eMERGENCY LATERAL TRANSSHIPMENT IN A TWO-RETAILER DECENTRALIZED INVENTORY SYSTEM WITH PARTIAL BACKORDERING

by

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**AUTHOR’S DECLARATION**

I, Siraphat Sukhobol, declare that the research work carried out for this thesis was in accordance with the regulations of the Asian Institute of Technology. The work presented in it are my own and has been generated by me as the result of my own original research, and if external sources were used, such sources have been cited. It is original and has not been submitted to any other institution to obtain another degree or qualification. This is a true copy of the thesis, including final revisions.

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Signature:

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# ABSTRACT

Emergency Lateral transshipment (ELT) is one type of lateral transshipment (LT). This method provided to face the customer demand that effective for a whole system and reduce the total cost in supply chain. The purpose of this research is to optimize the total expected profit of a two-retailer in supply chain for slow-moving products under decentralized inventory system. The slow-moving products is product that can stocks in the warehouse for a certain period of time. To make this research more realistic, partial backordering will be investigated. It is noted that demand of both retailers will follow a Poisson distribution. The formulated mathematical model is illustrated to find out the optimal value of order quantity for a two-retailer due to maximize the expected total profit of both retailer in the whole system. After formulating mathematical model, numerical experiment is conducted to see the response of mathematical model. Moreover, sensitivity analyses are illustrated the effect of input parameters compare with the base scenario.

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# LIST OF ABBREVIATIONS

|  |  |
| --- | --- |
| ELT | = Emergency Lateral Transshipment |
| PLT | = Preventive Lateral Transshipment |
| LT | = Lateral Transshipment |
| FMI | = Fast Moving Items |
| SMI | = Slow Moving Items |
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# INTRODUCTION

## Background of the Study

Supply chain is a whole process of creating and selling product, from the supply of material to manufacturer, then delivery to the end-user so as to satisfy the consumer demand. This process includes producer, vendors, warehouses, and retailers. Many companies need to develop their supply chain so as to reduce the cost and to build a faster production line. One of the general problems that is found in managing any supply chain is the control of inventory. If the companies hold too much items or products, the holding cost will be increased. In other words, if the companies are lack of inventory, the consumer satisfaction will be decreased.

However, the retailer in the supply chain have to face with an unpredictability demand of customer. This situation can be solved by use of emergency orders or lateral transshipment. To compare between two methods, the total cost in LT is usually lower than the emergency order. So, it is a good decision to use lateral transshipment in most supply chains.

The lateral transshipment might happen at a warehouse/retailer which provides stocked items to another warehouse/retailer to prevent out-of-stock situation by moving stock in the same level of the inventory system. Even though the transportation cost in lateral transshipment is high, it is still be better than if no lateral transshipment is employed in the system. Lateral transshipment can improve the effectiveness of the inventory system.

However, Lateral transshipments can be confined to occur at destined of the time before demand is perceived, or lateral transshipment can occur any time to react to the stockouts (Paterson et al., 2011). Lateral transshipment can be separated into two group based on the timing of transshipment which are Emergency lateral transshipment (ELT) and Preventive lateral transshipment (PLT).

* Emergency lateral transshipment (ELT)

Emergency lateral transshipment is used when the stockouts occur, and inventory movement should be considered immediately. In emergency lateral transshipment, the total transshipped amount can be realized at the end of season sale. In the inventory system with ELT, the transshipment cost is higher than in PLT because the companies use faster transportation in order to reduce waiting time of customer.

* Preventive lateral transshipment (PLT)

To prevent stockout in the future, preventive lateral transshipment can be rebalanced number of items at some point in time in the sale season. Using PLT in inventory system, the sale season is split off in two period of time. PLT is more complicated than ELT owing to impact between uncertain demand and replenish quantities.

Most of research with transshipment, the complete backordering is considered. This assumption completely ignores an impatient consumer when shortages occur. For this reason, partial backordering is a situation that occur normally in production’s retail. Both transshipment policies can apply partial backordering. In emergency lateral transshipment, partial backordering might be considered at the end of the sale season due to the fact that the amount of total shortages is observed at the end of sale season.

Comparing with preventive lateral transshipment, it already prevents stockout in the future, so shortage rarely occurs, and hence, backordering is no significantly affect on PLT. The occurrence of shortage is very small after the first period in PLT. Thus, this is agreeable that not consider a partial backordering in preventive lateral transshipment (Feng et al., 2018).

## Statement of the Problem

Establishing inventory policy is one of the most importance tasks in whole system. Nowadays, the technology is growing very fast, so many companies can employ both ELT and PLT. Even though multiple transshipments can be performed well in some situations, multiple transshipments need a good communicate for retailers and the handling cost. Thus, many retailers frequently use a singular transshipment within a sale season in order to get the easy operation (Tagaras & Vlachos, 2002). There are many researchers who studied on both ELT and PLT in their inventory system, but most of researchers considered ELT and PLT separately.

Feng et al. (2018) studied PLT and ELT in the same approach under partial backordering. Dealing with PLT in this research, the sale season is split off in two periods using uniformly distributed demand for each period. As a result, if the length of these two periods of time changes, the distribution of total demand will change. Also, the use of uniform distribution or other continuous distribution for demand is fixed only with fast moving items. To fulfill this gap, this research considers emergency lateral transshipment with slow-moving product in which the total demand follows a Poisson distribution.

## Objectives of the Study

This research aims at developing an ELT policy so as to maximize the expected total profit for a two-retailer in the system for slow-moving product. In this supply chain, transshipment occurs only one time at the end of the sale season and partial backordering that depends on the willing of customers to wait for the product, will be considered.

## Scope of the Study

In this research, an emergency lateral transshipment model will be derived for a supply chain of a slow-moving product with two retailers. The following assumptions are considered.

* Emergency transshipment occurs only one time at the end of the sale season when demands at two retailers have been realized.
* Demand follows a Poisson distribution.
* Partial backordering is allowed in which when a demand cannot be fulfilled during the selling season the customer is willing to wait until the end of sale season with a probability *p*.
* Part of demand that cannot be fulfilled will be lost.

# CHAPTER 2 LITERATURE REVIEWS

Lateral Transshipment (LT) is mentioned to move the stocks in the parallel level of the inventory system. Due to development of technology, many companies can access easily to get the real time database. Thus, lateral transshipment is frequently applied as a method that manage effectively inventory system in many industries. LT is classified into two group which are preventive lateral transshipment (PLT), and emergency lateral transshipment (ELT).

## Emergency Lateral Transshipment Under Partial Backordering

The circumstance of the inventory system with shortage is unavoidable in the real situation. First of all, this research is studied on a partial backordering of transshipment from Hadley & Whiten (1963). They determined an optimal order quantity, and rearrange of policy by using dynamic programming. Moreover, they used two types of transshipment that have non-negligible lead times. There are many papers studied on two-echelon inventory system. Hachicha et al. (2013) deal with emergency lateral transshipment model that involve two-echelon supply network. They derived an optimal approach and determined optimal solution for each retailer. In order to find the best policy, they tested three policies which are no backordering, complete backordering and partial backordering. As a result, the best policy in this research is partial pooling. Olsson (2015) also studied on a single-level of inventory system for spare parts with the same level of two locations. This study considered the circumstance of lateral transshipment in the middle of locations that allow backorders in the system. They developed a heuristic model by Poisson processes with doubly stochastic which rate of demand depends on the age of the products in whole system. Their results indicated that their heuristic can reduce the transshipment lead time. Dijkstra et al. (2017) proposed dynamic transshipment policies for items which are turned online channel to offline channel in cross channel. The result from the study is that the dynamic transshipment is more efficient than the stationary policies in term of unbalances in the beginning stock, and more efficient demand fulfillment in dual-channel companies.

## Emergency Lateral Transshipment in Decentralized System

When the information of technology is developed, Emergency lateral transshipment is applied and investigated in decentralized inventory system. The below studies focus on inventory system with ELT under decentralized system. Decentralized system considers multiple retailers who are self-operated the inventory system to maximize their own profit. Many of lateral transshipment paper focuses on two locations. Hu et al. (2007) studied a two-location inventory system model where the focus is to optimize its profit. They also optimized the inventory and transshipment decisions. Moreover, they studied the effect of demand, and changing in ability of the transshipment price. There are some researches that are similar with this research work. Rudi et al. (2001) studied the occurrence of transshipments in the middle of two liberty locations that affect on an optimal order inventory at each location. Purpose of each location is to maximize joint profit. They utilized the transshipment prices to select inventory levels to be match for the maximize joint-profit. Zhao et al. (2016) formulated an online to offline supply chain in the inventory transshipment. The results show that there is a unique Nash equilibrium of order quantity inventory in a decentralized inventory system. Moreover, transshipment price should be minimized in order to maximize the total profit in the supply chain. Feng et al. (2018) compared between ELT and PLT under partial backordering. The result shows that emergency lateral transshipment is more effective to the system with more patient consumer and low backordered cost. On the other side, preventive lateral transshipment will affect on the transfer point and the transshipment price.

## Emergency Lateral Transshipment with Fast Moving Items

From lateral transshipment literature, there are two groups of products which are fast-moving items (FMI), and slow-moving items (SMI). Fast-moving items have a short shelf life because of high consumer demand or they are perishable. Perishable products preserve a stable rate until their shelf life or date of expire i.e., packaged and fresh foods, blood, fashion, etc. Some researchers are considered in emergency lateral transshipment problem with perishable items. Dehghani and Abbasi (2018) proposed a new transshipment policy for perishable items in supply chain of blood based on the age of the oldest items. They developed a heuristic solution by using partial differential solution to calculate the cost, and then they can reduce the total inventory cost. Wang and Ma (2015) studied an age-based model with decision of select methods for transshipped blood. They compared the proposed policy with quantity-based policy. Additionally, sensitivity analyses of input parameters are analyzed under operating cases by various value of demand and supply. Nakandala et al. (2017) investigated emergency lateral transshipment with perishable product in the fresh food industry to optimize total inventory costs. To get more effective inventory management of the fresh foods, they develop a relevant decision model in supply chain. On the other hand, Seidscher & Minner (2013) applied a Semi Markov Decision Process (SMDP) to determine the optimal of Preventive lateral transshipment and Emergency lateral transshipment, and determine an optimal source for both transshipment policy. Cheong (2013) studied a main managing in many retail stores that combines the replenishment policies parallel with the applications of transshipments for the short shelf-life products so as to decrease the surplus inventory, and shortage inventory costs.

Although emergency lateral transshipment has been examined in many research works, most of those researches focused on fast-moving products. Thus, there exist gaps in emergency lateral transshipment for slow-moving product that needed to be filled. Thus, the summarization all of past research thesis is showed in Table 2.1.

**Table 2.1**

*The Summarization of Past Research Thesis of Lateral Transshipment*

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Authors | Transshipment Policy | Inventory System | Unsatisfied-Demand | Types of Product |
| Olsson (2015) | ELT | Centralized | Partial-backordering | Fast moving product |
| Tagarus and Vlachos (2002) | PLT | Centralized | Complete-backordering | Fast moving product |
| Zhao et al. (2016) | ELT | Decentralized | - | Fast moving  product |
| Wang and Ma (2015) | ELT | Decentralized | Partial-backordering | Fast moving  product |
| Feng et al. (2018) | ELT/PLT | Decentralized | Partial-backordering | Fast moving product |

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Authors | | Transshipment Policy | Inventory System | | Unsatisfied-Demand | | | Types of Product | |
| Cheong (2013) | | ELT | Centralized | | Replenish policy | | | Fast moving product | |
| Dehghani and Abbasi (2018) | | ELT | Centralized | | Partial-backordering | | | Fast moving  product | |
| Dijkstra et al. (2017) | | Dynamic transshipment | Centralized | | - | | | e-commerce | |
| Hachicha et al. (2013) | | ELT | Centralized | | Partial-backordering | | | - | |
| Hu et al. (2007)  8 | | ELT | Decentralized | | Complete-backordering | | | - | |
| Authors | Transshipment Policy | | | Inventory System | | Unsatisfied-Demand | Types of Product | |
| Nakandala et al. (2017) | ELT | | | Centralized | | Partial-backordering | Fast moving product | |
| Paterson et al. (2011) | ELT/PLT | | | Centralized/  Decentralized | | - | - | |
| Rudi et al. (2001) | ELT | | | Decentralized | | Partial-backordering | - | |
| Seidscher and Minner (2013) | ELT/PLT | | | Centralized | | Partial-backordering | A single product | |

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# CHAPTER 3 MATHMATICAL MODEL DEVELOPMENT

## 3.1 Introduction

In this chapter, a mathematical model of an emergency lateral transshipment with two-retailer in decentralized inventory system is formulated to find out an optimal order quantities and transshipment policy. The major aims of this chapter are to examine advantages of emergency lateral transshipment (ELT) under decentralized inventory system, and to maximize the total expected profit of a two-retailer for slow-moving product.

## 3.2 Notations

The general notations used in this chapter are presented below.

Parameters:

 Retailer index, , = 1,2

 Demand of the retailer  during period 

 The fraction of backordered demand

 Unit transshipment price

 Salvage value for unsold product per unit

 Unit order cost

 Unit penalty cost

 Unit retail price

 Unit backorder cost

 Unit transshipment handling cost

 The cycle length

 The expected total profit of retailer  for the system per cycle length for scenario 

 The expected total profit of retailer  for the system per cycle length for scenario 

 The expected total profit of both retailer per time unit

 The probability of occurrence for scenario 

 The probability that  at the end of period at retailer 

Decision Variables:

 The optimal order quantity of retailer 

 The optimal order quantity of retailer

## Model Framework and Assumptions

In the emergency lateral transshipment with two retailers, they have to order from another supplier to satisfy demand during a sale season. Both retailers have only one chance to receive a transshipped order to fulfill their demands. Since this research focus on emergency lateral transshipment, backlogging must be allowed due to ELT will satisfy the demand at the end of sale season. Therefore, part of customer demand in ELT is backordered. However, part of demand that cannot be fulfilled will be lost. Complete backordering is generally used in many researches of transshipment. On the other hand, partial backordering is more realistic than complete backordering. Thus, this research focus on the partial backordering.

Dong and Rudi (2004) proposed an assumption that retailer will pay the transshipment handling cost per unit  for each transshipped item other location. Each retailer has to order the items from supplier at a unit cost  and gets a revenue  for the sold product. Both retailers transshipped out receive a unit price  that the acceptor pays for the shipper. Moreover, the unit salvage value  is used for each unsold item. Then, a unit penalty costis used for each lost demand products. Due to the lead time in ELT is not considered, a unit backordered cost  is transported by the retailer receives the transshipment, and is also considered the length of waiting time.

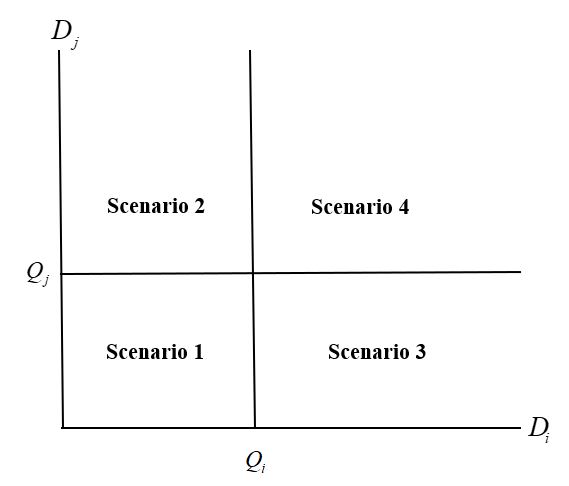
The backordered cost is taken by the retailer that received transshipped items. Therefore, it is supposed that  because penalty costs more than backordered demand in the practical utilization. In order to avoid this situation, this assumption will be used: , and . Moreover, transship price satisfies .

The two-retailer meet the independent demand with a Poisson distribution. It is notedis the demand of the retailer  at the end of the period . The , and  for  are  , and  , respectively.

In the lateral transshipment, each retailer will examine their inventory echelon in the inventory at the end of the period , and decides the number of transshipped items. When one retailer faces an item that out of stock, it can use the inventories from another retailer if another retailer has remaining inventory. Thus, there are four scenarios that need to be considered as follows. In this system, the demand follows a Poisson distribution. The four scenarios of two retailers’ demands are shown in Figure 3.1.

**Figure 3.1**

*Graphical Illustration of Four Scenarios*



### 3.3.1 Scenario 1: When and

In the scenario 1, the maximum inventory levels at the beginning of the period  of both retailers are lower than their demands. Therefore, there will be no transshipped items in this scenario.

### 3.3.2 Scenario 2: When and

In the scenario 2, at the end of sale season, the maximum inventory echelon of retailer  is higher than the demand of retailer  during period  , simultaneously the maximum inventory level of retailer  is lower than the demand of retailer  during period .Therefore, the situation that retailer  transshipped items to retailerwill occur.

When partial backordering is considered in the system, there is an ELT with the quantity of transshipment () from retailer  to.

 =  (1)

In contrast, there is no an ELT with the quantity of transshipment () from retailer  to .

 = 0 (2)

### 3.3.3 Scenario 3: When and

In the scenario 3, at the end of sale season, the maximum inventory echelon of retailer  is lower than the demand of retailer  during period  , simultaneously the maximum inventory echelon of retailer  is higher than the demand of retailer  during period . Therefore, the situation that retailer  transshipped items to retailerwill occur.

When partial backordering is considered in the system, there is an ELT with the quantity of transshipment () from retailer  to  .

 =  (3)

In contrast, there is no an ELT with the quantity of transshipment () from retailer  to.

 = 0 (4)

***3.3.4*** ***Scenario 4: When  and ***

In the scenario 4, the maximum inventory levels at the beginning of the period  of both retailers are higher than their demands. Therefore, there will be no transshipped items in this scenario.

Generally, we have

 =  (5)

 =  (6)

**3.4 Development of Mathematical Model**

The  of the demand in retailer  while time period  having defined as following equation:

 (7)

Then, the quantity of transshipment from retailer  to retailer  with partial backordering can be derived as the following equation:

 (8)

Quantity of sales for retailer  is specified as follows:

 (9)

While the quantity of unsold products at retailer  are,

 (10)

The unsatisfied demand at retailer  are,

 (11)

***3.4.1 Scenario 1: When  and ***

For scenario 1, the order quantities of both retailers are lower than their demand at the end of the sale season. There will be no transshipment in this scenario. The existence probability of scenario 1 is derived as:

 (12)

Profit function of the scenario 1 for retailer  is derived as:

 (13)

Profit function of the scenario 1 for retailer  is derived as:

 (14)

***3.4.2 Scenario 2: When  and ***

For scenario 2, the order quantity in retailer  is higher than his demand. Simultaneously, the order quantity in retailer  is lower than his demand. Therefore, the transshipped items from retailer  to retailerwill occur in this scenario. The existence probability of scenario 2 is expressed as:

 (15)

Number of transshipped items can be derived as the lowest value between surplus inventory of the retailer   and inventory requested by the receiving retailer ,  Thus, it can be expressed as:

 =  (16)

Profit function of the scenario 2 for retailer  is derived as:

 (17)

Profit function of the scenario 2 for retailer  is derived as:

 (18)

***3.4.3 Scenario 3: When  and ***

For scenario 3, the order quantity of retailer  is lower than his demand. Simultaneously, the order quantity of retailer  is higher than his demand. Therefore, the transshipped items from retailer  to retailer  will occur in this scenario. The existence probability of scenario 3 is expressed as:

 (19)

Number of transshipped items can be derived as the lowest value between surplus inventory of the retailer   and inventory requested by the receiving retailer  . Thus, it can be expressed as:

 =  (20)

Profit function of the scenario 3 for retailer  is derived as:

 (21)

Profit function of the scenario 3 for retailer is derived as:

 (22)

***3.4.4 Scenario 4: When  and ***

For scenario 4, the order quantities of both retailers are higher than their demands. Therefore, there will be no transshipped items in this scenario. The existence probability of scenario 4 is expressed as:

 (23)

Profit function of the scenario 4 for retailer  is derived as:

 (24)

Profit function of the scenario 4 for retailer is derived as:

 (25)

**3.5 Finalized of Mathematical Model**

After analyzing the profit function each scenario, we can be found out the total expected profit for both retailers.

For retailer , the expected total profit can be derived as:





(26)

For retailer , the expected profit can be derived as:





(27)

For retailer , let assume that the demand per unit time follows Normal distribution with mean  and variance . Then, demand of retailer  during time period  will follow Normal distribution with mean  , and variance . With this information, the functions  , and  can be determined. Similarly for retailer , the demand of retailer  during time period  will follow Normal distribution with mean  , and variance , and hence, functions  , and  can be determined.

***3.5.1******Scenario 1: When  and ***

The total expected of the profit function for retailer  in cycle length in scenario 1 will be computed as follows:



(28)

Then, the expected total profit function in cycle length for retailer  in scenario 1 will be expressed as,



(29)

***3.5.2 Scenario 2: When  and ***

In this scenario, the total expected of the profit function in cycle length of retailer  in scenario 2 will be computed as follows:



(30)

Then, the expected total profit function of retailer  in cycle length in scenario 2 will be expressed as,



(31)

***3.5.3 Scenario 3: When  and ***

In this scenario, the total expected of the profit function of retailer  in cycle length in scenario 3 will be computed as follows:



(32)

Then, the expected total profit function of retailer  in cycle length in scenario 3 will be expressed as,



(33)

***3.5.4******Scenario 4: When  and ***

In this scenario, the total expected of the profit function of retailer  in cycle length in scenario 4 will be computed as follows:



(34)

Then, the expected total profit function of retailer  in cycle length in scenario 3 will be expressed as,



(35)

After the analysis of 4 scenarios, the total expected of profit function of retailer  per a cycle time can be derived as,



(36)

Also, the expected total profit function of retailer  per a cycle time can be derived as,

(37)



Therefore, the expected total profit function of both retailer per time unit can be derived as,



(38)

**CHAPTER 4**

**NUMERICAL EXPERIMENTS AND SENSITIVITY ANALYSES**

**4.1 Numerical Experiments**

In this chapter, a variety of numerical experiments are conducted so as to evaluate the efficiency of the proposed emergency lateral transshipment model for both retailers. The optimal values of retailer ’s order quantity () and retailer ’s delivery quantity () will be determined so as to maximize expected total profit per unit time. MATLAB is employed to determine an optimal solution. Genetic Algorithm (GA) solver in MATLAB will be operated to determine the optimal values of two decision variables.

For finding optimal solution, the following inputs was applied for GA solver,

* Number of variables is 2
* Lower bound [1 1]
* Upper bound [100 100]

For the base scenario, the following values were used as input parameters,

**Table 4.1**

*Input Parameters for the Two Retailers Inventory System*

|  |  |
| --- | --- |
| Parameter | Value |
| Mean demand (, ) (unit per day) | 15, 10 |
| Standard deviation of demand ( (unit per day) | 3,2 |
| The fraction of backordered demand () | 0.8 |
| Unit transshipment price () ($ per unit) | 8 |
| Salvage value for unsold product per unit () ($ per unit) | 3 |
| Unit order cost () ($ per unit) | 4 |
| Unit penalty cost () ($ per unit) | 4 |
| Unit retail price () ($ per unit) | 10 |
| Unit backordered cost () ($ per unit) | 3 |
| Unit transshipment handing cost () ($ per unit) | 2 |
| The cycle length () (day) | 6 |

For the above input parameters, the optimal solutions are: the order quantity of retailer is 90 units, the order quantity of retailer  is 60 units. The total expected profit of both retailers is $ 675.87.

**4.2 Sensitivity Analyses**

Sensitivity Analyses is produced to examine the impacts of input parameters. There is 11 parameters that are examined in this session, i.e., the mean demand, the standard deviation of demand, the fraction of backordered demand, the transshipment price, the salvage value for unsold product, the unit order cost, the penalty cost, the retail price, the backordered cost, the transshipment handling cost, and the cycle length. However, it is noted that the cycle length is fixed, so the cycle length does not affect on other input parameters except the mean demand.

***4.2.1 The Effect of Mean Demand***

The effect of mean demand (, )is investigated in this part, and the other input parameters in the base scenario are fixed. The mean demand of retailer  () will be varied from 11 to 19, and the mean demand of retailer  () will be varied from 8 to 16. The results are shown and interpreted in Table 4.2 and Figure 4.1.

**Table 4.2**

*The Effect of Mean Demand*

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  |  |  |  |  |
| 11 | 8 | 48 | 48 | 658.32 |
| 10 | 48 | 60 | 589.56 |
| 12 | 48 | 72 | 284.24 |
| 14 | 48 | 84 | -369.68 |
| 16 | 48 | 96 | -1528.91 |
| 13 | 8 | 60 | 48 | 723.21 |
| 10 | 60 | 60 | 740.66 |
| 12 | 60 | 72 | 627.37 |
| 14 | 60 | 84 | 317.16 |
| 16 | 60 | 96 | -283.20 |
| 15 | 8 | 72 | 48 | 628.23 |
| 10 | 72 | 60 | 675.87 |
| 12 | 72 | 72 | 678.02 |
| 14 | 72 | 84 | 608.59 |
| 16 | 72 | 96 | 430.27 |
| 17 | 8 | 84 | 48 | 348.17 |
| 10 | 84 | 60 | 366.98 |
| 12 | 84 | 72 | 405.08 |
| 14 | 84 | 84 | 470.53 |
| 16 | 84 | 96 | 574.51 |
| 19 | 8 | 96 | 48 | -136.18 |
| 10 | 96 | 60 | -207.52 |
| 12 | 96 | 72 | -215.41 |
| 14 | 96 | 84 | -123.37 |
| 16 | 96 | 96 | 120.74 |

**Figure 4.1**

*The Expected Total Profit per Time Unit and Mean Demand*

From the results in Table 4.2 and Figure 4.1, it is noticed that when the mean demand of a retailer increases simultaneously the mean demand of the other retailer remains unchanged, order quantity of that retailer will rise up while the order quantity of the other retailer will not be changed. This trend is understandable due to the increase in order quantity will help to cover the increasing demand. However, it should be noted that the total profit might increase and then decrease when the mean demand of a retailer increases. The initial increase in profit is owing to the increase in demand, but when the demand is too high, demand might not be fulfilled even with transshipment, and hence, the total profit will be reduced due to the high penalty cost.

***4.2.2 The Effect of Standard Deviation of Demand***

The impact of standard deviation of demand ( is investigated, and the other parameters in the base scenario are fixed. Standard deviation of demand of retailer  ( will be varied 2 to 4, and standard deviation of demand of retailer  ( will be varied 1 to 3.5. The results of the expected total profit per time unit are shown and interpreted in Table 4.3 and Figure 4.2.

**Table 4.3**

*The Effect of Standard Deviation of Demand*

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  |  |  |  |  |
| 2 | 1 | 90 | 60 | 10361.46 |
| 1.5 | 90 | 60 | 1259.71 |
| 2 | 90 | 60 | 792.38 |
| 2.5 | 90 | 60 | 675.46 |
| 3 | 90 | 60 | 616.05 |
| 2.5 | 1 | 90 | 60 | 10279.72 |
| 1.5 | 90 | 60 | 1177.97 |
| 2 | 90 | 60 | 710.65 |
| 2.5 | 90 | 60 | 593.72 |
| 3 | 90 | 60 | 534.32 |
| 3 | 1 | 90 | 60 | 10244.93 |
| 1.5 | 90 | 60 | 1143.19 |
| 2 | 90 | 60 | 675.86 |
| 2.5 | 90 | 60 | 558.94 |
| 3 | 90 | 60 | 499.54 |
| 3.5 | 1 | 90 | 60 | 10225.61 |
| 1.5 | 90 | 60 | 1123.88 |
| 2 | 90 | 60 | 656.55 |
| 2.5 | 90 | 60 | 539.63 |
| 3 | 90 | 60 | 480.23 |
| 4 | 1 | 90 | 60 | 10213.22 |
| 1.5 | 90 | 60 | 1111.48 |
| 2 | 90 | 60 | 644.16 |
| 2.5 | 90 | 60 | 527.24 |
| 3 | 90 | 60 | 467.83 |

**Figure 4.2**

*The Expected Total Profit per Time Unit and Standard Deviation of Demand*

As shown in Table 4.3 and Figure 4.2, when the standard deviation of demand of retailer  ( remains fixed while the standard deviation of demand of retailer( changes or vice versa, both order quantities remain unchanged. This implies that the standard deviation of demand does not affect on order quantities. This is due to the effect of transshipment policy. However, the expected total profit per time unit decreases. The above trend is reasonable owing to the increase in shortage cost when standard deviation of demand rises up.

***4.2.3******The Effect of Fraction of Backordered Demand***

The effect of fraction of backordered demand () is investigated in this section. When the fraction of backordered demand () will be varied 0.2 to 1, other input parameters in the base scenario remain unchanged. The results are shown and interpreted in Table 4.4 and Figure 4.3.

**Table 4.4**

*The Effect of the Fraction of Backordered Demand*

|  |  |  |  |
| --- | --- | --- | --- |
|  |  |  |  |
| 0.2 | 90 | 60 | 377.93 |
| 0.4 | 90 | 60 | 509.04 |
| 0.6 | 90 | 60 | 601.73 |
| 0.8 | 90 | 60 | 675.87 |
| 1.0 | 90 | 60 | 741.60 |

**Figure 4.3**

*The Expected Total Profit per Time Unit and Fraction of Backorder Demand*

As shown in Table 4.4 and Figure 4.3, it can be seen clearly that when fraction of backordered demand ( increases, both order quantities remain unchanged. However, the expected total profit per time unit will increase. This trend can be explained as when the fraction of backordered demand increase, more demand shortage will be fulfilled, and hence, the expected total profit per time unit will increases.

***4.2.4 The Effect of Transshipment Price***

The effect of transshipment price () is investigated simultaneously other input parameters in the base scenario remain unchanged. Transshipment price ()will be varied 4 to 12. The results on the effect of transshipment price are presented are shown and interpreted in Table 4.5 and Figure 4.4.

**Table 4.5**

*The Effect of Transshipment Price*

|  |  |  |  |
| --- | --- | --- | --- |
|  |  |  |  |
| 4 | 90 | 60 | 300.35 |
| 6 | 90 | 60 | 488.11 |
| 8 | 90 | 60 | 675.87 |
| 10 | 90 | 60 | 863.62 |
| 12 | 90 | 60 | 1051.38 |

**Figure 4.4**

*The Expected Total Profit per Time Unit and the Transshipment Price*

According to Table 4.5 and Figure 4.4, it is noticed that when the transshipment price (****)increases, the expected total profit per time unit also increases. This trend is explainable because when transshipment price is high, the retailer is motivated to transship more, and hence, the expected total profit also increases. However, both order quantities remain unchanged when the transshipment price is changed.

***4.2.5 The Effect of Salvage Value for Unsold Product***

In this section, the effect of salvage value for unsold product ( is investigated simultaneously other input parameters in the base scenario are fixed. Salvage value for unsold product () is varied from 1.5 to 3.5. The results are shown and interpreted in Table 4.6 and Figure 4.5.

**Table 4.6**

*The Effect of Salvage Value for Unsold Product*

|  |  |  |  |
| --- | --- | --- | --- |
|  |  |  |  |
| 1.5 | 90 | 60 | 209.55 |
| 2 | 90 | 60 | 364.99 |
| 2.5 | 90 | 60 | 520.43 |
| 3 | 90 | 60 | 675.87 |
| 3.5 | 90 | 60 | 831.32 |

**Figure 4.5**

*The Expected Total Profit per Time Unit and Salvage Value for Unsold Product*

As shown in Table 4.6 and Figure 4.5, it is shown that when salvage value for unsold product ( increases, both order quantities remain unchanged. However, the expected total profit per time unit will increase. This trend is understandable. When salvage value increases, it means we can sell the unsold product with higher value. Thus, the expected total profit per time unit should be increased.

***4.2.6 The Effect of Order Cost***

The effect of order cost () is investigated simultaneously other input parameters in the base scenario are fixed. The order cost () will be varied 4 to 8. The results of the effect of order cost ()are shown and interpreted in Table 4.7 and Figure 4.6.

**Table 4.7**

*The Effect of Order Cost*

|  |  |  |  |
| --- | --- | --- | --- |
|  |  |  |  |
| 4 | 90 | 60 | 675.87 |
| 5 | 90 | 60 | 671.18 |
| 6 | 90 | 60 | 666.49 |
| 7 | 90 | 60 | 661.80 |
| 8 | 90 | 60 | 657.12 |

**Figure 4.6**

*The Expected Total Profit per Time Unit and Order Cost*

As shown in Table 4.7 and Figure 4.6, it is shown that when order cost ()increases, the order quantities remain unchanged. This trend is reasonable because the order quantities should be affected only by the demand. However, a decrease in the expected total profit per time unit is observed. This trend is reasonable.

***4.2.7 The Effect of Penalty Cost***

The effect of penalty cost () is investigated simultaneously other input parameters in the base scenario remain unchanged. The penalty cost () is varied from 1 to 5. The results the effect of penalty cost () are shown and interpreted in Table 4.8 and Figure 4.7.

**Table 4.8**

*The Effect of Penalty Cost*

|  |  |  |  |
| --- | --- | --- | --- |
|  |  |  |  |
| 1 | 90 | 60 | 1260.39 |
| 2 | 90 | 60 | 1065.55 |
| 3 | 90 | 60 | 870.71 |
| 4 | 90 | 60 | 675.87 |
| 5 | 90 | 60 | 481.03 |

**Figure 4.7**

*The Expected Total Profit per Time Unit and Penalty Cost*

From the result given by Table 4.8 and Figure 4.7, when the penalty cost () increases, the expected total profit per time unit keep decreases. This trend is reasonable. However, it seems that the increase in the penalty cost does not have impact on the order quantities.

***4.2.8 The Effect of Retail Price***

In this section, the effect of retail price () is investigated simultaneously other input parameters in the base scenario are fixed. The retail price () will be varied 9 to 13. The results of the effect of retail price () are shown and interpreted in Table 4.9 and Figure 4.8.

**Table 4.9**

*The Effect of Retail Price*

|  |  |  |  |
| --- | --- | --- | --- |
|  |  |  |  |
| 9 | 90 | 60 | 452.62 |
| 10 | 90 | 60 | 675.87 |
| 11 | 90 | 60 | 899.12 |
| 12 | 90 | 60 | 1122.37 |
| 13 | 90 | 60 | 1345.62 |

**Figure 4.8**

*The Expected Total Profit per Time Unit and Retail Price*

According to Table 4.9 and Figure 4.8, it is shown that when the retail price **()** increases, the expected total profit per time unit also increases. This trend is clearly reasonable.

***4.2.9 The Effect of Backordered Cost***

The effect of backordered cost () is investigated simultaneously other parameters in the base scenario remain unchanged. The backordered cost () is varied from 1 to 3. The results are presented in Table 4.10.

**Table 4.10**

*The Effect of Backordered Cost*

|  |  |  |  |
| --- | --- | --- | --- |
|  |  |  |  |
| 1 | 90 | 60 | 680.56 |
| 1.5 | 90 | 60 | 679.38 |
| 2 | 90 | 60 | 678.21 |
| 2.5 | 90 | 60 | 677.04 |
| 3 | 90 | 60 | 675.87 |

**Figure 4.9**

*The Expected Total Profit per Time Unit and Backordered Cost*

From Table 4.10 and Figure 4.9, it can be shown that when the backordered cost () increase, the expected total profit per time unit will decrease. The above trend is reasonable because when the backordered cost is high, the total cost should be increased which leads to the decrease in the expected total profit per time unit .

***4.2.10 The Effect of Transshipment Handling Cost***

The effect of transshipment handling cost () is investigated simultaneously the other input parameters in the base scenario remain unchanged. The transshipment handling cost () will be varied 1 to 5. The results of transshipment handling cost () are shown and interpreted in Table 4.11 and Figure 4.10.

**Table 4.11**

*The Effect of Transshipment Handling Cost*

|  |  |  |  |
| --- | --- | --- | --- |
|  |  |  |  |
| 1 | 90 | 60 | 967.84 |
| 2 | 90 | 60 | 675.87 |
| 3 | 90 | 60 | 383.89 |
| 4 | 90 | 60 | 91.91 |
| 5 | 90 | 60 | -200.06 |

**Figure 4.10**

*The Expected Total Profit per Time Unit and Transshipment Handling Cost*

As shown in Table 4.11 and Figure 4.10, when the transshipment handling cost ()increases, the expected total profit per time unit decreases. The decrease in the expected total profit per time unit is explainable because the total cost must increase when the transshipment handling cost () increases.

***4.2.11 The Effect of Cycle Length***

In this section, the effect of cycle length () is investigated simultaneously other input parameters in the base scenario remain unchanged. The cycle length () is varied from 4 to 8. The results are shown and interpreted in Table 4.12 and Figure 4.11.

**Table 4.12**

*The Effect of Cycle Length*

|  |  |  |  |
| --- | --- | --- | --- |
|  |  |  |  |
| 4 | 60 | 40 | 1189.53 |
| 5 | 75 | 50 | 1026.45 |
| 6 | 90 | 60 | 675.87 |
| 7 | 105 | 70 | 259.93 |
| 8 | 120 | 80 | -187.92 |

**Figure 4.11**

*The Expected Total Profit per Time Unit and Cycle Length*

From the result given by Table 4.12 and Figure 4.11, it is shown that when the cycle length () increases, the order quantity of retailer  and order quantity of retailer  increase. These trends are reasonable. However, the expected total profit per time unit will be decreased. This trend is understandable because when the cycle length is prolonged, the retailers have to stock more products which lead to the increase in holding cost. Therefore, the expected total profit per time unit will decrease.

# CHAPTER 5 CONCLUSIONS AND RECOMMENDATIONS

## Conclusions

This research studies emergency lateral transshipment under a decentralized two-retailer inventory system with slow-moving products. Emergency lateral transshipment (ELT) is analyzed assuming partial backordering that accepts some consumer who are willing to wait for the rest products until the end of the sale season. In this research, a mathematical model was derived so as to help maximize the expected total profit of a two-retailer transshipment system through the determination of optimal order quantities of the retailers. Numerical experiments and sensitivity analyses are then conducted to demonstrate the performance of the mathematical model and to investigate the impacts in various values of input parameters, i.e., the mean demand (), the standard deviation of demand (, the fraction of backordered demand (), the transshipment price (), salvage value for unsold product (), the unit order cost (), the penalty cost (), the retail price (), the backordered cost (), the transshipment handling cost(), and the cycle length (). From sensitivity analysis results, the following impacts of input parameters can be observed:

* The increase in the fraction of backordered demand (), the transshipment price (), the salvage value for unsold product (), and retailer price () will lead to the increases in the total expected profit per time unit .
* The decrease in the order cost (), the penalty cost (), the backordered cost (), the transshipment handling cost (), and the cycle length () will lead to the decreases in the total expected profit per time unit .
* Related to the increase in mean demand (), the expected total profit per time unit might increase, and then decrease due to the fact that when the demand is too high, it cannot be fulfilled through transshipment.
* Changes in the order quantities of the retailers are observed when the mean demand () or the cycle length () changes.
* Changes in backordered cost does not significantly affect the expected total profit per time unit .
  1. **Recommendations**

In the future, other studies on this issue could be performed in many ways as follows:

* In this research studies, only two-retailer decentralized inventory system with partial backordering is considered. In the future study, to make it more realistic, the multiple retailers centralized system should be considered.
* In this research, the cycle length is constant. To make the future study more realistic, the cycle length should be investigated as a decision variable.
* Last but not least, the lead time is not considered in this research. Thus, the future study should take this into consideration.

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APPENDIX   
COMPUTER PROGRAM (MATLAB)

function TPij = Profit(D) %{D(1) = qi, D(2) = qj}

mu\_i = 15; %Mean i

mu\_j = 10; %Mean j

sigma\_i = 3; %Standard deviation i

sigma\_j = 2; %Standard deviation j

del = 0.8; %The fraction of backordered demand

k = 8; %Unit transshipment price

s = 3; %Salvage value for unsold product per unit

w = 4; %Ordering cost per unit

v = 4; %Penalty cost per unit

r = 10; %Retail price per unit

b = 3; %Unit backordered cost

h = 2; %Unit transshipment handling cost paid by shipper

T = 6;

D(1) = mu\_i\*T;

D(2) = mu\_j\*T;

pi = 22/7;

syms x y;

A = 0; %P(Di =< qi)

for i1 = 0:D(1)

m1 = mu\_i\*T;

v1 = sqrt((sigma\_i)^2\*T);

b1 = int(exp(-0.5\*((x-m1)/v1)^2)/(sqrt(2\*pi)\*v1),'x',0,i1);

A = b1;

end

A;

B = 0; %P(Dj =< qj)

for i2 = 0:D(2)

m2 = mu\_j\*T;

v2 = sqrt((sigma\_j)^2\*T);

b2 = int(exp(-0.5\*((y-m2)/v2)^2)/(sqrt(2\*pi)\*v2),'y',0,i2);

B = b2;

end

B;

C = 0; %P(Dj > qj)

for i3 = 0:D(1)

for i4 = D(2):D(1)+D(2)-i3

m3 = mu\_j\*T;

v3 = sqrt((sigma\_j)^2\*T);

b3 = 1-(int(exp(-0.5\*((y-m3)/v3)^2)/(sqrt(2\*pi)\*v3),'y',0,i4));

C = b3;

end

C;

end

C;

E = 0; %P(Di > qi)

for i5 = 0:D(2)

for i6 = D(1):D(1)+D(2)-i5

m4 = mu\_i\*T;

v4 = sqrt((sigma\_i)^2\*T);

b4 = 1-(int(exp(-0.5\*((x-m4)/v4)^2)/(sqrt(2\*pi)\*v4),'x',0,i6));

E = b4;

end

E;

end

E;

%Probability for each scenario

P\_1 = (A\*B);

P\_2 = (A\*C);

P\_3 = (E\*B);

P\_4 = ((1-A)\*(1-B));

Total\_Prob = P\_1+P\_2+P\_3+P\_4;

%pdf and cdf of retaler i and retailer j

pdf\_i = (1/(sigma\_i\*sqrt(2\*pi\*T)))\*exp(-0.5\*((x-(mu\_i\*T))/((sigma\_i)^2\*T)));

pdf\_j = (1/(sigma\_j\*sqrt(2\*pi\*T)))\*exp(-0.5\*((y-(mu\_j\*T))/((sigma\_j)^2\*T)));

cdf\_i = (0.5\*(1+erf((D(1)-mu\_i\*T)/((sigma\_i)\*sqrt(2\*T)))));

cdf\_j = (0.5\*(1+erf((D(2)-mu\_j\*T)/((sigma\_j)\*sqrt(2\*T)))));

%Total Profit of retailer i for each case

%Case1

TPi1 = ((r-s)\*cdf\_j\*int(x\*pdf\_i,'x',0,D(1)))+((s-w)\*D(1)\*cdf\_j\*cdf\_i);

%Case2

TPi2 = ((r-s)\*(1-cdf\_j)\*int(x\*pdf\_i,'x',0,D(1)))+((s-w)\*D(1)\*(1-cdf\_j)\*cdf\_i)+(k-h-s)\*int((int((del\*(y-D(2))\*pdf\_j),'y',D(2),D(2)+((D(1)-x)/del)))+((D(1)-x)\*(1-(0.5\*(1+erf(((D(2)+(D(1)-x)/del)-mu\_j\*T)/sigma\_j\*sqrt(2\*T)))))\*pdf\_i),'x',0,D(1));

%Case3

TPi3 = ((r-v)\*D(1)\*(1-cdf\_i)\*cdf\_j)+(v\*cdf\_j\*int(x\*pdf\_i,'x',D(1),100))+(r-k-v)\*int((int((del\*(x-D(1))\*pdf\_j),'y',0,D(2)-(del\*(x-D(1))))+(int((D(2)-y)\*pdf\_j,'y',D(2)-del\*(x-D(1)),D(2)))\*pdf\_i),'x',D(1),100);

%Case4

TPi4 = ((r-v-w)\*D(1)\*(1-cdf\_j)\*(1-cdf\_i))+(v\*(1-cdf\_j)\*(int(x\*pdf\_i,'x',D(1),100)));

%Total Profit of retailer i

TPi = ((P\_1)\*TPi1)+((P\_2)\*TPi2)+((P\_3)\*TPi3)+((P\_4)\*TPi4);

%Total Profit of retailer j for each case

%Case1

TPj1 = ((r-s)\*cdf\_i\*(int(y\*pdf\_j,'y',0,D(2))))+((s-w)\*D(2)\*cdf\_i\*cdf\_j);

%Case2

TPj2 = (r-k-v)\*int((int((del\*(y-D(2))\*pdf\_j),'y',D(2),D(2)+((D(1)-x)/del))+(1-(0.5\*(1+erf(((D(2)+(D(1)-x)/del)-mu\_j\*T)/sigma\_j\*sqrt(2\*T)))))\*pdf\_i),'x',0,D(1))+(D(2)\*(r-v-(del\*b))\*(1-cdf\_j)\*cdf\_i)+((v+(del\*b))\*cdf\_i\*int(y\*pdf\_j,'y',D(2),100));

%Case3

TPj3 = ((r-s)\*(1-cdf\_i)\*int(y\*pdf\_j,'y',0,D(2)))+((s-w)\*D(2)\*cdf\_j\*(1-cdf\_i))+((k-h-s)\*int((int((del\*(x-D(2))\*pdf\_j),'y',0,D(2)-(del\*(x-D(1)))))+((int((D(2)-y)\*pdf\_j,'y',D(2)-del\*(x-D(1)),D(2))\*pdf\_i)),'x',D(1),100));

%Case4

TPj4 = ((r-v-w)\*D(2)\*(1-cdf\_j)\*(1-cdf\_i))+(v\*(1-cdf\_i)\*int(y\*pdf\_j,'y',D(1),100));

%Total Profit of retailer j

TPj = ((P\_1)\*TPj1)+((P\_2)\*TPj2)+((P\_3)\*TPj3)+((P\_4)\*TPj4);

%Total Profit of both retailers

TPij = (TPi+TPj)/T;

end