Midterm Examination **Control Theory AT74.02** **September 26, 2017**

Time: 9:00-11:00 hrs. Open Book
Marks: 100

Attempt all questions.

In a piezo actuator, voltage across the piezo electrodes, V is used to control deflection of the actuator, x. The relation between the voltage (in Volt) and deflection (in micro meter) is expressed by the following transfer function.

$$\frac{x}{v} = \frac{0.15}{0.001s + 1}$$

(a) Determine the function of the voltage, $v(t)$, as a function of time, *t*, (in seconds), if the desired deflection is expressed by $x(t) = 15sin(80\pi t)$. (25)

Solution

Magnitude ratio of frequency response is determined from

$$\left|\frac{x}{v}\right| = \left|\frac{0.15}{0.001\omega j + 1}\right| = \frac{0.15}{\sqrt{(0.001\omega)^2 + 1}}$$

Substitute magnitude of deflection and frequency of the desired output.

$$\left|\frac{15}{v}\right| = \frac{0.15}{\sqrt{(0.001 \times 80\pi)^2 + 1}}$$

$$|v| = 103.11$$

Phase difference of frequency response is determined from

$$\angle(x) - \angle(v) = \angle\left(\frac{0.15}{0.001\omega j + 1}\right) = -atan(0.001\omega)$$

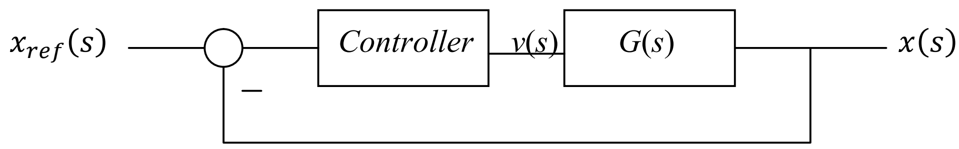
Substitute magnitude of deflection and frequency of the desired output.

$$\angle(x) - \angle(v) = -atan(0.001 \times 80\pi) = -0.25$$

Thus, the function of the voltage is expressed by

$$v(t) = 103.11sin(80\pi t + 0.25)$$

(b) In order to control the deflection, PI controller is applied as shown in the below block diagram. Determine the gains that make time constant of the controlled system become 0.5 millisecond and damping ratio of 0.5. Determine steady-state deflection when the reference deflection is 20 micrometers. (25)



Solution

The closed loop transfer function with PI controller is determined.

$$G_c = \frac{x}{x_{ref}} = \frac{0.15K_P s + 0.15K_I}{0.001s^2 + (0.15K_P + 1)s + 0.15K_I}$$

Characteristic equation that makes 0.5 mill seconds time constant and 0.5 damping ratio.

$$s^2 + 4000s + 16000000 = 0$$

The desired characteristic equations follows

$$s^2 + 4000s + 16000000 = s^2 + (150K_P + 1000)s + 150K_I = 0$$

$$K_P = 20$$

$$K_I = 106666.67$$

The steady-state deflection with 20 micrometer reference deflection is determined from final value theorem.

$$x_{ss} = \lim_{s\to 0} s \frac{0.15K_P s + 0.15K_I}{0.001s^2 + (0.15K_P + 1)s + 0.15K_I}\frac{20}{s} = 20$$

(c) If a lead compensator, expressed by $K\frac{(s+z)}{(s+p)}$ with $K = 1$, replaces the PI controller in (b) in order to control the deflection, determine the zero, z, and the pole, p, of the lead compensator, that make time constant of the controlled system become 0.5 milliseconds and damping ratio of 0.5. Determine the steady-state deflection when the reference deflection is 20 micrometers. (25)

Solution

The closed loop transfer function with lead compensator is determined.

$$G_c = \frac{x}{x_{ref}} = \frac{0.15Ks + 0.15Kz}{0.001s^2 + (1 + 0.001p + 0.15K)s + p + 0.15Kz}$$

With $K = 1$,

$$G_c = \frac{x}{x_{ref}} = \frac{0.15s + 0.15z}{0.001s^2 + (1.15 + 0.001p)s + p + 0.15z}$$

Characteristic equation that makes 0.5 mill seconds time constant and 0.5 damping ratio.

$$s^2 + 4000s + 16000000 = 0$$

The desired characteristic equations follows

$$s^2 + 4000s + 16000000 = s^2 + (1150 + p)s + 1000p + 150z = 0$$

$$p = 2850$$

$$z = 87333.33$$

The steady-state deflection with 20 micrometer reference deflection is determined from final value theorem.

$$x_{ss} = \lim_{s \to 0} s \frac{0.15s + 0.15z}{0.001s^2 + (1.15 + 0.001p)s + p + 0.15z} \frac{20}{s} = \frac{0.15z(20)}{p + 0.15z} = 16.43$$

(d) If a lag compensator, expressed by $K\frac{(s+z)}{(s+p)}$, replaces the lead compensator designed in (c) in order to reduce steady-state error, determine the zero, z, and the gain, K, of the lag compensator, that make steady-state offset error be 1% and a time constant of the controlled system become 0.5 milliseconds when $p = 0.0001$. Determine the remaining pole of the controlled system. (25)

Solution

The closed loop transfer function with lag compensators is determined.

$$G_c = \frac{x}{x_{ref}} = \frac{0.15Ks + 0.15Kz}{0.001s^2 + (1 + 0.001p + 0.15K)s + p + 0.15Kz}$$

With $p = 0.0001$,

$$G_c = \frac{x}{x_{ref}} = \frac{0.15Ks + 0.15Kz}{0.001s^2 + (1 + 0.0000001 + 0.15K)s + 0.0001 + 0.15Kz}$$

The steady-state error of 1% is determined from final value theorem.

$$x_{ss} = \lim_{s \to 0} s \frac{0.15Ks + 0.15Kz}{0.001s^2 + (1 + 0.0000001 + 0.15K)s + 0.0001 + 0.15Kz} \frac{1}{s} = \frac{0.15Kz}{0.0001 + 0.15Kz} = 0.99$$

$$Kz = 0.066$$

Characteristic equation of the controlled system.

$$s^2 + (1000.0001 + 150K)s + 10 = 0$$

The desired characteristic equations follows

$$(s + 2000)(s + m) = s^2 + (2000 + m)s + 2000m = s^2 + (1000.0001 + 150K)s + 10 = 0$$

$$m = 0.005$$

$$K = 6.67$$

$$z = 0.01$$