**OPTIMIZING AN INTEGRATED VENDOR-MANAGED INVENTORY SYSTEM FOR A SINGLE SUPPLIER AND TWO RETAILERS WITH DEPENDENT STOCHASTIC DEMAND**

by

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**AUTHOR’S DECLARATION**

As the author of this thesis, I, Linxuan Deng, affirm that the research conducted aligns with all the regulations stipulated by the Asian Institute of Technology. The content presented herein is a direct result of my personal research efforts; it is authentic and entirely self-originated. All utilized external resources have been duly acknowledged and cited. This work is distinctive and has not been previously submitted for credit towards any other degree at another institution. This document is a precise copy of the original thesis, inclusive of the final amendments.

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**ABSTRACT**

In the dynamic realm of supply chain management, Vendor Managed Inventory (VMI) systems are pivotal for synchronizing stock levels amidst unpredictable demand. This study delves into a VMI model operating under a base-stock level approach, coordinating between a sole supplier and dual retailers, and contending with variable demands that are contingent on multiple factors. The cornerstone of this research is the formulation of a mathematical model that ascertains the most efficient ordering cycle length, a crucial element for the seamless flow of inventory and an essential metric for the enhancement of supply chain management.

The fundamental goal of the model is to determine the most favorable ordering period that corresponds with the retailers' erratic demand fluctuations, while guaranteeing the aggregate expenses of the supply chain are minimized, preserving service excellence. A detailed sensitivity analysis is also a cornerstone of this study, providing a thorough analysis of how various supply chain parameters, such as lead times and cost components, influence the optimal cycle length. The outcomes of this analysis offer strategic insights, and illustrate how slight perturbations in input parameters can affect the cost-efficiency.

**Keywords:** Vendor managed inventory, Stochastic demand, Dependent demand,base-stock level approach

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**LIST OF ABBREVIATIONS**

VMI = Vendor Managed Inventory

VAR(1) = First Order Vector Autoregressive Model

AVG = Average Daily Demand

STD = Standard Deviation of Daily Demand

**CHAPTER 1**

**INTRODUCTION**

**1.1 Background of the Study**

Vendor managed inventory (VMI) is becoming a prevalent strategy for streamlining supply chain operations. Within a VMI framework, inventory levels from the retailer are disclosed to the vendor, who then assumes the responsibility for determining the quantity of orders. This stands in contrast to conventional inventory control practices, where the retailer independently dictates the size of the order. Reducing inventory-related expenses for both sides is the aim of VMI. Inventory is renewed only when needed when suppliers handle it, which lowers overstock and retailer expenses.

The major goals of putting a VMI system in place are to lower inventory costs for both retailers and vendors and to raise customer service level. In VMI arrangements, it is imperative that the systems overseeing sales, inventory, and order processing are interconnected, facilitating seamless information exchange between the involved parties. The consumer demand and the point of replenishment are the key pieces of information. A supply network that has mutual understanding of one another's processes can become stronger and obtain a competitive edge over other supply networks. Enhanced information clarity in a VMI supply chain bolsters decision-making acumen and mitigates inventory optimization risks, thereby diminishing overall costs. Consequently, this transparency cultivates a more agile supply chain with reduced inventory levels.

The application of VMI has been the subject of some research works. Various configurations have been examined, such as single-vendor to single-retailer, single-vendor to dual-retailers, and single-vendor to multiple-retailers systems, under both deterministic and stochastic demand scenarios. Initial models for constructing VMI frameworks with stochastic demand have been rooted in deterministic demand models. These deterministic models are crucial for dissecting the basic vendor-retailer dynamics. Nevertheless, to accommodate consumer preferences, stochastic models are indispensable. One of the prevalent problems with VMI is the demand uncertainty that retailers must deal with.

In a vendor-managed inventory system, it is the supplier who assumes the role of maintaining and regulating the inventory of the buyer's goods. This entails strategizing on the timing and volume of replenishment to align with the buyer's requirements while minimizing costs and maximizing service levels. Govindan (2015) explores a particular scenario where the demand for the product fluctuates stochastically over time and remains uncertain. The main goal is to identify the optimal replenishment strategy for this type of VMI system while taking into account the stochastic and time-varying characteristics of the demand. The goal is to find the best approach to replenishing inventory that balances the costs of holding excess inventory with potential inventory costs.

**1.2 Problem Statement**

Inventory control procedures that are cooperative between vendors and retailers are known as vendor-controlled inventory. VMI boosts the responsiveness and efficiency of the supply chain by reducing the overall system costs. Many large and well-known companies have used VMI in the past, including Walmart, Campbell Soup, Intel, and others, with positive results. As a result, numerous scholars investigated and created VMI models in a variety of contexts. Researchers have created VMI models for stochastic needs in a variety of scenarios during the last few years, including stochastic demands in single-vendor two-retailer supply chains, joint single vendor-single buyer supply chains, stochastic demands in two-level supply chains, etc. To make VMI more realistic and useful, work still needs to be done on various cases.

Although the models developed by Mateen and Chatterjee (2015) reasonably reflect various supply networks, the assumptions used in VMI introduce certain limitations. When the vendor assumes obligation for the replenishment decisions under VMI, assumptions commonly were made about the predictable nature of demand and stable order costs. Based on previous research, specialists who investigated single vendor multiple retailers systems under vendor managed inventory policy generally assumed that the demands of all retailers are independent.

Therefore, to address this gap, this research will investigate a system involving a single supplier and two retailers, where the demands of the retailers are stochastic variables that depend on each other.

**1.3 Objective of Study**

The primary aim of this research is to formulate a mathematical VMI model for a system with one supplier and two retailers in which the demands of retailers are dependent variables. The ultimate goal is to help reduce the total inventory costs across the entire supply chain.

**1.4 Scopes and Limitation**

In this research, the assumptions and limitations used to derive the mathematical model are considered below.

* Single vendor and two retailers with only one product.
* The supplier will use a periodic review policy.
* Holding cost of the vendor is higher than the retailer's holding cost.
* Shortages will be lost.
* The demands of retailers are dependent.
* The same replenishment cycle will be employed for both retailers
* Lead times to deliver products to retailers are constant.

**CHAPTER 2**

**LITERATURE REVIEW**

**2.1** **Vendor Managed Inventory**

VMI has emerged as a fascinating topic in the management of inventory systems in contemporary supply chain networks. VMI differs from traditional inventory management methods. In traditional inventory systems, a retailer places orders based on personal interest, and the vendor fulfills these orders by delivering the items. The vendor makes the decision on replenishment in VMI. As a result, the seller keeps an eye on the retailer’s inventory level and decides when to replenish it. By using VMI, the vendor will be able to identify the true demand and not rely on retailer orders that might not accurately reflect the true need, hence preventing the bullwhip effect.

Vendor Managed Inventory (VMI) was thoroughly reviewed by Marquès et al. (2010) by tracing the development of VMI from a conceptual framework to its practical implementation procedures. In a VMI setup, the supplier is responsible for managing the inventory levels at the customer's location throughout the supply chain. Huang and Li (2012) found that under VMI, the production decision needs only to consider the customers' market information (such as demand), not their operational information (such as inventory/logistic costs).

Several studies have been carried out on VMI systems involving one vendor and multiple retailers. Siajadi et al. (2006) developed a model featuring a single vendor and multiple buyers. The seller is the only supplier, and the demand is deterministic. Zavanella and Zanoni (2009) suggested a model for a system with one vendor and several buyers, incorporating a collective inventory management approach to decrease or stabilize holding costs. In the study by Hariga et al. (2013), the focus is on a supply chain featuring a single vendor and multiple retailers, operating under a VMI agreement that imposes limits on the retailers' inventory levels. They tackled the challenge of aligning the vendor’s cycle time with the buyers’ irregular ordering cycles by developing a mixed integer nonlinear program that optimizes the combined relevant inventory costs subject to storage constraints

Wang (2013) introduced a periodic-review inventory control strategy for a two-level supply chain involving multiple vendors and facing stochastic demand. In order to balance inventory replenishment decisions among different vendors while taking uncertain demand trends into account, Wang's research tries to design a policy. The research makes a contribution to improve supply chain performance and efficiency in circumstances where several retailers are involved, demand is uncertain, and coordination is essential by introducing this periodic-review approach.

**2.2 Stochastic Demand and Different Replenishment Cycles for Retailers under VMI**

Given that stochastic demand and replenishment cycles are a significant issue for vendor managed inventory (VMI), research on VMI has risen, and greater attention has been paid to them. Mateen et al. (2015) explored the interactions between a vendor and multiple retailers within a vendor managed inventory (VMI) system operating under stochastic demand. It is assumed that the vendor replenishes all stores simultaneously and that the vendor's replenishment cycle is an integer multiple of the retailers' replenishment cycle. Besides, Darwish and Odah (2010) proposed a vendor managed inventory (VMI) model specifically designed for supply chains with a single vendor and multiple retailers. VMI enhances supply chain efficiency by having the supplier take responsibility for inventory management. Suppliers are advantaged in the VMI system by dictating the restocking schedule and volumes, coupled with insights into the retailer’s stock levels and consumer demand patterns. Moreover, they have devised a strategy that ensures consistent supply chain replenishment. Additionally, they presented a plan that offers uniform replenishment periods for every retailer.

However, an alternative method for a system involving a single vendor and several retailers within a VMI framework was presented by Verma et al. (2014). This strategy permits the adoption of distinct replenishment cycles catered to each retailer's needs. In 2013, Taleizadeh et al. investigated a unified vendor-buyer supply chain dilemma under variable demand and uncertain lead times, focusing on reducing the forecasted overall costs through the calculation of both the reorder point and the order quantity. Taleizadeh et al. (2013) set their research apart by considering the lead-time for each product as a fuzzy variable, unlike earlier studies which treated lead-time as varying linearly with the lot size.

In addition, some models are built and aspects of the research are presented. In Mateen and Chatterjee (2015) work, they provided analytical models for a variety of strategies that can be utilized to coordinate a single vendor-multiple retailer system via VMI. To offer effective inventory management solutions, mathematical models are created, optimization techniques and simulations are run while accounting for the uncertainty of demand and delivery times. Meanwhile, Mateen and Chatterjee (2015) conducted an operational analysis of the VMI system from the perspective of ideal replenishment policy. Sajadieh and Larsen (2015) directed their research towards a dual-stage supply chain framework, featuring a manufacturer and a retailer each grappling with unpredictable demand and production yield. To establish the best coordinated decision strategy, which acts as a benchmark, a thorough Markov chain model was created. Sirikasemsuk and Luong (2017) examined a Vendor Managed Inventory system within a supply chain consisting of a single supplier and two retailers, utilizing a first-order bivariate vector auto regression (VAR(1)) demand model to account for the interrelated demands of the retailers.

In conclusion, the literature review shows that there has been a lot of research studies on vendor managed inventory done in the past. However, these study efforts still have certain limitations, for example, demands of retailers are assumed to be independent. As a result, this study will investigate a system that includes a single supplier and two retailers, in which the retailers' demands are considered dependent random variables.

**CHAPTER 3**

**MATHEMATICAL MODEL DEVELOPMENT**

The primary objective of this study is to develop a Vendor Managed Inventory (VMI) system under a base-stock level policy for a network consisting of one supplier and two retailers. This chapter will introduce a mathematical model designed to manage scenarios in which the demands of the two retailers are interdependent variables.

**3.1 Establishing the Mathematical Framework**

The two retailers in this study's VMI system have equal replenishment cycles. For the entire system, the supplier will place the order. Utilizing the notations below, the total cost function for the supplier and the two retailers has been formulated.

 = Index of retailers (K = 1, K = 2)

 = Expected cycle length (Decision Variable)

 = Demand faced by retailer K in time period t

 = Total demand of two retailers in time period t

 = Order lead time between retailer K and the supplier

 = Order-up-to-level of retailer K at the beginning of period t

 = Base-stock level at two retailers

 = Order quantity placed by retailer K to the supplier in time period t

 = Total order quantity received by the supplier at the beginning of
 period t

 = Mean of the autoregressive process which is used to describe

 the demand process at retailer K

 = Total mean of the autoregressive process which is used to

 describe the demand process at two retailers

 = Variance of demand at retailer K

 = Total variance of demand at two retailers

 = Probability density function during lead time for a normal

distribution with mean and the standard

deviation

 = The expected shortage amount at two retailers

 = Holding cost at the centralized warehouse per time unit

 = Ordering cost at the supplier per cycle

 = Shortage cost of the two retailers per unit

 = Total cost at both retailers

**Figure. 3.1**

*A Two-Stage Supply Chain Model.*

The VAR(1) demand model, characterized as a first-order vector autoregressive model, serves to delineate the linear dependencies that exist across various time series. The VAR(1) demand model is used in this research because the demand of the two retailers is correlated and random.

From the basic equation of bivariate VAR (1) demand model, demands faced by retailer 1 and retailer 2 in time period t can be expressed as:

 (1)

Then, the total demand of two retailers in time period t is:

 (2)

Let be a vector representing the demands of the two retailers. We can express that:

 is a vector representing mean of the autoregressive process which is used to describe the demand process at retailer 1 and retailer 2

 is 2x2 covariance matrix which is expressed as , where and are variance of demand at retailer 1 and retailer 2, is the covariance of and

It is noted that:

 (3)

 is constant vector of the VAR (1) model

 is autoregressive parameter matrix

 is white noise vector following the jointly-distributed normal distribution with

zero mean vector, which represents the random factors affecting the demand of

retailer 1 and retailer 2

Then:

 (4)

The distribution of is stationary, so:

 (5)

We also know:

 (6)

From the equations (4) (5) (6) imply:

Where: is 2x2 identity matrix

 (7)

We have:

+

 (8)

Where and follow the same distribution, so

 is 2x2 covariance matrix which is expressed as , where is the covariance between and , and are variance of and

So:

 (9)

From equation (7):

Therefore:

 (10)

From equation (9):

Therefore:

 (11)

From equations (11),

 (12)

 (13)

 (14)

From Cramer's rule, rewrite the equations (12) (13) (14) and represent it as a matrix:

# Calculate , and g:

# The final results of, and g are shown below:

# (15)

#  (16)

 (17)

In conclusion:

Therefore, total demand

 And

Where:

 (18)

And:

 (19)

Then calculate :

Then, the final result of is:

 (20)

**3.2 Base-Stock Level Policy**

A base-stock level policy is a method often employed in inventory management for stock regulation. The aim within this framework is to keep the inventory at an established,
designated level, termed the base-stock level.

**Figure. 3.2**

*****Base-Stock Policy*

Assume:

S = Base-stock level

 = Length of the review period

 = Lead Time

 = Average daily demand

 = Standard deviation of this daily demand

 = Service level

In the base-stock level policy, the target inventory level, denoted by the base-stock level S, is reviewed at each interval T. Orders are then placed as necessary to elevate the inventory position to match the base-stock level.

The average demand during an interval of T+ L days is:

 (21)

Safety stock is:

 (22)

Then, the base-stock level is:

 (23)

We know: The average daily demand:

 (24)

The standard deviation of this daily demand:

 (25)

Then, from equation (10), the base-stock level is:

 (26)

**3.3 Total Cost**

The total cost will consist of the holding cost incurred at the centralized warehouse, the ordering cost incurred at the supplier, and the shortage cost incurred at the two retailers. When an order is placed for inventory products or supplies, there are costs involved that are referred to as ordering costs. The costs related to maintaining and keeping inventory for a particular period of time are referred to as the holding cost. The costs a company incurs when its inventory runs out and it is unable to satisfy consumer demand are referred to as the shortage cost.

**Figure. 3.3**

*Base-Stock Level Policy*

The graph presented illustrates the inventory distribution of the retailer utilizing a base-stock level policy.

At point A:

The order is released at the vendor, and the average inventory level at retailer is

 (27)

At point B:

The order is received at the vendor, and the average inventory level at retailer is

 (Safety stock) (28)

At point C:

The order is delivered from the warehouse to the retailer, and the average inventory

level at retailer is

(29)

***3.3.1 Total Cost at the Supplier***

Ordering cost:

The ordering cost per cycle at the supplier is:

Therefore, total cost at the supplier per unit of time is:

***3.3.2 Total Cost at the Centralized Warehouse***

Holding cost:

The average inventory level is shown as below:

**Figure. 3.4**

*The Average Inventory Level for Holding Cost*

From the graph above, the average inventory level = the area under the inventory curve = the shadow area:

 (30)

The holding cost per cycle at the centralized warehouse is:

Therefore, total cost at the centralized warehouse per unit of time is:

***3.3.3 Total Cost at the Retailers***

Shortage cost:

The expected shortage amount before the next order arrives is:

(31)

Where: is the demand of two retailers during (T+L)

is probability density function for a normal distribution

And:

The total mean during (T + L) is:

The total standard deviation during (T + L) is:

 (32)

So:

(33)

From the lost sales policy:

The shortage cost per unit of time at two retailers is:

***3.3.4 Total Cost at the VMI System***

The aggregate cost within the VMI system encompasses the holding cost at the central warehouse, the ordering cost at the supplier, and the shortage cost at both retailers.

Therefore, the total cost per unit of time is:

**CHAPTER 4**

**NUMERICAL EXPERIMENTS**

**4.1 Numerical Example**

To demonstrate the practicality of the created mathematical model, MATLAB's fmincon toolbox is utilized to conduct numerical experiments. Utilizing the fmincon solver, it will ascertain the optimal cycle length (T) aimed at diminishing the overall inventory expenses across the entire system.

For the optimization model, the following options are employed.

Bounds: Lower – 0.01

 Upper – Infinity

The subsequent values were employed as input parameters of the base case.

 = 100$ per cycle

 = 5$ per time unit

 = 40$ per unit,

z = 1.645 (95% service level)

L = 2 days

 = 0.2

 = -0.4

 = -0.4

 = 0.2

 = 10

 = 15

 = 10

 = 20

 = -12

The optimal values derived from the fmincon solver for these input parameters are:

Optimal cycle length T = 1.336, and the total inventory cost is 184.846$.

**4.2 Sensitivity Analysis**

This section will scrutinize the impacts of the input parameters. Investigations are conducted on parameters including service level, delivery time, holding cost, ordering cost, shortage cost, constant parameter of VAR(1), autoregressive parameter of VAR(1), variance of demand error term, and covariance between the error terms of demands.

***4.2.1 Effect of Ordering Cost of the Two Retailers***

In this section, the ordering cost will vary from 80 to 120 while maintaining the initial values for the remaining parameters. Table 4.2.1 displays the results.

**Table 4.1**

*Effect of Ordering Cost*

|  |  |  |
| --- | --- | --- |
|  | Optimal T | Total cost |
| 80 | 1.1982 | 169.06 |
| 90 | 1.2691 | 177.17 |
| 100 | 1.3363 | 184.85 |
| 110 | 1.4003 | 192.15 |
| 120 | 1.4616 | 199.14 |

Based on the findings, it is clear that with an increase in ordering cost, both the optimal cycle length and the total cost also increase. The above trends look reasonable. In fact, it is understandable that the total cost increases when the ordering cost increases. Also, when the ordering cost increases, the cycle length must also increases to help reduce ordering cost per unit of time, which is one component in the total cost function.

***4.2.2 Effect of Holding Cost of the Two Retailers***

Within this section, the holding cost will vary from 2 to 10 while maintaining the starting point values for the remaining parameters. Table 4.2.2 provides the results.

Based on the findings, it is clear that as the holding cost increases, the optimal cycle length decreases while the total cost rises. The above trends are reasonable. It is understandable that the total cost increases as the holding cost increases. Additionally, when the holding cost increases, the cycle length must decrease to avoid accumulating excessive holding cost over time, which is one component in the total cost function.

**Table 4.2**

*Effect of Holding Cost*

|  |  |  |
| --- | --- | --- |
|  | Optimal T | Total cost |
| 2 | 2.1243 | 110.03 |
| 3 | 1.7301 | 138.03 |
| 4 | 1.4958 | 162.55 |
| 5 | 1.3363 | 184.85 |
| 6 | 1.2187 | 205.57 |
| 7 | 1.1274 | 225.09 |
| 8 | 1.0539 | 243.67 |
| 9 | 0.9931 | 261.47 |
| 10 | 0.9417 | 278.62 |

***4.2.3 Effect of Shortage Cost of the Two Retailers***

Within this section, the shortage cost will vary from 20 to 100 while maintaining the starting values for the others parameters. Table 4.2.3 displays the results.

**Table 4.3**

*Effect of Shortage Cost*

|  |  |  |
| --- | --- | --- |
| Cs | Optimal T | Total cost |
| 20 | 1.3262 | 183.41 |
| 30 | 1.3313 | 184.13 |
| 40 | 1.3363 | 184.85 |
| 50 | 1.3413 | 185.56 |
| 60 | 1.3462 | 186.27 |
| 70 | 1.3512 | 186.98 |
| 80 | 1.3561 | 187.68 |
| 90 | 1.3611 | 188.39 |
| 100 | 1.3660 | 189.09 |

With regard to the findings, it is evident that with an increase in the shortage cost, both the optimal cycle length and the total cost are prone to rise. The above trends are reasonable. It is understandable that as the shortage cost increases, the total cost rises as well. Additionally, the cycle length shows a slight increase as the shortage cost rises to help reduce the shortage cost per unit of time in the total cost function.

***4.2.4 Effect of Lead Time of the Two Retailers***

In this section, the lead time ranges from 1 to 10, with all other input parameters held constant. Table 4.2.4 provides the results.

**Table 4.4**

*Effect of Lead Time*

|  |  |  |
| --- | --- | --- |
| L | Optimal T | Total cost |
| 1 | 1.3185 | 178.24 |
| 2 | 1.3363 | 184.85 |
| 3 | 1.3485 | 190.50 |
| 4 | 1.3579 | 195.52 |
| 5 | 1.3654 | 200.09 |
| 6 | 1.3718 | 204.31 |
| 7 | 1.3774 | 208.24 |
| 8 | 1.3823 | 211.95 |
| 9 | 1.3867 | 215.46 |
| 10 | 1.3907 | 218.81 |

Based on the findings, it is apparent that as the lead time increases, both the optimal cycle length and the total cost increase. The above trends are reasonable. As the lead time increases, it is understandable that the total cost would also rise, since a longer lead time typically heightens the risk of holding excess inventory or encountering shortages, both of which can drive up the total cost. Furthermore, as the lead time extends, the cycle length increases to mitigate the increase of shortage cost due to longer lead times.

***4.2.5 Effect of Service Level of the Two Retailers***

The Z-score represents a percentile indicating the service level, which is the likelihood that demand will not surpass the stock level throughout the lead time. In this section, the service level varies from 90% to 97% (corresponds z-score varies from 1.28 to 1.88) while keeping the remaining input parameters constant. Table 4.2.5 showcases the results.

**Table 4.5**

*Effect of Service Level*

|  |  |  |
| --- | --- | --- |
| z | Optimal T | Total cost |
| 1.28 | 1.3766 | 180.08 |
| 1.33 | 1.3699 | 180.59 |
| 1.38 | 1.3637 | 181.15 |
| 1.43 | 1.3578 | 181.75 |
| 1.48 | 1.3523 | 182.40 |
| 1.53 | 1.3471 | 183.10 |
| 1.58 | 1.3422 | 183.84 |
| 1.63 | 1.3376 | 184.61 |
| 1.68 | 1.3333 | 185.41 |
| 1.73 | 1.3291 | 186.25 |
| 1.78 | 1.3253 | 187.12 |
| 1.83 | 1.3216 | 188.01 |
| 1.88 | 1.3181 | 188.93 |

Considering the data gathered, it is evident that as the service level time extends, it is evident that the optimal cycle length decreases while the total cost rises. The above trends are reasonable. As the service level increases, it leads to higher inventory levels to prevent fluctuations in demand, which leads to the increase in the total cost. Correspondingly, the cycle length decreases slightly as service level increases, indicating a strategy to compensate for the higher inventory levels by reducing the interval between orders, which helps to manage the holding cost and mitigate the risk of overstocking.

***4.2.6 Effect of the Autoregressive Parameter Phi 11 of the Retailer 1***

In this section, the autoregressive parameter phi 11will vary from -0.3 to 0.3 while maintaining the baseline values for the remaining parameters. Table 4.2.6 illustrates the results.

**Table 4.6**

*Effect of the Autoregressive Parameter Phi 11*

|  |  |  |
| --- | --- | --- |
|  | Optimal T | Total cost |
| -0.3 | 1.3538 | 180.13 |
| -0.25 | 1.3537 | 180.53 |
| -0.2 | 1.3535 | 180.95 |
| -0.15 | 1.3530 | 181.39 |
| -0.1 | 1.3522 | 181.84 |
| -0.05 | 1.3511 | 182.31 |
| 0 | 1.3495 | 182.78 |
| 0.05 | 1.3474 | 183.28 |
| 0.1 | 1.3446 | 183.78 |
| 0.15 | 1.3409 | 184.30 |
| 0.2 | 1.3363 | 184.85 |
| 0.25 | 1.3303 | 185.42 |
| 0.3 | 1.3229 | 186.04 |

Based on the findings, it becomes apparent that with an increase in phi11, there is a decrease in the optimal cycle length while an increase in the total cost. The above trends are reasonable. As phi11 increases, there is a corresponding rise in the total cost. This is understandable because a higher phi11 suggests stronger positive autocorrelation in demand, which could lead to larger fluctuates in demand over time, thus increasing both potential holding and shortage costs. However, the cycle length decreases slightly, indicating a shift toward more frequent ordering to mitigate the risks associated with these larger demand fluctuations on the total cost function.

***4.2.7 Effect of the Autoregressive Parameter Phi 12 of the Two Retailers***

In this section, the autoregressive parameter phi12will vary from -0.6 to -0.2 with the remaining parameters remaining unchanged from the base case. Table 4.2.7 showcases the findings.

**Table 4.7**

*Effect of the Autoregressive Parameter Phi 12*

|  |  |  |
| --- | --- | --- |
|  | Optimal T | Total cost |
| -0.6 | 1.4374 | 172.21 |
| -0.55 | 1.4061 | 175.93 |
| -0.5 | 1.3796 | 179.25 |
| -0.45 | 1.3566 | 182.22 |
| -0.4 | 1.3363 | 184.85 |
| -0.35 | 1.3178 | 187.15 |
| -0.3 | 1.3008 | 189.15 |
| -0.25 | 1.2850 | 190.88 |
| -0.2 | 1.2702 | 192.37 |

Based on the findings, it is obvious that when as phi12 experiences an increase, it results in a reduction of the optimal cycle length, while leading to an escalation in the total cost. The trend illustrates that as phi12 becomes less negative, there has been an escalation in the total cost. This pattern is reasonable because as a less negative phi12 could imply a decrease in the competitive effect between two retailers, leading to less advantage taken from the competitor's demand fluctuations. The decrease in cycle length could be a strategy to maintain service levels and avoid stock outs in a less competitive environment. On the other hand, the total cost increases due to higher total demand.

***4.2.8 Effect of the Autoregressive Parameter Phi 21 of the Two Retailers***

In this section, theautoregressive parameter phi21will vary from -0.6 to -0.2 while maintaining the initial values for the remaining parameters. Table 4.2.8 outlines the results.

Based on the findings, it is obvious that when with an increase in phi21, there is a corresponding decrease in the optimal cycle length while an increase in the total cost. The trends observed here are the same as the trends observed on the effects of phi12.

**Table 4.8**

*Effect of the Autoregressive Parameter Phi 21*

|  |  |  |
| --- | --- | --- |
|  | Optimal T | Total cost |
| -0.6 | 1.3582 | 181.61 |
| -0.55 | 1.3524 | 182.62 |
| -0.5 | 1.3469 | 183.49 |
| -0.45 | 1.3415 | 184.23 |
| -0.4 | 1.3363 | 184.85 |
| -0.35 | 1.3310 | 185.34 |
| -0.3 | 1.3258 | 185.74 |
| -0.25 | 1.3206 | 186.05 |
| -0.2 | 1.3155 | 186.28 |

Based on the findings, it is obvious that when with an increase in phi21, there is a corresponding decrease in the optimal cycle length while an increase in the total cost. The trends observed here are the same as the trends observed on the effects of phi12.

***4.2.9 Effect of the Autoregressive Parameter Phi 22 of the Retailer 2***

In this section, theautoregressive parameter phi22will vary from -0.3 to 0.3 while maintaining the starting point values for the remaining parameters. Table 4.2.9 contains the results.

Based on the findings, it is evident that when as phi22 experiences an upward trend, the optimal cycle length decreases, while the total cost shows an upward trajectory. The trends observed here are the same as the trends observed on the effects of phi11.

**Table 4.9**

*Effect of the Autoregressive Parameter Phi 22*

|  |  |  |
| --- | --- | --- |
|  | Optimal T | Total cost |
| -0.3 | 1.4435 | 169.24 |
| -0.25 | 1.4366 | 170.17 |
| -0.2 | 1.4293 | 171.21 |
| -0.15 | 1.4216 | 172.37 |
| -0.1 | 1.4132 | 173.67 |
| -0.05 | 1.4041 | 175.11 |
| 0 | 1.3939 | 176.70 |
| 0.05 | 1.3824 | 178.45 |
| 0.1 | 1.3693 | 180.38 |
| 0.15 | 1.3541 | 182.50 |
| 0.2 | 1.3363 | 184.85 |
| 0.25 | 1.3152 | 187.44 |
| 0.3 | 1.2904 | 190.36 |

***4.2.10 Effect of the Constant Parameter Delta 1 of the Retailer 1***

In this section, the constant parameter delta 1will vary from 5 to 15 while maintaining base values for the remaining parameters. The results are showcased in Table 4.2.10.

From the results, it can be seen that as delta 1 increases, there is a corresponding decrease in the optimal cycle length while leading to an increase in the total cost. The above trends are reasonable because an increase in mean demand would typically lead to a higher total cost due to the need for more inventory and the potential for increased holding costs. Furthermore, the reduction in cycle length suggests a strategy to order more frequently to match the higher rate of demand.

**Table 4.10**

*Effect of the Constant Parameter Delta 1*

|  |  |  |
| --- | --- | --- |
|  | Optimal T | Total cost |
| 5 | 1.478 | 170.23 |
| 6 | 1.446 | 173.27 |
| 7 | 1.416 | 176.25 |
| 8 | 1.388 | 179.17 |
| 9 | 1.361 | 182.04 |
| 10 | 1.336 | 184.85 |
| 11 | 1.313 | 187.60 |
| 12 | 1.290 | 190.32 |
| 13 | 1.269 | 192.98 |
| 14 | 1.248 | 195.60 |
| 15 | 1.229 | 198.18 |

***4.2.11 Effect of the Constant Parameter Delta 2 of the Retailer 2***

In this section, the constant parameter delta 2will vary from 15 to 25 while maintaining the initial values for the remaining parameters. The results are showcased in Table 4.2.11.

**Table 4.11**

*Effect of the Constant Parameter Delta 2*

|  |  |  |
| --- | --- | --- |
|  | Optimal T | Total cost |
| 15 | 1.3363 | 184.85 |
| 16 | 1.3125 | 187.60 |
| 17 | 1.2900 | 190.32 |
| 18 | 1.2687 | 192.98 |
| 19 | 1.2483 | 195.60 |
| 20 | 1.2290 | 198.18 |
| 21 | 1.2105 | 200.72 |
| 22 | 1.1928 | 203.23 |
| 23 | 1.1759 | 205.69 |
| 24 | 1.1596 | 208.13 |
| 25  | 1.1441 | 210.53 |

From the results, it can be seen that as delta 2 increases, there is a corresponding decrease in the optimal cycle length while leading to an increase in the total cost. The trends here are the same as the trends observed on the effects of delta 1.

***4.2.12 Effect of the Variance of Demand Error Term of the Retailer 1***

In this section, the variance of demand error term will vary from 5 to 30 while maintaining the starting point values for the remaining parameters. Table 4.2.12 outlines the results.

**Table 4.12**

*Effect of the Variance of Demand Error Term Sigma 1 Squared*

|  |  |  |
| --- | --- | --- |
|  | Optimal T | Total cost |
| 5 | 1.3646 | 160.90 |
| 10 | 1.3363 | 184.85 |
| 15 | 1.3204 | 199.14 |
| 20 | 1.3085 | 210.39 |
| 25 | 1.2987 | 219.97 |
| 30 | 1.2902 | 228.46 |

From the results, it can be seen that with an increase in , there is a decrease in the optimal cycle length while an increase in the total cost. The above trends are reasonable. This is understandable because higher variability in demand leads to increase in shortage cost, and cycle length should be reduced to mitigate the risk of stock outs.

***4.2.13 Effect of the Variance of Demand Error Term of the Retailer 2***

In this section, the variance of demand error term will vary from 15 to 40 while maintaining the initial values for the remaining parameters. The outcomes can be found in Table 4.2.13.

From the results, it can be seen that with an increase in , there is a corresponding decrease in the optimal cycle length while an increase in the total cost. These trends are similar to the trends observed on the effects of .

**Table 4.13**

*Effect of the Variance of Demand Error Term Sigma 2 Squared*

|  |  |  |
| --- | --- | --- |
|  | Optimal T | Total cost |
| 15 | 1.3646 | 160.90 |
| 20 | 1.3363 | 184.85 |
| 25 | 1.3204 | 199.14 |
| 30 | 1.3085 | 210.39 |
| 35 | 1.2987 | 219.97 |
| 40 | 1.2902 | 228.46 |

***4.2.14 Effect of the Covariance between the Error Terms******of the Two Retailers***

In this section, the covariance of demand between the error terms will vary from -15 to -1 while maintaining the starting point values for the remaining parameters. The outcomes can be found in Table 4.2.14.

Based on the findings, it is apparent that with an increase in there is a corresponding decrease in the optimal cycle length while an increase in the total cost. The above trends are reasonable. The increase of covariance term indicates a stronger relationship in demand fluctuations between retailers. The increase in total costs reflects the additional cost of maintaining inventories. Consequently, the reduction in cycle length is due to the potential overstock caused by competition.

**Table 4.14**

*Effect of the Covariance of Demand between the Error terms GammaEpsilon*

|  |  |  |
| --- | --- | --- |
|  | Optimal T | Total cost |
| -15 | 1.3856 | 144.34 |
| -14 | 1.3563 | 167.75 |
| -13 | 1.3448 | 177.43 |
| -12 | 1.3363 | 184.85 |
| -11 | 1.3292 | 191.10 |
| -10 | 1.3232 | 196.60 |
| -9 | 1.3178 | 201.57 |
| -8 | 1.3129 | 206.14 |
| -7 | 1.3085 | 210.39 |
| -6 | 1.3044 | 214.38 |
| -5 | 1.3005 | 218.16 |
| -4 | 1.2969 | 221.75 |
| -3 | 1.2934 | 225.17 |
| -2 | 1.2902 | 228.46 |
| -1 | 1.2871 | 231.62 |

**CHAPTER 5**

**CONCLUSIONS AND RECOMMENDATIONS**

**5.1 Conclusions**

The aim of this research is to develop a Vendor Managed Inventory (VMI) system, operating on a base-stock level policy, for a supply network comprised of a single supplier and two retailers, where the demands of the two retailers are not only stochastic but also dependent. Using a first-order vector auto regression (VAR (1)) model, this interdependency can be captured.

Numerical experiments were carried out with the fmincon toolbox in MATLAB to verify the mathematical framework, which assist in determining the expected cycle length to minimize the total inventory cost. The main results of sensitivity analyses show that:

* Ordering Cost: As the ordering cost rises, there is a corresponding increase in both the optimal cycle length and the overall cost.
* Holding Cost: A rise in holding cost results in a shorter optimal cycle length and an increased total cost.
* Shortage Cost: Higher shortage cost yielded a slight increase in optimal cycle length and a steady rise in total cost.
* Lead Time: Extended lead times resulted in increases in both the optimal cycle length and total cost, reflecting the need for additional safety stock.
* Service Level: An increase in the service level (z-score) led to a reduction in the optimal cycle length and an increase in total costs, as higher service levels typically require higher stock levels.
* Mean demand: The surge in mean demand has led to a reduction in the optimal cycle length and an increase in total cost, consistent with the principle that higher demand requires more frequent ordering.
* Standard deviation of demand: Greater demand variability leads to the fact that the optimal cycle length nearly unchanged while total cost increases.

**5.2 Recommendations**

Considering the knowledge acquired from the sensitivity testing and model analyses, the following recommendations are proposed for supply chain practitioners and future research:

Real-time Data Integration: Establish integrated systems for real-time data sharing between the supplier and retailers to better anticipate and respond to stochastic demand.

Flexible Inventory Policy Design: Develop and implement inventory policies that can dynamically adjust to changes in ordering, holding, and shortage costs.

Lead Time Management: Implement strategies to manage lead times effectively, which may include supplier diversification or investment in faster transportation methods.

Service Level Optimization: Carefully consider service level increases, as they can significantly impact total costs. A balance must be struck between service quality and cost management.

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**APPENDIX**

**COMPUTER PROGRAM (MATLAB)**

% Define variables with actual values

Co = 100;

Ch = 5;

Cs = 40;

z = 1.645;

L = 2;

phi11 = 0.2;

phi12 = -0.4;

phi21 = -0.4;

phi22 = 0.2;

delta1 = 10;

delta2 = 15;

sigma\_1\_squared = 10;

sigma\_2\_squared = 20;

gammaEpsilon = -12;

% Objective Function

function CT = objectiveFunction(Co, Ch, Cs, T, L, z, mu\_x, sigma\_x, sigma\_x\_squared)

 % Compute the total mean (mu) and total standard deviation (sigma) for (T+L)

 mu = mu\_x \* (T + L);

 sigma = sigma\_x \* sqrt(T + L);

 % Check and ensure mu and sigma are finite

 if isnan(mu) || isinf(mu) || isnan(sigma) || isinf(sigma)

 error('Mu or sigma is non-finite, aborting calculation.');

 end

 % Calculate the base-stock level S

 S = max((T + L) \* mu\_x + z \* sqrt(sigma\_x\_squared) \* sqrt(T + L), 0);

 % Ensure S is finite

 if isinf(S) || isnan(S)

 error('Calculated S is non-finite, cannot proceed with calculation.');

end

orderingCost = Co / T;

 Area = 1/2 \* (z \* sigma\_x \* sqrt(T + L) + mu\_x \* L + z \* sigma\_x \* sqrt(T + L)) \* L + ...

 1/2 \* (mu\_x \* T + z \* sigma\_x \* sqrt(T + L) + mu\_x \* L + z \* sigma\_x \* sqrt(T + L)) \* (T - L);

 holdingCost = Ch \* Area / T;

 % Define the normal distribution probability density function

 normal\_pdf = @(x) (1 / (sigma \* sqrt(2 \* pi))) \* exp(-((x - mu).^2) / (2 \* sigma^2));

 % Calculate the expected shortage amount (ES) using integration

 integrand\_ES = @(x) max(x - S, 0) .\* normal\_pdf(x);

 ES = integral(integrand\_ES, S, Inf);

 shortageCost = Cs \* ES / T;

 CT = orderingCost + holdingCost + shortageCost;

end

% Optimal cycle length T

 mu\_1d = ((1 - phi22) \* delta1 + phi12 \* delta2) / ((1 - phi11) \* (1- phi22) - phi12 \* phi21);

 mu\_2d = (phi21 \* delta1 + (1 - phi11) \* delta2) / ((1 - phi11) \* (1- phi22) - phi12 \* phi21);

sigma\_1d\_squared = ((phi11\*phi22-phi11\*phi22^3 + phi22^2 + phi12\*phi21 + phi12\*phi21\*phi22^2 - 1) \* sigma\_1\_squared +...

 (phi12^3\*phi21-phi12^2-phi11\*phi12^2\*phi22) \* sigma\_2\_squared +...

 (2\*phi11\*phi12\*phi22^2-2\*phi11\*phi12-2\*phi12^2\*phi21\*phi22)\* gammaEpsilon) /(phi11^3\*phi22^3-phi11^3\*phi22 -...

 3\*phi11^2\*phi12\*phi21\*phi22^2+phi11^2\*phi12\*phi21-phi11^2\*phi22^2+phi11^2+3\*phi11\*phi12^2\*phi21^2\*phi22 -...

 phi11\*phi22^3+phi11\*phi22-phi12^3\*phi21^3+phi12^2\*phi21^2+phi12\*phi21\*phi22^2+phi12\*phi21+phi22^2-1);

sigma\_2d\_squared = ((phi12\*phi21^3-phi11\*phi21^2\*phi22-phi21^2) \* sigma\_1\_squared +...

 (phi11\*phi22+phi12\*phi21+phi11^2+phi11^2\*phi12\*phi21-phi11^3\*phi22-1) \* sigma\_2\_squared +...

 (2\*phi11^2\*phi21\*phi22-2\*phi11\*phi12\*phi21^2-2\*phi21\*phi22) \* gammaEpsilon) / (phi11^3\*phi22^3-phi11^3\*phi22 -...

 3\*phi11^2\*phi12\*phi21\*phi22^2+phi11^2\*phi12\*phi21-phi11^2\*phi22^2+phi11^2+3\*phi11\*phi12^2\*phi21^2\*phi22 -...

 phi11\*phi22^3+phi11\*phi22-phi12^3\*phi21^3+phi12^2\*phi21^2+phi12\*phi21\*phi22^2+phi12\*phi21+phi22^2-1);

g = ((phi11\*phi21\*phi22^2-phi12\*phi21^2\*phi22-phi11\*phi21) \* sigma\_1\_squared +...

 (phi11^2\*phi12\*phi22-phi12\*phi22-phi11\*phi12^2\*phi21) \* sigma\_2\_squared +...

 (phi12^2\*phi21^2-phi11^2\*phi22^2+phi11^2+phi22^2-1) \* gammaEpsilon) / (phi11^3\*phi22^3-phi11^3\*phi22 -...

 3\*phi11^2\*phi12\*phi21\*phi22^2+phi11^2\*phi12\*phi21-phi11^2\*phi22^2+phi11^2+3\*phi11\*phi12^2\*phi21^2\*phi22 -...

 phi11\*phi22^3+phi11\*phi22-phi12^3\*phi21^3+phi12^2\*phi21^2+phi12\*phi21\*phi22^2+phi12\*phi21+phi22^2-1);

 mu\_x = mu\_1d + mu\_2d; % Total mean of demand at two retailers

 sigma\_x\_squared = sigma\_1d\_squared + 2\*g + sigma\_2d\_squared;

 % Total variance of demand at two retailers

 sigma\_x = sqrt(sigma\_x\_squared);

% Optimize cycle length T

% Define an anonymous function for the objective function that takes only T as input

objectiveFun = @(T) objectiveFunction(Co, Ch, Cs, T, L, z, mu\_x, sigma\_x,sigma\_x\_squared);

% Define the initial guess, lower and upper bounds for T

T0 = 1; % Initial guess for T

T\_lb = 0.01; % Lower bound for T

T\_ub = Inf; % Upper bound for T

% Set optimization options, if necessary

options = optimoptions('fmincon', 'Display', 'iter', 'Algorithm', 'sqp');

% Test the objective function at the initial point

try

 testVal = objectiveFun(T0);

 fprintf('The objective function value at the initial point is: %f\n', testVal);

catch ME

 error('The objective function failed at the initial point with error: %s', ME.message);

end

% Run the optimization

[T\_optimal, cost\_optimal] = fmincon(objectiveFun, T0, [], [], [], [], T\_lb, T\_ub, [], options);

% Display the optimal T and corresponding cost

fprintf('Optimal T is: %f\n', T\_optimal);

fprintf('Optimal cost is: %f\n', cost\_optimal);

% Sensitivity Co

% Fixed parameters

Ch = 5;

Cs = 40;

L = 2;

z = 1.645;

phi11 = 0.2;

phi12 = -0.4;

phi21 = -0.4;

phi22 = 0.2;

delta1 = 10;

delta2 = 15;

sigma\_1\_squared = 10;

sigma\_2\_squared = 20;

gammaEpsilon = -12;

% Variable range of Co

Co\_values = 80:10:120; % Assuming we analyze Co in increments of 10 from 80 to 120

% Initialize the result variable, storing the optimal T-value and minimum CT value for each Co

T\_optimal = zeros(size(Co\_values));

CT\_min = zeros(size(Co\_values));

% Optimization option

options = optimoptions('fmincon', 'Display', 'off', 'Algorithm', 'sqp');

% Iterate over the different values of Co

for i = 1:length(Co\_values)

 Co = Co\_values(i);

 sigma\_x = sqrt(sigma\_x\_squared); % Total standard deviation of demand

 % Define an objective function that takes T as its variable only

 objectiveFun = @(T) objectiveFunction(Co, Ch, Cs, T, L, z, mu\_x, sigma\_x, sigma\_x\_squared);

 % Set the initial guess, lower bound, and upper bound for T

 T0 = 1;

 T\_lb = 0.01;

 T\_ub = 10;

 % Optimization is performed to find the optimal T and the corresponding minimum CT

 [T\_optimal(i), CT\_min(i)] = fmincon(objectiveFun, T0, [], [], [], [], T\_lb, T\_ub, [], options);

end

% Create result table

T\_Co\_CT\_table = table(Co\_values', T\_optimal', CT\_min', 'VariableNames', {'Co', 'Optimal\_T', 'Min\_CT'});

% Display result table

disp(T\_Co\_CT\_table);

% Visualized result

figure;

plot(Co\_values, CT\_min, '-o');

title('Effect of Ordering Cost Co on Minimum Total Cost CT');

xlabel('Ordering Cost Co ($)');

ylabel('Minimum Total Cost CT ($)');

grid on;

% Sensitivity Ch

% Fixed parameters

Co = 100;

Cs = 40;

L = 2;

z = 1.645;

phi11 = 0.2;

phi12 = -0.4;

phi21 = -0.4;

phi22 = 0.2;

delta1 = 10;

delta2 = 15;

sigma\_1\_squared = 10;

sigma\_2\_squared = 20;

gammaEpsilon = -12;

% Variable range of Ch

Ch\_values = 2:1:10; % Assuming we analyze Ch in increments of 1 from 2 to 10

% Initialize the result variable, storing the optimal T-value and minimum CT value for each Ch

T\_optimal = zeros(size(Ch\_values));

CT\_min = zeros(size(Ch\_values));

% Optimization option

options = optimoptions('fmincon', 'Display', 'off', 'Algorithm', 'sqp');

% Iterate over the different values of Ch

for i = 1:length(Ch\_values)

 Ch = Ch\_values(i);

sigma\_x = sqrt(sigma\_x\_squared);

% Define an objective function that takes T as its variable only

 objectiveFun = @(T) objectiveFunction(Co, Ch, Cs, T, L, z, mu\_x, sigma\_x, sigma\_x\_squared);

 % Set the initial guess, lower bound, and upper bound for T

 T0 = 1;

 T\_lb = 0.01;

 T\_ub = 10;

 % Optimization is performed to find the optimal T and the corresponding minimum CT

 [T\_optimal(i), CT\_min(i)] = fmincon(objectiveFun, T0, [], [], [], [], T\_lb, T\_ub, [], options);

end

% Create result table

T\_Ch\_CT\_table = table(Ch\_values', T\_optimal', CT\_min', 'VariableNames', {'Ch', 'Optimal\_T', 'Min\_CT'});

% Display result table

disp(T\_Ch\_CT\_table);

% Visualized result

figure;

plot(Ch\_values, CT\_min, '-o');

title('Effect of Holding Cost Ch on Minimum Total Cost CT');

xlabel('Holding Cost Ch ($ per unit per time unit)');

ylabel('Minimum Total Cost CT ($ per time unit)');

grid on;

% Sensitivity Cs

% Fixed parameters

Co = 100;

Ch = 5;

L = 2;

z = 1.645;

phi11 = 0.2;

phi12 = -0.4;

phi21 = -0.4;

phi22 = 0.2;

delta1 = 10;

delta2 = 15;

sigma\_1\_squared = 10;

sigma\_2\_squared = 20;

gammaEpsilon = -12;

% Variable parameter Cs range

Cs\_values = 20:10:100; % Assuming we analyze Cs in increments of 10 from 20 to 100

% Initialize result variables to store optimal T and minimum CT for each Cs

T\_optimal = zeros(size(Cs\_values));

CT\_min = zeros(size(Cs\_values));

% Optimization options

options = optimoptions('fmincon', 'Display', 'off', 'Algorithm', 'sqp');

% Loop over different Cs values

for i = 1:length(Cs\_values)

 Cs = Cs\_values(i);

 % Define the objective function that only varies with T

 objectiveFun = @(T) objectiveFunction(Co, Ch, Cs, T, L, z, mu\_x, sigma\_x, sigma\_x\_squared);

 % Set initial guess, lower and upper bounds for T

 T0 = 1;

 T\_lb = 0.01;

 T\_ub = 10;

 % Perform optimization to find the optimal T and corresponding minimum CT

 [T\_optimal(i), CT\_min(i)] = fmincon(objectiveFun, T0, [], [], [], [], T\_lb, T\_ub, [], options);

end

% Create result table

T\_Cs\_CT\_table = table(Cs\_values', T\_optimal', CT\_min', 'VariableNames', {'Cs', 'Optimal\_T', 'Min\_CT'});

% Display the result table

disp(T\_Cs\_CT\_table);

% Visualize the results

figure;

plot(Cs\_values, CT\_min, '-o');

title('Effect of Shortage Cost Cs on Minimum Total Cost CT');

xlabel('Shortage Cost Cs ($ per unit short)');

ylabel('Minimum Total Cost CT ($ per time unit)');

grid on;

% Sensitivity L

% Fixed parameters

Co = 100;

Ch = 5;

Cs = 40;

z = 1.645;

phi11 = 0.2;

phi12 = -0.4;

phi21 = -0.4;

phi22 = 0.2;

delta1 = 10;

delta2 = 15;

sigma\_1\_squared = 10;

sigma\_2\_squared = 20;

gammaEpsilon = -12;

% Variable parameter L range

L\_values = 1:1:10; % Analyzing L from 1 to 10 days

% Initialize result variables to store optimal T and minimum CT for each L

T\_optimal = zeros(size(L\_values));

CT\_min = zeros(size(L\_values));

% Optimization options

options = optimoptions('fmincon', 'Display', 'off', 'Algorithm', 'sqp');

for i = 1:length(L\_values)

 L = L\_values(i);

 % Define the objective function that only varies with T

 objectiveFun = @(T) objectiveFunction(Co, Ch, Cs, T, L, z, mu\_x, sigma\_x, sigma\_x\_squared);

 % Set initial guess, lower and upper bounds for T

 T0 = 1;

 T\_lb = 0.01;

 T\_ub = 10;

 % Perform optimization to find the optimal T and corresponding minimum CT

 [T\_optimal(i), CT\_min(i)] = fmincon(objectiveFun, T0, [], [], [], [], T\_lb, T\_ub, [], options);

end

% Create result table

T\_L\_CT\_table = table(L\_values', T\_optimal', CT\_min', 'VariableNames', {'L', 'Optimal\_T', 'Min\_CT'});

% Display the result table

disp(T\_L\_CT\_table);

% Visualize the results

figure;

plot(L\_values, CT\_min, '-o');

title('Effect of Lead Time L on Minimum Total Cost CT');

xlabel('Lead Time L (days)');

ylabel('Minimum Total Cost CT ($ per time unit)');

grid on;

% Sensitivity z

% Fixed parameters

Co = 100;

Ch = 5;

Cs = 40;

L = 2;

phi11 = 0.2;

phi12 = -0.4;

phi21 = -0.4;

phi22 = 0.2;

delta1 = 10;

delta2 = 15;

sigma\_1\_squared = 10;

sigma\_2\_squared = 20;

gammaEpsilon = -12;

% Variable parameter z range

z\_values = 1.28:0.05:1.88; % Analyzing z from 90% to 97% service level in increments of 0.05

% Initialize result variables to store optimal T and minimum CT for each z

T\_optimal = zeros(size(z\_values));

CT\_min = zeros(size(z\_values));

% Optimization options

options = optimoptions('fmincon', 'Display', 'off', 'Algorithm', 'sqp');

for i = 1:length(z\_values)

 z = z\_values(i);

 % Define the objective function that only varies with T

 objectiveFun = @(T) objectiveFunction(Co, Ch, Cs, T, L, z, mu\_x, sigma\_x, sigma\_x\_squared);

 % Set initial guess, lower and upper bounds for T

 T0 = 1;

 T\_lb = 0.01;

 T\_ub = 10;

 % Perform optimization to find the optimal T and corresponding minimum CT

 [T\_optimal(i), CT\_min(i)] = fmincon(objectiveFun, T0, [], [], [], [], T\_lb, T\_ub, [], options);

end

% Create result table

T\_z\_CT\_table = table(z\_values', T\_optimal', CT\_min', 'VariableNames', {'z', 'Optimal\_T', 'Min\_CT'});

% Display the result table

disp(T\_z\_CT\_table);

% Visualize the results

figure;

plot(z\_values, CT\_min, '-o');

title('Effect of Safety Factor z on Minimum Total Cost CT');

xlabel('Safety Factor z');

ylabel('Minimum Total Cost CT ($ per time unit)');

grid on;

% Sensitivity phi11

% Fixed parameters

Co = 100;

Ch = 5;

Cs = 40;

L = 2;

z = 1.645;

phi12 = -0.4;

phi21 = -0.4;

phi22 = 0.2;

delta1 = 10;

delta2 = 15;

sigma\_1\_squared = 10;

sigma\_2\_squared = 20;

gammaEpsilon = -12;

% Variable parameter phi11 range

phi11\_values = -0.3:0.05:0.3; % Analyzing phi11 in increments of 0.05 from -0.3 to 0.3

% Initialize result variables to store optimal T and minimum CT for each phi11

T\_optimal = zeros(size(phi11\_values));

CT\_min = zeros(size(phi11\_values));

% Optimization options

options = optimoptions('fmincon', 'Display', 'off', 'Algorithm', 'sqp');

% Loop over different phi11 values

for i = 1:length(phi11\_values)

 phi11 = phi11\_values(i);

 % Define the objective function that only varies with T

 objectiveFun = @(T) objectiveFunction(Co, Ch, Cs, T, L, z, mu\_x, sigma\_x, sigma\_x\_squared);

 % Set initial guess, lower and upper bounds for T

 T0 = 1; % Initial guess

 T\_lb = 0.01; % Lower bound

 T\_ub = 10; % Upper bound

 % Perform optimization to find the optimal T and corresponding minimum CT

 [T\_optimal(i), CT\_min(i)] = fmincon(objectiveFun, T0, [], [], [], [], T\_lb, T\_ub, [], options);

end

% Create result table

T\_phi11\_CT\_table = table(phi11\_values', T\_optimal', CT\_min', 'VariableNames', {'phi11', 'Optimal\_T', 'Min\_CT'});

% Display the result table

disp(T\_phi11\_CT\_table);

% Visualize the results

figure;

plot(phi11\_values, CT\_min, '-o');

title('Effect of Parameter phi11 on Minimum Total Cost CT');

xlabel('Parameter phi11');

ylabel('Minimum Total Cost CT ($ per time unit)');

grid on;

% Sensitivity phi12

% Fixed parameters

Co = 100;

Ch = 5;

Cs = 40;

L = 2;

z = 1.645;

phi11 = 0.2;

phi21 = -0.4;

phi22 = 0.2;

delta1 = 10;

delta2 = 15;

sigma\_1\_squared = 10;

sigma\_2\_squared = 20;

gammaEpsilon = -12;

% Variable parameter phi12 range

phi12\_values = -0.6:0.05:-0.2; % Analyzing phi12 from -0.6 to -0.2 in decrements of 0.05, ensuring negative values

% Initialize result variables to store optimal T and minimum CT for each phi12

T\_optimal = zeros(size(phi12\_values));

CT\_min = zeros(size(phi12\_values));

% Optimization options

options = optimoptions('fmincon', 'Display', 'off', 'Algorithm', 'sqp');

% Loop over different phi12 values

for i = 1:length(phi12\_values)

 phi12 = phi12\_values(i);

 % Define the objective function that only varies with T

 objectiveFun = @(T) objectiveFunction(Co, Ch, Cs, T, L, z, mu\_x, sigma\_x, sigma\_x\_squared);

 % Set initial guess, lower and upper bounds for T

 T0 = 1;

 T\_lb = 0.01;

 T\_ub = 10;

 % Perform optimization to find the optimal T and corresponding minimum CT

 [T\_optimal(i), CT\_min(i)] = fmincon(objectiveFun, T0, [], [], [], [], T\_lb, T\_ub, [], options);

end

% Create result table

T\_phi12\_CT\_table = table(phi12\_values', T\_optimal', CT\_min', 'VariableNames', {'phi12', 'Optimal\_T', 'Min\_CT'});

% Display the result table

disp(T\_phi12\_CT\_table);

% Visualize the results

figure;

plot(phi12\_values, CT\_min, '-o');

title('Effect of Parameter phi12 on Minimum Total Cost CT');

xlabel('Parameter phi12');

ylabel('Minimum Total Cost CT ($ per time unit)');

grid on;

% Sensitivity phi21

% Fixed parameters

Co = 100;

Ch = 5;

Cs = 40;

L = 2;

z = 1.645;

phi11 = 0.2;

phi12 = -0.4;

phi22 = 0.2;

delta1 = 10;

delta2 = 15;

sigma\_1\_squared = 10;

sigma\_2\_squared = 20;

gammaEpsilon = -12;

% Variable parameter phi21 range

phi21\_values = -0.6:0.05:-0.2; % Analyzing phi21 from -0.6 to -0.2 in increments of 0.05, ensuring negative values

% Initialize result variables to store optimal T and minimum CT for each phi21

T\_optimal = zeros(size(phi21\_values));

CT\_min = zeros(size(phi21\_values));

% Optimization options

options = optimoptions('fmincon', 'Display', 'off', 'Algorithm', 'sqp');

% Loop over different phi21 values

for i = 1:length(phi21\_values)

 phi21 = phi21\_values(i);

 % Define the objective function that only varies with T

 objectiveFun = @(T) objectiveFunction(Co, Ch, Cs, T, L, z, mu\_x, sigma\_x, sigma\_x\_squared);

 % Set initial guess, lower and upper bounds for T

 T0 = 1;

 T\_lb = 0.01;

 T\_ub = 10;

 % Perform optimization to find the optimal T and corresponding minimum CT

 [T\_optimal(i), CT\_min(i)] = fmincon(objectiveFun, T0, [], [], [], [], T\_lb, T\_ub, [], options);

end

% Create result table

T\_phi21\_CT\_table = table(phi21\_values', T\_optimal', CT\_min', 'VariableNames', {'phi21', 'Optimal\_T', 'Min\_CT'});

% Display the result table

disp(T\_phi21\_CT\_table);

% Visualize the results

figure;

plot(phi21\_values, CT\_min, '-o');

title('Effect of Parameter phi21 on Minimum Total Cost CT');

xlabel('Parameter phi21');

ylabel('Minimum Total Cost CT ($ per time unit)');

grid on;

% Sensitivity phi22

% Fixed parameters

Co = 100;

Ch = 5;

Cs = 40;

L = 2;

z = 1.645;

phi11 = 0.2;

phi12 = -0.4;

phi21 = -0.4;

delta1 = 10;

delta2 = 15;

sigma\_1\_squared = 10;

sigma\_2\_squared = 20;

gammaEpsilon = -12;

% Variable parameter phi22 range

phi22\_values = -0.3:0.05:0.3; % Analyzing phi22 from -0.3 to 0.3 in increments of 0.05

% Initialize result variables to store optimal T and minimum CT for each phi22

T\_optimal = zeros(size(phi22\_values));

CT\_min = zeros(size(phi22\_values));

% Optimization options

options = optimoptions('fmincon', 'Display', 'off', 'Algorithm', 'sqp');

% Loop over different phi22 values

for i = 1:length(phi22\_values)

 phi22 = phi22\_values(i);

 % Define the objective function that only varies with T

 objectiveFun = @(T) objectiveFunction(Co, Ch, Cs, T, L, z, mu\_x, sigma\_x, sigma\_x\_squared);

 % Set initial guess, lower and upper bounds for T

 T0 = 1;

 T\_lb = 0.01;

 T\_ub = 10;

 % Perform optimization to find the optimal T and corresponding minimum CT

 [T\_optimal(i), CT\_min(i)] = fmincon(objectiveFun, T0, [], [], [], [], T\_lb, T\_ub, [], options);

end

% Create result table

T\_phi22\_CT\_table = table(phi22\_values', T\_optimal', CT\_min', 'VariableNames', {'phi22', 'Optimal\_T', 'Min\_CT'});

% Display the result table

disp(T\_phi22\_CT\_table);

% Visualize the results

figure;

plot(phi22\_values, CT\_min, '-o');

title('Effect of Parameter phi22 on Minimum Total Cost CT');

xlabel('Parameter phi22');

ylabel('Minimum Total Cost CT ($ per time unit)');

grid on;

% Sensitivity delta1

% Fixed parameters

Co = 100;

Ch = 5;

Cs = 40;

L = 2;

z = 1.645;

phi11 = 0.2;

phi12 = -0.4;

phi21 = -0.4;

phi22 = 0.2;

delta2 = 15;

sigma\_1\_squared = 10;

sigma\_2\_squared = 20;

gammaEpsilon = -12;

% Variable parameter delta1 range

delta1\_values = 5:1:15; % Analyzing delta1 from 5 to 15 in increments of 1

% Initialize result variables to store optimal T and minimum CT for each delta1

T\_optimal = zeros(size(delta1\_values));

CT\_min = zeros(size(delta1\_values));

% Optimization options

options = optimoptions('fmincon', 'Display', 'off', 'Algorithm', 'sqp');

% Loop over different delta1 values

for i = 1:length(delta1\_values)

 delta1 = delta1\_values(i);

 % Define the objective function that only varies with T

 objectiveFun = @(T) objectiveFunction(Co, Ch, Cs, T, L, z, mu\_x, sigma\_x, sigma\_x\_squared);

 % Set initial guess, lower and upper bounds for T

 T0 = 1;

 T\_lb = 0.01;

 T\_ub = 10;

 % Perform optimization to find the optimal T and corresponding minimum CT

 [T\_optimal(i), CT\_min(i)] = fmincon(objectiveFun, T0, [], [], [], [], T\_lb, T\_ub, [], options);

end

% Create result table

T\_delta1\_CT\_table = table(delta1\_values', T\_optimal', CT\_min', 'VariableNames', {'delta1', 'Optimal\_T', 'Min\_CT'});

% Display the result table

disp(T\_delta1\_CT\_table);

% Visualize the results

figure;

plot(delta1\_values, CT\_min, '-o');

title('Effect of Baseline Demand delta1 on Minimum Total Cost CT');

xlabel('Baseline Demand delta1');

ylabel('Minimum Total Cost CT ($ per time unit)');

grid on;

% Sensitivity delta2

% Fixed parameters

Co = 100;

Ch = 5;

Cs = 40;

L = 2;

z = 1.645;

phi11 = 0.2;

phi12 = -0.4;

phi21 = -0.4;

phi22 = 0.2;

delta1 = 10;

sigma\_1\_squared = 10;

sigma\_2\_squared = 20;

gammaEpsilon = -12;

% Variable parameter delta2 range

delta2\_values = 15:1:25; % Analyzing delta2 from 15 to 25 in increments of 1

% Initialize result variables to store optimal T and minimum CT for each delta2

T\_optimal = zeros(size(delta2\_values));

CT\_min = zeros(size(delta2\_values));

% Optimization options

options = optimoptions('fmincon', 'Display', 'off', 'Algorithm', 'sqp');

% Loop over different delta2 values

for i = 1:length(delta2\_values)

 delta2 = delta2\_values(i);

 % Define the objective function that only varies with T

 objectiveFun = @(T) objectiveFunction(Co, Ch, Cs, T, L, z, mu\_x, sigma\_x, sigma\_x\_squared);

 % Set initial guess, lower and upper bounds for T

 T0 = 1;

 T\_lb = 0.01;

 T\_ub = 10;

 % Perform optimization to find the optimal T and corresponding minimum CT

 [T\_optimal(i), CT\_min(i)] = fmincon(objectiveFun, T0, [], [], [], [], T\_lb, T\_ub, [], options);

end

% Create result table

T\_delta2\_CT\_table = table(delta2\_values', T\_optimal', CT\_min', 'VariableNames', {'delta2', 'Optimal\_T', 'Min\_CT'});

% Display the result table

disp(T\_delta2\_CT\_table);

% Visualize the results

figure;

plot(delta2\_values, CT\_min, '-o');

title('Effect of Baseline Demand delta2 on Minimum Total Cost CT');

xlabel('Baseline Demand delta2');

ylabel('Minimum Total Cost CT ($ per time unit)');

grid on;

% Sensitivity sigma\_1\_squared

% Fixed parameters

Co = 100;

Ch = 5;

Cs = 40;

L = 2;

z = 1.645;

phi11 = 0.2;

phi12 = -0.4;

phi21 = -0.4;

phi22 = 0.2;

delta1 = 10;

delta2 = 15;

sigma\_2\_squared = 20;

gammaEpsilon = -12;

% Variable parameter sigma\_1\_squared range

sigma\_1\_squared\_values = 5:5:30; % Analyzing sigma\_1\_squared from 5 to 30 in increments of 5

% Initialize result variables to store optimal T and minimum CT for each sigma\_1\_squared

T\_optimal = zeros(size(sigma\_1\_squared\_values));

CT\_min = zeros(size(sigma\_1\_squared\_values));

% Optimization options

options = optimoptions('fmincon', 'Display', 'off', 'Algorithm', 'sqp');

% Loop over different sigma\_1\_squared values

for i = 1:length(sigma\_1\_squared\_values)

 sigma\_1\_squared = sigma\_1\_squared\_values(i);

% If sigma\_x\_squared is not positive, log a warning and use NaN for this iteration's results

 if sigma\_x\_squared <= 0

 warning('Negative or zero sigma\_x\_squared encountered for gammaEpsilon = %f. Skipping this iteration.', gammaEpsilon);

 T\_optimal(i) = NaN; % Indicate that this iteration's result is invalid

 CT\_min(i) = NaN; % Indicate that this iteration's result is invalid

 continue; % Skip to the next iteration of the loop

 end

 % Proceed with the remaining calculations knowing sigma\_x\_squared is positive

 sigma\_x = sqrt(sigma\_x\_squared);

 % Define the objective function that only varies with T

 objectiveFun = @(T) objectiveFunction(Co, Ch, Cs, T, L, z, mu\_x, sigma\_x, sigma\_x\_squared);

 % Set initial guess, lower and upper bounds for T

 T0 = 1;

 T\_lb = 0.01;

 T\_ub = 10;

 % Perform optimization to find the optimal T and corresponding minimum CT

 [T\_optimal(i), CT\_min(i)] = fmincon(objectiveFun, T0, [], [], [], [], T\_lb, T\_ub, [], options);

end

% Create result table

T\_sigma1\_CT\_table = table(sigma\_1\_squared\_values', T\_optimal', CT\_min', 'VariableNames', {'sigma\_1\_squared', 'Optimal\_T', 'Min\_CT'});

% Display the result table

disp(T\_sigma1\_CT\_table);

% Visualize the results

figure;

plot(sigma\_1\_squared\_values, CT\_min, '-o');

title('Effect of Demand Variance sigma\_1\_squared on Minimum Total Cost CT');

xlabel('Demand Variance sigma\_1\_squared');

ylabel('Minimum Total Cost CT ($ per time unit)');

grid on;

% Sensitivity sigma\_2\_squared

% Fixed parameters

Co = 100;

Ch = 5;

Cs = 40;

L = 2;

z = 1.645;

phi11 = 0.2;

phi12 = -0.4;

phi21 = -0.4;

phi22 = 0.2;

delta1 = 10;

delta2 = 15;

sigma\_1\_squared = 10;

gammaEpsilon = -12;

% Variable parameter sigma\_2\_squared range

sigma\_2\_squared\_values = 15:5:40; % Analyzing sigma\_2\_squared from 15 to 40 in increments of 5

% Initialize result variables to store optimal T and minimum CT for each sigma\_2\_squared

T\_optimal = zeros(size(sigma\_2\_squared\_values));

CT\_min = zeros(size(sigma\_2\_squared\_values));

% Optimization options

options = optimoptions('fmincon', 'Display', 'off', 'Algorithm', 'sqp');

% Loop over different sigma\_2\_squared values

for i = 1:length(sigma\_2\_squared\_values)

 sigma\_2\_squared = sigma\_2\_squared\_values(i);

% If sigma\_x\_squared is not positive, log a warning and use NaN for this iteration's results

 if sigma\_x\_squared <= 0

 warning('Negative or zero sigma\_x\_squared encountered for gammaEpsilon = %f. Skipping this iteration.', gammaEpsilon);

 T\_optimal(i) = NaN; % Indicate that this iteration's result is invalid

 CT\_min(i) = NaN; % Indicate that this iteration's result is invalid

 continue; % Skip to the next iteration of the loop

 end

 % Proceed with the remaining calculations knowing sigma\_x\_squared is positive

 sigma\_x = sqrt(sigma\_x\_squared);

 % Define the objective function that only varies with T

 objectiveFun = @(T) objectiveFunction(Co, Ch, Cs, T, L, z, mu\_x, sigma\_x, sigma\_x\_squared);

 % Set initial guess, lower and upper bounds for T

 T0 = 1;

 T\_lb = 0.01;

 T\_ub = 10;

 % Perform optimization to find the optimal T and corresponding minimum CT

 [T\_optimal(i), CT\_min(i)] = fmincon(objectiveFun, T0, [], [], [], [], T\_lb, T\_ub, [], options);

end

% Create result table

T\_sigma2\_CT\_table = table(sigma\_2\_squared\_values', T\_optimal', CT\_min', 'VariableNames', {'sigma\_2\_squared', 'Optimal\_T', 'Min\_CT'});

% Display the result table

disp(T\_sigma2\_CT\_table);

% Visualize the results

figure;

plot(sigma\_2\_squared\_values, CT\_min, '-o');

title('Effect of Demand Variance sigma\_2\_squared on Minimum Total Cost CT');

xlabel('Demand Variance sigma\_2\_squared');

ylabel('Minimum Total Cost CT ($ per time unit)');

grid on;

% Sensitivity gammaEpsilon

% Fixed parameters

Co = 100;

Ch = 5;

Cs = 40;

L = 2;

z = 1.645;

phi11 = 0.2;

phi12 = -0.4;

phi21 = -0.4;

phi22 = 0.2;

delta1 = 10;

delta2 = 15;

sigma\_1\_squared = 10;

sigma\_2\_squared = 20;

% Variable parameter gammaEpsilon range

gammaEpsilon\_values = -15:1:-1; % Analyzing gammaEpsilon from -15 to -1 in increments of 1

% Initialize result variables to store optimal T and minimum CT for each gammaEpsilon

T\_optimal = zeros(size(gammaEpsilon\_values));

CT\_min = zeros(size(gammaEpsilon\_values));

% Optimization options

options = optimoptions('fmincon', 'Display', 'off', 'Algorithm', 'sqp');

% Loop over different gammaEpsilon values

for i = 1:length(gammaEpsilon\_values)

 gammaEpsilon = gammaEpsilon\_values(i);

% If sigma\_x\_squared is not positive, log a warning and use NaN for this iteration's results

 if sigma\_x\_squared <= 0

 warning('Negative or zero sigma\_x\_squared encountered for gammaEpsilon = %f. Skipping this iteration.', gammaEpsilon);

 T\_optimal(i) = NaN; % Indicate that this iteration's result is invalid

 CT\_min(i) = NaN; % Indicate that this iteration's result is invalid

 continue; % Skip to the next iteration of the loop

 end

 % Proceed with the remaining calculations knowing sigma\_x\_squared is positive

 sigma\_x = sqrt(sigma\_x\_squared);

 % Define the objective function that only varies with T

 objectiveFun = @(T) objectiveFunction(Co, Ch, Cs, T, L, z, mu\_x, sigma\_x, sigma\_x\_squared);

 % Set initial guess, lower and upper bounds for T

 T0 = 1;

 T\_lb = 0.01;

 T\_ub = 10;

 % Perform optimization to find the optimal T and corresponding minimum CT

 [T\_optimal(i), CT\_min(i)] = fmincon(objectiveFun, T0, [], [], [], [], T\_lb, T\_ub, [], options);

end

% Create result table

T\_gammaEpsilon\_CT\_table = table(gammaEpsilon\_values', T\_optimal', CT\_min', 'VariableNames', {'gammaEpsilon', 'Optimal\_T', 'Min\_CT'});

% Display the result table

disp(T\_gammaEpsilon\_CT\_table);

% Visualize the results

figure;

plot(gammaEpsilon\_values, CT\_min, '-o');

title('Effect of Covariance gammaEpsilon on Minimum Total Cost CT');

xlabel('Covariance gammaEpsilon (negative values)');

ylabel('Minimum Total Cost CT ($ per time unit)');

grid on;