined as,

 (3.44)

Let be the expected requested amount of retailer during then

 (3.45)

And, the expected excess inventory of retailer during is given by,

 (3.46)

Then, the expected transshipped amount from to is given by,

 (3.47)

Also, the shortage amount at retailer can be determined as,

 (3.48)

In this case, there is the lateral transshipment cost. Furthermore, there are fixed cost components, i.e., ordering costs to the two retailers at the beginning of the cycle. Therefore, the expected total minimum cost in a cycle of case 3, i.e., , can be expressed as,

 (3.49)

### Case 4: when and

**Figure 3.4**

*Inventory Level of Retailers when and*

**T**

**Inventory Level**

**Time**

**Retailer**

**T**

**Inventory Level**

**Time**

**Retailer**

At the lateral transshipment point of this case, the remaining quantity inventory level of both retailers is lower than their demand. Therefore, there are no the emergency lateral transshipment in this case.

The probability of case 4 is given by,

 (3.50)

Let be the conditional average demand at retailer in case 4 then

 (3.51)

We have

 (3.52)

So,

 (approximately)

 (3.53)

Similarly, let be the conditional average demand at retailer in case 4 then

 (3.54)

We have

 (3.55)

So,

 (approximately)

 (3.56)

The expected holding cost in a cycle can be expressed as,

 (3.57)

For case 4, there are no the lateral transshipment because the whole inventory of retailer and are also fully consumed before the end of the cycle.

The shortage amounts at retailer and retailer j can be determined as,

 (3.58)

And,

 (3.59)

In this case, there are no the lateral transshipment cost. Furthermore, there are fixed cost components, i.e., ordering costs to the two retailers at the beginning of the cycle. Therefore, the expected total minimum cost in a cycle of case 4, i.e., , can be derived as,

 (3.60)

According to the analyses of all 4 cases above, the expected total minimum inventory cost of the whole system per time unit, i.e., , can be expressed as

 (3.61)

# NUMERICAL EXPERIMENTS AND SENSITIVITY ANALYSES

## Numerical Experiments

Numerical experiments are demonstrated in order to illustrate the mathematical model for the two retailers under the centralized inventory system with emergency lateral transshipment. The genetic solver (GA) of MATLAB is employed to determine the optimal values of three decision variables, i.e., retailer ’s order quantity (), retailer ’s order quantity () and the cycle length () so as to minimize the expected total cost of the system.

It is noted that the constraint was included in the model to obtain more realistic values for the model.

The following options in the optimization toolbox are used in order to determine the optimal values in the model.

**Table 4.1**

*The Values in the Optimization Toolbox*

|  |  |
| --- | --- |
| Optimization toolbox | Value |
| Number of variables | 3 variables ( |
| Bounds | Lower bound– [1 1 1], Upper bound– [500 500 10] |
| Population size | max (min (10\*number Of Variables, 100), 40) = 40 |
| Reproduction | 0.05\*max (min (10\*number Of Variables, 100), 40) = 40 |
| Stopping criteria | 100\*number Of Variables = 300 |

For the base case, the values of input parameters which are used to determines the optimal values are presented below.

**Table 4.2**

*Input Parameters for the Two Retailers Inventory System*

|  |  |
| --- | --- |
| Parameter | Value |
| Retailer  | Retailer  |
| Average arrival rate of order ( (unit)Mean demand in one order ( (unit)Standard deviation of demand in one order ( (unit)Ordering cost ( ($ per order)Shortage cost ( ($ per unit per day)Holding cost ( ($ per unit per day)Transshipment cost ( ($ per unit) | 85325075 | 67420086 |
| 8 |

According to the input parameters shown in Table 4.2, the genetic solver (GA) of MATLAB determines the optimal values which the three decision variables are, retailer ’s order quantity ( is 200 units, retailer ’s order quantity ( is 210 units, the cycle length ( is 5 days and the expected minimum total inventory cost per day is 2667.7$.

## Sensitivity Analyses

Sensitivity analyses is conducted in order to examine the effects on developed mathematical model when the input parameters in the base case are changed. i.e., effect of average arrival rate of order, effect of mean demand in one order, effect of standard deviation of demand in one order, effect of retailers holding cost, effect of shortage cost, effect of ordering cost and effect of transshipment cost.

### Effect of Average Arrival Rate of Order

In this section, the value of average arrival rate of order of a retailer is varied from 4 to 12 units, while keeping other input parameters as same as in the base case. The optimal results for retailer ’s order quantity (), retailer ’s order quantity (), the cycle length () and the expected total cost of the whole system are interpreted in the following table.

**Table 4.3**

*Effect of Average Arrival Rate of Order*

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  |
| 4 | 4 | 140 | 196 | 7 | 2104.7 |
| 6 | 120 | 252 | 6 | 2469.7 |
| 8 | 120 | 336 | 6 | 2825.0 |
| 10 | 100 | 350 | 5 | 3309.8 |
| 12 | 100 | 420 | 5 | 4077.1 |
| 6 | 4 | 210 | 196 | 7 | 2637.8 |
| 6 | 180 | 252 | 6 | 2898.4 |
| 8 | 150 | 280 | 5 | 2936.4 |
| 10 | 150 | 350 | 5 | 3646.1 |
| 12 | 150 | 420 | 5 | 4413.4 |
| 8 | 4 | 240 | 168 | 6 | 2857.8 |
| 6 | 240 | 252 | 6 | 3272.6 |
| 8 | 200 | 280 | 5 | 3312.5 |
| 10 | 200 | 350 | 5 | 4023.7 |
| 12 | 200 | 420 | 5 | 4794.3 |
| 10 | 4 | 300 | 168 | 6 | 3191.4 |
| 6 | 300 | 252 | 6 | 3426.1 |
| 8 | 250 | 280 | 5 | 3727.7 |
| 10 | 250 | 350 | 5 | 4438.9 |
| 12 | 250 | 420 | 5 | 5209.4 |
| 12 | 4 | 300 | 140 | 5 | 3364.7 |
| 6 | 300 | 210 | 5 | 3533.0 |
| 8 | 300 | 280 | 5 | 4177.8 |
| 10 | 300 | 350 | 5 | 4889.0 |
| 12 | 300 | 420 | 5 | 5659.6 |

**Figure 4.1**

*Expected Total Inventory Cost of the System vs. Value*

According to the results shown in Table 4.3 and Figure 4.1, when the average arrival rate of order of one retailer increases while the average arrival rate of order of the other retailer remains unchanged, the order quantity of that retailer increases to cope with higher demand. However, the cycle length will decrease in order to reduce inventory holding cost. This decreasing trend in the cycle length will lead to the decrease in the order quantity of the other retailer. Related to the total cost, it can be seen that the expected total cost will increase when the average arrival rates of order increase. This trend is reasonable due to the increase in holding cost for the whole system.

### Effect of Mean Demand in One Order

In this section, the value of mean demand in one order of a retailer is varied from 4 to 8 units, while keeping other input parameters as same as in the base case. The optimal results for retailer ’s order quantity (), retailer ’s order quantity (), the cycle length () and the expected total cost of the whole system are interpreted in the following table.

**Table 4.4**

*Effect of Mean Demand in One Order*

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  |
| 4 | 4 | 128 | 96 | 4 | 1302.7 |
| 5 | 128 | 120 | 4 | 1469.4 |
| 6 | 128 | 144 | 4 | 1637.8 |
| 7 | 160 | 210 | 5 | 2387.8 |
| 8 | 160 | 240 | 5 | 2616.7 |
| 5 | 4 | 160 | 96 | 4 | 1506.6 |
| 5 | 200 | 150 | 5 | 2208.0 |
| 6 | 200 | 180 | 5 | 2437.9 |
| 7 | 200 | 210 | 5 | 2667.7 |
| 8 | 240 | 288 | 6 | 3681.5 |
| 6 | 4 | 240 | 120 | 5 | 2259.0 |
| 5 | 240 | 150 | 5 | 2486.5 |
| 6 | 240 | 180 | 5 | 2716.4 |
| 7 | 240 | 210 | 5 | 2946.2 |
| 8 | 288 | 288 | 6 | 4042.0 |
| 7 | 4 | 280 | 120 | 5 | 2535.5 |
| 5 | 280 | 150 | 5 | 2763.1 |
| 6 | 280 | 180 | 5 | 2992.9 |
| 7 | 336 | 252 | 6 | 4104.8 |
| 8 | 336 | 288 | 6 | 4399.7 |
| 8 | 4 | 320 | 120 | 5 | 2810.0 |
| 5 | 320 | 150 | 5 | 3037.6 |
| 6 | 384 | 216 | 6 | 4163.6 |
| 7 | 384 | 252 | 6 | 4459.9 |
| 8 | 384 | 288 | 6 | 4754.8 |

**Figure 4.2**

*Expected Total Inventory Cost of the System vs. Value*

According to the results given in Table 4.4 and Figure 4.2, it can be observed that when the mean demand in one order of one retailer increases while the mean demand in one order of the other retailer remains unchanged, the expected total minimum cost of the system and the cycle length increase. The increasing trend in the cycle length leads to the increase in the order quantities of the retailers. This is because of the fact that, in order to avoid too high shortage cost, the retailers should increase order quantities.

### Effect of Standard Deviation of Demand in One Order

In this section, the value of standard deviation of demand in one order of a retailer is varied from 2 to 6 units, while keeping other input parameters the same as in the base case. The optimal results for retailer ’s order quantity (), retailer ’s order quantity (), the cycle length () and the expected total cost of the whole system are interpreted in the following table.

**Table 4.5**

*Effect of Standard Deviation of Demand in One Order*

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  |
| 2 | 2 | 120 | 126 | 3 | 1498.4 |
| 3 | 160 | 168 | 4 | 1773.9 |
| 4 | 160 | 168 | 4 | 2056.7 |
| 5 | 160 | 168 | 4 | 2218.9 |
| 6 | 200 | 210 | 5 | 2626.4 |
| 3 | 2 | 160 | 168 | 4 | 1924.9 |
| 3 | 160 | 168 | 4 | 2175.9 |
| 4 | 200 | 210 | 5 | 2667.7 |
| 5 | 200 | 210 | 5 | 2913.4 |
| 6 | 240 | 252 | 6 | 3242.5 |
| 4 | 2 | 160 | 168 | 4 | 2026.2 |
| 3 | 200 | 210 | 5 | 2447.6 |
| 4 | 200 | 210 | 5 | 2732.5 |
| 5 | 200 | 210 | 5 | 3081.2 |
| 6 | 240 | 252 | 6 | 3457.6 |
| 5 | 2 | 200 | 210 | 5 | 2276.3 |
| 3 | 200 | 210 | 5 | 2580.0 |
| 4 | 200 | 210 | 5 | 2875.6 |
| 5 | 240 | 252 | 6 | 3128.9 |
| 6 | 240 | 252 | 6 | 3526.4 |
| 6 | 2 | 200 | 210 | 5 | 2562.4 |
| 3 | 200 | 210 | 5 | 2781.3 |
| 4 | 200 | 210 | 5 | 2922.9 |
| 5 | 240 | 252 | 6 | 3356.8 |
| 6 | 280 | 294 | 7 | 3635.7 |

**Figure 4.3**

*Expected Total Inventory Cost of the System vs. Value*

According to the results given in Table 4.5 and Figure 4.3, it is noticed that when the standard deviation of demand in one order of one retailer increases while the standard deviation of demand in one order of the other retailer remains unchanged, the expected total inventory cost of the system increases. Also, both order quantities increase in response to the increase in standard deviation of demand in one order. Meanwhile, when the standard deviation of demand in one order reduces, the cycle length reduces. This trend is reasonable to prevent shortages when the standard deviation of demand in one order increases.

### Effect of Holding Cost

In this section, the value of a retailer’s holding cost is varied from 4 to 6 $ per unit per day, while keeping other input parameters the same as in the base case. The optimal results for retailer ’s order quantity (), retailer ’s order quantity (), the cycle length () and the expected total cost of the whole system are interpreted in the following table.

**Table 4.6**

*Effect of Holding Cost*

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  |
| 4 | 4 | 240 | 252 | 6 | 2014.2 |
| 4.5 | 240 | 252 | 6 | 2103.8 |
| 5 | 200 | 210 | 5 | 2175.3 |
| 5.5 | 200 | 210 | 5 | 2297.1 |
| 6 | 200 | 210 | 5 | 2418.9 |
| 4.5 | 4 | 200 | 210 | 5 | 2056.0 |
| 4.5 | 200 | 210 | 5 | 2177.8 |
| 5 | 200 | 210 | 5 | 2299.6 |
| 5.5 | 200 | 210 | 5 | 2421.4 |
| 6 | 200 | 210 | 5 | 2543.3 |
| 5 | 4 | 200 | 210 | 5 | 2180.4 |
| 4.5 | 200 | 210 | 5 | 2302.2 |
| 5 | 200 | 210 | 5 | 2424.0 |
| 5.5 | 200 | 210 | 5 | 2545.8 |
| 6 | 200 | 210 | 5 | 2667.7 |
| 5.5 | 4 | 200 | 210 | 5 | 2304.6 |
| 4.5 | 200 | 210 | 5 | 2426.5 |
| 5 | 200 | 210 | 5 | 2548.3 |
| 5.5 | 200 | 210 | 5 | 2670.1 |
| 6 | 160 | 168 | 4 | 2778.6 |
| 6 | 4 | 200 | 210 | 5 | 2429.0 |
| 4.5 | 200 | 210 | 5 | 2550.8 |
| 5 | 200 | 210 | 5 | 2672.6 |
| 5.5 | 160 | 168 | 4 | 2745.7 |
| 6 | 160 | 168 | 4 | 2864.2 |

**Figure 4.4**

*Expected Total Inventory Cost of the System vs. Value*

According to the results shown in Table 4.6 and Figure 4.4, it is noticed that when the holding cost of one retailer increases while the holding cost of the other retailer remains unchanged, the expected total inventory cost of the system also increases. Moreover, it can be observed that the increase in the holding cost leads to the decrease in the cycle length. Also, the increase in value of holding cost will discourage the retailer to increase order quantities because of the retailer’s avoidance of too high inventory holding cost.

### Effect of Shortage Cost

In this section, the value of a retailer’s shortage cost is varied from 5 to 13$ per unit per day, while keeping other input parameters the same as in the base case. The optimal results for retailer ’s order quantity (), retailer ’s order quantity (), the cycle length () and the expected total cost of the whole system are interpreted in the following table.

**Table 4.7**

*Effect of Shortage Cost*

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  |
| 5 | 5 | 160 | 168 | 4 | 2463.8 |
| 7 | 160 | 168 | 4 | 2564.7 |
| 9 | 160 | 168 | 4 | 2687.3 |
| 11 | 200 | 210 | 5 | 2731.2 |
| 13 | 200 | 210 | 5 | 2797.6 |
| 7 | 5 | 160 | 168 | 4 | 2567.6 |
| 7 | 160 | 168 | 4 | 2612.6 |
| 9 | 200 | 210 | 5 | 2747.7 |
| 11 | 200 | 210 | 5 | 2797.7 |
| 13 | 200 | 210 | 5 | 2816.6 |
| 9 | 5 | 160 | 168 | 4 | 2670.5 |
| 7 | 160 | 168 | 4 | 2782.4 |
| 9 | 200 | 210 | 5 | 2812.3 |
| 11 | 200 | 210 | 5 | 2884.9 |
| 13 | 200 | 210 | 5 | 2911.2 |
| 11 | 5 | 200 | 210 | 5 | 2683.5 |
| 7 | 200 | 210 | 5 | 2807.6 |
| 9 | 200 | 210 | 5 | 2956.6 |
| 11 | 200 | 210 | 5 | 2989.5 |
| 13 | 200 | 210 | 5 | 3016.5 |
| 13 | 5 | 200 | 210 | 5 | 2734.7 |
| 7 | 200 | 210 | 5 | 2847.8 |
| 9 | 200 | 210 | 5 | 2991.2 |
| 11 | 200 | 210 | 5 | 3095.8 |
| 13 | 240 | 252 | 6 | 3394.7 |

**Figure 4.5**

*Expected Total Inventory Cost of the System vs. Value*

According to the results given in Table 4.7 and Figure 4.5, when the shortage cost of one retailerincreases while the shortage cost of the other retailer remains unchanged, the expected total inventory cost of the system also increases. This is because of the fact that, the retailers should increase order quantities in order to avoid too high shortage cost. Related to the total expected cost, it can be noticed that the expected total cost will increase due to the increase in shortage cost of the system.

### Effect of Ordering Cost

In this section, the value of a retailer’s ordering cost is varied from 100 to 300$ per order, while keeping other input parameters the same as in the base case. The optimal results for retailer ’s order quantity (), retailer ’s order quantity (), the cycle length () and the expected total cost of the whole system are interpreted in the following table.

**Table 4.8**

*Effect of Ordering Cost*

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  |
| 100 | 100 | 120 | 126 | 3 | 1344.9 |
| 150 | 160 | 168 | 4 | 1960.1 |
| 200 | 160 | 168 | 4 | 1972.6 |
| 250 | 200 | 210 | 5 | 2647.7 |
| 300 | 200 | 210 | 5 | 2657.7 |
| 150 | 100 | 160 | 168 | 4 | 1985.0 |
| 150 | 160 | 168 | 4 | 1997.5 |
| 200 | 200 | 210 | 5 | 2667.6 |
| 250 | 200 | 210 | 5 | 2677.6 |
| 300 | 200 | 210 | 5 | 2687.6 |
| 200 | 100 | 160 | 168 | 4 | 1992.6 |
| 150 | 200 | 210 | 5 | 2687.7 |
| 200 | 200 | 210 | 5 | 2697.7 |
| 250 | 240 | 252 | 6 | 3386.5 |
| 300 | 240 | 252 | 6 | 3394.9 |
| 250 | 100 | 200 | 210 | 5 | 2647.7 |
| 150 | 200 | 210 | 5 | 2707.7 |
| 200 | 200 | 210 | 5 | 2717.7 |
| 250 | 240 | 252 | 6 | 3394.9 |
| 300 | 240 | 252 | 6 | 3403.2 |
| 300 | 100 | 200 | 210 | 5 | 2657.7 |
| 150 | 240 | 252 | 6 | 3386.5 |
| 200 | 240 | 252 | 6 | 3394.9 |
| 250 | 240 | 252 | 6 | 3403.2 |
| 300 | 280 | 294 | 7 | 4180.0 |

**Figure 4.6**

*Expected Total Inventory Cost of the System vs. Value*

According to the results given in Table 4.8 and Figure 4.6, it is noticed that when the ordering cost of one retailer increases while the ordering cost of the other retailer remains unchanged, the order quantity of both retailers and the cycle length increase. Both order quantities trend to rise in response to the increase in ordering cost. This is understandable because when the ordering cost is high, the system has to order more products in one cycle in order to reduce the ordering cost per unit. As a result, more items will be stocked at the retailers.

### Effect of Transshipment Cost

In this section, the value of transshipment cost is varied from 6 to 10$ per unit, while keeping other input parameters the same as in the base case. The optimal results for retailer ’s order quantity (), retailer ’s order quantity (), the cycle length () and the expected total cost of the whole system are interpreted in the following table.

**Table 4.9**

*Effect of Transshipment Cost*

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  |  |  |  |  |
| 6 | 200 | 210 | 5 | 2433.1 |
| 7 | 200 | 210 | 5 | 2550.4 |
| 8 | 200 | 210 | 5 | 2667.7 |
| 9 | 240 | 252 | 6 | 2984.9 |
| 10 | 240 | 252 | 7 | 3351.5 |

**Figure 4.7**

*Expected Total Inventory Cost of the System vs. Transshipment Cost*

According to the results shown in Table 4.9 and Figure 4.7, it can be noticed that when the emergency lateral transshipment cost increases, the optimal order quantities and the cycle length also increase accordingly. These trends can be expected as when the transshipment cost increases, the emergency lateral transshipment amount will be reduced, and hence, the retailer’s order quantities should be increased accordingly. The expected total cost increases as the inventory holding cost increases because of the increase in order quantities.

# CONCLUSIONS AND RECOMMENDATIONS

## Conclusions

In highly competitive market, supply chain management becomes very significant in order to maintain a satisfactory customer service level. Utilizing an appropriate inventory strategy is crucial for the archive of any supply chain in cases of supply chain improvement. The implementation of this research is the development of an emergency lateral transshipment model between two retailers where the retailers follow compound Poisson demand in which the elapsed time between two consecutive demand requests exponential distribution.

The developed mathematical model was examined to determine the optimal of three decision variables, i.e., retailer ’s order quantity (), retailer ’s order quantity () and the cycle length () such that the expected total inventory cost of the whole system is minimized. According to the results of sensitivity analyses, following interesting characteristics of the optimal solutions have been found.

* The optimal retailer’s order quantity is affected by the average arrival rate of order, the mean demand in one order, the standard deviation of demand in one order, the ordering cost, the holding cost, the shortage cost and the transshipment cost. The optimal retailer’s order quantity trends to rise in response to the increase of the above input parameters except the holding cost.
* The optimal cycle length is affected by the average arrival rate of order, the mean demand in one order, the standard deviation of demand in one order, the ordering cost, the holding cost, the shortage cost and the transshipment cost. The optimal cycle length trends to rise in response to the increase of the above input parameters except the average arrival rate of order and the holding cost.

## Recommendations

In this research, the mathematical model was developed for the emergency lateral transshipment in the case of a single supplier – two retailers under the centralized inventory system. For further research directions, this can be extended to the case of more than two retailers where the preventive lateral transshipment can occur in the future in response to a stockout risk. Also, future research works could be considering the case of decentralized inventory system.

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# APPENDIX COMPUTER PROGRAM

**MATLAB Code**

function WTC = transshipment(Q) %{Q(1) = qi, Q(2) = qj, Q(3) = T}

L\_i = 8; %Lambda\_i

L\_j = 6; %Lambda\_j

m\_i = 5; %Mean\_i

m\_j = 7; %Mean\_i

sm\_i = 3; %Standard deviation\_i

sm\_j = 4; %Standard deviation\_j

C\_e = 8; %Cost of emergency lateral transshipment

O\_i = 250; %Ordering cost\_i

O\_j = 200; %Ordering cost\_j

Cs\_i = 7; %Shortage cost\_i

Cs\_j = 8; %Shortage cost\_j

h\_i = 5; %Holding cost\_i

h\_j = 6; %Holding cost\_j

pi = 22/7;

syms x y;

%Formulating Probabilities

A = 0; %P(Qi <= qi)

for i1 = 0:Q(1)

 m1 = L\_i\*Q(3)\*m\_i;

 v1 = sqrt(L\_i\*Q(3)\*((m\_i^2)+(sm\_i^2)));

 b1 = int(exp(-0.5\*((x-m1)/v1)^2)/(sqrt(2\*pi)\*v1),'x',0,i1);

 A = A+b1;

end

A;

B = 0; %P(Qj <= qj)

for i2 = 0:Q(2)

 m2 = L\_j\*Q(3)\*m\_j;

 v2 = sqrt(L\_j\*Q(3)\*((m\_j^2)+(sm\_j^2)));

 b2 = int(exp(-0.5\*((y-m2)/v2)^2)/(sqrt(2\*pi)\*v2),'y',0,i2);

 B = B+b2;

end

B;

C = 0; %P(Qj > qj)

for i3 = 0:Q(1)

 for i4 = Q(2):Q(1)+Q(2)-i3

 m3 = L\_j\*Q(3)\*m\_j;

 v3 = sqrt(L\_j\*Q(3)\*((m\_j^2)+(sm\_j^2)));

 b3 = 1-(int(exp(-0.5\*((y-m3)/v3)^2)/(sqrt(2\*pi)\*v3),'y',0,i4));

 C = C+b3;

 end

 C;

end

C;

E = 0; %P(Qi > qi)

for i5 = 0:Q(2)

 for i6 = Q(1):Q(1)+Q(2)-i5

 m4 = L\_i\*Q(3)\*m\_i;

 v4 = sqrt(L\_i\*Q(3)\*((m\_i^2)+(sm\_i^2)));

 b4 = 1-(int(exp(-0.5\*((x-m4)/v4)^2)/(sqrt(2\*pi)\*v4),'x',0,i6));

 E = E+b4;

 end

 E;

end

E;

%Probability for each case

P1 = (A\*B);

P2 = (A\*C);

P3 = (E\*B);

P4 = ((1-A)\*(1-B));

Total\_Prob = P1+P2+P3+P4;

%Formulating Expectations

%Case1

F = 0; %E[Qi]

for i7 = 0:Q(1)

 m5 = L\_i\*Q(3)\*m\_i;

 v5 = sqrt(L\_i\*Q(3)\*((m\_i^2)+(sm\_i^2)));

 f1 = (int(x\*exp(-0.5\*((x-m5)/v5)^2)/(sqrt(2\*pi)\*v5),'x',0,i7))/(A);

 F = F+f1;

end

F;

G = 0; %E[Qj]

for i8 = 0:Q(2)

 m6 = L\_j\*Q(3)\*m\_j;

 v6 = sqrt(L\_j\*Q(3)\*((m\_j^2)+(sm\_j^2)));

 f2 = (int(y\*exp(-0.5\*((y-m6)/v6)^2)/(sqrt(2\*pi)\*v6),'y',0,i8))/(B);

 G = G+f2;

end

G;

%Case2

H = 0; %E[Qi]

for i9 = 0:Q(1)

 m7 = L\_i\*Q(3)\*m\_i;

 v7 = sqrt(L\_i\*Q(3)\*((m\_i^2)+(sm\_i^2)));

 f3 = (int(x\*exp(-0.5\*((x-m7)/v7)^2)/(sqrt(2\*pi)\*v7),'x',0,i9))/(A);

 H =H+f3;

end

H;

I = 0; %E[Qj]

for i10 = Q(2):max(0,Q(1)+Q(2))

 for i11 = 0:max(0,Q(1)+Q(2)-i10)

 m8 = (L\_j\*Q(3)\*m\_j);

 v8 = sqrt(L\_j\*Q(3)\*((m\_j^2)+(sm\_j^2)));

 f4 = (m8-(int(y\*exp(-0.5\*((y-m8)/v8)^2)/(sqrt(2\*pi)\*v8),'y',0,i11))/(C));

 I = I+f4;

 end

 I;

end

I;

%Case3

J = 0; %E[Qi]

for i12 = Q(1):max(0,Q(1)+Q(2))

 for i13 = 0:max(0,Q(1)+Q(2)-i12)

 m9 = (L\_i\*Q(3)\*m\_i);

 v9 = sqrt(L\_i\*Q(3)\*((m\_i^2)+(sm\_i^2)));

 f5 = (m9-(int(x\*exp(-0.5\*((x-m9)/v9)^2)/(sqrt(2\*pi)\*v9),'x',0,i13))/(E));

 J = J+f5;

 end

 J;

end

J;

K = 0; %E[Qj]

for i14 = 0:Q(2)

 m10 = L\_j\*Q(3)\*m\_j;

 v10 = sqrt(L\_j\*Q(3)\*((m\_j^2)+(sm\_j^2)));

 f6 = (int(y\*exp(-0.5\*((y-m10)/v10)^2)/(sqrt(2\*pi)\*v10),'y',0,i14)/(B));

 K = K+f6;

end

K;

%Case4

L = 0; %E[Qi]

for i15 = Q(1):500

 m11 = (L\_i\*Q(3)\*m\_i);

 v11 = sqrt(L\_i\*Q(3)\*((m\_i^2)+(sm\_i^2)));

 f7 = (m11-(int(x\*exp(-0.5\*((x-m11)/v11)^2)/(sqrt(2\*pi)\*v11),'x',0,i15)))/(1-A);

 L = L+f7;

end

L;

M = 0; %E[Qj]

for i16 = Q(2):500

 m12 = (L\_j\*Q(3)\*m\_j);

 v12 = sqrt(L\_j\*Q(3)\*((m\_j^2)+(sm\_j^2)));

 f8 = (m12-(int(y\*exp(-0.5\*((y-m12)/v12)^2)/(sqrt(2\*pi)\*v12),'y',0,i16))/(1-B));

 M = M+f8;

end

M;

%Formulating Lambda for each case

%Case1

L\_1i = F/Q(3);

L\_1j = G/Q(3);

%Case2

L\_2i = H/Q(3);

L\_2j = I/Q(3);

%Case3

L\_3i = J/Q(3);

L\_3j = K/Q(3);

%Case4

L\_4i = L/Q(3);

L\_4j = M/Q(3);

%Total Inventory Cost for each case

%Case1

TC1 = O\_i+O\_j+(((2\*Q(1)-L\_1i\*Q(3))/2)\*Q(3)\*h\_i)+(((2\*Q(2)-L\_1j\*Q(3))/2)\*Q(3)\*h\_j);

%Case2

Nij = min((L\_2j\*Q(3)-Q(2)),(Q(2)-L\_2i\*Q(3)));

S\_j2 = L\_2j\*Q(3)-Q(2)-Nij;

TC2 = O\_i+O\_j+(((2\*Q(1)-L\_2i\*Q(3))/2)\*Q(3)\*h\_i)+(((Q(2)^2)/(2\*L\_2j))\*h\_j)+(C\_e\*Nij)+(Cs\_j\*S\_j2);

%Case3

Nji = min((L\_3i\*Q(3)-Q(1)),(Q(1)-L\_3j\*Q(3)));

S\_i3 = L\_3i\*Q(3)-Q(1)-Nji;

TC3 = O\_i+O\_j+(((Q(1)^2)/(2\*L\_3i))\*h\_i)+(((2\*Q(2)-L\_3j\*Q(3))/2)\*Q(3)\*h\_j)+(C\_e\*Nji)+(Cs\_i\*S\_i3);

%Case4

S\_i4 = (L\_4i\*Q(3))-Q(1);

S\_j4 = (L\_4j\*Q(3))-Q(2);

TC4 = O\_i+O\_j+(((Q(1)^2)/(2\*L\_4i))\*h\_i)+(((Q(2)^2)/(2\*L\_4j))\*h\_j)+(Cs\_i\*S\_i4)+(Cs\_j\*S\_j4);

%Total Inventory Cost of the whole system

WTC = ((P1\*TC1)+(P2\*TC2)+(P3\*TC3)+(P4\*TC4))/Q(3);

**Result**

 