AN OPTIMAL STOCK LEVEL AND PRICING STRATEGY FOR PERISHABLE PRODUCTS UNDER THE VMI SYSTEM

by

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AUTHOR'S DECLARATION

I, Janthratip Khamwarut, declare that the research work carried out for this thesis was in accordance with the regulations of the Asian Institute of Technology. The work presented in it are my own and has been generated by me as the result of my own original research, and if external sources were used, such sources have been cited. It is original and has not been submitted to any other institution to obtain another degree or qualification. This is a true copy of the thesis, including final revisions.

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ABSTRACT

Vendor-managed inventory (VMI) is an approach used to ensure that perishable items are always available while also improving operational efficiency and compliance with quality standards. In this study, we study a new decision scenario concerning a deteriorating item, focusing on the case of a single vendor and single retailer under VMI system. We develop the mathematical model considering the base stock policy, which will be implemented with a maximum stock level (S), with adjustments made to discount prices based on shelf life to encourage a retailer to buy products. Demand rates are influenced by pricing, assuming that the rate of deterioration for a deteriorating item follows a Weibull distribution and a fully backlogged shortage. To assess the efficacy of the proposed models, we conduct numerical experiments and sensitivity analyses. These analyses aim to evaluate how changes in input variables impact experimental outcomes and shed light on the effects of alterations in input parameters on optimal solutions. Finally, we present the conclusions drawn from our study, along with recommendations for future research.

Keywords: Vendor-managed inventory, Perishable product, base stock policy, adjustments in discounts, Weibull distribution

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LIST OF ABBREVIATIONS

CIRP	= Completely Independent Replenishment
CPFR	= Collaborative Planning Forecasting and Replenishment
EOQ	= Economic Order Quantity Model
EPQ	= Economic Production Quantity
MRs	= Multiple Retailers
RSC	= Revenue Sharing Contract
SR	= Single Retailer
SV	= Single Vendor
VMI	= Vendor Managed Inventory

CHAPTER 1
INTRODUCTION

1.1 Background of the Study

In contemporary research landscapes, as competition intensifies, companies and sellers are compelled to refine their strategies to foster competitiveness. One pivotal strategy in this regard is optimizing the supply chain, which holds significant importance in modern commerce. The supply chain encompasses a multifaceted framework comprising various procedures, organizations, and networks dedicated to the production, procurement, and dissemination of goods and services. Supply chain management seeks to seamlessly integrate suppliers, manufacturers, warehouses, and retailers to ensure the efficient and timely delivery of products. Commencing with the acquisition of raw materials, the process extends through production, shipping, and warehousing, culminating in the distribution of the final product to end-users. Supply chain management aims to optimize the movement of commodities and services from the source to the final recipient. This helps organizations streamline their operations, reduce risks, and gain a competitive edge in the marketplace. By effectively coordinating all of the processes in the supply chain, businesses can ensure the smooth functioning of their procedures.

Effective inventory management plays a crucial role in supply chain operations. It entails monitoring inventory levels, guaranteeing stock availability when required, and ensuring timely inventory replenishment. The strategy of inventory management always e ensures the correct amount of inventory is available at the right time and place, enabling companies to fulfill consumer requirements efficiently while minimizing the risks and costs associated with high stock levels. Efficient inventory management has the potential to save operating costs, minimize supply chain risks, and foster better supplier relationships.

A critical approach in the implementation of inventory management is the use of vendor-managed inventory (VMI). VMI specifies that the vendor assumes the role of managing and deciding on the replenishment of inventory placed at the retailer's point of deal. VMI moves inventory control decisions from the retailer to the supplier, relying on sharing information about customer demand and inventory. This approach depends on the exchange of information to effectively relay customer demand and inventory data. With the assistance provided by VMI, suppliers can overcome supply-demand challenges while satisfying customer expectations, remaining competitive, and maintaining profit margins. Another valuable VMI feature is item stratification, which focuses on fast-moving items while maintaining an appropriate mix of slow-moving items to ensure a diverse product catalog. Extensive research has demonstrated that Vendor Managed Inventory (VMI) is a valuable method for managing products. VMI can significantly lower supply chain expenses by reducing shipments, aligning objectives, and facilitating total cost savings, reduced inventory levels, and fewer stockouts. Therefore, implementing VMI is an important approach to product management.

Perishable goods refer to items that possess a limited shelf life and are at risk of decay or deterioration over a period of time. This category includes a diverse range of things, including perishable food items such as fruits, vegetables, medications, chemicals, flower products, beauty products, and other related goods. The maintenance of the quality and safety of these items requires attention to appropriate protocols for handling, storage, and transportation. Managing perishable goods is a difficult task as their shelf life is limited. It is crucial to ensure that the products are of good quality and that losses are minimized. To overcome the challenges associated with the limited shelf life of perishable goods, suppliers and retailers can adopt VMI policies that are tailored to their specific needs. VMI ensures that critical perishable items are always available, while also improving operational efficiency and compliance with quality standards. VMI policies can be customized to include an inventory review policy, which maintains a predetermined minimum level of stock on hand. By using an inventory policy, essential perishable items are readily available, reducing the risk of product decay. However, a pricing strategy linked to the deterioration rate should be employed with the VMI policy to further optimize sales and minimize product decay or waste. (Bramorski, 2008) proposed an approach for the development of perishable product inventory policies, considering the inventory stock and the time remaining until the products expire are important factors to consider when making decisions. Demand stimulation was employed as a strategy to reduce deterioration by applying price reductions to the products experiencing deterioration. The result shows that the offering of discounts on older products will enhance customers' willingness to buy.

1.2 Statement of the Problem

As an approach to supply chain management, vendor-managed inventory (VMI) is a strategy in which vendor is tasked with deciding and managing the retailer's inventory levels. This entails vendor making decisions regarding when to replenish stock, effectively relieving the retailer of this task and ensuring optimal inventory levels are maintained. (Verma & Chatterjee, 2017a). Many studies have explored implementing VMI for perishable goods in different situations. Two primary factors are crucial in examining this topic. Firstly, the demand for perishable products can be deterministic, which includes time-dependent demand, constant demand, or demand influenced by pricing. The other factor is stochastic demand, characterized by a known demand distribution and arbitrary demand patterns. Secondly, understanding how quickly perishable products deteriorate is essential. Studies have examined deterioration rates under various conditions like constant rate, time-dependent rate, and Weibull distribution. Furthermore, some previous studies have delved into diverse methods for effectively managing perishable goods by integrating various strategies with VMI. such as the implementation of discounts or adopting inventory review policies. Research consistently shows that a tailored VMI policy for perishable goods ensures product availability, quality maintenance, and minimizes overall supply chain costs. However, the study highlights specific scenarios that necessitate further consideration to enhance the practical implementation and real-world effectiveness of VMI.

The existing researchers mostly focused on studying inventory management under the VMI strategy of perishable products within a one-vendor, one-retailer system, employing a continuous review policy. There still exists a gap in managing perishable products within a one-vendor, one-retailer VMI system where a base stock inventory policy is employed, and discounts are adjusted based on product shelf life.

The purpose of this study is to fill the gap in the existing literature concerning the VMI policy to manage a perishable product within one vendor and one retailer system, which adopts a base stock inventory policy and includes adjustments in discounts according to the shelf life of the products.

1.3 Objective of Study

The main goal of this study is to determine the optimal stock level and determine at what point in the lifetime of an item a price-reducing strategy should be employed to encourage customers to buy aims to maximize the total profit.

1.4 Scope and Limitation

The scope and limitations of this study concern the management of deteriorating items within a vendor-managed inventory model for a supply chain, exactly within the context of one vendor and one retailer.

The research will consider the following assumptions.

1. A price-dependent demand rate is deterministic.
2. At a specific time, t (which is less than the lifespan of item T), when the rate of deterioration meets a predetermined value, the price will be decreased to encourage customers to buy. Two demand functions will be used, one for the time interval [0, t_1] and [t_2,T], and another for the time interval [t_1, t_2], as the prices vary between these periods.
3. The probability distribution of the time it takes for one finished perishable good to deteriorate adheres to a Weibull distribution.
4. The inventory policy adopted will be the base stock policy, consider a maximum stock level (S) at the start of the review period.
5. Shortage is allowed and fully backlogged.

CHAPTER 2
LITERATURE REVIEWS

The objective of this research is to ascertain the optimal stock level and determine the opportune moment in an item's lifespan to implement a price-reduction strategy, thereby enticing customers to purchase and maximizing total profit. Vendor Managed Inventory (VMI) is a supply chain method where the vendor assumes responsibility for managing the retailer's inventory to ensure the highest level of customer service by maintaining appropriate inventory levels (Tat et al., 2014). VMI helps the goal of cost reductions by the optimization of inventory levels and the prevention of stockouts. The benefits help to supply chain that is more efficient and cost-effective. However, there are still some issues that past research papers have not been able to address. Based on past research papers, most studies on inventory management of deteriorating items have focused on one supplier and one retailer system under VMI utilizing a continuous review policy. However, there is still a gap in the application of VMI that relates to the inventory management of a deteriorating good using a base stock policy and adjustments in discounts based on the shelf life of the products.

To fulfill this gap, this research focuses on the management of perishable products under the VMI policy in case one vendor and one retailer system, which implements a base stock inventory policy and includes adjustments in discounts based on the shelf life of the products.

2.1 Vendor Managed Inventory

Numerous studies have demonstrated that VMI can emerge as a strategic collaboration between retailers and vendors. Vendor Managed Inventory (VMI) is an effective supply chain management method. This strategy entails a partnership between retailers and vendors, with the vendor assuming responsibility for inventory management at the retail level. This collaboration helps to minimize overall costs, increase profitability, improve service efficiency, and enhance coordination between the two parties. Many researchers have studied vendor-managed inventory in the past. It is a coordinated strategy between retailers and vendors, where the vendor controls inventory levels within predetermined boundaries (Darwish & Odah, 2010). A study by (Razmi et al., 2010) was conducted to compare traditional inventory management systems with VMI. The researchers used advanced mathematical models and the total inventory cost as the main performance metric. They found strong evidence that VMI systems consistently outperformed traditional methods in various scenarios, even considering backorders. This emphasizes the significant benefits of adopting VMI systems for efficient coordination in supply chain management. The exploration of the vendor-managed inventory (VMI) concept was conducted by (J. Y. Lee & Ren, 2011) through the utilization of a periodic-review stochastic inventory model. This study took into account not only exchange rate uncertainty but also varied fixed ordering costs for both the supplier and the retailer. Results found that implementing VMI led to a reduction in overall supply chain costs and the study highlighted that state-dependent (s, S) are most appropriate for suppliers, which was supported by analytical results. A study conducted (Kazemi & Zhang, 2013) aimed to compare the efficiency of two inventory management models: vendor-managed inventory (VMI) systems and collaborative planning, forecasting, and replenishment (CPFR). The study incorporated production costs in a mathematical model and factored in demand under price-sensitive and uncertain conditions to optimize the overall profit of a solitary vendor and a lone retailer. (Mateen & Chatterjee, 2015) research delves into the analysis of stochastic demand within a VMI context that involves one vendor and multiple retailers. The authors emphasize the importance of a mutual framework in VMI to address benefit distribution issues and propose negotiation-based solutions. Additionally, VMI should be considered a vital component of a continuous improvement process, and any solution that enhances the supply chain for all parties involved should be implemented. The model developed by (Verma & Chatterjee, 2017b) considers the uses of nonlinear mixed-integer programming to determine an inventory replenishment in a situation with a single vendor and many retailers under vendor-managed inventory (VMI) policy. By considering retailer heterogeneity, they develop a complex model that optimizes replenishment decisions and the quantity for each of its locations.

2.2 Deteriorating Products

Perishable goods refer to items that possess a limited shelf life and are at risk of decay or deterioration over time. However, managing such products can be challenging, mainly due to factors like inventory control, transportation logistics, and demand forecasting. Therefore, it becomes necessary to optimize inventory management strategies to minimize waste, ensure product quality, and meet consumer demands. (Indrajitsingha et al., 2021) devised a model aimed at mitigating the repercussions of COVID-19 on consumer behavior and economic stability. This model introduces an inventory system for perishable items to an optimal ordering quantity, strategically designed to decrease the total average cost per unit time. Considering variables such as selling price and the management of shortages after the COVID-19 pandemic (De et al., 2024), a framework was established to address the non-instantaneous deterioration of goods. This model enhances the overall average profit by modifying both the selling price and the total cycle time, especially in situations of inflation and partial backlogging. The findings underscore the substantial influence of basic demand on total profit sensitivity, underscoring the model's relevance for post-disruption recovery strategies.

2.3 Vendor Managed Inventory for Deteriorating Products

Researchers consider and implement vendor-managed inventory (VMI) as an approach to effectively manage perishable goods inventory. VMI enables vendors to take control of inventory management, including replenishment cycles and order quantities. This reduces response time and minimizes product deterioration for retailers. (Lan et al., 2011) proposed two inventory control models specifically designed for managing deteriorating products within the framework of vendor-managed inventory (VMI). They developed and tested bi-level programming models using a genetic algorithm to optimize delivery strategies for managing deteriorating products in a VMI system. (Yu et al., 2011) created a model that integrates pricing and deterioration factors to analyze the impact on profit in vendor-managed inventory (VMI) systems. This strategy used a combination of genetic algorithms and analytical tools to achieve optimization. The study examined the specific scenario of a single vendor and many retailers operating under a Vendor Managed Inventory (VMI) system. One year after that, (Yu et al., 2012) used the golden search algorithm to analyze how deterioration rates impact total costs and determine how important it is to manage finished products in comparison to raw materials. An inventory management model has been developed by (S. Lee & Kim, 2014) in the case of one vendor and one retailer system. It considers perishable and defective items to optimize delivery quantities and improve supply chain performance. (Setak & Daneshfar, 2014) proposed EOQ model for constant deterioration and inventory-dependent demand with the shortage is backlogging, considering both VMI and non-VMI supply, whose results show the VMI system consistently reduces total costs. (Tat et al., 2014) a comprehensive economic order quantity model (EOQ) has been developed to address the challenges posed by non-instantaneous and perishable goods, in both cases with and without shortages under VMI using the EOQ model. They extended their research to an item with instantaneous deterioration. (Tat et al., 2015), comparing a case involving a single vendor and multiple retailers operating under VMI with and without shortages, both studies concluded that VMI is more effective, resulting in lower costs. (Taleizadeh et al., 2015) apply VMI system to consideration while taking into account different times of deterioration for both raw materials and completed goods by using the Stackelberg method, with the vendor leading and the retailers following, the aim is to optimize retail prices, raw material replenishment frequency, replenishment cycles, and production rates to optimize overall profit across the supply chain. (Akbari Kaasgari et al., 2017), they considered VMI with case one vendor and multiple retailers for managing perishable items, considering the probability of product perishability and the use of discounts to stimulate demand. (Rabbani et al., 2018) developed an inventory model by considering and applying EOQ that deals with multiple items and multiple limitations for deteriorating items under VMI, considering factors including a shortage allowed with partial backorder, area for storage, duration of time, and budgetary constraints. (Chen, 2018) studied to compare the centralized model with the VMI approach and the decentralized approach with the completely independent replenishment (CIRP) in the decision inventory model for perishable goods, which includes one vendor and multiple retailers, considering pricing, and promotion. The results show that VMI has a consistent economic benefit. However, the impacts are different for the vendor and the retailer. Finding the optimal sales level for deteriorating goods under VMI was studied by (Salehi Amiri et al., 2020) to maximize profits, we aim to fine-tune sales strategies using accurate and meta-heuristic methods, all while considering retailers' requests across various timeframes. The study conducted by (Sojithamporn, 2021) focused on developing an inventory model developed for perishable goods. This model takes into account one supplier and many retailers, shortage allowance with full backlogging, and price-sensitive demand. This model enables well-informed decision-making for all members of the supply chain through offering an understanding of the overall cost and the optimal value of decision-making variables. (Sakrujiratham, 2023) comparative analysis between two models focusing on centralized and decentralized vendor-managed inventory (VMI). Subsequently, a revenue-sharing contract (RSC) was introduced to facilitate coordination within the decentralized VMI supply chain. The findings indicate that the RSC effectively harmonizes the decentralized VMI supply chain, particularly concerning deteriorating items. It markedly enhances supply chain profitability and coordination efficiency while ensuring equitable distribution of total profits among all stakeholders.

In conclusion, based on a thorough literature review of existing research, many research studies have focused on VMI to reduce costs and make the supply chain as efficient as possible in different situations and conditions. These studies also highlight the need to use VMI to maintain product quality and cost savings, especially for perishable goods. However, there are still some research gaps. Therefore, this research will study the case of a single vendor and a single retailer under VMI supply chain for perishable products. The base stock inventory policy and discounts are adjusted based on the product shelf life that will be employed in this model.

2.4 Identify the Gaps in the Research

Many past studies have constantly concentrated on approaches to maximize the total profit in the supply chain under vendor-managed inventory for perishable goods in cases of demand, and the lifetime of the products is constant. In reality, it is normal for products to have various rates of deterioration, which means that the rate at which they deteriorate is not constant and may change over time or under different conditions. Therefore, this research will study VMI in the case of one vendor and one retailer, in which price-sensitive demand and deteriorating rates follow Weibull distribution, a base stock inventory policy is employed, and discounts are adjusted based on product shelf life. A summary of previous research is provided in Table 2.1

Table 2.1

The Summary of Previous Research

Year	Author	SC level	Demand	Deterioration rate	VMI		Shortages		Price strategy	Inventory review policy
					No VMI	VMI	Not allow	Allow		
2008	Bramorski		Constant		•				•	
2010	Darwish & Odah	SV + MRs	Constant			•				
2010	Razmi et al.	SV + SR	Stochastic			•		•		
2011	J. Y. Lee & Ren	SV + SR	Stochastic			•				Periodic review (s, S)
2011	Lan et al.	SV + MRs	Stochastic	Constant		•				
2011	Yu et al.	SV + MRs	Depending on unit price	Constant		•			•	
2012	Yu et al.	SV + MRs	Constant	Constant						
2013	Kazemi & Zhang	SV + SR	Stochastic and price sensitive			•	•			

Year	Author	SC level	Demand	Deterioration rate	VMI		Shortages		Price strategy	Inventory review policy
					No VMI	VMI	Not allow	Allow		
2014	S. Lee & Kim	SV + SR	Constant	Constant		•	•			
2014	Setak & Daneshfar,	SV + SR	Stock dependent	Constant		•		•		Continuous review (Q, R)
2014	Tat et al.	SV + SR	Constant	Constant		•	•	•		Continuous review (Q, R)
2015	Tat et al	SV + SR	Constant	Constant		•	•	•		Continuous review (Q, R)
2015	Taleizadeh et al.	SV + MRs	Constant	Constant		•	•			
2015	Mateen & Chatterjee	SV + MRs	Constant			•				
2017	Verma & Chatterjee	SV + MRs	Constant			•				Continuous review (Q, R)
2017	Akbari Kaasgari et al.	SV + MRs	Uncertain	Dependent on keeping conditions		•			•	

Year	Author	SC level	Demand	Deterioration rate	VMI		Shortages		Price strategy	Inventory review policy
					No VMI	VMI	Not allow	Allow		
2018	Rabbani et al.	SV + SR	Uncertain (fuzzy numbers)	Constant		•		•		Continuous review (Q, R)
2018	Chen	SV + MRs	Price sensitive	Constant		•		•	•	
2020	Salehi Amiri et al.	SV + MRs	Different and definite demands in different periods	Constant		•				
2021	Indrajitsingha et		Selling-Price	No deterioration was observed in [0, m]. while a deterioration with constant, in [m, T]	•			•		Continuous review (Q, R)

Year	Author	SC level	Demand	Deterioration rate	VMI		Shortages		Price strategy	Inventory review policy
					No VMI	VMI	Not allow	Allow		
2021	(Sojithamporn, 2021)	SV + MRs	Price sensitive	Constant		•		•		
2023	(Sakrujiratham, 2023)	SV + SR	Price sensitive	Weibull distribution.		•		•		
2024	De et al.		Price and Stock	No deterioration was observed in $[0, t_1]$. while a deterioration with constant, in $[t_1, t_2]$.	•			•		

CHAPTER 3
MODEL FORMULATION

This chapter will dive into the details of developing a vendor-managed inventory (VMI) system that involves a single vendor and a single retailer for a perishable item in order to maximize supply chain total profit. Under the VMI policy, vendors utilize inventory data from retailers to anticipate customer demand and strategically deliver products based on price-dependent demand. Furthermore, the model also accounts for shortages with full backlog, wherein retailers are willing to wait for fulfillment during periods of scarcity. Product deterioration is followed by a Weibull distribution, with discounts tailored to product shelf life to encourage customer purchases.

3.1 Assumptions

These assumptions will be made before a mathematical model of a finite planning horizon is determined.

1. Consider the following supply chain of one vendor and one retailer, which carries a single perishable finished product.
2. Demand rate is considered as price dependent.
3. Consider adjusting product discount based on their remaining shelf life, both for vendors and retailers.
4. The duration of deterioration rate of the product follows Weibull distribution with an increasing deterioration rate $\theta(t) = \alpha\beta t^{\beta-1}$ where $0 < \alpha < 1$ and $\beta > 1$.
5. The base stock policy will be implemented with the maximum stock level (S) at the start of a review period.
6. Shortage with full backlogged is allowed.

3.2 Notations

In order to develop the model, the subsequent notations are employed.

$D_1(p_1)$: The demand rate in the interval $[0,t_1]$ and the interval $[t_1,t_2]$ which the demand function is $D_1(p_1) = a_1 - b_1 p_1$ and function of the selling price p_1.

t_1: The time duration in a replenishment cycle where the deterioration rate reaches the value where the price will be reduced to encourage the customer to buy.

$D_2(p_2)$: The demand rate in the interval $[t_1, t_2]$ which the demand function is $D_2(p_2) = a_2 - b_2 p_2$ and function of the selling price p_2.

a: Market size factor. a_1 is market size factor of $D_1(p_1)$ and a_2 is market size factor of $D_2(p_2)$.

b: Price sensitivity factor. b_1 is price sensitivity factor of $D_1(p_1)$ and b_2 is price sensitivity of $D_2(p_2)$.

p: The selling price per unit item (in which $p_2 < p_1$).

$I_1(t)$: The inventory level of the retailer at the time t, $0 \leq t \leq t_1$.

t_2: The time duration in a replenishment cycle where the inventory level decreases to zero.

$I_2(t)$: The inventory level of the retailer at the time t, $t_1 \leq t \leq t_2$.

$I_3(t)$: The negative inventory level of the retailer at the time t, $t_2 \leq t \leq T$.

I_0: The maximum inventory level.

I_b: The maximum backlog level.

$\theta(t)$: The rate of deterioration for the finished product.

Q: Order quantity.

A: The purchasing cost per order.

TC: The total cost.

T: The length of a replenishment cycle.

3.3 Mathematical Model of VMI Supply Chain

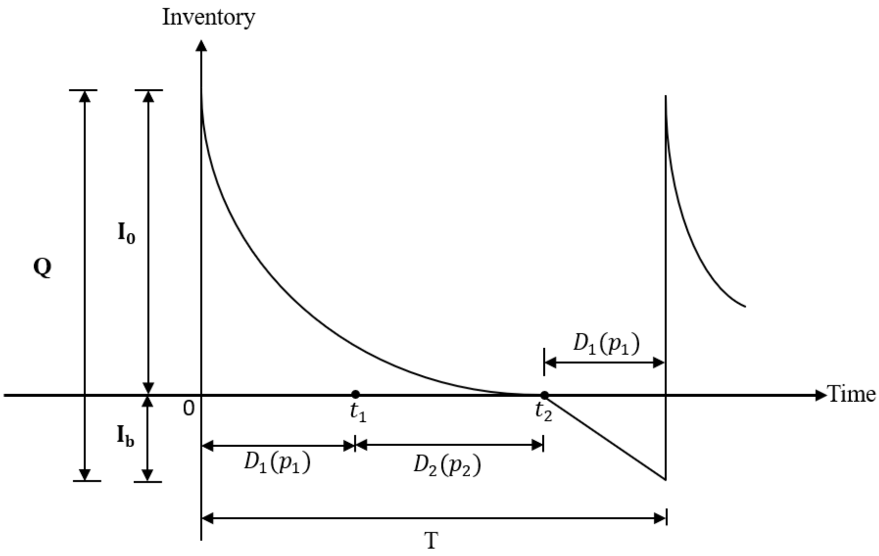
A mathematical model will be constructed to determine the optimal quantity of stock and determine at what point in the lifetime of an item a price-reducing strategy should be employed to encourage customers to buy, hence optimizing the total profit of the supply chain. Given a perishable completed product with demand that is influenced by price, we will assume that the time it takes for the product to deteriorate follows a Weibull distribution. The rate at which the product deteriorates at any given time t is denoted by $\boldsymbol{\theta(t)}$.

$$\theta(t) = \alpha\beta t^{\beta-1}$$

Where $0 < \alpha < 1$ is the scale parameter and $\beta > 0$ is the shape paramrter.

Figure 3.1

Inventory of Retailer for Perishable Products.



According to Figure 3.1, the replenishment duration of the cycle is T. Assuming that the inventory is replenished at time t = 0, the inventory level reaches its maximum level I_0. During time $[0, t_1]$ inventory is depleted due to deterioration and demand of the products and the demand rate is $D_1(p_1)$ in the interval $[0,t_1]$. At the point in time t_1, when the rate of deterioration meets a specific value, there will be a reduction in the price. The demand rate is $D_2(p_2)$ in the interval $[t_1, t_2,]$ (in which $p_2 < p_1$) and the inventory level decreases to zero at time t = t_2. The shortages begin to appear in the fully backlogged period $[t_2, T]$ till it reaches maximum I_b at t = T and the demand rate is $D_1(p_1)$.

It is noted that the replenishment duration of the cycle, T is divided into three sub-intervals: $[0, t_1]$, $[t_1, t_2]$ and $[t_2, T]$. $I_1(t)$ represents the inventory level during $[0, t_1]$, $I_2(t)$ represents the inventory level during $[t_1, t_2]$ and $I_3(t)$ represents the inventory level during $[t_2, T]$.

Related to the inventory level at time t, ($0 \le t \le T$). Difference inventory functions will be derived as follows:

For the inventory function $I_2(t)$ during time $t \in [t_1, t_2]$, we have

$$\frac{dI_2(t)}{dt} + \alpha\beta t^{\beta-1} I_2(t) = -D_2(p_2) \quad (t_1 \le t \le t_2) \tag{1}$$

Multiply $e^{\alpha t^\beta}$ both above equations of sides, we have

$$e^{\alpha t^\beta}\frac{dI_2(t)}{dt} + e^{\alpha t^\beta}\alpha\beta t^{\beta-1} I_2(t) = -D_2(p_2)e^{\alpha t^\beta}$$

$$\frac{d}{dt}\left(e^{\alpha t^\beta} I_2(t)\right) = -D_2(p_2)e^{\alpha t^\beta} \tag{2}$$

From equation (2), we have

$$e^{\alpha t^\beta} I_2(t) = -D_2(p_2)\int_{t_1}^{t} e^{\alpha t^\beta}\, dt + C_2 \tag{3}$$

From the Taylor series expansion

$$e^{\alpha t^\beta} = 1 + \alpha t^\beta + \frac{(\alpha t^\beta)^2}{2!} + \frac{(\alpha t^\beta)^3}{3!} + \cdots \tag{4}$$

Neglecting high-power components of α, we have

$$\int_{t_1}^{t} e^{\alpha t^\beta}\, dt = \int_{t_1}^{t} (1 + \alpha t^\beta) dt$$

$$\int_{t_1}^{t} e^{\alpha t^\beta}\, dt = \left(t + \frac{\alpha t^{\beta+1}}{\beta+1}\right) - \left(t_1 + \frac{\alpha {t_1}^{\beta+1}}{\beta+1}\right) \tag{5}$$

From equation (3) and (5), we have

$$e^{\alpha t^\beta} I_2(t) = -D_2(p_2)\left[\left(t + \frac{\alpha t^{\beta+1}}{\beta+1}\right) - \left(t_1 + \frac{\alpha {t_1}^{\beta+1}}{\beta+1}\right)\right] + C_2 \tag{6}$$

With boundary condition $I_2(t_2) = 0$, we have

$$0 = -D_2(p_2)\left[\left(t_2 + \frac{\alpha t_2{}^{\beta+1}}{\beta+1}\right) - \left(t_1 + \frac{\alpha t_1{}^{\beta+1}}{\beta+1}\right)\right] + C_2$$

$$C_2 = D_2(p_2)\left[\left(t_2 + \frac{\alpha t_2{}^{\beta+1}}{\beta+1}\right) - \left(t_1 + \frac{\alpha t_1{}^{\beta+1}}{\beta+1}\right)\right] \quad (7)$$

So, the solution can be derived as

$$I_2(t) = D_2(p_2)e^{-\alpha t^{\beta}}\left[\left(t_2 + \frac{\alpha t_2{}^{\beta+1}}{\beta+1}\right) - \left(t + \frac{\propto t^{\beta+1}}{\beta+1}\right)\right] \quad (8)$$

For the inventory function $I_3(t)$ during time $t \in [t_2\,, T]$, It is observed that there is a complete backlog in the shortage demand. Hence, we have the different equation as follows:

$$\frac{dI_3(t)}{dt} = -D_1(p_1) \qquad (t_2 \le t \le T) \quad (9)$$

From equation (9), we have

$$I_3(t) = -D_1(p_1)(t - t_2) + C_3 \quad (10)$$

With boundary condition $I_3(\,t_2)$= 0, we have

$$0 = -D_1(p_1)(t_2 - t_2) + C_3$$
$$C_3 = 0 \quad (11)$$

So, the solution can be derived as

$$I_3(t) = -D_1(p_1)(t - t_2) \quad (12)$$

For the inventory function $I_1(t)$ during time $t \in [0, t_1]$, we have

$$\frac{dI_1(t)}{dt} + \alpha\beta t^{\beta-1} I_1(t) = -D_1(p_1) \quad (0 \le t \le t_1) \quad (13)$$

Multiply $e^{\alpha t^{\beta}}$ both above equations of sides, we have

$$e^{\alpha t^{\beta}} \frac{dI_1(t)}{dt} + e^{\alpha t^{\beta}} \alpha\beta t^{\beta-1} I_1(t) = -D_1(p_1) e^{\alpha t^{\beta}}$$

$$\frac{d}{dt}\left(e^{\alpha t^{\beta}} I_1(t)\right) = -D_1(p_1) e^{\alpha t^{\beta}} \quad (14)$$

From equation (14), we have

$$e^{\alpha t^{\beta}} I_1(t) = -D_1(p_1) \int_0^t e^{\alpha t^{\beta}}\, dt + C_1 \quad (15)$$

From the Taylor series expansion

$$e^{\alpha t^{\beta}} = 1 + \alpha t^{\beta} + \frac{(\alpha t^{\beta})^2}{2!} + \frac{(\alpha t^{\beta})^3}{3!} + \cdots \quad (16)$$

Neglecting high-power components of α, we have

$$\int_0^t e^{\alpha t^{\beta}}\, dt = \int_0^t (1 + \alpha t^{\beta}) dt$$

$$\int_0^t e^{\alpha t^{\beta}}\, dt = \left(t + \frac{\alpha t^{\beta+1}}{\beta+1}\right) \quad (17)$$

From equation (15) and (17), we have

$$e^{\alpha t^{\beta}} I_1(t) = -D_1(p_2)\left(t + \frac{\alpha t^{\beta+1}}{\beta+1}\right) + C_1 \quad (19)$$

From (19), the solution can be derived as

$$I_1(t) = -D_1(p_1) e^{-\alpha t^{\beta}}\left(t + \frac{\alpha t^{\beta+1}}{\beta+1}\right) + C_1 \quad (20)$$

At $t = t_1$ $I_1(t_1) = I_2(t_1)$, we have

$$-D_1(p_1)e^{-\alpha t^{\beta}}\left(t_1+\frac{\alpha {t_1}^{\beta+1}}{\beta+1}\right)+\ C_1 = D_2(p_2)e^{-\alpha t^{\beta}}\left[\left(t_2+\frac{\alpha {t_2}^{\beta+1}}{\beta+1}\right)-\left(t_1+\frac{\alpha {t_1}^{\beta+1}}{\beta+1}\right)\right]$$

$$C_1 = \ D_2(p_2)\left[\left(t_2+\frac{\alpha t_2{}^{\beta+1}}{\beta+1}\right)-\left(t_1+\frac{\alpha t_1{}^{\beta+1}}{\beta+1}\right)\right]+D_1(p_1)\left(t_1+\frac{\alpha t_1{}^{\beta+1}}{\beta+1}\right) \qquad (21)$$

So, the solution can be derived as

$$I_1(t) = -D_1(p_1)e^{-\alpha t^{\beta}}\left(t+\frac{\propto t^{\beta+1}}{\beta+1}\right)+D_2(p_2)\left[\left(t_2+\frac{\alpha {t_2}^{\beta+1}}{\beta+1}\right)-\left(t_1+\frac{\alpha {t_1}^{\beta+1}}{\beta+1}\right)\right]$$
$$+\,D_1(p_1)\left(t_1+\frac{\alpha t_1{}^{\beta+1}}{\beta+1}\right) \qquad (22)$$

By setting t equal to 0, we can find that $I_1(0)$ is equal to I_0, which is the maximum inventory level (S) determined from equation (22).

$$I_0 = I_1(0) = \ D_2(p_2)\left[\left(t_2+\frac{\alpha t_2{}^{\beta+1}}{\beta+1}\right)-\left(t_1+\frac{\alpha t_1{}^{\beta+1}}{\beta+1}\right)\right]+D_1(p_1)\left(t_1+\frac{\alpha t_1{}^{\beta+1}}{\beta+1}\right) \qquad (23)$$

The maximum quantity of backlogged demand at time t = T as $I_3(T) = I_b$ is determined using equation (14)

$$I_b = \ -I_3(T) = \ D_1(p_1)\,(\,T-\ t_2) \qquad (24)$$

Therefore, the order quantity can be derived from the inventory level from time t = 0 until t = T, as shown in equation (25)

$$Q = \ I_0 + \ I_b$$

$$Q = \ D_2(p_2)\left[\left(t_2+\frac{\alpha t_2{}^{\beta+1}}{\beta+1}\right)-\left(t_1+\frac{\alpha t_1{}^{\beta+1}}{\beta+1}\right)\right]+D_1(p_1)\left(t_1+\frac{\alpha t_1{}^{\beta+1}}{\beta+1}\right)+$$
$$D_1(p_1)[\,T-t_2] \qquad (25)$$

For the total cost per replenishment cycle, we derive the ordering cost, holding cost, shortage cost, deteriorating cost, and production cost. The total cost comprises the following cost components:

1. Ordering cost = A

2. Holding cost

There is an inventory on hand during the time interval $[0,t_1]$ and $[0,t_2]$. Difference holding cost functions will be derived as follows:

For inventory on hand during the time interval $[0,t_1]$. The equation (22) can approximately compute the holding cost.

$$I_1(t) = -D_1(p_1)e^{-\alpha t^\beta}\left(t + \frac{\propto t^{\beta+1}}{\beta+1}\right) + D_2(p_2)\left[\left(t_2 + \frac{\alpha {t_2}^{\beta+1}}{\beta+1}\right) - \left(t_1 + \frac{\alpha {t_1}^{\beta+1}}{\beta+1}\right)\right] + D_1(p_1)\left(t_1 + \frac{\alpha {t_1}^{\beta+1}}{\beta+1}\right)$$

From the Taylor series expansion

$$e^{-\alpha t^\beta} = 1 - \alpha t^\beta + \frac{(\alpha t^\beta)^2}{2!} - \frac{(\alpha t^\beta)^3}{3!} + \frac{(\alpha t^\beta)^4}{4!} + \cdots \quad (26)$$

Neglecting high-power components of α, we have

$$e^{-\alpha t^\beta} = 1 - \alpha t^\beta \quad (27)$$

So, from the equation (22) we have

$$I_1(t) = -D_1(p_1)(1 - \alpha t^\beta)\left(t + \frac{\propto t^{\beta+1}}{\beta+1}\right) + D_2(p_2)\left[\left(t_2 + \frac{\alpha {t_2}^{\beta+1}}{\beta+1}\right) - \left(t_1 + \frac{\alpha {t_1}^{\beta+1}}{\beta+1}\right)\right] + D_1(p_1)\left(t_1 + \frac{\alpha {t_1}^{\beta+1}}{\beta+1}\right) \quad (28)$$

To approximate the holding cost, we can expand equation (28) and ignore the terms with α^2, we have

$$I_1(t) = D_1(p_1)\left[\alpha t^{\beta+1} - (t + \frac{\alpha t^{\beta+1}}{\beta+1}) + (t_1 + \frac{\alpha {t_1}^{\beta+1}}{\beta+1})\right] + D_2(p_2)\left[\left(t_2 + \frac{\alpha {t_2}^{\beta+1}}{\beta+1}\right) - \left(t_1 + \frac{\alpha {t_1}^{\beta+1}}{\beta+1}\right)\right] \tag{29}$$

From equation (29), holding cost during the time interval $[0,t_1]$ can be determined as:

$$\text{Holding cost } = c_h \int I_1(t)\, dt$$

$$\text{Holding cost } = c_h \int_0^{t_1} \left\{D_1(p_1)\left[\alpha t^{\beta+1} - \left(t + \frac{\alpha t^{\beta+1}}{\beta + 1}\right) + \left(t_1 + \frac{\alpha {t_1}^{\beta+1}}{\beta + 1}\right)\right]\right\} dt$$
$$+ c_h \int_0^{t_1} \left\{D_2(p_2)\left[\left(t_2 + \frac{\alpha {t_2}^{\beta+1}}{\beta+1}\right) - \left(t_1 + \frac{\alpha {t_1}^{\beta+1}}{\beta+1}\right)\right]\right\} dt \tag{30}$$

So, the holding cost during the time interval $[0,t_1]$ can be derived as

$$\text{Holding cost } = c_h \left\{c_h D_1(p_1)\left[\frac{\alpha {t_1}^{\beta+2}}{\beta+2} + \frac{1}{2}{t_1}^2 + \frac{\alpha {t_1}^{\beta+2}}{\beta+1} - \frac{\alpha {t_1}^{\beta+2}}{(\beta+1)(\beta+2)}\right] + D_2(p_2)\left[t_1 t_2 + \frac{\alpha {t_2}^{\beta+1}}{\beta+1} t_1 - {t_1}^2 - \frac{\alpha {t_1}^{\beta+1}}{\beta+1} t_1\right]\right\} \tag{31}$$

For inventory on hand during the time interval $[t_1,t_2]$. The equation (8) can approximately compute the holding cost.

$$I_2(t) = D_2(p_2) e^{-\alpha t^{\beta}}\left[\left(t_2 + \frac{\alpha {t_2}^{\beta+1}}{\beta + 1}\right) - \left(t + \frac{\propto t^{\beta+1}}{\beta + 1}\right)\right]$$

From the Taylor series expansion

$$e^{-\alpha t^{\beta}} = 1 - \alpha t^{\beta} + \frac{(\alpha t^{\beta})^2}{2!} - \frac{(\alpha t^{\beta})^3}{3!} + \frac{(\alpha t^{\beta})^4}{4!} + \cdots \tag{32}$$

Neglecting high-power components of α, we have

$$e^{-\alpha t^{\beta}} = 1 - \alpha t^{\beta} \tag{33}$$

So, from the equation (8) we have

$$I_2(t) = D_2(p_2)(1 - \alpha t^{\beta})\left[\left(t_2 + \frac{\alpha t_2{}^{\beta+1}}{\beta+1}\right) - \left(t + \frac{\propto t^{\beta+1}}{\beta+1}\right)\right] \tag{34}$$

To approximate the holding cost, we can expand equation (34) and ignore the terms with α^2, we have

$$I_2(t) = D_2(p_2)\left(t_2 + \frac{\alpha t_2{}^{\beta+1}}{\beta+1} - t - \frac{\alpha t^{\beta+1}}{\beta+1} - \alpha t_2 t^{\beta} + \alpha t^{\beta+1}\right) \tag{35}$$

From equation (35), holding cost during the time interval $[t_1,t_2]$ can be determined as:

$$\text{Holding cost} = c_h \int I_2(t)\,dt$$

$$\text{Holding cost} = c_h \int_{t_1}^{t_2}\left\{D_2(p_2)\left(t_2 + \frac{\alpha t_2{}^{\beta+1}}{\beta+1} - t - \frac{\alpha t^{\beta+1}}{\beta+1} - \alpha t_2 t^{\beta} + \alpha t^{\beta+1}\right)\right\}dt \tag{36}$$

So, the holding cost during the time interval $[t_1,t_2]$ can be derived as

$$\text{Holding cost} = c_h D_2(p_2)\left[\frac{1}{2}t_2{}^2 - t_1 t_2 + \frac{\alpha t_2{}^{\beta+1}}{\beta+1}(t_2 - t_1) + \frac{1}{2}t_1{}^2 + \frac{\alpha}{(\beta+1)(\beta+2)}(t_1{}^{\beta+2} - t_2{}^{\beta+2}) + \frac{\alpha t_2}{\beta+1}(t_1{}^{\beta+1} - t_2{}^{\beta+1}) + \frac{\alpha}{\beta+2}(t_2{}^{\beta+2} - t_1{}^{\beta+2})\right] \tag{37}$$

From equation (31) and (37) the total holding cost can be derived as

$$\text{Total Holding cost} = c_h\left\{D_1(p_1)\left[\frac{\alpha t_1{}^{\beta+2}}{\beta+2} + \frac{1}{2}t_1{}^2 + \frac{\alpha t_1{}^{\beta+2}}{\beta+1} - \frac{\alpha t_1{}^{\beta+2}}{(\beta+1)(\beta+2)}\right] + D_2(p_2)\left[t_1 t_2 + \frac{\alpha t_2{}^{\beta+1}}{\beta+1}t_1 - t_1{}^2 - \frac{\alpha t_1{}^{\beta+1}}{\beta+1}t_1\right] + D_2(p_2)\left[\frac{1}{2}t_2{}^2 - t_2 + \frac{\alpha t_2{}^{\beta+1}}{\beta+1}(t_2 - t_1) + \frac{1}{2}t_1{}^2 + \frac{\alpha}{(\beta+1)(\beta+2)}(t_1{}^{\beta+2} - t_2{}^{\beta+2}) + \frac{\alpha t_2}{\beta+1}(t_1{}^{\beta+1} - t_2{}^{\beta+1}) + \frac{\alpha}{\beta+2}(t_2{}^{\beta+2} - t_1{}^{\beta+2})\right]\right\} \tag{38}$$

3. Shortage cost

There is a shortage due to stock out during the interval $[t_2,T]$. We have

$$\text{Shortage cost} = -c_s \int_{t_2}^{T} I_3(t)\, dt$$

$$\text{Shortage cost} = c_s \int_{t_2}^{T} D_1(p_1)(t - t_2)\, dt \tag{39}$$

From equation (39) the total shortage cost can be derived as

$$\text{Shortage cost} = \frac{1}{2} c_s D_1(p_1)[T^2 - 2Tt_2 + t_2{}^2] \tag{40}$$

4. Deterioration cost

Deterioration occurs in the intervals $[0, t_1]$ and $[t_1, t_2]$. The total cost of deterioration can be calculated as follows:

$$\text{Deterioration cost} = c_d \left[I_0 - \int_0^{t_1} D_1(p_1)\, dt - \int_{t_1}^{t_2} D_2(p_2)\, dt \right]$$

$$\text{Deterioration cost} = c_d \left[\left(D_2(p_2) \left[\left(t_2 + \frac{\alpha t_2{}^{\beta+1}}{\beta+1} \right) - \left(t_1 + \frac{\alpha t_1{}^{\beta+1}}{\beta+1} \right) \right] + D_1(p_1) \left(t_1 + \frac{\alpha t_1{}^{\beta+1}}{\beta+1} \right) - \int_0^{t_1} D_1(p_1)\, dt - \int_{t_1}^{t_2} D_2(p_2)\, dt \right) \right] \tag{41}$$

From equation (41) the total deterioration cost can be derived as

$$\text{Deterioration cost} = c_d \left[D_1(p_1) \left(\frac{\alpha t_1{}^{\beta+1}}{\beta+1} \right) + D_2(p_2) \left(\frac{\alpha t_2{}^{\beta+1}}{\beta+1} - \frac{\alpha t_1{}^{\beta+1}}{\beta+1} \right) \right] \tag{42}$$

5. Production cost

The total production cost can be derived as follows:

$$\text{Production cost} = cQ$$

$$\text{Production cost} = c\left[D_2(p_2)\left[\left(t_2 + \frac{\alpha t_2{}^{\beta+1}}{\beta+1}\right) - \left(t_1 + \frac{\alpha t_1{}^{\beta+1}}{\beta+1}\right)\right] + D_1(p_1)\left(t_1 + \frac{\alpha t_1{}^{\beta+1}}{\beta+1}\right) + D_1(p_1)[T - t_2]\right] \quad (43)$$

6. Total cost

The total cost per unit of time can be calculated using the following method:

$$TC_{VMI} = TC_{vendor}$$
$$= \frac{1}{T}\Bigg(A + c_h\Bigg\{D_1(p_1)\left[\frac{\alpha t_1{}^{\beta+2}}{\beta+2} + \frac{1}{2}t_1{}^2 + \frac{\alpha t_1{}^{\beta+2}}{\beta+1} - \frac{\alpha t_1{}^{\beta+2}}{(\beta+1)(\beta+2)}\right] + D_2(p_2)\left[t_1 t_2 + \frac{\alpha t_2{}^{\beta+1}}{\beta+1}t_1 - t_1{}^2 - \frac{\alpha t_1{}^{\beta+1}}{\beta+1}t_1\right] + D_2(p_2)\left[\frac{1}{2}t_2{}^2 - t_2 + \frac{\alpha t_2{}^{\beta+1}}{\beta+1}(t_2 - t_1) + \frac{1}{2}t_1{}^2 + \frac{\alpha}{(\beta+1)(\beta+2)}(t_1{}^{\beta+2} - t_2{}^{\beta+2}) + \frac{\alpha t_2}{\beta+1}(t_1{}^{\beta+1} - t_2{}^{\beta+1}) + \frac{\alpha}{\beta+2}(t_2{}^{\beta+2} - t_1{}^{\beta+2})\right]\Bigg\} + \frac{1}{2}c_s D_1(p_1)[T^2 - 2Tt_2 + t_2{}^2] + c_d\left[D_1(p_1)\left(\frac{\alpha t_1{}^{\beta+1}}{\beta+1}\right) + D_2(p_2)\left(\frac{\alpha t_2{}^{\beta+1}}{\beta+1} - \frac{\alpha t_1{}^{\beta+1}}{\beta+1}\right)\right] + c\left[D_2(p_2)\left[\left(t_2 + \frac{\alpha t_2{}^{\beta+1}}{\beta+1}\right) - \left(t_1 + \frac{\alpha t_1{}^{\beta+1}}{\beta+1}\right)\right] + D_1(p_1)\left(t_1 + \frac{\alpha t_1{}^{\beta+1}}{\beta+1}\right) + D_1(p_1)[T - t_2]\right]\Bigg) \quad (44)$$

7. Total revenue

the total revenue per time unit can be determined as follows:

$$\text{Total revenue} = \frac{1}{T}\left[\int_0^{t_1} p_1 D_1(p_1)dt + \int_{t_1}^{t_2} p_2 D_2(p_2)\,dt - \int_{t_2}^{T} p_1 I_3(t)dt\right] \quad (45)$$

From equation (45) the total revenue can be derived as

$$\text{Total revenue} = \frac{1}{T}\{p_1 D_1(p_1)[T + t_1 - t_2] + p_2 D_2(p_2)[t_2 - t_1]\} \quad (46)$$

8. Total profit (TP)

the total profit per time unit can be determined as follows:

$$\text{Total profit} = \text{toal revenue} - \text{total cost} \quad (47)$$

From equation (47) the total profit can be derived as

Total profit

$$= \frac{1}{T}\{ p_1 D_1(p_1)[T + t_1 - t_2] + p_2 D_2(p_2)[t_2 - t_1]\} - \frac{1}{T}\Big\{A + c_h \Big\{D_1(p_1)\Big[\frac{\alpha {t_1}^{\beta+2}}{\beta+2} + \frac{1}{2}{t_1}^2 + \frac{\alpha {t_1}^{\beta+2}}{\beta+1} - \frac{\alpha {t_1}^{\beta+2}}{(\beta+1)(\beta+2)}\Big] + D_2(p_2)\Big[t_1 t_2 + \frac{\alpha {t_2}^{\beta+1}}{\beta+1} t_1 - {t_1}^2 - \frac{\alpha {t_1}^{\beta+1}}{\beta+1} t_1\Big] + D_2(p_2)\Big[\frac{1}{2}{t_2}^2 - t_2 + \frac{\alpha {t_2}^{\beta+1}}{\beta+1}(t_2 - t_1) + \frac{1}{2}{t_1}^2 + \frac{\alpha}{(\beta+1)(\beta+2)}({t_1}^{\beta+2} - {t_2}^{\beta+2}) + \frac{\alpha t_2}{\beta+1}({t_1}^{\beta+1} - {t_2}^{\beta+1}) + \frac{\alpha}{\beta+2}({t_2}^{\beta+2} - {t_1}^{\beta+2})\Big]\Big\} + \frac{1}{2}c_s D_1(p_1)[T^2 - 2Tt_2 + {t_2}^2] + c_d\Big[D_1(p_1)\Big(\frac{\alpha {t_1}^{\beta+1}}{\beta+1}\Big) + D_2(p_2)\Big(\frac{\alpha {t_2}^{\beta+1}}{\beta+1} - \frac{\alpha {t_1}^{\beta+1}}{\beta+1}\Big)\Big] + c\Big[D_2(p_2)\Big[\Big(t_2 + \frac{\alpha {t_2}^{\beta+1}}{\beta+1}\Big) - \Big(t_1 + \frac{\alpha {t_1}^{\beta+1}}{\beta+1}\Big)\Big] + D_1(p_1)\Big(t_1 + \frac{\alpha {t_1}^{\beta+1}}{\beta+1}\Big) + D_1(p_1)[T - t_2]\Big]\Big\} \tag{48}$$

CHAPTER 4

NUMERICAL EXPERIMENTS AND SENSITIVITY ANALYSES

In this chapter, to ensure the validation of the mathematical models formulated in Chapter 3, we will conduct numerical experiments. Additionally, sensitivity analyses will be performed to assess the impact of various input parameters on decision variables and optimal solutions. The inclusion of relevant data will enable a comprehensive investigation into these influences.

4.1 Numerical Experiment for the Base Case.

In order to demonstrate the feasibility of the proposed methodology, numerical experiments are conducted using MATLAB. The input parameters that manage the vendor-managed inventory (VMI) in the case of one-vendor and one-retailer systems are provided in Table 4.1.

Table 4.1

The Values of Input Parameters in the Base Case.

Parameters	Value
A	12,000
c_h	100
c_s	400
c_d	350
c	340
a_1	3,000
b_1	2
p_1	870
a_2	3,000
b_2	2
p_2	860
β	0.9
α	3

The input parameters listed in Table 4.1 are utilized for numerical analysis in the base case. With these input parameters, the optimal solution is determined and presented in Table 4.2.

Table 4.2

The Optimal Solution for the Base Case

t_1	t_2	T	Q	Total profit
0.2401	0.321	0.4243	537	615,730

The subsequent sections will delve into the discussion of how alterations in the input parameters impact the optimal solutions. Sensitivity studies will be carried out by changing certain input parameters while keeping the remaining parameters unchanged.

4.2 Sensitivity Analysis.

4.2.1 The Effect of Ordering Cost (A)

Adjustments were implemented for the ordering cost within this subsection. Table 4.3 contains the detailed results.

The results from Table 4.3 suggest that changes in ordering cost have a minimal impact on the specific timing of price adjustments (t_1) and total profit. However, these changes significantly affect the point in time when inventory reaches zero (t_2), the length of the replenishment cycle (T), and the quantity of the order (Q).

When ordering costs increase, it leads to longer replenishment cycles and larger quantities ordered. This trend is logical because higher ordering cost often prompt adjustments, such as ordering larger quantities to increase the duration of the point in time when the inventory level reaches zero and the replenishment cycle of a product. The adjustments may result in higher expenses for inventory holding and dealing with deterioration. However, the increase in ordering costs will result in a slight decline in total profit

Table 4.3

Analysis of Sensitivity with Regard to Ordering Cost (A)

Ordering Cost (A)										
Value	$\mathbf{t_1}$	**% $\mathbf{t_1}$ change**	$\mathbf{t_2}$	**% $\mathbf{t_2}$ change**	**T**	**% T change**	**Q**	**% Q change**	**Total profit**	**%Total profit change**
6,000	0.2408	0.29%	0.2413	-24.83%	0.3124	-26.37%	393	-26.82%	631,930	2.63%
9,000	0.2404	0.12%	0.2872	-10.53%	0.3757	-11.45%	474	-11.73%	623,230	1.22%
15,000	0.2398	-0.12%	0.3477	8.32%	0.4644	9.45%	588	9.50%	608,990	-1.09%
18,000	0.2396	-0.21%	0.3697	15.17%	0.4988	17.56%	632	17.69%	602,760	-2.11%

Table 4.4

Analysis of Sensitivity with regard to Holding Cost (c_h)

Holding Cost (c_h)										
Value	$\mathbf{t_1}$	**% $\mathbf{t_1}$ change**	$\mathbf{t_2}$	**% $\mathbf{t_2}$ change**	**T**	**% T change**	**Q**	**% Q change**	**Total profit**	**%Total profit change**
50	0.2355	-1.92%	0.3793	18.16%	0.4652	9.64%	590	9.87%	624,510	1.43%
75	0.2377	-1.00%	0.3493	8.82%	0.4445	4.76%	563	4.84%	619,850	0.67%
125	0.2424	0.96%	0.2950	-8.10%	0.4055	-4.43%	512	-4.66%	612,100	-0.59%
150	0.2448	1.96%	0.2716	-15.39%	0.3884	-8.46%	490	-8.75%	608,900	-1.11%

4.2.2 *The Effect of Holding Cost (c_h)*

Adjustments were implemented for the holding cost within this subsection. Table 4.4 contains the detailed results.

The findings from Table 4.4 indicate that increases in inventory holding costs have little effect on the specific timing of price adjustments, and total profit. However, they do significantly impact the point in time when inventory reaches zero, the replenishment cycle, and the quantity of the order.

Shorter replenishment cycles and reduced order quantities are typically implemented by retailers due to rising holding costs. This is because they aim to minimize inventory to reduce holding cost, leading to more frequent restocks from the vendor. Consequently, at the point in time when the inventory level reaches zero, the replenishment cycle and order quantities decrease. The lower inventory levels may result in shortages. However, when holding cost increases that leads to the total profit will slightly decrease.

4.2.3 *The Effect of Shortage Cost (c_s)*

Adjustments were implemented for the shortage cost within this subsection. Table 4.5 contains the detailed results.

The numerical outcomes from Table 4.9 suggest that a higher shortage cost has little effect on the point in time when the inventory level reaches zero but does not significantly affect the specific timing of price adjustments and total profit. However, it does have a significant impact on the replenishment cycle and quantity.

When shortage cost rises, it results in shorter replenishment cycles and smaller order quantities. This trend is logical because retailers aim to minimize the risk of inventory shortages by placing more frequent but smaller orders. While these adjustments can reduce expenses related to inventory holding and deterioration costs, they may also lead to increased ordering costs. However, when shortage cost increases that leads to the total profit will slightly decrease.

Table 4.5

Analysis of Sensitivity with egard to Shortage Cost (c_s)

Shortage Cost (c_s)										
Value	t_1	**% t_1 change**	t_2	**% t_2 change**	**T**	**% T change**	**Q**	**% Q change**	**Total profit**	**%Total profit change**
200	0.2403	0.08%	0.2975	-7.32%	0.4830	13.83%	610	13.59%	621,050	0.86%
300	0.2401	0%	0.3125	-2.65%	0.4450	4.88%	562	4.66%	617,720	0.32%
500	0.2400	-0.04%	0.3265	1.71%	0.4112	-3.09%	520	-3.17%	614,410	-0.21%
600	0.2400	-0.04%	0.3303	2.90%	0.4022	-5.21%	509	-5.21%	613,470	-0.37%

Table 4.6

Analysis of Sensitivity with Regard to Deterioration Cost (c_d)

Deterioration Cost (c_d)										
Value	t_1	**% t_1 change**	t_2	**% t_2 change**	**T**	**% T change**	**Q**	**% Q change**	**Total profit**	**%Total profit change**
175	0.2397	-0.17%	0.3294	2.62%	0.4320	1.81%	547	1.86%	616,080	0.06%
262.5	0.2399	-0.08%	0.3250	1.25%	0.4280	0.87%	541	0.74%	615,900	0.03%
437.5	0.2402	0.04%	0.3174	-1.12%	0.4210	-0.78%	532	-0.93%	615,580	-0.02%
525	0.2404	0.12%	0.3140	-2.18%	0.4179	-1.51%	528	-1.68%	615,430	-0.05%

4.2.4 *The Effect of Deterioration Cost (c_d)*

Adjustments were implemented for the deterioration cost within this subsection. Table 4.6 contains the detailed results.

The numerical outcomes from Table 4.9 suggest that changes in deterioration costs have no significant impact on optimal solutions and total profit.

When the deteriorating cost increases, retailers typically reduce the order quantity of goods and increase the frequency of ordering to mitigate the risk of higher deterioration costs. While these adjustments can decrease expenses related to inventory holding and deterioration costs, they may also result in increased ordering costs. However, the effect of change in deterioration cost on total profit is insignificant because the adjustments in quantity and replenishment cycle length are small.

4.2.5 *The Effect of Unit Production Cost (c)*

Adjustments were implemented for the unit production cost within this subsection. Table 4.7 contains the detailed results.

The numerical outcomes from Table 4.9 suggest that a higher unit production cost significantly impacts the specific timing of price adjustments and total profit. However, it does not significantly impact the point in time when inventory reaches zero, the length of the replenishment cycle, and the quantity of the order.

This trend is logical because when production cost rise, the total cost will increase. causing total profits to decrease. However, the replenishment cycle of the product and the amount quantity are influenced by demand parameters rather than the unit production cost. For instance, if there is high demand for a product, the retailer may need to replenish their inventory more frequently to meet customer needs, regardless of changes in production cost.

Table 4.7

Analysis of Sensitivity with Regard to Unit Production Cost (c)

Unit Production Cost (c)										
Value	$\mathbf{t_1}$	**% $\mathbf{t_1}$ change**	$\mathbf{t_2}$	**% $\mathbf{t_2}$ change**	**T**	**% T change**	**Q**	**% Q change**	**Total profit**	**%Total profit change**
238	0.1720	-28.36%	0.3321	3.46%	0.4338	2.24%	550	2.42%	745,040	21.00%
306	0.2281	-5.00%	0.3247	1.15%	0.4276	0.78%	541	0.74%	658,810	7.00%
374	0.2486	3.54%	0.3174	-1.12%	0.4211	-0.75%	532	-0.93%	572,680	-6.99%
442	0.2602	8.37%	0.3101	-3.40%	0.4146	-2.29%	524	-2.42%	486,610	-20.97%

Table 4.8

Analysis of Sensitivity with Regard to Market Sizeable Factor (a_1 and a_2)

Market Sizeable Factor ($\mathbf{a_1}$ and $\mathbf{a_2}$)										
Value	$\mathbf{t_1}$	**% $\mathbf{t_1}$ change**	$\mathbf{t_2}$	**% $\mathbf{t_2}$ change**	**T**	**% T change**	**Q**	**% Q change**	**Total profit**	**%Total profit change**
2,800	0.1674	-30.28%	0.3465	7.94%	0.4589	8.15%	582	8.38%	514,120	-16.50%
2,900	0.2164	-9.87%	0.3334	3.86%	0.4411	3.96%	559	4.10%	564,840	-8.26%
3,100	0.2556	6.46%	0.3093	-3.64%	0.4086	-3.70%	516	-3.91%	666,760	8.29%
3,200	0.2670	11.20%	0.2982	-7.10%	0.3939	-7.16%	497	-7.45%	717,910	16.59%

4.2.6 *The Effect of Market Sizeable Factor (a_1 and a_2)*

Adjustments were implemented for the market sizeable factor (a_1 and a_2) within this subsection. Table 4.8 contains the detailed results.

The numerical outcomes from Table 4.8 suggest that a higher market size factor significantly impacts the specific timing of price adjustments and total profit. However, they have little effect on the point at which inventory is depleted, the replenishment cycle, and the quantity ordered.

The market size factor, a significant parameter in the price-dependent demand function: $D_i = a_i - b_i p_i$ (where i = 1,2), holds a pivotal role in demand. An increase in the market size factor correlates with a heightened demand for the product. Consequently, this surge in demand extends the duration during which product discounts are in effect. This prolonged duration suggests that immediate reductions in product prices are unnecessary, thereby resulting in amplified total profits. However, the extended discounted price duration may lead retailers to adjust their ordering practices by ordering smaller quantities.

4.2.7 *The Effect of the Price Sensitiveness Factor (b_1 and b_2)*

Adjustments were implemented for the price-sensitiveness factor (b_1 and b_2) within this subsection. Table 4.9 contains the detailed results.

The numerical outcomes from Table 4.9 suggest that a higher price-sensitivity factor significantly impacts the specific timing of price adjustments and total profit. However, it has minimal impact on the depletion point of inventory, the replenishment cycle, and the ordered quantity.

According to the price-dependent demand function $D_i = a_i - b_i p_i$ (where i = 1,2), an increase in price sensitivity leads to a decrease in demand. Consequently, the duration of the replenishment cycle, during which the deterioration rate reaches the threshold for price reduction to stimulate customer purchases, decreases (where $p_1 > p_2$). This reduction in the length of the cycle results in decreased total profits, despite an increase in the quantity of orders placed.

Table 4.9

Analysis of Sensitivity with Regard to Price Sensitiveness Factor (b_1 and b_2)

Price Sensitiveness Factor (b_1 *and* b_2)										
Value	$\mathbf{t_1}$	**% $\mathbf{t_1}$ change**	$\mathbf{t_2}$	**% $\mathbf{t_2}$ change**	**T**	**% T change**	**Q**	**% Q change**	**Total profit**	**%Total profit change**
1.84	0.2725	13.49%	0.3020	-5.92%	0.4000	-5.73%	505	-5.96%	686,730	11.53%
1.92	0.2587	7.75%	0.3114	-2.99%	0.4120	-2.90%	520	-3.17%	651,180	5.76%
2.08	0.2115	-11.91%	0.3308	3.05%	0.4369	2.97%	553	2.98%	580,400	-5.74%
2.16	0.1403	-41.57%	0.3406	6.11%	0.4494	5.92%	571	6.33%	545,220	-11.45%

Table 4.10

Analysis of Sensitivity with Regard to Selling Price (p_1)

Selling Price (p_1)										
Value	$\mathbf{t_1}$	**% $\mathbf{t_1}$ change**	$\mathbf{t_2}$	**% $\mathbf{t_2}$ change**	**T**	**% T change**	**Q**	**% Q change**	**Total profit**	**%Total profit change**
860	0.0111	-95.38%	0.3281	2.21%	0.4288	1.06%	547	1.86%	614,040	-0.27%
865	0.2019	-15.91%	0.3249	1.21%	0.4271	0.66%	541	0.74%	614,820	-0.15%
875	0.2650	10.37%	0.3170	-1.25%	0.4212	-0.73%	532	-0.93%	616,670	0.15%
880	0.2837	18.16%	0.3130	-2.49%	0.4179	-1.51%	527	-1.86%	617,580	0.30%

4.2.8 The Effect of Selling Price (p_1)

Adjustments were implemented to the selling price (p_1) within this subsection. Table 4.10 contains the detailed results.

The numerical outcomes from Table 4.10 suggest that higher selling prices (p_1) significantly impact the specific timing of price adjustments. However, it has little effect on the point in time when inventory is depleted, the duration of the replenishment cycle, the quantity of the order, and total profit.

When the selling price (p_1) is lower than the reference price (p_2), the duration of the specific timing of price adjustments decreases This decrease in the length of the cycle leads to an increase in the quantity of orders placed. Conversely, when the selling price (p_1) is higher than the reference price (p_2), the duration of the specific timing of price adjustments increases. An extension of the duration of the cycle leads to a reduction in the quantity of items ordered. However, it's noteworthy that the total profit experiences a slight increase due to the rise in the selling price (p_1).

4.2.9 The Effect of Selling Price (p_2)

Adjustments were implemented to the selling price (p_2) within this subsection. Table 4.11 contains the detailed results.

The numerical outcomes from Table 4.11 suggest that higher selling prices (p_2), significantly impact the specific timing of price adjustments. However, it has little effect on the point in time when inventory reaches zero, the length of replenishment cycle, quantity of the order, and total profit.

When the selling price (p_2) is lower than the reference price (p_1), the duration of the specific timing of price adjustments increases This increase in the length of the cycle leads to a reduction in the quantity of orders placed. Conversely, when the selling price (p_2) is higher than the reference price (p_1), the duration of the specific timing of price adjustments decreases. This reduction in cycle length increases the order quantity. However, the total profit experiences a slight increase due to the rise in the selling price (p_2).

Table 4.11

Analysis of Sensitivity with regard to Selling Price (p_2)

Selling Price (p_2)										
Value	$\mathbf{t_1}$	**%** $\mathbf{t_1}$ **change**	$\mathbf{t_2}$	**%** $\mathbf{t_2}$ **change**	**T**	**% T change**	**Q**	**% Q change**	**Total profit**	**%Total profit change**
850	0.2935	22.24%	0.3081	-4.02%	0.4121	-2.88%	520	-3.17%	615,370	-0.06%
855	0.2696	12.29%	0.3150	-1.87%	0.4188	-1.30%	529	-1.49%	615,500	-0.04%
865	0.1983	-17.41%	0.3260	1.56%	0.4286	1.01%	543	1.12%	616,090	0.06%
870	0.0058	-97.58%	0.3299	2.77%	0.4315	1.70%	551	2.61%	616,610	0.14%

CHAPTER 5
CONCLUSIONS AND RECOMMENDATIONS

5.1 Conclusions

Vendor-managed inventory is an effective strategy in the supply chain for helping to manage inventory between vendor and retailer. This approach allows for better coordination among supply chain members and can reduce response times to customers' demands. In this research, the VMI of a two-tier supply chain is employed to manage the inventory of one deteriorating finished item at one vendor and one retailer. Under the assumption that the deterioration rate of the finished product follows a Weibull distribution, with an increasing deterioration rate when there is a price-sensitive demand, the inventory policy adopted will be the base stock policy, and a maximum stock level (S) will be applied at the start of the period for review, the shortage is permitted, and all backlogged orders are fully fulfilled. We developed a vendor-managed inventory (VMI) inventory model to maximize total profit across the supply chain. This model aids in determining optimal solutions for various parameters, considering the time at which price reduction decisions are made to encourage customer purchases, the inventory depletion to zero and replenishing cycle duration, and the quantity replenished. Costs considered in the analysis encompass ordering costs, unit deteriorating costs, unit shortage costs per time unit, holding costs per unit per time unit, and unit production costs per time unit. The results of the sensitivity analyses have identified the following:

- The point in time where the deterioration rate reaches, the value at which the price will be reduced to encourage the customer to buy increases with higher values of c_h, c_d, c, a_1, a_2 and p_1. Conversely, this point in time decreases when values of A, c_s, b_1, b_2 and p_2 increase.
- The points in time when the inventory reaches zero (t_2) increases with higher values of A, c_s, b_1, b_2 and p_2. Conversely, this point in time decreases when c_h, c_d, c, a_1, a_2 and p_1 increase.
- The impacts on the replenishment cycle (T) are equivalent to the point in time when inventory reaches zero (t_2). Nevertheless, the effect of unit shortage cost per time unit on T differs from its impact on t_2. In particular, an increase in c_s leads to a decrease in the replenishment cycle duration because retailers aim to

minimize the risk of inventory shortages by placing more frequent but smaller orders.

- Total profit increases with increments in both the market size factor and selling price parameters due to the price-dependent demand function $D_i = a_i - b_i p_i$ (where i = 1,2), a rise in the market size factor correlates with heightened demand for the product, while the impact of increasing the selling price on profit augmentation is relatively modest.

The research makes a valuable contribution by elucidating how individual unit costs impact optimal decision variables, to maximize total profit in the supply chain system. This helps in effectively managing deteriorating products, thereby enabling members of the supply chain to make informed decisions.

5.2 Recommendations

To explore further research directions, we can consider various extensions, such as:

- Investigating VMI models to diverse scenarios, including multiple retailers or vendors.
- Exploring alternative probability distribution functions to model deterioration rates.
- Examining various demand functions, such as stochastic demand or time-dependent demand, to enhance model accuracy.
- Investigating different types of shortages, including backorders, complete backorders, or lost sales.
- It would be helpful to take into account other limitations, such as financial constraints and the spatial boundaries of warehouse facilities. Additionally, it would be worth exploring the feasibility of inventory management strategies, such as Economic Order Quantity (EOQ).

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APPENDIX
COMPUTER PROGRAM (MATLAB)

```
function TP = Cal_New (x)
    A = 12000;
    ch = 100;
    cs = 400;
    cd = 350;
    c = 340;
    a1 = 3000;
    b1 = 2;
    p1 = 870;
    a2 = 3000;
    b2 = 2;
    p2 = 860;
    alpha = 0.9;
    beta = 4;
    D_1 = a1 - b1*p1;
    beta1 = beta + 1;
    D_2 = a2 - b2*p2;
    beta2 = beta + 2;
    x_1b1 = x(1)^beta1;
    beta1_2 = beta1*beta2;
    x_1b2  = x(1)^beta2;
    x_2b1 = x(2)^beta1;
    x_2b2  = x(2)^beta2;
    HC_1 = ch*D_1*(((alpha*x_1b2)/beta2)+(0.5*x(1)^2)+((alpha*x_1b2)/beta1)-
((alpha*x_1b2)/beta1_2));
    HC_12 = ch*D_2*((x(1)*x(2))+((alpha*x_2b1*x(1)/beta1))-((x(1)^2))-
((alpha*x_1b2)/beta1));
    HC_2 = ch*D_2*((0.5*x(2)^2)-(x(1)*x(2))+(((alpha*x_2b1)/beta1)*(x(2)-
x(1)))+(0.5*x(1)^2));
```

```
    HC_21 = ch*D_2*(((alpha)/beta1_2)*((x_1b2)-
(x_2b2))+((alpha*x(2))/beta1)*((x_1b1)-(x_2b1))+((alpha/beta2)*((x_2b2)-
(x_1b2))));
    HC_all = HC_1+HC_12+HC_2+HC_21;
    SC = 0.5*cs*D_1*((x(3)^2)-(2*x(3)*x(2))+((x(2)^2)));
    DC = cd*D_1*((alpha*x_1b1)/beta1)+cd*D_2*(((alpha*x_2b1)/beta1)-
((alpha*x_1b1)/beta1));
    PC_1 = c*D_2*((x(2))+((alpha*x_2b1)/beta1)-((x(1))-((alpha*x_1b1)/beta1)));
    PC_2 = c*D_1*((x(1))+((alpha*x_1b1)/beta1)+x(3)-x(2));
    PC_all = PC_1+PC_2;
    TC_all = (A+HC_all+SC+DC+PC_all)/x(3);
    RV_V1 = (p1*D_1*(x(3)+x(1)-x(2)))/(x(3));
    RV_V2 = (p2*D_2*(x(2)-x(1)))/(x(3));
    RV_all = RV_V1+RV_V2;
    TP = RV_all-TC_all;
End

% Define the modified objective function to maximize
     fun = @(x) -Cal_New(x); %profit
     x0 = [1; 1; 1]; % initial
     % Optimization options
     options = optimoptions('fminunc', 'Display', 'iter', 'Algorithm', 'quasi-newton');
     % Perform optimization
    [x_optimal, negative_profit] = fminunc(fun, x0, options);
    disp('Optimal solution (x):'); % show the optimal solution
    disp(x_optimal);
    % Calculate and display the total profit
    total_profit = -negative_profit;
    disp('Total profit:');
    disp(total_profit);
```

```
% For quantity
function Q = Quantity()
    a1 = 3000;
    b1 = 2;
    p1 = 870;
    a2 = 3000;
    b2 = 2;
    p2 = 860;
    alpha = 0.9;
    beta = 4;
    D_1 = a1 - b1*p1;
    D_2 = a2 - b2*p2;
    beta1 = beta + 1;
    x(1)= 0.2401;
    x(2)= 0.3210;
    x(3) = 0.4243;
    x_1b1 = x(1)^beta1;
    x_2b1 = x(2)^beta1;
    I_max = (D_2*((x(2))+((alpha*x_2b1)/beta1)-(x(1))-
((alpha*x_1b1)/beta1)))+(D_1*(x(1)+((alpha*x_1b1)/beta1)));
    I_sh = D_1*(x(3)-x(2));
    Q = I_max+I_sh;
end

% Find order quantity
Q = Quantity()
```