OPTIMIZING TWO-ECHELON SUPPLY CHAIN UNDER VENDOR-MANAGED INVENTORY FOR A DETERIORATING PRODUCT

by

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AUTHOR'S DECLARATION

I, Phanumas Sojithamporn, declare that the research work carried out for this thesis was in accordance with the regulations of the Asian Institute of Technology. The work presented in it are my own and has been generated by me as the result of my own original research, and if external sources were used, such sources have been cited. It is original and has not been submitted to any other institution to obtain another degree or qualification. This is a true copy of the thesis, including final revisions.

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ABSTRACT

Vendor Managed Inventory (VMI) is an outstanding method which helps to boost efficiency of supply chain and improve cooperation between members in the supply chain. This research work focuses on developing an inventory model of one vendor and multiple retailers under VMI for deteriorating products. The considered deteriorating product, such as fruits or vegetables, perishes at a constant rate. The demand distribution is supposed to be linearly dependent on retail price. Also, shortage with full backlogging is considered in this inventory model. This study aims to optimize the total cost of the whole system by deriving the optimal replenishment cycle of the product and the point in time that the inventory is zero at the retailers. After developing the inventory model, numerical experiments are provided to demonstrate the applicability of the model. Sensitivity analyses are also performed to illustrate the impact of all input parameters on optimal solutions and total cost. Last but not least, future research directions are presented.

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LIST OF ABBREVIATIONS

GA-AIS	= Genetic Algorithm and Artificial Immune System
GA	= Genetic Algorithm
IRP	= Inventory-Routing Problem
PSO	= Particle Swarm Optimization
VMI	= Vendor Managed Inventory

CHAPTER 1
INTRODUCTION

1.1 Background of the Study

A network of buyers and its vendors who manufacture and distribute the specific product to those buyers is called a supply chain. Generally, the supply chain includes a set of activities and people between each entities (Kenton, 2020). The supply chain also display the processes in which products or services flow from the suppliers to final customers (Figure 1.1). The processes include moving and producing the finished product, transporting the product, holding the inventory, and distributing to final customers. To integrate those processes, supply chain management is required. Supply chain management is an essential process that every enterprises or companies have to deal with because they can lower their supply chain cost and faster their production cycle if supply chain is managed well.

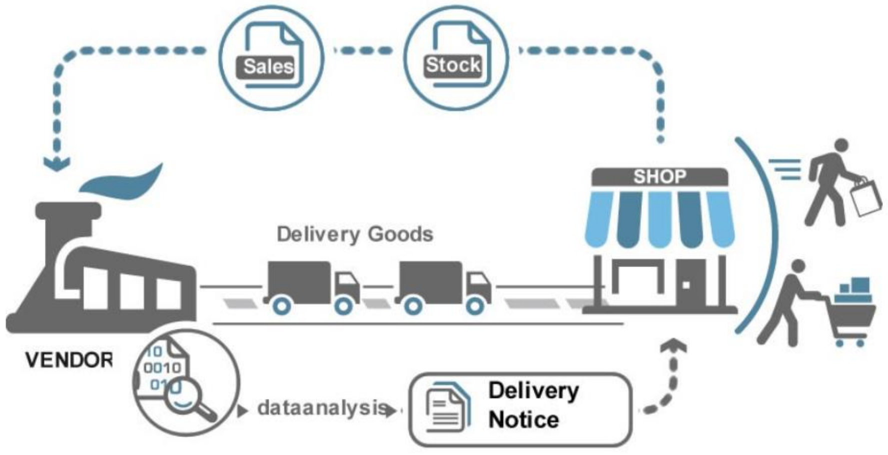
Figure 1.1

The General Entities in Supply Chain

Inventory management means keeping the right inventory at the appropriate level in the right location at the proper time and at a reasonable cost. Therefore, inventory management is one crucial issue of supply chain management. With inventory management, there are multiple strategies and policies have been proposed. One of the well-known inventory management policies is Vendor managed inventory (VMI) in which the responsibilities of vendor are their inventory and retailers' inventory. In VMI, to track retailers' inventory, the vendors visit their store or pull reports from retailers' systems (see Figure 1.2). Therefore, information sharing is very important in VMI in order to check the sold or unsold items and fills the area allocated accordingly (EasyEcom, 2021).

Figure 1.2

The Vendor Managed Inventory Model



The reasons why VMI is well-known and many large companies such as Walmart adopted this policy are given as follows;

The advantages for retailers

- The retailers' inventory will never go out of stock.

One of the biggest problems of inventory management is running out of stock and it can have impact on the service level of retailers. But with VMI, the vendor has prior insight into retailers' inventory levels and responsible to keep retailers' inventory stocked. Therefore, the vendor will know when their retailers want products.

- Lower carrying cost

Normally, there is a situation that the vendor requires a minimum order quantity but the optimum order quantity of retailers is less than the minimum required. Therefore, retailers have to carry those cost because of excessive stock. On the other hand, the vendor can supply the required quantity whenever retailers need with VMI which help retailers manage inventory more efficiently by reducing excess carrying stock resulting lower carrying cost.

The advantages for vendors

- Vendors are able to modulate output (finished product) to fit demand

With VMI, vendors will get insight to the customer demand for their product because of information sharing from their retailers. Therefore, they can estimate accurately how many items they will have in retailers' warehouses and know how much products they have to supply. That helps vendors to reduce safety stock and also reduce their carrying cost.

- Vendors can keep the relationship with retailers

After retailers agree to adopt VMI policy with the vendor, it is a very hard chance that retailers will move to another vendor because the vendor has already got the insight into retailers' information. Therefore, it seems like VMI policy is the long-term guarantee of cooperating between retailers and suppliers.

Among the research studies conducted on VMI policy, dealing with perishable products was a very interesting topic. A perishable product is a product with expired date such as chemicals, pharmaceuticals, fruits and vegetables. In order to reduce the losses on these types of product in a warehouse or in our supply chain, an effective way to manage inventory is required. Managing perishable product inventory is a crucial issue in many companies. Therefore, there are a lot of researchers interested on studying VMI with perishable product to help reduce the losses which resulted in reducing supply chain cost also.

1.2 Statement of the Problem

Vendor managed inventory can be known as an inventory management policy which is collaborative among vendors and retailers. VMI improves supply chain efficiency and responsiveness by reducing total cost of the whole system. In the past, there are many large and famous enterprises adopted VMI i.e., Walmart, Cambell Soup, Intel, etc. and they get successful outcomes. With that, many researchers studied and developed VMI in many scenarios. In the past few years, researchers developed VMI for deteriorating items in many scenarios such as different distributions of demand, different deteriorating rates, shortage allowance, additional condition, etc. However, there still exist other scenarios that need to work on to make VMI more realistic or practicable.

In fact, the existing research works mostly study on single vendor-single retailer system with shortage and without shortage. Only some research works were conducted on single vendor-multiple retailers system without shortage allowance or with shortage. There still exists a gap in which the VMI is applied for one vendor and multiple retailers system where shortages are allowed but fully backlogged. To fulfill this gap, this research considers one vendor and multiple retailers supply chain under VMI for deteriorating product in which shortages are allowed with full backlogging.

1.3 Objectives of the Study

A two-level supply chain of a deteriorating item which consists of one vendor and multiple retailers under vendor managed inventory (VMI) is studied in this research work. This research aims to develop an inventory model such that the total cost of the system is optimized.

1.4 Scopes and Limitations

In this research, the scope is to derive an inventory model under VMI for one vendor and multiple retailers supply chain on deteriorating product.

The research will be conducted under the following assumptions.

1. Costs and deteriorating rates of the retailers are different
2. Shortage is allowed with full backlogging
3. Deterioration happens only at retailers because of stocking condition.
4. The market demand for the product depends linearly on price.
5. Deterioration rate of the product (e.g., fruits, vegetables, food stuffs etc.) is constant.

CHAPTER 2
LITERATURE REVIEWS

This research aims to develop an inventory model and get the minimum costs of the supply chain which includes one vendor-multiple retailers under vendor managed inventory on deteriorating products and considering shortage with full backlogging. The inventory management approach that helps relieve costs and improve efficiency of the supply chain is VMI. However, there still have some points or issues which have not been addressed in previous research works. By addressing those points, the model will become more realistic and practicable. According to the studies in the past, researchers who studied one vendor and multiple retailers systems under VMI on deteriorating product assume that all retailers are homogenous.

To fill this gap, this research studies on one vendor and multiple retailers system in which their costs and deterioration rate are different under vendor managed inventory on deteriorating products and considering shortage with full backlogging.

2.1 Vendor Managed Inventory

An inventory management policy that has been used to reduce the total cost of the system is Vendor managed inventory (VMI). The concept of VMI is the incorporation between vendors and retailers in which the retailers share inventory information such as customer demand to their vendors. In the past, there are many studies on vendor managed inventory in order to minimize total cost or maximize total profit of the chain in various aspects. Sue-Ann et al. (2012) studied on one vendor and multiple retailers system under VMI. The objective of their research is maximizing profit model. The algorithms that are used to solve the problem are Particle Swarm Optimization (PSO) and hybrid heuristic (e.g. GA-AIS). In 2013, the efficiency of vendor managed inventory (VMI) and collaborative planning forecasting and replenishment (CPFR) was compared (Kazemi and Zhang, 2013). They developed a model under a price-dependent stochastic demand and getting the optimal retailer charges and amount of order of both VMI and CPFR is their main purpose. Moreover, Diabat (2014) also optimizing the profit of system under the VMI model by proposing a hybrid genetic or simulated annealing algorithm. According to above studies, both proposed models tried to use a hybrid heuristic algorithm in order to optimize the profit of the system and both are not consider shortage. However, most past literatures of vendor managed inventory aimed to minimize total cost with different circumstances. In 2013, Sadeghi et al. determined the optimal number of order and amount of shipments of multiple vendor multiple retailers and single warehouse with limited capacity and number of orders per year to get the minimized total cost. The near optimal solution is found by applying Particle Swarm Optimization (PSO). One year after that, Sadeghi et al. (2014) also proposed a system with one producer and many retailers under VMI model. The deterministic demand rate and not allow shortage are considered. This research mean to optimize total chain cost and improve the system reliability of producing items by applying meta-heuristic optimization algorithm. The economic order quantity of multiple product under VMI of one vendor and one buyer model is studied (Nia et al., 2014). The demand, availability of inventory, and the amount of order are considered to be fuzzy and shortage with backorder is allowed. The ant colony optimization was applied to find an approximate optimal result. In 2014, Rad et al. compared mathematical models of a single vendor- two buyers system between VMI and RMI. Their outcomes indicate that VMI can relieve total cost and boost the capability of the supply chian more than retailer-managed inventory. In 2015, a stochastic demand of one vendor and multiple retailers supply chain under VMI model was discussed by Mateen et al. A vendor replenishes simultaneously to all retailers. Optimizing the expected total cost of the whole supply chain is their objective.

2.2 Vendor Managed Inventory for Deteriorating Product

The research works on vendor managed inventory have been increased and researchers considered more about deteriorating product because it is a major problem for many companies. Vendors can provide deteriorating product to their retailers or customers as good as possible by adopting VMI (Yu et al., 2012). Therefore, some researchers interested in study on this issue and there are various aspects of the research have been presented. In 2011, Hongjie et al. studies on vendor managed inventory model of deteriorating items for suppliers and developed bi-level programming models. They considered demand as a poison process and also considered deterioration rate of inventory in stock to be constant. Yu et al. (2012) research on how to handle the system-wide inventory of fast and slow perishable products under VMI supply chain. They assume deteriorating rate as constant and demand is deterministic for every retailers.

Their results indicate that deteriorating rate on product is more considerable than raw material. Then, Xiao and XU (2013) measured system efficiency and investigate the way to integrate price and service level in VMI supply chain of one supplier and one retailer on perishable product. VMI model with no shortage for single vendor-multiple retailers was developed by Taleizadeh et al. in 2015. Raw material and finished product are assumed to deteriorate in different rate, and demand is deterministic and price sensitive. Maximizing total profit is the objective of this research. A Stackelberg game model where vendor acts as captain and retailers act as crews was considered in this research. Kaasgari (2017) studied on one vendor-multiple retailers for perishable product under VMI policy with no shortage. Discount is used to encourage customer demand. To find the optimal solution, some meta-heuristics algorithms are applied in order to get minimized total cost of supply chain. Chen (2018) studied production inventory problem of one vendor and multiple retailers system under VMI which considers pricing and promotion. They also proposed the model under just-in-time shipment policy and considered two promotion strategies. Furthermore, there are shortages with partial backlogging in this paper. Amiri et al. (2020) indicated that applying VMI policy for perishable product is an effective policy to get maximized profit of the chain. They presented a model of one vendor and multiples buyers of deteriorating product under VMI policy in order to optimize sales level. They compared results from exact method and meta-heuristics. A cyclic inventory-routing problem (IRP) under VMI policy was studied by Dai in 2020. The loss cost due to deteriorating of products is considered and the demand is assumed to be dependent on price and stock. In this paper, three situations including model ending, starting with shortage, and no shortage are considered. A hybrid heuristic was proposed so as to the total cost is minimized.

In conclusion, according to literature review, there are many past research studies on VMI and most of them aimed to reduce cost so that the total cost of the system is optimized in various situations and various assumption such as different customer demand rates. Moreover, the importance of deteriorating product on inventory management is considerable. Therefore, researchers studied on VMI with consideration of deteriorating items. However, there still have some gaps from those research works. Therefore, this research will study one vendor-multiple retailers supply chain in which the retailers' costs and deterioration rates are different under vendor managed inventory on deteriorating products and considering shortage with full backlogging.

2.3 Identifying the Gaps for the Research

According to the research in the past, they mostly studied on vendor managed inventory policy of one vendor- one retailer supply chain, and some studies on vendor managed inventory supply chain of one vendor-multiple retailers. There are various conditions that can be considered to make the studies more practicable. Many studies consider deteriorating rate of the products while adopting vendor managed inventory. However, the shortage with full backlogging along with one vendor and multiple retailers under VMI for deteriorating product has not been considered in the past. Therefore, the summarization of past research works is stated in Table 2.1.

Table 2.1

The Summarization of Past Research Works of Vendor Managed Inventory (VMI)

Authors	Research topic	Model structure	Deteriorating product	Shortage
Sue-Ann et al. (2012)	Evolutionary algorithms for optimal operating parameters of vendor managed inventory systems in a two-echelon supply chain	one vendor and many retailers		
Diabat (2014)	Hybrid algorithm for a vendor managed inventory system in a two-echelon supply chain	one vendor and many retailers		
Sadeghi et al. (2013)	Optimizing a multi-vendor multi-retailer vendor managed inventory problem: Two tuned meta-heuristic algorithms	Multiple vendors and multiple retailers		
Kazemi and Zhang (2013)	Optimal decisions and comparison of VMI and CPFR under price-sensitive uncertain demand	one vendor and one retailer		

Authors	Research topic	Model structure	Deteriorating product	Shortage
Nia et al. (2014)	A fuzzy vendor managed inventory of multi-item economic order quantity model under shortage: An ant colony optimization algorithm	A vendor and a retailer		• (Backorder)
Rad et al. (2014)	Optimizing an integrated vendor-managed inventory system for a single-vendor two-buyer supply chain with determining weighting factor for vendor's ordering cost	one vendor and two retailers		• (Backorder)
Mateen et al. (2015)	VMI for single-vendor multi-retailer supply chains under stochastic demand	A vendor and many retailers		
Yu et al. (2012)	A vendor managed inventory supply chain with deteriorating raw materials and products	one vendor and many retailers	• (Constant rate)	

Authors	Research topic	Model structure	Deteriorating product	Shortage
Taleizadeh et al. (2015)	Joint optimization of price, replenishment frequency, replenishment cycle and production rate in vendor managed inventory system with deteriorating items	A single vendor and multiple retailers	• (Constant rate)	
Kaasgari et al. (2017)	Optimizing a vendor managed inventory (VMI) supply chain for perishable products by considering discount: Two calibrated meta-heuristic algorithms	A vendor and many retailers	• (Depend on keeping condition)	
Chen (2018)	Optimization of production inventory with pricing and promotion effort fora single-vendor multi-buyer system of perishable products	A single vendor and many retailers	• (Constant rate)	
Amiri et al. (2020)	Determination of the optimal sales level of perishable goods in a two-echelon supply chain network	one vendor and many retailers	•	

Authors	Research topic	Model structure	Deteriorating product	Shortage
Dai et al. (2020)	A hybrid heuristic algorithm for cyclic inventory-routing problem with perishable products in VMI supply chain	Single vendor and many retailers	• (Constant rate)	• (Not backlogged)
Sadeghi et al. (2014)	A hybrid vendor managed inventory and redundancy allocation optimization problem in supply chain management: An NSGA-II with tuned parameters	A vendor and many retailers		
Xiao and XU (2013)	Coordinating price and service level decisions for a supply chain with deteriorating item under vendor managed inventory	Single vendor and one retailer	• (Constant rate)	
Hongjie et al. (2011)	Study on the inventory control of deteriorating items under VMI model based on bi-level programming	A single vendor and many retailers	• (Constant rate)	

CHAPTER 3
MODEL FORMULATION

The development of inventory model is presented in this chapter. The model considers one vendor and multiple retailers supply chain under vendor managed inventory (VMI) for deteriorating product. In VMI supply chain, after vendors get retailers' inventory level, the vendor then predicts customers' demand and deliver the deteriorating product according to retailers' demand which is price-dependent. The deteriorating product in a batch will be distributed from a vendor to each retailer simultaneously. Only the product at retailers deteriorate through time due to stocking condition. The shortage is considered with complete backlogging which means customers or retailers are willing to wait for their demand to be fulfilled when shortage occurs. To optimize the total cost of the system, the mathematical inventory model is established under following assumptions and notation.

3.1 Assumptions

Before setting up a mathematical model over a finite planning horizon, following assumptions will be used.

1. Customers' demand is linearly dependent on retailers' price.
2. Shortage with full backlogged is allowed.
3. The deterioration occurs only at retailers due to stocking condition. The deterioration rates of the product (e.g. fruits, vegetables, food stuffs, etc.) is constant and known ($0 < \theta < 1$).
4. The model is formulated with a generalized demand function.
5. The costs between each retailer are different.

3.2 Notations

To derive the optimum policy, the total cost of the related members in the supply chain are minimized. The length of replenishment cycle (T), and point in time at which the inventory is zero at the i^{th} retailer (t_i^r) are the decision variables in this inventory model.

To develop the model, the following notations are used.

$D_i(s)$: The demand rate of the i^th retailer per unit time which is a function of the retail price.

$I_i^r(t)$: The inventory level of the i^th retailer at time t, $0 \le t \le t_i^r$.

$I_{bi}^r(t)$: The negative inventory level of the i^th retailer at time t, $t_i^r \le t \le T$.

θ_i: Deteriorating rate of finished product at the i^th retailer.

B_i: The maximum amount of demand backlogged of the i^th retailer per cycle.

$B_i(t)$: Backlogging rate at retailer i for negative inventory.

Q_i^r: Order quantity of the i^th retailer.

T: Common replenishment cycle of the retailers.

t_i^r: Point in time at which the inventory is zero at the i^th retailer.

O_i^r: The ordering cost of the i^th retailer for finished product.

h_i^r: The holding cost of the i^th retailer per unit per time unit.

c_{pi}^r: The unit purchasing cost of the i^th retailer.

c_{di}^r: The unit deteriorating cost for finish product of the i^th retailer.

c_{si}^r: The unit shortage cost of the i^th retailer per time unit.

TC_i^r: The total cost of the i^th retailer per unit time.

3.3 Mathematical Model of VMI Supply Chain

The demand function of customer is linearly price-dependent. The demand function can be express as $D_i(s) = a_i + b_i s_i$ in which a_i is a positive parameter of the i^th retailer, b_i is price sensitivity coefficient of the i^th retailer, and s_i is the selling price of the i^th retailer.

By applying vendor managed inventory model, total cost of supply chain is total cost of retailers which is the responsibility of the vendor only. Therefore, the model starts with developing of the total cost of a retailer.

3.3.1 The Total Cost of A Retailer

The total cost of a retailer ith is developed in this subsection. The retailer gets the batch of product from the vendor which can be deteriorated at constant rate (θ_i) due to stocking condition. Furthermore, the perishable products are removed from the inventory on-hand. Therefore, the variation of ith retailer's inventory with respect to time can be expressed as follows.

$$\frac{dI_i^r(t)}{dt} = -\theta_i I_i^r(t) - D_i(s), 0 \le t \le t_i^r \qquad (1)$$

From (1), the solution can be derived as

$$I_i^r(t) = -\frac{D_i(s)}{\theta_i} + \frac{c}{e^{\theta_i t}}, 0 \le t \le t_i^r \qquad (2)$$

With boundary condition $I_i^r(t_i^r) = 0$, i = 1, 2,…., n, we have $c = \frac{D_i(s)}{\theta_i}(e^{\theta_i t_i^r})$ Hence,

$$I_i^r(t) = \frac{D_i(s)}{\theta_i}\left[e^{\theta_i(t_i^r - t)} - 1\right], 0 \le t \le t_i^r \qquad (3)$$

By substituting t equal to 0, the maximum inventory level of the ith retailer at time t is obtained as in the following equation (4).

$$I_i^t(0) = \frac{D_i(s)}{\theta_i}\left[e^{\theta_i t_i^r} - 1\right] \qquad (4)$$

Between the time intervals $\left(t_i^r, T\right)$, the level of inventory will be negative because of demand accumulation and demand shortage is fully backlogged. So, the differential equation of inventory level during $\left(t_i^r, T\right)$ can be derived in the following equation.

$$\frac{dI_{bi}^r(t)}{dt} = -D_i(s) \text{ , } t_i^r \le t \le T \qquad (5)$$

From (5), the solution can be derived as

$$I_{bi}^{r}(t) = -tD_i(s) + c \tag{6}$$

With boundary condition $I_{bi}^{r}(t_i^r) = 0$, we have $c = t_i^r D_i(s)$. Hence,

$$I_{bi}^{r}(t) = D_i(s)\left[t_i^r - t\right] \tag{7}$$

At time t=T, we obtain the maximum amount of demand backlogged as

$$-I_{bi}^{r}(T) = B_i = D_i(s)\left[T - t_i^r\right] \tag{8}$$

Therefore, the order quantity of the ith retailer (Q_i^r) can be derived by adding the maximum level of inventory of the ith retailer at time t and the maximum quantity of demand backlogged of the ith retailer per cycle as showed in equation (9).

$$Q_i^r = I_i^r(0) + B_i = \frac{D_i(s)}{\theta_i}\left[e^{\theta_i t_i^r} - 1 + \theta_i T - \theta_i t_i^r\right] \tag{9}$$

To develop the total cost of ith retailer, the holding cost, shortage cost, deteriorating cost, and purchasing cost are derived. Firstly, the holding cost is shown as in equation (10).

$$\begin{aligned}\int_0^{t_i^r} h_i^r I_i^r(t)dt &= h_i^r \int_0^{t_i^r} \frac{D_i(s)}{\theta_i}\left[e^{\theta_i (t_i^t - t)} - 1\right]dt \\ &= \frac{(h_i^r D_i(s))(e^{\theta_i t_i^r} - t_i^r \theta_i - 1)}{\theta_i^2}\end{aligned} \tag{10}$$

Secondly, the purchasing cost is as in equation (11).

$$c_{pi}^{r}Q_{i}^{r} = c_{pi}^{r}\left[\frac{D_i(s)}{\theta_i}\left(e^{\theta_i t_i^r} - 1 + \theta_i T - \theta_i t_i^r\right)\right] \tag{11}$$

Next, the deteriorating cost which occurs only during $[0, t_i^r]$ can be derived as in equation (12).

$$\begin{aligned}\int_0^{t_i^r} c_{di}^r\theta_i \cdot I_i^r(t)dt &= c_{di}^r D_i(s)\int_0^{t_i^r}(e^{\theta_i(t_i^r - t)} - 1)dt \\ &= \frac{(C_{di}^r D_i(s))(e^{\theta_i t_i^r} - t_i^r\theta_i - 1)}{\theta_i}\end{aligned} \tag{12}$$

Lastly, the shortage cost which occurs during $[t_i^r, T]$ can be derived as in equation (13).

$$\begin{aligned}\int_{t_i^r}^{T} c_{si}^r \cdot \left(t - t_i^r\right)\cdot\left(-I_{bi}^r(t)\right)dt &= c_{si}^r\int_{t_i^r}^{T}\left(t_i^r - t\right)\left(D_i(\mathrm{s})\left[t_i^r - t\right]\right)dt \\ &= \frac{(c_{si}^r D_i(s))\left(T - t_i^r\right)^3}{3}\end{aligned} \tag{13}$$

Therefore, the total cost per unit time of a retailer includes the ordering cost, the holding cost, the purchasing cost, the deteriorating cost, and the shortage can be derived as in equation (14)

$$TC_i^r = \frac{1}{T}\left[\begin{aligned}&O_i^r + \frac{(h_i^r D_i(s))(e^{\theta_i t_i^r} - t_i^r\theta_i - 1)}{\theta_i^2} + \frac{\left(C_{pi}^r D_i(s)\right)\left(e^{\theta_i t_i^r} - 1 + \theta_i T - \theta_i t_i^r\right)}{\theta_i} \\ &+ \frac{\left(C_{di}^r D_i(s)\right)\left(e^{\theta_i t_i^r} - t_i^r\theta_i - 1\right)}{\theta_i} + \frac{\left(C_{si}^r D_i(s)\right)\left(T - t_i^r\right)^3}{3}\end{aligned}\right] \tag{14}$$

To minimize the total cost (TC_i^r) per unit time of a retailer, the optimum value of the point in time that the inventory is zero at the i[th] retailer (t_i^r) and the replenishment cycle of the retailers (T) can be found by solving the following equations (15) and (16).

$$\frac{\partial TC_i^r}{\partial t_i^r} = 0 \text{ and } \frac{\partial TC_i^r}{\partial T} = 0$$

$$\frac{\partial TC_i^r}{\partial t_i^r} = \frac{1}{T}\left[\begin{array}{l} \frac{\left(h_i^r D_i(s)\right)}{\theta_i^2}\left(\theta_i e^{\theta_i t_i^r} - \theta_i\right) + \frac{c_{pi}^r D_i(s)}{\theta_i}\left(\theta_i e^{\theta_i t_i^r} - \theta_i\right) \\ + \frac{C_{di}^r D_i(s)(\theta_i e^{\theta_i t_i^r} - \theta)}{\theta_i} - c_{si}^r \mathrm{D}_i(\mathrm{s}) \cdot (\mathrm{T} - t_i^r)^2 \end{array}\right] = 0 \tag{15}$$

$$\frac{\partial TC_i^r}{\partial T} = \left[\begin{array}{l} -\frac{O_i^r}{T^2} - \frac{\left(h_i^r D_i(s)\right)\left(e^{\theta_i t_i^r} - t_i^r \theta_i - 1\right)}{\theta_i^2 T^2} + \frac{c_{pi}^r D_i(s)}{\theta_i}\left(\frac{-e^{\theta_i t_i^r} + 1 + \theta_i t_i^r}{T^2}\right) \\ -\frac{c_{di}^r D_i(s)(e^{\theta_i t_i^r} - \theta_i t_i^r - 1)}{\theta_i T^2} + \frac{c_{si}^r D_i(s)}{3}\left(\frac{2T^3 - 3T^2 t_i^r + (t_i^r)^3}{T^2}\right) \end{array}\right] = 0 \tag{16}$$

3.3.2 The Total Cost of Multiple Retailers

This model considers multiple retailers which amount of retailers is n and retailer index is i = 1, 2, …, n. Since the costs are different between each retailer, the total cost of multiple retailers will be the sum of each retailer's total costs. In this problem replenishment cycle of the retailers (T) will be the same for all retailers.The total cost of multiple retailers is given by (17).

$$\sum_{i=1}^{n} TC_i^r = \frac{1}{T}\left[\sum_{i=1}^{n}\left(\begin{array}{l} O_i^r + \frac{\left(h_i^r D_i(s)\right)\left(e^{\theta_i t_i^r} - t_i^r \theta_i - 1\right)}{\theta_i^2} + \frac{\left(C_{pi}^r D_i(s)\right)\left(e^{\theta_i t_i^r} - 1 + \theta_i T - \theta_i t_i^r\right)}{\theta_i} \\ + \frac{\left(C_{di}^r D_i(s)\right)\left(e^{\theta_i t_i^r} - t_i^r \theta_i - 1\right)}{\theta_i} + \frac{\left(C_{si}^r D_i(s)\right)\left(T - t_i^r\right)^3}{3} \end{array}\right)\right] \tag{17}$$

Similarly, to optimize the total cost, the points in time that the inventory is zero at each retailer (t_i^r) and the replenishment cycle of the retailers (T) should be determined. Then, the optimal solution of total cost will be determined. The applicability of the proposed model is presented through numerical experiments and sensitivity analyses will be conducted in the next chapter.

CHAPTER 4

NUMERICAL EXPERIMENTS AND SENSITIVITY ANALYSES

Numerical experiments related to the developed mathematical models in chapter 3 will be presented in this section in order to present the applicability of the developed models. Moreover, sensitivity analyses are presented in this chapter also. The needed data for computational experiments are provided. Then, the impacts of important input parameters on decision variables and optimal solution can be examined by conducting sensitivity analyses.

4.1 Numerical Experiment for the Case of One Retailer

In this section, the input parameters of one vendor and one retailer system under vendor managed inventory (VMI) policy are concluded in Table 4.1.

Table 4.1

The Value of Input Parameters in the Base Case for One Retailer System

Parameters	Value
θ_i	0.03
O_i^r	10,000
c_{pi}^r	140
c_{di}^r	145
c_{si}^r	150
h_i^r	6
a	2,000
b	11
s_i	178.67

Firstly, the values of input parameters in Table 4.1 are used to solve the value of optimal decision variables which are the point in time at which the inventory is zero at the i^{th} retailer (t_i^r), and replenishment cycle of the retailers (T). Then, the optimal solution of total cost of supply chain is determined. The outcomes are presented in Table 4.2.

Table 4.2

The Optimal Solutions of Single Vendor and One Retailers

t_i^r	T	Total cost
5.4883	6.2491	7,854.30

In next part, the impacts of input parameters on decision variables and total cost for the case of one retailer can be examined by conducting sensitivity analyses.

4.2 Sensitivity Analysis for the Case of One Retailer

In this part, sensitivity analyses in the case of one retailer are applied to examine the impact of input parameters on decision variables and optimal solutions. To conduct sensitivity analyses, the input parameters are changed one at a time.

4.2.1 Sensitivity Analysis With Respect to the Deteriorating Rate

This section investigate the deteriorating rate of product at the i^{th} retailer (θ_i) while other parameters are kept unchanged. The value of deteriorating rate of product at the i^{th} retailer (θ_i) is changed by 75%, 50%, 25%, -75%, -50%, and 25%, respectively, from the base value, and the outcomes are shown in Table 4.3.

According to Table 4.3, the point in time at which the inventory is zero at the i^{th} retailer (t_i^r) and replenishment cycle of the retailer (T) gradually decrease when θ_i increases. These trends are understandable. The values of t_i^r and T decrease because when the product deteriorates faster the retailers should hold the product in a shorter time. However, the total cost slightly increases when θ_i increases. Practically, the increase in deteriorating cost results in an increase in total cost.

Table 4.3

Sensitivity Analysis With Respect to the Deteriorating Rate (θ_i)

Parameter	Change rates

	Change rates	t_i^r	% t_i^r change	T	% T change	Total cost	% Total cost change
	-75	7.836	42.77%	8.497	35.97%	7122.40	-9.32%
	-50	6.801	23.92%	7.501	20.04%	7395.90	-5.84%
	-25	6.056	10.35%	6.789	8.64%	7647.20	-2.76%
θ_i	0	5.488	0.00%	6.249	0.00%	7854.30	0.00%
	25	5.037	-8.22%	5.822	-6.83%	8052.00	2.52%
	50	4.667	-14.96%	5.475	-12.39%	8234.00	4.83%
	75	4.358	-20.60%	5.185	-17.03%	8402.70	6.98%

4.2.2 Sensitivity Analysis With Respect to the Ordering Cost

In this subsection, the ordering cost of the i^th^ retailer (O_i^r) is changed and the outcomes are presented in Table 4.4.

From Table 4.4, the point in time at which the inventory is zero at the i^th^ retailer (t_i^r) and replenishment cycle of the retailers (T) moderately increase. Similarly, total cost slightly increases when ordering cost increases. It is obvious that t_i^r and T should increase to help reduce the component of ordering cost per unit of time in the total cost function.

Table 4.4

Sensitivity Analysis With Respect to the Ordering Cost of the i^th^ Retailer (O_i^r)

Parameter	Change rates

	Change rates	t_i^r	% t_i^r change	T	% T change	Total cost	% Total cost change
	-75	2.738	-50.11%	3.264	-47.76%	6286.10	-19.97%
	-50	3.890	-29.12%	4.523	-27.62%	6927.40	-11.80%
	-25	4.762	-13.23%	5.467	-12.52%	7424.60	-5.43%
O_i^r	0	5.488	0.00%	6.249	0.00%	7854.30	0.00%
	25	6.121	11.53%	6.928	10.87%	8233.60	4.83%
	50	6.687	21.84%	7.534	20.57%	8679.30	9.23%
	75	7.203	31.24%	8.086	29.39%	8899.40	13.31%

4.2.3 Sensitivity Analysis With Respect to the Unit Purchasing Cost

This subsection investigate the purchasing cost of the ith retailer (c_{pi}^r) per unit with change rates 75%, 50%, 25%, -75%, -50%, and 25% from the base value. The outcomes are presented in Table 4.5.

From Table 4.5, the point in time at which the inventory is zero at the ith retailer (t_i^r) and replenishment cycle of the retailers (T) decline when the unit purchasing cost of the ith retailer (c_{pi}^r) increases. In contrast, total cost significantly increases when c_{pi}^r increases. These trends are reasonable because when c_{pi}^r increases the replenishment cycle *T* should be reduced to help alleviate the increase in deterioration cots. The decrease in *T* will then result in a decrease in t_i^r. However, the total cost definitely increases when c_{pi}^r increases.

Table 4.5

Sensitivity Analysis With Respect to the Unit Purchasing Cost of the ith retailer (c_{pi}^r)

Parameter	Change rates	Change rates					
		t_i^r	% t_i^r change	T	% T change	Total cost	% Total cost change
	-75	6.234	13.59%	6.956	11.31%	3918.20	-50.11%
	-50	5.956	8.51%	6.691	7.08%	5235.50	-33.34%
	-25	5.709	4.02%	6.458	3.34%	6547.30	-50.11%
c_{pi}^r	0	5.488	0.00%	6.249	0.00%	7854.30	0.00%
	25	5.289	-3.63%	6.061	-3.00%	9156.90	16.58%
	50	5.109	-6.92%	5.891	-5.72%	10456.00	33.12%
	75	4.943	-9.93%	5.736	-8.20%	11751.00	49.61%

4.2.4 Sensitivity Analysis With Respect to the Unit Deteriorating Cost

Next input parameter to be investigated is the unit deteriorating cost of the ith retailer (C_{di}^r). The change rates are similar to previous investigations and the outcomes are presented in Table 4.6.

From Table 4.6, the point in time at which the inventory is zero at the ith retailer (t_i^r) and replenishment cycle of the retailers (*T*) slightly drop but total cost slightly rises when the unit deteriorating cost of the ith retailer (C_{di}^r) increases. The decreases in t_i^r and *T* are reasonable to help reduce the deterioration cost component in the total cost function.

Table 4.6

Sensitivity Analysis With Respect to the Unit Deteriorating Cost for Product of the ith Retailer (C_{di}^r)

Parameter	Change rates	Change rates					
		t_i^r	% t_i^r change	T	% T change	Total cost	% Total cost change
	-75	6.266	14.17%	6.986	11.79%	7542.80	-3.97%
	-50	5.974	8.86%	6.709	7.36%	7652.30	-2.57%
	-25	5.717	4.17%	6.466	3.46%	7755.90	-1.25%
c_{di}^r	0	5.488	0.00%	6.249	0.00%	7854.30	0.00%
	25	5.283	-3.75%	6.055	-3.10%	7048.00	1.19%
	50	5.096	-7.14%	5.880	-5.91%	8037.50	2.33%
	75	4.927	-10.23%	5.721	-8.46%	8123.30	3.42%

4.2.5 Sensitivity Analysis With Respect to the Unit Shortage Cost

The effects of shortage cost of the ith retailer per time unit (c_{si}^r) on decision variables are examined in this subsection. The change rates will be the same as previous investigation. The outcomes are illustrated in Table 4.7.

As shown in Table 4.7, when unit shortage cost of the ith retailer per time unit (c_{si}^r) increases, the point in time that the inventory is zero at the ith retailer (t_i^r) and total cost increase. The increase in total cost is understandable. However, it is noticed that the replenishment cycle (*T*) slightly drops when c_{si}^r increases. The reduction of *T* is to help reduce the impact of increasing shortage cost per unit on the total cost function.

Table 4.7

Sensitivity Analysis With Respect to the Unit Shortage Cost of the ith Retailer per Time Unit (c_{si}^r)

Parameter	Change rates	Change rates					
		t_i^r	% t_i^r change	T	% T change	Total cost	% Total cost change
	-75	5.108	-6.92%	6.572	5.16%	7629.80	-2.86%
	-50	5.324	-3.00%	6.382	2.13%	7756.70	-1.24%
	-25	5.426	-1.14%	6.298	0.79%	7817.00	-0.47%
c_{si}^r	0	5.488	0.00%	6.249	0.00%	7854.30	0.00%
	25	5.532	0.80%	6.215	-0.54%	7880.30	0.33%
	50	5.565	1.39%	6.191	-0.94%	7899.80	0.58%
	75	5.591	1.86%	6.171	-1.24%	7915.10	0.77%

4.2.6 Sensitivity Analysis With Respect to the Unit Holding Cost

This subsection investigate the effect of the holding cost of the ith retailer per unit per time unit (h_i^r) on the optimal solutions and keep other input parameters intact. The outcomes are shown in Table 4.8.

As illustrated in Table 4.8, the point in time at which the inventory is zero at the ith retailer (t_i^r) and retailer's replenishment cycle (T) are moderately reduced while total cost slightly increases when the holding cost of the ith retailer per unit per time unit (h_i^r) increases. In this case, the shorter time the products are held, the less holding cost the system has to cover. Therefore, to minimize total cost, the decrease in t_i^r and T are expected.

Table 4.8

Sensitivity Analysis With Respect to the Unit Holding Cost of the ith Retailer per Unit per Time Unit (h_i^r)

Parameter	Change rates	Change rates					
		t_i^r	% t_i^r change	T	% T change	Total cost	% Total cost change
	-75	6.650	21.17%	7.353	17.66%	7409.80	-5.66%
	-50	6.192	12.82%	6.916	10.67%	7569.80	-3.62%
	-25	5.811	5.88%	6.554	17.66%	7717.30	-1.74%
h_i^r	0	5.488	0.00%	6.249	0.00%	7854.30	0.00%
	25	5.210	-5.07%	5.987	-4.20%	7982.40	1.63%
	50	4.966	-9.51%	5.758	-7.86%	8102.90	3.17%
	75	4.751	-13.44%	5.556	-11.09%	8216.70	4.61%

4.2.7 Sensitivity Analysis With Respect to the Unit Retail Price

This section will examine the effect of retail price (s_i) which is another important input parameter. It is noted that the minimum retail price must be greater than the unit purchasing cost of the i^th retailer (c_{pi}^r). The value of the retail price will be changed from 179 to 181.50. The outcomes are shown in Table 4.9.

As in table 4.9, the point in time at which the inventory is zero at the i^th retailer (t_i^r), and replenishment cycle of the retailer (T) dramatically increase when selling price increases. In contrast, the total cost decrease when the selling price increases. The increase in t_i^r and T are understandable because the selling price directly affects to the demand. When selling price increases, the demand will be reduced, and hence, the replenishment cycle should increase. This also leads to the increase in t_i^r. However, the total cost will be reduced due to the fact that the deterioration cost will be reduced because of the system does not keep much inventory to satisfy demand.

Table 4.9

Sensitivity Analysis With Respect to the Retail price (s_i)

Parameter	Change rates	Change rates					
		t_i^r	% t_i^r change	T	% T change	Total cost	% Total cost change
	178.67	5.488	0.00%	6.249	0.00%	7854.30	0.00%
	179.00	5.794	5.58%	6.578	5.26%	7194.30	-8.40%
	179.50	6.373	16.11%	7.198	15.18%	6175.50	-21.37%
s_i	180.00	7.166	30.57%	8.047	28.77%	5126.40	-34.73%
	180.50	8.355	52.23%	9.3148	49.06%	4033.30	-48.65%
	181.00	10.448	90.36%	11.538	84.64%	2866.80	-63.50%
	181.50	15.988	191.31%	17.399	178.42%	1534.80	-80.46%

4.3 Numerical Experiments for the Case of Multiple Retailers

This section presents the numerical experiments of multiple retailers case. To illustrate the applicability of the developed model in this case, numerical experiments for one vendor-three retailers system in which each retailer has different costs and deteriorating rate due to stocking condition are illustrated. The data that need for computational experiments is provided in Table 4.10.

The optimal decision variables which are the point in time at which the inventory is zero for each retailer (t_i^r), the common replenishment cycle (T), and the optimal solutions of total cost of the system are determined. The outcomes are illustrated in Table 4.11.

Table 4.10

The Value of Base Case Parameters for the Case of Three Retailers

Parameters	Base case value		
	Retailer 1	Retailer 2	*Retailer 3*
θ_i	0.03	0.025	0.04
O_i^r	10,000	9,000	10,500
c_{pi}^r	140	135	135
c_{di}^r	145	160	125
c_{si}^r	150	160	130
h_i^r	6	10	5
a	2,000	2,000	2,000
b	11	11	11
s_i	178.67	180	175

Table 4.11

The Optimal Solutions of Single Vendor and Three retailers

	t_i^r	T	Total cost
Retailer 1	4.720	5.421	7,888.90
Retailer 2	4.686	5.421	5,170.50
Retailer 3	4.644	5.421	14,790.00
Total			27,849.00

The sensitivity analyses of one vendor and three retailers case are examined in the next part.

4.4 Sensitivity Analysis for the Case of Multiple Retailers

In this part, sensitivity analyses of multiple retailers case (i.e., three retailers) are conducted to investigate the impact of input parameters on decision variables and optimum solutions. To conduct sensitivity analyses, the input parameters for two retailer are fixed, and the input parameters for the other retailers are changed from the base case value. For instance, the input parameters of retailer 2 and 3 are fixed, and the input parameters of retailer 1 are changed. After that, the impact on the decision variables and total cost are discussed.

4.4.1 Sensitivity Analysis With Respect to the Deteriorating Rate of Retailer 1

In this section, the deteriorating rate of product at retailer 1 (θ_1) is investigated and other input parameters are kept unchanged. The value of deteriorating rate of product at retailer 1 (θ_1) is changed by 75%, 50%, 25%, -75%, -50%, and 25%, respectively, from the base value, and the outcomes are shown in Table 4.12.

From Table 4.12, the common replenishment cycle (*T*) increases when the deteriorating rate of product at retailer 1 (θ_1) decreases. The increase in *T* is understandable because the product perishes slower so retailer can hold the product longer. Similarly, the points in time at which the inventory are zero at the retailer 1 (t_1^r), retailer 2 (t_2^r) and retailer 3 (t_3^r) increase when θ_1 decrease. In contrast, the total cost increases when θ_1 increase because of the increase in deteriorating cost.

4.4.2 Sensitivity Analysis With Respect to the Ordering Cost of Retailer 1

For this section, the ordering cost of retailer 1 (O_1^r) is changed from the base case value and the outcomes are shown in Table 4.13.

As in Table 4.13, the common replenishment cycle (*T*) and the points in time at which the inventory is zero at retailer 1 (t_1^r), retailer 2 (t_2^r) and retailer 3 (t_3^r) increase when the ordering cost of retailer 1 (O_1^r) increases. Since the total cost per time unit should be minimized, this is reasonable that *T*, t_1^r, t_2^r, and t_3^r increase when O_1^r increases. Similarly, total cost increases when O_1^r increases.

Table 4.12

The Sensitivity Analysis With Respect to the Deteriorating Rate of Retailer 1 (θ_1)

Parameter	Change rates	Change rates									
		T	% T change	t_1^r	% t_1^r change	t_2^r	% t_2^r change	t_3^r	% t_3^r change	Total cost	% Total cost change
	-75	5.7387	5.86%	5.2022	10.22%	4.9798	6.27%	4.9348	6.27%	27,312.00	-1.93%
	-50	5.6291	3.84%	5.0308	6.59%	4.8785	4.10%	4.8343	4.11%	27,495.00	-1.27%
	-25	5.5231	1.89%	4.8705	3.20%	4.7806	2.01%	4.7371	2.02%	27,674.00	-0.63%
θ_1	0	5.4209	0.00%	4.7197	0.00%	4.6862	0.00%	4.6435	0.00%	27,849.00	0.00%
	25	5.3226	-1.81%	4.5773	-3.02%	4.5954	-1.94%	4.5534	-1.94%	28,020.00	0.61%
	50	5.2281	-3.56%	4.4425	-5.87%	4.5083	-3.80%	4.4669	-3.80%	28,186.00	1.21%
	75	5.1374	-5.23%	4.3148	-8.58%	4.4246	-5.58%	4.3840	-5.59%	28,349.00	1.80%

Table 4.13

The Sensitivity Analysis With Respect to the Ordering Cost of Retailer 1 (O_1^r *)*

Parameter	Change rates	Change rates									
		T	% T change	t_1^r	% t_1^r change	t_2^r	% t_2^r change	t_3^r	% t_3^r change	Total cost	% Total cost change
	-75	4.7319	-12.71%	4.0828	-13.49%	4.0515	-13.54%	4.0138	-13.56%	26,372.00	-5.30%
	-50	4.9743	-8.24%	4.3066	-8.75%	4.2744	-9.43%	4.2350	-8.80%	26,887.00	-3.45%
	-25	5.2034	-4.01%	4.5183	-4.27%	4.4855	-4.96%	4.4443	-4.29%	27,379.00	-1.69%
O_1^r	0	5.4209	0.00%	4.7197	0.00%	4.6862	-0.71%	4.6435	0.00%	27,849.00	0.00%
	25	5.6284	3.83%	4.9119	4.07%	4.8779	3.35%	4.8336	4.09%	28,302.00	1.63%
	50	5.8270	7.49%	5.0962	7.98%	5.0616	7.24%	5.0159	8.02%	28,738.00	3.19%
	75	6.0177	11.01%	5.2733	11.73%	5.2381	10.98%	5.1910	11.79%	29,160.00	4.71%

4.4.3 Sensitivity Analysis With Respect to the Unit Purchasing Cost of Retailer 1

This section investigates the unit purchasing cost of retailer 1 (c^r_{p1}) with change rates 75%, 50%, 25%, -75%, -50%, and 25% from the base value. The outcomes are illustrated in Table 4.14.

From Table 4.14, the replenishment cycle (T), the points in time at which the inventory is zero at retailer 1 (t^r_1), retailer 2 (t^r_2) and retailer 3 (t^r_3) decrease when the unit purchasing cost of retailer 1 (c^r_{p1}) increases. These trends are reasonable because T should be decrease to avoid the increase in deteriorating cost, and the decrease in T will result in the decreases in t^r_1, t^r_2 and t^r_3. In contrast, the total cost increases when c^r_{p1} increases due to the increase in unit purchasing cost.

4.4.4 Sensitivity Analysis With Respect to the Unit Deteriorating Cost of Retailer 1

The unit deteriorating cost of retailer 1 (C^r_{d1}) is the next input parameter to be investigated. The change rates are similar to previous investigations and the outcomes are shown in Table 4.15.

As illustrated in Table 4.15, the replenishment cycle (T), the points in time at which the inventory are zero at retailer 1 (t^r_1), retailer 2 (t^r_2) and retailer 3 (t^r_3) slightly decrease when the unit deteriorating cost of retailer 1 (C^r_{d1}) increases. In contrast, total cost of the system slightly increases when C^r_{d1} increases. These trends are obviously understandable because the decrease in T helps to reduce the deteriorating cost in total cost function which lead to the decreases in t^r_1, t^r_2 and t^r_3.

Table 4.14

The Sensitivity Analysis With Respect to the Unit Purchasing Cost of Retailer 1 (c_{p1}^{r})

Parameter	Change rates	Change rates									
		T	% T change	t_1^r	% t_1^r change	t_2^r	% t_2^r change	t_3^r	% t_3^r change	Total cost	% Total cost change
	-75	5.5548	2.47%	4.7661	0.98%	4.8098	2.64%	4.9200	5.95%	23,971.00	-13.93%
	-50	5.5087	1.62%	4.7239	0.09%	4.7673	1.73%	4.8504	4.46%	25,265.00	-9.28%
	-25	5.4642	0.80%	4.7837	1.36%	4.7261	0.85%	4.6831	0.85%	26,558.00	-4.64%
c_{p1}^{r}	0	5.4209	0.00%	4.7197	0.00%	4.6862	0.00%	4.6435	0.00%	27,849.00	0.00%
	25	5.3791	-0.77%	4.6580	-1.31%	4.6475	-0.83%	4.6051	-0.83%	29,139.00	4.63%
	50	5.3384	-1.52%	4.5986	-2.57%	4.6100	-1.63%	4.5679	-1.63%	30,427.00	9.26%
	75	5.2990	-2.25%	4.5413	-3.78%	4.5736	-2.40%	4.5318	-2.41%	31,714.00	13.88%

Table 4.15

The Sensitivity Analysis With Respect to the Unit Deteriorating Cost for the Product of Retailer 1 (c^r_{d1})

Parameter	Change rates	Change rates									
		T	% T change	t^r_1	% t^r_1 change	t^r_2	% t^r_2 change	t^r_3	% t^r_3 change	Total cost	% Total cost change
	-75	5.5598	2.56%	4.9277	4.41%	4.8145	2.74%	4.7707	2.74%	27,598.00	-0.90%
	-50	5.5120	1.68%	4.8553	2.87%	4.7703	1.79%	4.7269	1.80%	27,683.00	-0.60%
	-25	5.4657	0.83%	4.7861	1.41%	4.7275	0.88%	4.6845	0.88%	27,767.00	-0.29%
c^r_{d1}	0	5.4209	0.00%	4.7197	0.00%	4.6862	0.00%	4.6435	0.00%	27,849.00	0.00%
	25	5.3776	-0.80%	4.6558	-1.35%	4.6461	-0.86%	4.6038	-0.85%	27,929.00	0.29%
	50	5.3356	-1.57%	4.5944	-2.65%	4.6074	-1.68%	4.5653	-1.68%	28,008.00	0.57%
	75	5.2948	-2.33%	4.5353	-3.91%	4.5698	-2.48%	4.5280	-2.49%	28,086.00	0.85%

4.4.5 Sensitivity Analysis With Respect to the Unit Shortage Cost of Retailer 1

This subsection examines effects of shortage cost of retailer 1 per time unit (c_{s1}^r) on decision variables. The change rates will be the same as previous investigation. The outcomes are shown in Table 4.16.

As shown in Table 4.16, the replenishment cycle (*T*), the points in time at which the inventory are zero at retailer 2 (t_2^r) and retailer 3 (t_3^r) decrease when unit shortage cost per time unit of retailer 1 (c_{s1}^r) increases. The decrease in *T* is reasonable because it helps to reduce shortage cost in total cost function which result in reductions of t_2^r and t_3^r. In the opposite direction, the points in time at which the inventory are zero at retailer 1 (t_1^r) and total cost increases when c_{s1}^r increases due to the increase in unit shortage cost.

4.4.6 Sensitivity Analysis With Respect to the Unit Holding Cost of Retailer 1

This section investigate the effect of the holding cost of retailer 1 per unit per time unit (h_1^r) on the optimal solutions and keep other input parameters intact. The outcomes are shown in Table 4.17.

As shown in Table 4.17, when the holding cost of retailer 1 per unit per time unit (h_1^r) increases, the replenishment cycle (*T*), the points in time that the inventory is zero at retailer 1(t_1^r), retailer 2 (t_2^r) and retailer 3 (t_3^r) decrease. In this case, due to the reason that the total cost should be optimized, the reductions in *T*, t_1^r, t_2^r, and t_3^r help to reduce the time of holding the product which results in the decrease in holding cost. However, total cost rises when h_1^r increases because of the increase in unit holding cost.

4.4.7 Sensitivity Analysis With Respect to the Unit Retail Price of Retailer 1

In this section, the effect of retail price of retailer 1 (s_1) will be examined. It is noted that the minimum retail price must be greater than the unit purchasing cost of retailer 1 (c_{p1}^r). The value of the retail price will be changed from 179 to 181.50 whereas other input parameters remain unchanged. The outcomes are presented in Table 4.18.

As presented in Table 4.18, it is obvious that when retail price rises, the replenishment cycle (*T*), the points in time at which the inventory is zero at retailer 1 (t_1^r), retailer 2 (

t_2^r) and retailer 3 (t_3^r) increase but total cost decreases. The retail price has the effect on demand directly. Therefore, the increase in *T* is reasonable because when s_1 increases, demand will decrease which lead to the increase in *T*. The increase in *T* results in the increases in t_1^r, t_2^r and t_3^r.

Table 4.16

The Sensitivity Analysis With Respect to the Unit Shortage Cost of Retailer 1 (c_{s1}^r)

Parameter	Change rates	Change rates									
		T	% T change	t_1^r	% t_1^r change	t_2^r	% t_2^r change	t_3^r	% t_3^r change	Total cost	% Total cost change
	-75	5.4905	1.28%	4.1766	-11.51%	4.7504	1.37%	4.7072	1.37%	27,650.00	-0.71%
	-50	5.4504	0.54%	4.4853	-4.97%	4.7133	0.58%	4.6704	0.58%	27,762.00	-0.31%
	-25	5.4320	0.20%	4.6305	-1.89%	4.6964	0.22%	4.6536	0.22%	27,816.00	-0.12%
c_{s1}^r	0	5.4209	0.00%	4.7197	0.00%	4.6862	0.00%	4.6435	0.00%	27,849.00	0.00%
	25	5.4133	-0.14%	4.7816	1.31%	4.6791	-0.15%	4.6365	-0.15%	27,873.00	0.09%
	50	5.4076	-0.25%	4.8280	2.29%	4.6739	-0.26%	4.6312	-0.26%	27,890.00	0.15%
	75	5.4032	-0.33%	4.8644	3.07%	4.6698	-0.35%	4.6272	-0.35%	27,904.00	0.20%

Table 4.17

The Sensitivity Analysis With Respect to the Unit Holding Cost of Retailer 1 (h_1^r)

Parameter	Change rates	Change rates									
		T	% T change	t_1^r	% t_1^r change	t_2^r	% t_2^r change	t_3^r	% t_3^r change	Total cost	% Total cost change
	-75	5.6163	3.60%	5.0142	6.24%	4.8667	3.85%	4.8226	3.86%	27,498.00	-1.26%
	-50	5.5481	2.35%	4.9099	4.03%	4.8037	2.51%	4.7600	2.51%	27,619.00	-0.83%
	-25	5.4831	1.15%	4.8120	1.96%	4.7436	1.22%	4.7004	1.23%	27,736.00	-0.41%
h_1^r	0	5.4209	0.00%	4.7197	0.00%	4.6862	0.00%	4.6435	0.00%	27,849.00	0.00%
	25	5.3615	-1.10%	4.6323	-1.85%	4.6313	-1.17%	4.5890	-1.17%	27,960.00	0.40%
	50	5.3045	-2.15%	4.5493	-3.61%	4.5787	-2.29%	4.5369	-2.30%	28,067.00	0.78%
	75	5.2499	-3.15%	4.4704	-5.28%	4.5283	-3.37%	4.4869	-3.37%	28,172.00	1.16%

Table 4.18

The Sensitivity Analysis With Respect to the Unit Retail Price of Retailer 1 (s_1)

Parameter	Change rates	Change rates									
		T	% T change	t_1^r	% t_1^r change	t_2^r	% t_2^r change	t_3^r	% t_3^r change	Total cost	% Total cost change
	178.67	5.4209	0.00%	4.7197	0.00%	4.6862	0.00%	4.6435	0.00%	27849.00	0.00%
	179.00	5.4878	1.23%	4.7815	1.31%	4.7479	1.32%	4.7047	1.32%	27215.00	-2.28%
	179.50	5.5940	3.19%	4.8800	3.40%	4.8460	3.41%	4.8021	3.42%	26250.00	-5.74%
s_1	180.00	5.7068	5.27%	4.9847	5.61%	4.9504	5.64%	4.9056	5.64%	25281.00	-9.22%
	180.50	5.8270	7.49%	5.0962	7.98%	5.0615	8.01%	5.0159	8.02%	24307.00	-12.72%
	181.00	5.9554	9.86%	5.2153	10.50%	5.1804	10.55%	5.1337	10.56%	23328.00	-16.23%
	181.50	6.0928	12.39%	5.3430	13.21%	5.3077	13.26%	5.2600	13.28%	22344.00	-19.77%

CHAPTER 5
CONCLUSIONS AND RECOMMENDATIONS

A two-echelon supply chain under vendor managed inventory for deteriorating products is considered in this research. We investigate one vendor and multiple retailers supply chain. A price sensitive demand of deteriorating products such as fruit, vegetables, and food stuff, etc. is examined. The deteriorating rate for these types of product is consider to be at constant rate. The products will be distributed in a batch to each retailer simultaneously after the vendor anticipates the demand from retailers' inventory information. It is assumed that the product deteriorates only at retailers due to stocking condition. Also, this research considers the case of shortage allowance with full backlogging which has not been considered in any previous research works. An inventory model under Vendor Managed Inventory (VMI) strategy is developed in order to minimize the total cost of the whole supply chain. The optimal solutions of the points in time at which the inventory is zero at the retailers (t_i^r's), and replenishment cycle (*T*) are examined. The impacts of the deteriorating rates of product at the retailers (θ_i's), the ordering costs of the retailers (O_i^r's), the unit purchasing costs of the retailers (c_{pi}^r's), the unit deteriorating costs of the retailers (C_{di}^r's), the unit shortage costs of the retailers per time unit (c_{si}^r's),the holding costs of the retailers per unit per time unit (h_i^r's), and the retail prices (s_i's) on the optimal decision variables and total cost of the whole supply chain are examined. According to the sensitivity analyses, the following results have been observed.

- The points in time at which the inventory are zero at the retailers (t_i^r's) increases when O_i^r, c_{si}^r, and s_i increase. On the other hand, the points in time at which the inventory are zero at the retailers (t_i^r's) decrease when θ_i, c_{pi}^r, C_{di}^r, and h_i^r increase.
- The effects on the replenishment cycle (*T*) are similar to the effects on the points in time at which the inventory are zero at the retailers (t_i^r's). Only the impact of unit shortage cost per time unit (c_{si}^r) on *T* is different from the impact on t_i^r. Specifically, the replenishment cycle decreases when c_{si}^r increases.

- Total cost increases when all input parameters increase except retail price because when the unit cost of any cost component increases, it is obvious that total cost will increase. However, for retail price, total cost reduces due to the reduction of deteriorating cost.

Based on the findings, it can be confirmed that this research can help the members of the supply chain make the right decision and know how each unit cost has the impact on the optimal value of decision variables and total cost function.

5.1 Recommendations

For further research directions, there are various extensions that can be explored as follows.

- In this study, the inventory model consider demand as a deterministic and linearly price-sensitive function. For future study, this can be extended to stochastic demand or uncertain demand in order to make the model more realistic and more practicable.
- Deteriorating rate is considered as a constant in this research but it can be considered as an increasing function with time in the future work because there are many types of perishable product which have increasing deteriorating rate.
- Other conditions such as limitation of warehouse space, budget, discount, etc. can be incorporated in future research work.

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APPENDIX

COMPUTER PROGRAM (MATLAB)

```
function TC_all = objfun1(x)
theta1 = 0.03;
Cs1r = 150;
Cp1r = 140;
O1r = 10000;
h1r = 6;
Cd1r = 145;
a = 2000;
b = 11;
s1 = 178.67;
HC_1 = (h1r*(a-b*s1)*(exp(theta1*x(2))-1-(x(2)*theta1)))/theta1^2;
SC_1 = (Cs1r*(a-b*s1)*(x(1)-x(2))^3)/3;
DC_1 = (Cd1r*(a-b*s1)*(exp(theta1*x(2))-(x(2)*theta1)-1))/theta1;
PC_1 = (Cp1r*(a-b*s1)*(exp(x(2)*theta1)-1+(x(1)*theta1)-(x(2)*theta1)))/theta1;
TC1 =(O1r+HC_1+SC_1+DC_1+PC_1)/x(1);
theta2 = 0.025;
Cs2r = 160;
Cp2r = 135;
O2r = 9000;
h2r = 10;
Cd2r = 160;
a = 2000;
b = 11;
s2 = 180;
HC_2 = (h2r*(a-b*s2)*(exp(theta2*x(3))-1-(x(3)*theta2)))/theta2^2;
SC_2 = (Cs2r*(a-b*s2)*(x(1)-x(3))^3)/3;
DC_2 = (Cd2r*(a-b*s2)*(exp(theta2*x(3))-(x(3)*theta2)-1))/theta2;
PC_2 = (Cp2r*(a-b*s2)*(exp(x(3)*theta2)-1+(x(1)*theta2)-(x(3)*theta2)))/theta2;
TC2 =(O2r+HC_2+SC_2+DC_2+PC_2)/x(1);
theta3 = 0.04;
```

```
Cs3r = 130;
Cp3r = 135;
O3r = 10500;
h3r = 5;
Cd3r = 125;
a = 2000;
b = 11;
s3 = 175;
HC_3 = (h3r*(a-b*s3)*(exp(theta3*x(4))-1-(x(4)*theta3)))/theta3^2;
SC_3 = (Cs3r*(a-b*s3)*(x(1)-x(4))^3)/3;
DC_3 = (Cd3r*(a-b*s3)*(exp(theta3*x(4))-(x(4)*theta3)-1))/theta3;
PC_3 = (Cp3r*(a-b*s3)*(exp(x(4)*theta3)-1+(x(1)*theta3)-(x(4)*theta3)))/theta3;
TC3 =(O3r+HC_3+SC_3+DC_3+PC_3)/x(1);
TC_all = TC1 + TC2 + TC3;
end
% % find optimal solution for 3 retailers
x0 = [1;1;1;1]
f = objfun1(x0);
options = optimoptions(@fminunc,'Display','iter','Algorithm','quasi-newton');
[x,fval,exitflag,output] = fminunc(@objfun1,x0,options)

% % run numerical experiment for three retailers >> retailer1, retailer2, and retailer3
syms O2r s2 Cs2r Cd2r Cp2r h2r theta2 t2r O3r s3 Cs3r Cd3r Cp3r h3r theta3 t3r
theta1 = 0.03;
Cs1r = 150;
Cp1r = 140;
O1r = 10000;
h1r = 6;
Cd1r = 145;
a = 2000;
b = 11;
s1 = 178.67;
t1r = 4.7197;
T = 5.4209;
```

```
HC_1 = (h1r*(a-b*s1)*(exp(theta1*t1r)-1-(t1r*theta1)))/theta1^2;
SC_1 = (Cs1r*(a-b*s1)*(T-t1r)^3)/3;
DC_1 = (Cd1r*(a-b*s1)*(exp(theta1*t1r)-(t1r*theta1)-1))/theta1;
PC_1 = (Cp1r*(a-b*s1)*(exp(t1r*theta1)-1+(T*theta1)-(t1r*theta1)))/theta1;
TC1 =(O1r+HC_1+SC_1+DC_1+PC_1)/T;
theta2 = 0.025;
Cs2r = 160;
Cp2r = 135;
O2r = 9000;
h2r = 10;
Cd2r = 160;
a = 2000;
b = 11;
s2 = 180;
t2r = 4.6862;
HC_2 = (h2r*(a-b*s2)*(exp(theta2*t2r)-1-(t2r*theta2)))/theta2^2;
SC_2 = (Cs2r*(a-b*s2)*(T-t2r)^3)/3;
DC_2 = (Cd2r*(a-b*s2)*(exp(theta2*t2r)-(t2r*theta2)-1))/theta2;
PC_2 = (Cp2r*(a-b*s2)*(exp(t2r*theta2)-1+(T*theta2)-(t2r*theta2)))/theta2;
TC2 =(O2r+HC_2+SC_2+DC_2+PC_2)/T;
theta3 = 0.04;
Cs3r = 130;
Cp3r = 135;
O3r = 10500;
h3r = 5;
Cd3r = 125;
a = 2000;
b = 11;
s3 = 175;
t3r = 4.6435;
HC_3 = (h3r*(a-b*s3)*(exp(theta3*t3r)-1-(t3r*theta3)))/theta3^2;
SC_3 = (Cs3r*(a-b*s3)*(T-t3r)^3)/3;
DC_3 = (Cd3r*(a-b*s3)*(exp(theta3*t3r)-(t3r*theta3)-1))/theta3;
PC_3 = (Cp3r*(a-b*s3)*(exp(t3r*theta3)-1+(T*theta3)-(t3r*theta3)))/theta3;
```

```
TC3 =(O3r+HC_3+SC_3+DC_3+PC_3)/T;
TC_all = TC1 + TC2 + TC3;
```