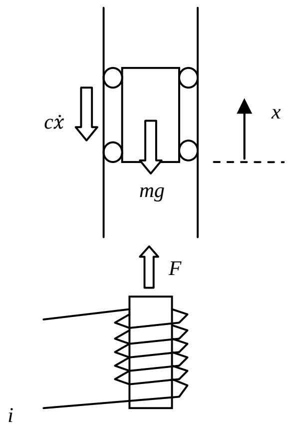
Midterm Examination **Control Theory AT74.02** **October 1, 2013**

Time: 10:00-12:00 hrs. Open Book
Marks: 100

Attempt all questions.

Repulsive electromagnetic force, *F*, which linearly varies with the supplied current, *i*, is used to lift a magnetic slider with mass, *m*, under the viscous friction with viscosity coefficient, *c*, as shown in the below figure.



The relation between the supplied current, *i*, and the slider position, *x*, can be expressed by

$$k_f i - mg - c\dot{x} = m\ddot{x}$$

when k_f denotes the electromagnetic force constant.

At the equilibrium reference point where $x = 0$, $k_f i_0 = mg$. By defining $i = i_1 + i_0$, where i_1 denotes additional current from the reference point, then the relation is simplified to

$$k_f i_1 - c\dot{x} = m\ddot{x}$$

From system identification, m = 400 grams, g = 10 m/s^2, c = 20 Ns/m, and k_f = 500 N/A.

(a) Determine transfer function, $G(s)$, from additional current, i_1, to slider position, *x*, of this system. What are the roots of the characteristic equation? Is this system asymptotically stable, neutrally stable, or unstable? (10)

$$k_f i_1 = m\ddot{x} + c\dot{x} \quad (1)$$

By taking Laplace transformation and neglect all the initial conditions,

$$k_f I_1 = (ms^2 + cs)X \quad (2)$$

$$\frac{X}{I_1} = G = \frac{k_f}{ms^2+cs} = \frac{500}{0.4s^2+20s} = \frac{1250}{s^2+50s} = \frac{1250}{(s+50)s} \quad (3)$$

Roots of the characteristic equation,

$$s = 0, -50 \qquad (4)$$

The system is neutrally stable.

(b) Determine the slider position in meter as a function of time, $x(t)$, when the additional current is represented by $i_1(t) = 2e^{-4t}\ mA$, assume the slider rests at the equilibrium reference point at the beginning. Then determine the slider position at the steady state. (10)

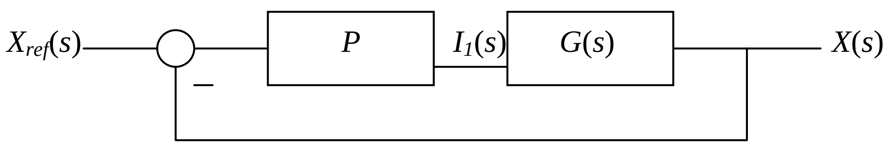
$$X(s) = \frac{1250}{(s+50)s} \cdot \frac{2\times10^{-3}}{(s+4)} = \left[\frac{12.500}{s} + \frac{1.087}{s+50} - \frac{13.587}{s+4}\right] \times 10^{-3} \qquad (5)$$

Take the inverse Laplace transformation,

$$x(t) = [12.500 + 1.087e^{-50t} - 13.587e^{-4t}] \times 10^{-3}\ m \qquad (6)$$

$$x_{ss}(t) = 12.500 \times 10^{-3}\ m \qquad (7)$$

(c) If a P controller is applied to control the slider position as shown in the below block diagram. Plot the root-locus diagram, Nyquist diagram, and Bode diagram of the system in order to check system stability. In the root-locus diagram determine open-loop zero, pole, slopes of asymptotic lines and their intersection point, break-away/in point and its gain, cross over point and its gain. In the Nyquist diagram, determine points O, A and direction that the Nyquist diagram approaches origin point. In the Bode diagram, determine all the zero, pole, and dc gain. Roughly plot Bode diagram of each zero, pole, and dc gain of the open loop transfer function. (30)



$$G = \frac{1250}{(s+50)s} \qquad (8)$$

Root locus diagram,

$$Zero = \emptyset \qquad (9)$$

$$Pole = 0, -50 \qquad (10)$$

$$e = 2 \qquad (11)$$

$$\measuredangle = \pm 90° \qquad (12)$$

$$Intersection\ point = -25 \qquad (13)$$

Closed loop transfer function, $T_c(s)$, is determined.

$$T_c(s) = \frac{1250K}{s^2+50s+1250K} \qquad (14)$$

Break-away point is considered from characteristic polynomial.

$$P = s^2 + 50s + 1250K \tag{15}$$

$$\acute{P} = 2s + 50 = 0 \tag{16}$$

$$s = -25 \tag{17}$$

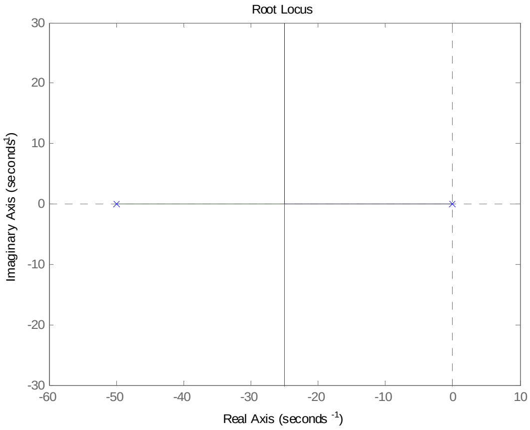
$$0 = (-25)^2 + 50(-25) + 1250K \tag{18}$$

$$K = 0.5 \tag{19}$$

Cross-over point is obtained by substitution of s with $j\omega$ in the characteristic equation.

$$0 = 1250K - \omega^2 + 50\omega j \tag{20}$$

$$\omega = 0, K = 0 \tag{21}$$



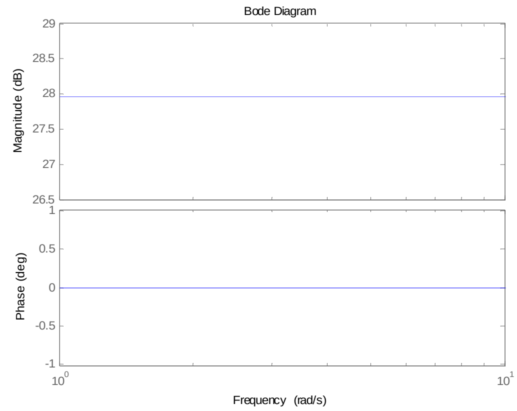
Nyquist diagram

Point	*s*	*G(s)*
O	*0,+0j*	$\frac{1250}{50s} = \infty, -\infty j$
A	ωj	$\frac{1250}{-\omega^2+50\omega j} = \frac{-1250(\omega^2+50\omega j)}{\omega^4+(50\omega)^2}$
	∞j	$-0 - 0j$

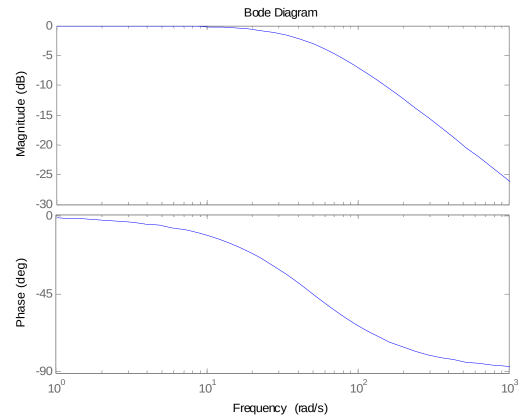
Bode diagram,

$$G = \frac{1250}{(s+50)s} = \frac{25}{(s/50+1)s} = \frac{G_1}{G_2 G_3} \tag{22}$$

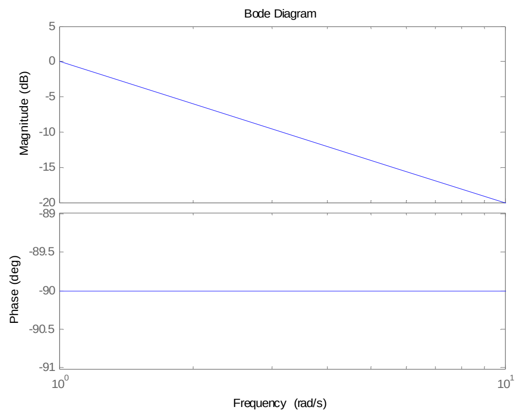
$$G_1 = 25 \tag{23}$$



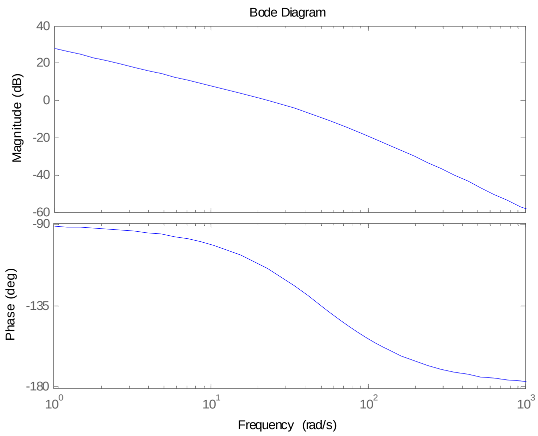
$$G_2 = \frac{1}{(s/50+1)} \tag{24}$$



$$G_3 = \frac{1}{s} \tag{25}$$



$$G = \frac{25}{(s/50+1)s} = \frac{G_1}{G_2 G_3} \tag{26}$$



(d) If a PID controller is applied to control the slider as shown in the below block diagram. Design all the gains that place one pole at -100, and the remaining conjugate poles to have 0.5 damping ratio with natural frequency of 100 rad/s. (15)

$X_{ref}(s)$ *PID* $I_1(s)$ $G(s)$ $X(s)$

Closed loop transfer function, $T_c(s)$, is determined.

$$T_c(s) = \frac{1250K_D s^2 + 1250K_P s + 1250K_I}{s^3 + (1250K_D + 50)s^2 + 1250K_P s + 1250K_I} \quad (27)$$

Characteristic equation,

$$s^3 + (1250K_D + 50)s^2 + 1250K_P s + 1250K_I = 0 \quad (28)$$

The desired characteristic equation,

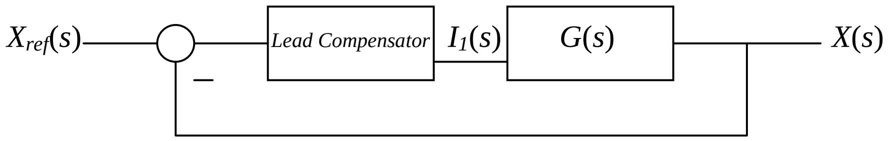
$$(s + 100)(s^2 + 100s + 10000) = s^3 + 200s^2 + 20000s + 1000000 = 0 \quad (29)$$

$$K_I = 800 \quad (30)$$

$$K_P = 16 \quad (31)$$

$$K_D = 0.12 \quad (32)$$

(e) If a lead compensator replaces PID controller to control the slider position as shown in the below block diagram. Design all the parameters of the lead compensator and its gain that achieve the same characteristic equation as PID controller has achieved in question (d). (15)



Closed loop transfer function, $T_c(s)$, is determined.

$$T_c(s) = \frac{1250Ks + \frac{1250K}{aT}}{s^3 + \left(\frac{1}{T} + 50\right)s^2 + (1250K + \frac{50}{T})s + \frac{1250K}{aT}} \tag{33}$$

Characteristic equation,

$$s^3 + \left(\frac{1}{T} + 50\right)s^2 + (1250K + \frac{50}{T})s + \frac{1250K}{aT} = 0 \tag{34}$$

The desired characteristic equation,

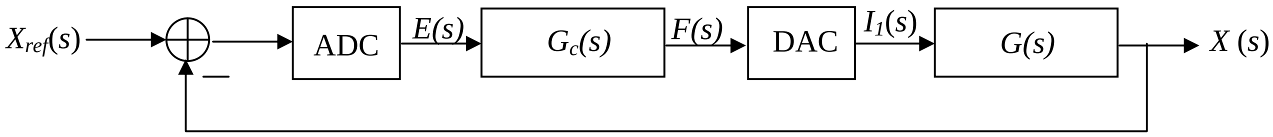
$$(s + 100)(s^2 + 100s + 10000) = s^3 + 200s^2 + 20000s + 1000000 = 0 \tag{35}$$

$$T = 0.0067 \tag{36}$$

$$K = 10 \tag{37}$$

$$a = 1.875 \tag{38}$$

(f) If a digital controller, $G_c(s)$, as shown in the block diagram below is used in the system, design the controller by direct design method when the desired closed loop transfer function, T_c, is represented in s domain by $\frac{1-e^{-0.001s}}{s}\cdot\frac{100}{s+100}$, which is a first order transfer function with zero order hold circuit having unity sensitivity, and time constant of 0.01 sec. The sampling time, T, is 0.001 sec. Then determine the control signal at step k, $f(k)$, as a function of control signal at previous steps and error, e, at the current and previous steps. (20)



Firstly determine plant in combination with zero-order hold circuit DAC,

$$G_2(s) = \frac{1}{s}\cdot G(s) = \frac{1250}{s^3 + 50s^2} = \frac{0.5}{s+50} - \frac{0.5}{s} + \frac{25}{s^2} \tag{39}$$

$$G_2(z) = \frac{0.5z}{z-0.9512} - \frac{0.5z}{z-1} + \frac{0.025z}{(z-1)^2} \tag{40}$$

$$G = \frac{z-1}{z}G_2(z) = \frac{0.5(z-1)}{z-0.9512} - 0.5 + \frac{0.025}{z-1} = \frac{0.0006z + 0.0006}{(z-0.9512)(z-1)} \tag{41}$$

$$T_c(s) = \frac{1-e^{-0.001s}}{s} \cdot \frac{100}{s+100} \quad (42)$$

$$\frac{100}{s(s+100)} = \frac{1}{s} - \frac{1}{s+100} \quad (43)$$

Take Z transformation,

$$\frac{1}{s} - \frac{1}{s+100} \Rightarrow \frac{z}{z-1} - \frac{z}{z-0.9048} \quad (44)$$

$$T_c(z) = 1 - \frac{z-1}{z-0.9048} = \frac{0.0952}{z-0.9048} \quad (45)$$

$$G_D = \frac{T_c}{G(1-T_c)} = \frac{\dfrac{0.0952}{z-0.9048}}{\dfrac{0.0006z+0.0006}{(z-0.9512)(z-1)}\left(1-\dfrac{0.0952}{z-0.9048}\right)} \quad (46)$$

$$G_D = \frac{f}{e} = \frac{0.0952z-0.0906}{0.0006z+0.0006} = \frac{158.67z-151}{z+1} \quad \underline{(47)}$$

$$(z+1)f = (158.67z-151)e \quad (48)$$

$$(1+\frac{1}{z})f = (158.67-\frac{151}{z})e \quad (49)$$

$$f(k) = -f(k-1) + 158.67e(k) - 151e(k-1) \quad \underline{(50)}$$