Final Examination **Control Theory AT74.02** **November 24, 2017**

Time: 9:00-11:00 hrs. Open Book

Marks: 100

Attempt all questions.

Consider again the piezo actuator given in the mid semester examination, voltage across the piezo electrodes, v, is used to control deflection of the actuator, x. The relation between the voltage (in Volt) and deflection (in micro meter) is expressed by the following transfer function.

$$\frac{x}{v} = \frac{0.15}{0.001s + 1}$$

(a) Determine a state-space representation of the system in the Jordan form having the input matrix $B = [1]$ and when the actuator deflection is measured directly. (20)

Solution

$$\frac{x}{v} = \frac{y}{u} = \frac{0.15}{0.001s+1} = \frac{150}{s+1000} \quad (1)$$

Thus,

$$[\dot{x}_1] = [-1000][x_1] + [1][u] \quad \underline{(2)}$$

$$[y] = [150][x_1] \quad \underline{(3)}$$

(b) If the actuator is controlled to deflect at a constant distance, x_r, remodel the state-space system by taking into consideration the references. (20)

Solution

With exogenous input, the system is remodeled as,

$$\dot{e} = Ae + Bu + (A - A_r)x_r + Fx_d = Ae + Bu + Ex_0 \quad (1)$$

When

$$[y] = [x_r] = [150][x_{1r}] \quad (2)$$

$$[x_{1r}] = \left[\frac{x_r}{150}\right] \quad (3)$$

$$[e_1] = [x_1 - x_{1r}] = \left[x_1 - \frac{x_r}{150}\right] \quad (4)$$

$$E = A - A_r = [-1000] - [0] = [-1000] \quad (5)$$

Thus,

$$[\dot{e}_1] = [-1000][e_1] + [1][u] + [-1000]\left[\frac{x_r}{150}\right] \quad \underline{(6)}$$

$$[y] = [150][e_1] \quad \underline{(7)}$$

(c) If the voltage across the electrode, v, composes of the voltage used to keep the actuator at the desired deflection, v_1, and the remaining voltage, v_2, and the cost function is expressed by $V = \int_0^\infty (Q(x - x_r)^2 +$ $v22dt$, determine the constant voltage, $v1$, as a function of the desired deflection and the varied voltage, $v2$, as a function of the deflection error that minimizes the cost function and places the regulated pole to have the time constant at 0.5 millisecond. Determine the weight Q. (20)

Solution

At steady state

$$0 = Bv_1 + Ex_0 \tag{1}$$

$$v_1 = -B^{-1}Ex_0 = 1000x_{1r} = 1000\frac{x_r}{150} = 6.67x_r \tag{2}$$

$$V = \int_0^\infty (Q(x - x_r)^2 + v_2^2)dt = \int_0^\infty (150^2 Q(e_1)^2 + v_2^2)dt \tag{3}$$

$$Q = [22500Q] \tag{4}$$

$$R = [1] \tag{5}$$

Control signal is determined from

$$v_2 = -Ge_1 = -R^{-1}B^t\overline{M}e_1 \tag{6}$$

$$G = R^{-1}B^t\overline{M} = [1]^{-1}[1]^t[m_1] = [m_1] \tag{7}$$

When

$$0 = -\dot{\overline{M}} = \overline{M}A + A^t\overline{M} - \overline{M}BR^{-1}B^t\overline{M} + Q \tag{8}$$

Substitute all the concerned matrices,

$$[0] = [m_1][-1000] + [-1000][m_1] - [m_1][1][m_1] + [22500Q] \tag{9}$$

$$[0] = [-2000m_1] - [m_1^2] + [22500Q] \tag{10}$$

Thus,

$$[m_1] = \left[100\sqrt{100 + 2.25Q} - 1000\right] \tag{11}$$

$$G = R^{-1}B^t\overline{M} = \left[100\sqrt{100 + 2.25Q} - 1000\right] \tag{12}$$

$$|sI - A_c| = \left|[s] - [-1000] + [1]\left[100\sqrt{100 + 2.25Q} - 1000\right]\right| \tag{13}$$

$$|sI - A_c| = s + 1000 + 100\sqrt{100 + 2.25Q} - 1000 \tag{14}$$

The desired pole has 0.5 millisecond time constant.

$$|sI - A_c| = s + 1000 + 100\sqrt{100 + 2.25Q} - 1000 = s + 2000 \tag{15}$$

$$Q = 133.33 \tag{16}$$

$$G = R^{-1}B^t\overline{M} = [1000] \tag{17}$$

$$v_2 = -1000e_1 = -1000 \times 150(x - x_r) = -150000(x - x_r) \tag{18}$$

(d) If the voltage across the piezo electrode contains Gaussian white noise disturbance, d, with power spectral density of V, and the output reading of the deflection sensor is contaminated by Gaussian white noise, w, with power spectral density of W, Determine Kalman filter gain to estimate the state error, characteristic equation of the optimal observer, and the pole as functions of V and W. (20)

Solution

$$[\dot{e}_1] = [-1000][e_1] + [1][u] + [-1000]\left[\frac{x_r}{150}\right] + [1][d] \quad (1)$$

$$[y] = [150][e_1] + [w] \quad (2)$$

$$K = \bar{P}C^t W^{-1} \quad (3)$$

$$K = [p_1][150]^t[W]^{-1} = \left[\frac{150p_1}{W}\right] \quad (4)$$

When

$$0 = \dot{\bar{P}} = A\bar{P} + \bar{P}A^t - KC\bar{P} + FVF^t \quad (5)$$

Substitute all the concerned matrices,

$$[0] = [-1000][p_1] + [p_1][-1000] - \left[\frac{150\ _1}{W}\right][150][p_1] + [1][V][1] \quad (6)$$

$$[0] = \left[\frac{-22500p_1^2}{W} - 2000p_1 - 2000p_2 + V\right] \quad (7)$$

Thus,

$$[p_1] = \left[\left[\sqrt{4000000 + 90000(\frac{V}{W})} - 2000\right]\frac{W}{45000}\right] \quad (8)$$

$$K = \left[\sqrt{\frac{400}{9} + (\frac{V}{W})} - \frac{20}{3}\right] \quad \underline{(9)}$$

Determine characteristic equation of the Kalman filter,

$$\left|sI - \hat{A}\right| = |sI - A + KC| = \left|[s] - [-1000] + \left[\sqrt{\frac{400}{9} + (\frac{V}{W})} - \frac{20}{3}\right][150]\right| \quad (10)$$

$$\left|sI - \hat{A}\right| = \left|s + 150\sqrt{\frac{400}{9} + (\frac{V}{W})}\right| = s + 150\sqrt{\frac{400}{9} + (\frac{V}{W})} = 0 \quad \underline{(11)}$$

(e) Draw the block diagram in the state-space form showing the detail of compensator and the piezo actuator system. Indicate the voltage across the electrode, the desired deflection, the actual deflection, disturbance, noise, all connections of signals, all LQR and Kalman filter gains in the diagram. (20)

Solution