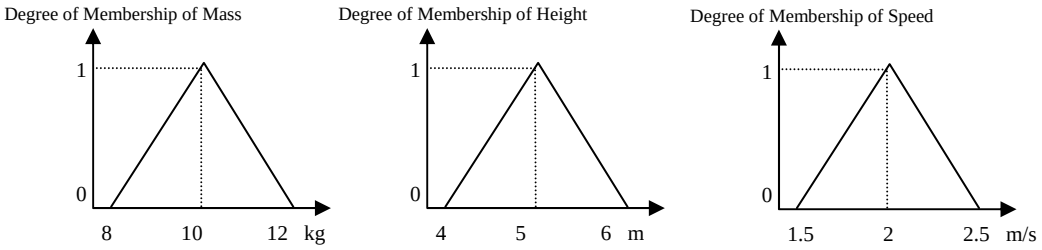
Final Examination AI and Neuro-Fuzzy Theory AT07.24 May 10, 2011

Time: 9:00-10:30 h. Open Book
Marks: 100

Attempt all questions.

Q.1 Determine the range of alpha cut operation as a function of alpha varying from 0 to 1 of the total energy of a falling object with mass about 10 kg from the height of about 5 m and initial speed of about 2 m/s in downward direction. Assume the membership functions of the mass, the height and the speed are given below. Use gravitational acceleration of 10 m/s^2. (30)



Solution

$$E = mgh + \tfrac{1}{2}mv^2 \tag{1}$$

$$m(x) = \begin{cases} \frac{x}{2} - 4 & \text{for } 8 \le x < 10 \\ -\frac{x}{2} + 6 & \text{for } 10 \le x < 12 \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

$$^{\alpha}m = [2\alpha + 8 \quad 12 - 2\alpha] \tag{3}$$

$$^{\alpha}(mg) = [20\alpha + 80 \quad 120 - 20\alpha] \tag{4}$$

$$h(y) = \begin{cases} y - 4 & \text{for } 4 \le y < 5 \\ -y + 6 & \text{for } 5 \le x < 6 \\ 0 & \text{otherwise} \end{cases} \tag{5}$$

$$^{\alpha}h = [\alpha + 4 \quad 6 - \alpha] \tag{6}$$

$$^{\alpha}(mgh) = [20\alpha^2 + 160\alpha + 320 \quad 20\alpha^2 - 240\alpha + 720] \tag{7}$$

$$^{\alpha}\tfrac{1}{2}m = [\alpha + 4 \quad 6 - \alpha] \tag{8}$$

$$v(z) = \begin{cases} 2z - 3 & \text{for } 1.5 \le z < 2 \\ -2z + 5 & \text{for } 2 \le z < 2.5 \\ 0 & \text{otherwise} \end{cases} \tag{9}$$

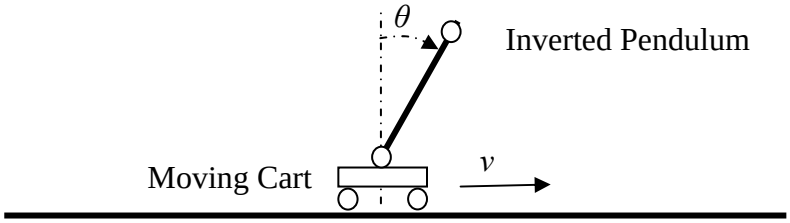
$$^{\alpha}v = [0.5\alpha + 1.5 \quad 2.5 - 0.5\alpha] \tag{10}$$

$$^{\alpha}(\tfrac{1}{2}mv) = [0.5\alpha^2 + 3.5\alpha + 6 \quad 0.5\alpha^2 - 5.5\alpha + 15] \tag{11}$$

$$^{\alpha}(\tfrac{1}{2}mv^2) = [0.25\alpha^3 + 2.5\alpha^2 + 8.25\alpha + 9 \quad -0.25\alpha^3 + 4\alpha^2 - 21.25\alpha + 37.5] \tag{12}$$

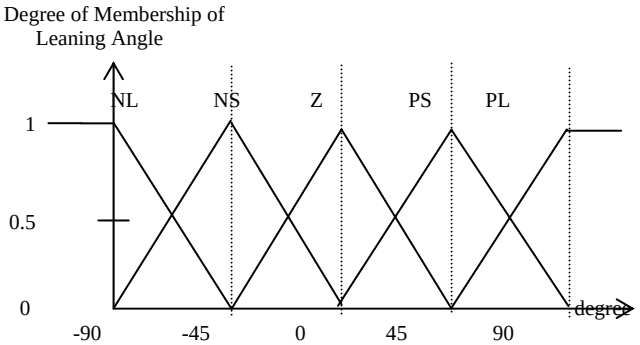
$$^{\alpha}E = [0.25\alpha^3 + 22.5\alpha^2 + 168.25\alpha + 329 \quad -0.25\alpha^3 + 24\alpha^2 - 261.25\alpha + 757.5] \tag{13}$$

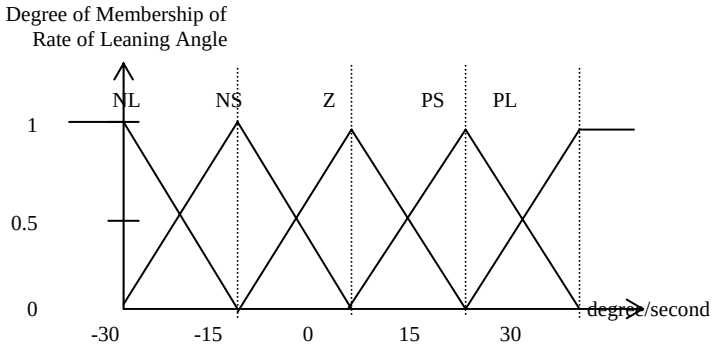
Q.2 A singleton-output fuzzy controller is applied to balance an inverted pendulum of a moving-cart type inverted pendulum system as shown in the figure below. The two inputs of the controller are leaning angle, θ, and the rate of leaning angle, $\dot{\theta}$, of the pendulum. The output of the controller is cart speed, v.



If membership functions of the leaning angle, and the rate of leaning angle are given as shown below. When N represents Negative, P represents Positive, L represents Large, S represents Small, and Z represents Zero. The membership function of the cart speed is singleton type. Determine the cart speed if the leaning angle of the pendulum is detected at 25˚ and the rate of leaning angle is detected at -5˚/s. Apply center of gravity of singleton output in defuzzification.

(30)





Assume the following rules are applied to determine the cart speed.

Leaning Angle Rate / Leaning Angle	NL	NS	Z	PS	PL
NL	-4	-3	-2	-1	0
NS	-3	-2	-1	0	1
Z	-2	-1	0	1	2
PS	-1	0	1	2	3
PL	0	1	2	3	4

Solution

Determine degree of memberships of leaning angle of the inverted pendulum,

$$NL(25^{\circ}) = NS(25^{\circ}) = PL(25^{\circ}) = 0 \quad (1)$$

$$Z(25^{\circ}) = 4/9 = 0.44 \quad (2)$$

$$PS(25^{\circ}) = 5/9 = 0.56 \quad (3)$$

Determine degree of memberships of rate of leaning angle of the inverted pendulum,

$$NL(-5^{\circ}/s) = PS(-5^{\circ}/s) = PL(-5^{\circ}/s) = 0 \quad (4)$$

$$Z(-5^{\circ}/s) = 2/3 = 0.67 \quad (5)$$

$$NS(-5^{\circ}/s) = 1/3 = 0.33 \quad (6)$$

Determine degree of memberships of the result of each rule by using the minimum degree of membership

Leaning Angle Rate / Leaning Angle	NL (0)	NS (1/3)	Z (2/3)	PS (0)	PL(0)
NL (0)	-4 (0)	-3 (0)	-2 (0)	-1 (0)	0 (0)
NS (0)	-3 (0)	-2 (0)	-1 (0)	0 (0)	1 (0)
Z (4/9)	-2 (0)	-1 (1/3)	0 (4/9)	1 (0)	2 (0)
PS (5/9)	-1 (0)	0 (1/3)	1 (5/9)	2 (0)	3 (0)
PL (0)	0 (0)	1 (0)	2 (0)	3 (0)	4 (0)

The cart speed by center of gravity method, thus, becomes

$$\frac{[(-1\times\frac{1}{3})+(0\times\frac{4}{9})+(1\times\frac{5}{9})]}{[\frac{1}{3}+\frac{4}{9}+\frac{5}{9}]} = 0.17\ m/s \qquad (7)$$

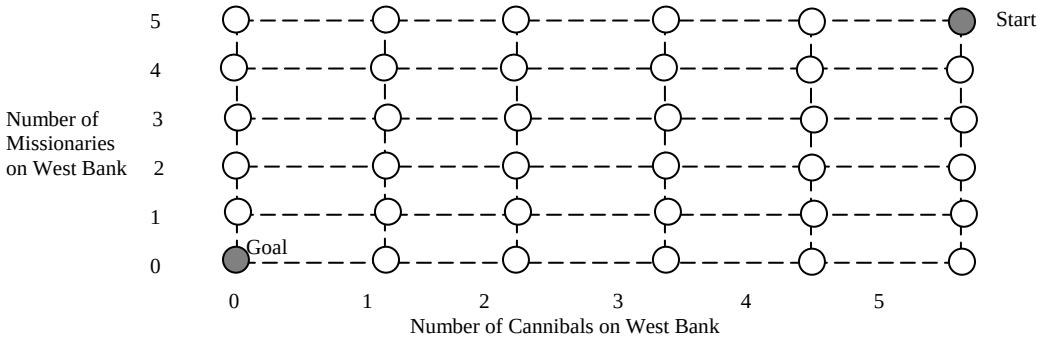
Q.3 Consider the missionaries-cannibals problem represented in the state space representation as shown in the figure below when (*m*, *n*) shows *m* missionaries and *n* cannibals on the west bank of a river. Initially, there are 5 missionaries and 5 cannibals on the west bank of the river. A small boat can carry at most of 3 persons. If the number of cannibals is larger than the number of missionaries either on the west bank or on the east bank of the river, the missionaries will be eaten by the cannibals. However, the boat is designed so that the cannibals cannot eat the missionaries when they are on the boat. The boat without passenger cannot move. Determine the steps that all missionaries and cannibals could cross the river to the east bank safely, draw its tree, and identify the order of nodes opening in the tress by using

(a) hill-climbing search

(b) beam search with beam width of 2

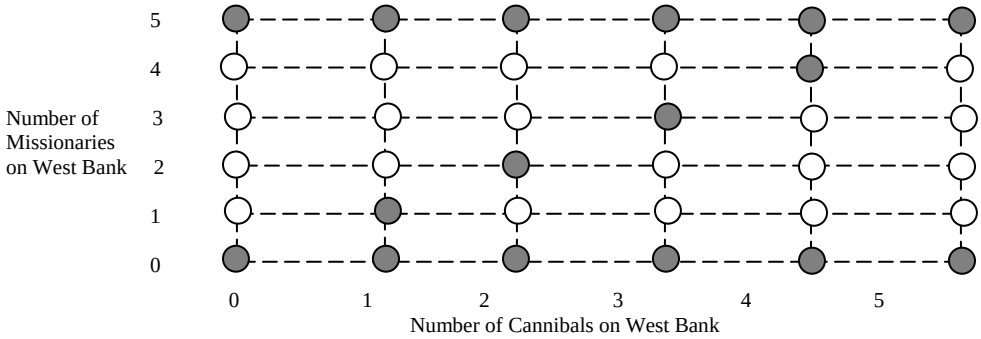
(c) best-first search

In (a) - (c), sort the opened nodes by using less number of missionaries on the west bank as first priority then less number of cannibals on the west bank as second priority then using the same criteria of their parent nodes for the following priorities. (30)

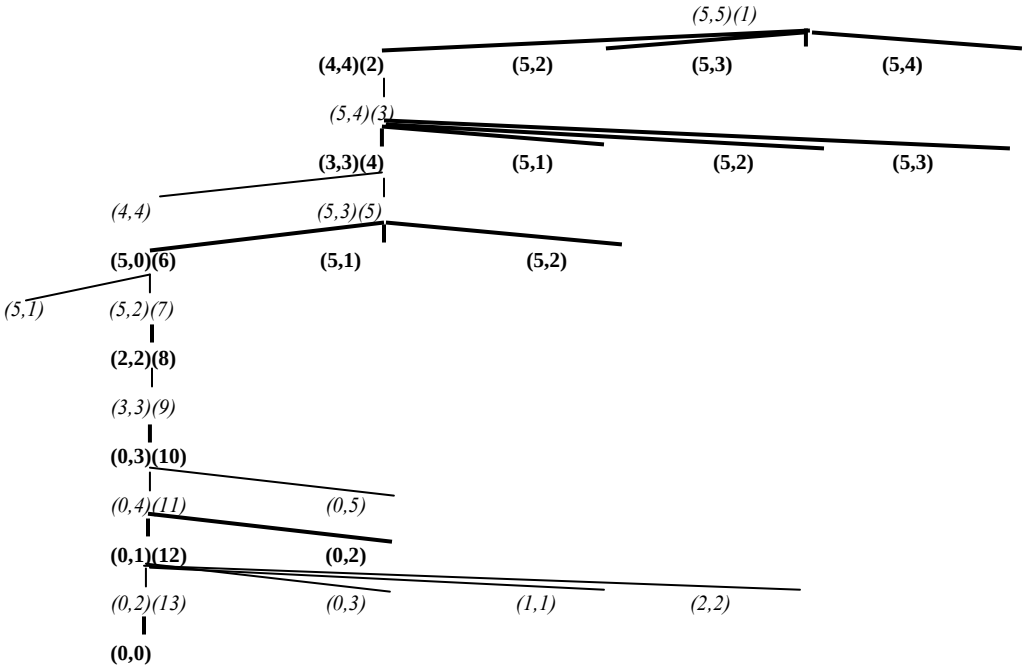


Solution

Label the safe nodes,

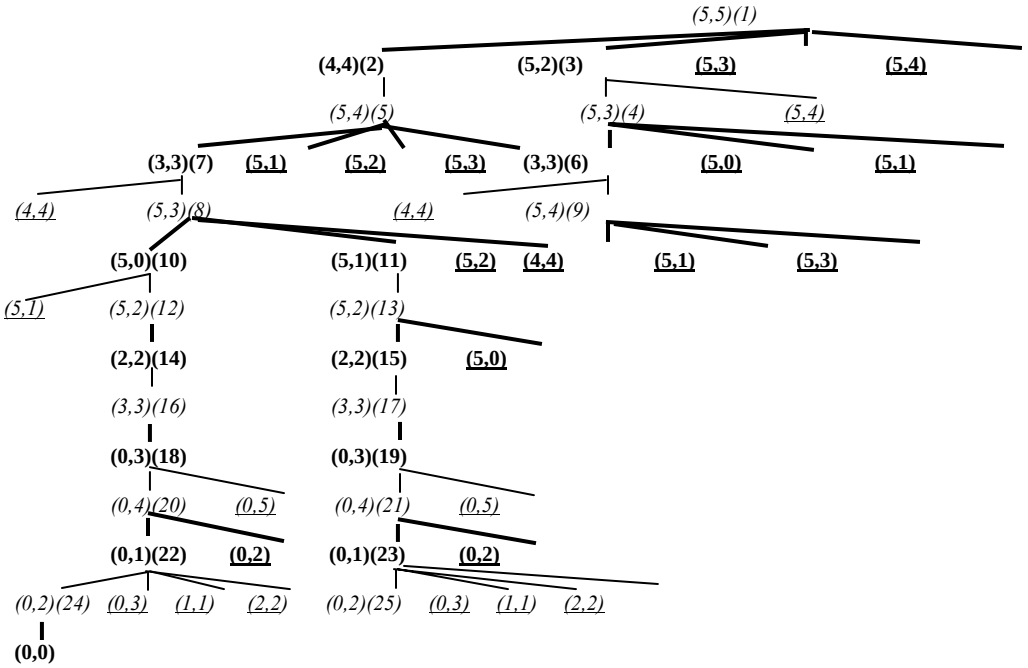


(a) hill-climbing search



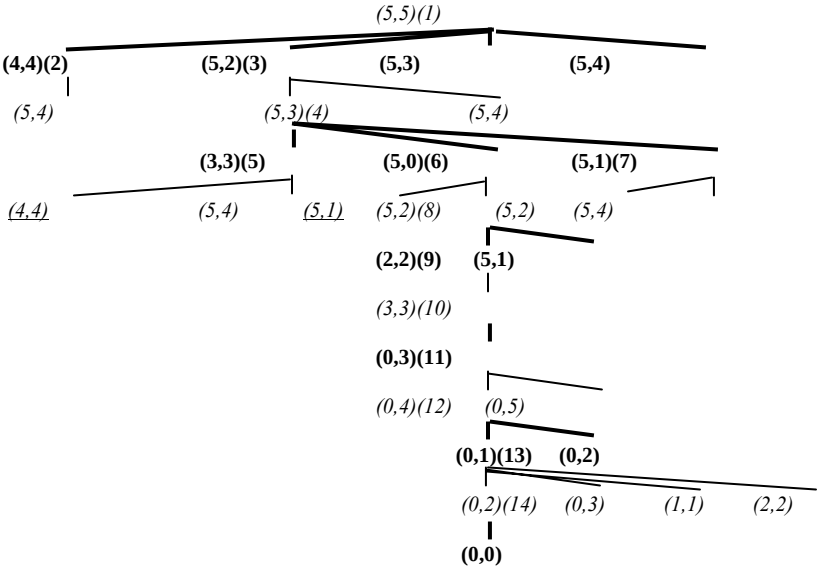
The steps are (5,5)→(4,4)-(5,4)→(3,3)-(5,3)→(5,0)-(5,2)→(2,2)-(3,3)→(0,3)-(0,4)→(0,1)-(0,2)→(0,0)

(b) beam search with beam width of 2



The steps are (5,5)→(4,4)-(5,4)→(3,3)-(5,3)→(5,0)-(5,2)→(2,2)-(3,3)→(0,3)-(0,4)→(0,1)-(0,2)→(0,0)

(c) best-first search



The steps are (5,5)→(5,2)-(5,3)→(5,0)-(5,2)→(2,2)-(3,3)→(0,3)-(0,4)→(0,1)-(0,2)→(0,0)

Q.4 Apply the optimization method, either GA, PSO, or SA, to your thesis research. Discuss how to define the objective function, and how to code the problem into the problem of GA, PSO, or SA. Explain the operation in the selected optimization method. (10)