**Final Examination AI and Neuro-Fuzzy Theory AT07.24 May 6, 2013**

Time: 13:00-15:00 h. Open Book

Marks: 100

Attempt all questions.

Q.1 Torque generated from a DC motor, $T$, varies with the supplied armature current, $I_a$, as expressed by the equation, $T = K_T I_a$, when $K_T$ is called the torque constant of the motor. When the armature current about 2 A is supplied to the motor, the generated torque is measured about 16 Nm. Assume fuzzy number of the armature current, $A(x)$, and fuzzy number of the generated torque, $B(x)$, are represented by the following membership functions.

$$A(x) = e^{-\frac{(x-2)^2}{0.5}}$$

$$B(x) = e^{-\frac{(x-16)^2}{2}}$$

Determine membership functions of the torque constant when $\alpha \geq 0.1$. (25)

<u>Solution</u>

When $\alpha \geq 0.1$, all intervals are in positive range.

$$A(x) = \alpha = e^{-\frac{(x-2)^2}{0.5}} \tag{1}$$

$$x = 2 \pm \sqrt{-0.5(\ln \alpha)} \tag{2}$$

$$^{\alpha}A = [2 - \sqrt{-0.5(\ln \alpha)}, 2 + \sqrt{-0.5(\ln \alpha)}] \tag{3}$$

$$B(x) = \alpha = e^{-\frac{(x-16)^2}{2}} \tag{4}$$

$$x = 16 \pm \sqrt{-2(\ln \alpha)} \tag{5}$$

$$^{\alpha}B = [16 - \sqrt{-2(\ln \alpha)}, 16 + \sqrt{-2(\ln \alpha)}] \tag{6}$$

$$^{\alpha}(B/A) = [16 - \sqrt{-2(\ln \alpha)}, 16 + \sqrt{-2(\ln \alpha)}] / [2 - \sqrt{-0.5(\ln \alpha)}, 2 + \sqrt{-0.5(\ln \alpha)}] \tag{7}$$

$$^{\alpha}(B/A) = {}^{\alpha}(B \times (1/A)) = [16 - \sqrt{-2(\ln \alpha)}, 16 + \sqrt{-2(\ln \alpha)}] \times / [1/(2 + \sqrt{-0.5(\ln \alpha)}), 1/(2 - \sqrt{-0.5(\ln \alpha)})] \tag{8}$$

$$^{\alpha}(B/A)=[\frac{16-\sqrt{-2(\ln\alpha)}}{2+\sqrt{-0.5(\ln\alpha)}},\frac{16+\sqrt{-2(\ln\alpha)}}{2-\sqrt{-0.5(\ln\alpha)}}] \tag{9}$$

For the relation, $x=\frac{16-\sqrt{-2(\ln\alpha)}}{2+\sqrt{-0.5(\ln\alpha)}}$, when α = 0.1, x = 4.51, when α = 1, x = 8.

For the relation, $x=\frac{16+\sqrt{-2(\ln\alpha)}}{2-\sqrt{-0.5(\ln\alpha)}}$, when α = 0.1, x = 19.57, when α = 1, x = 8.

Define $\sqrt{-(\ln\alpha)}=m$.

When $4.51\le x\le 8$,

$$x=\frac{16-\sqrt{2}m}{2+\sqrt{0.5}m} \tag{10}$$

$$x(2+\sqrt{0.5}m)=16-\sqrt{2}m \tag{11}$$

$$m=\frac{16-2x}{\sqrt{0.5}x+\sqrt{2}} \tag{12}$$

$$\alpha=e^{-\left(\frac{16-2x}{\sqrt{0.5}x+\sqrt{2}}\right)^2} \tag{13}$$

When $8\le x\le 19.57$,

$$x=\frac{16+\sqrt{2}m}{2-\sqrt{0.5}m} \tag{14}$$

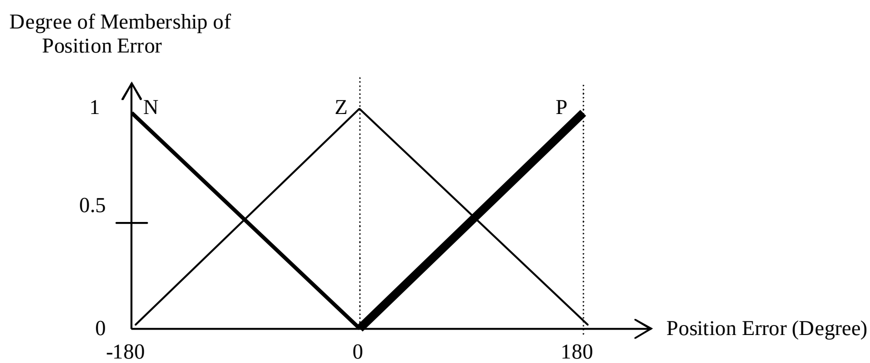
$$x(2-\sqrt{0.5}m)=16+\sqrt{2}m \tag{15}$$

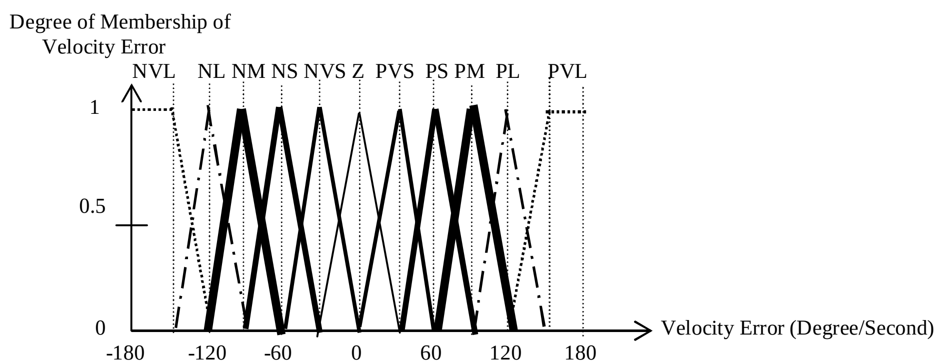
$$m=\frac{16+2x}{\sqrt{0.5}x-\sqrt{2}} \tag{16}$$

$$\alpha=e^{-\left(\frac{16+2x}{\sqrt{0.5}x-\sqrt{2}}\right)^2} \tag{17}$$

$$A/B=\begin{cases} e^{-\left(\frac{16-2x}{\sqrt{0.5}x+\sqrt{2}}\right)^2} & \text{when } 4.51\le x\le 8 \\ e^{-\left(\frac{16+2x}{\sqrt{0.5}x-\sqrt{2}}\right)^2} & \text{when } 8\le x\le 19.57 \end{cases} \tag{18}$$

Q.2 In a leg exoskeleton system, fuzzy is applied to adjust the proportional gain of the controller. The inputs of the system are position error and velocity error of the system from the position and velocity profiles. The output is a number that will be multiplied with the original proportional gain. The position error is categorized as Negative (N), Zero (Z), and Positive (P). Likewise, the velocity error is categorized as Very Large Negative (VLN), Large Negative (LN), Medium Negative (MN), Small Negative (SN), Very Small Negative (VSN), Zero (Z), Very Small Positive (VSP), Small Positive (SP), Medium Positive (MP), Large Positive (LP), and Very Large Positive (VLP). The output is a singleton type membership function. All the membership functions are shown below.




Fuzzy Inference Rule is given below.

| | NVL | NL | NM | NS | NVS | Z | PVS | PS | PM | PL | PVL |
|---|---|---|---|---|---|---|---|---|---|---|---|
| N | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1.0 | 1.1 | 1.2 |
| Z | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1.0 | 1.1 | 1.2 | 1.3 | 1.4 | 1.5 |
| P | 0.8 | 0.9 | 1.0 | 1.1 | 1.2 | 1.3 | 1.4 | 1.5 | 1.6 | 1.7 | 1.8 |

Determine the number that will be multiplied with the original proportional gain by using center of gravity defuzzification method when the position error is 15 degree and velocity error is 25 degree/second. (25)

**Solution**

| | NVL (0) | NL (0) | NM (0) | NS (0) | NVS (0) | Z (0.17) | PVS (0.83) | PS (0) | PM (0) | PL (0) | PVL (0) |
|---|---|---|---|---|---|---|---|---|---|---|---|
| N(0) | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1.0 | 1.1 | 1.2 |
| Z(0.92) | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | **1.0** | **1.1** | 1.2 | 1.3 | 1.4 | 1.5 |
| P(0.08) | 0.8 | 0.9 | 1.0 | 1.1 | 1.2 | **1.3** | **1.4** | 1.5 | 1.6 | 1.7 | 1.8 |

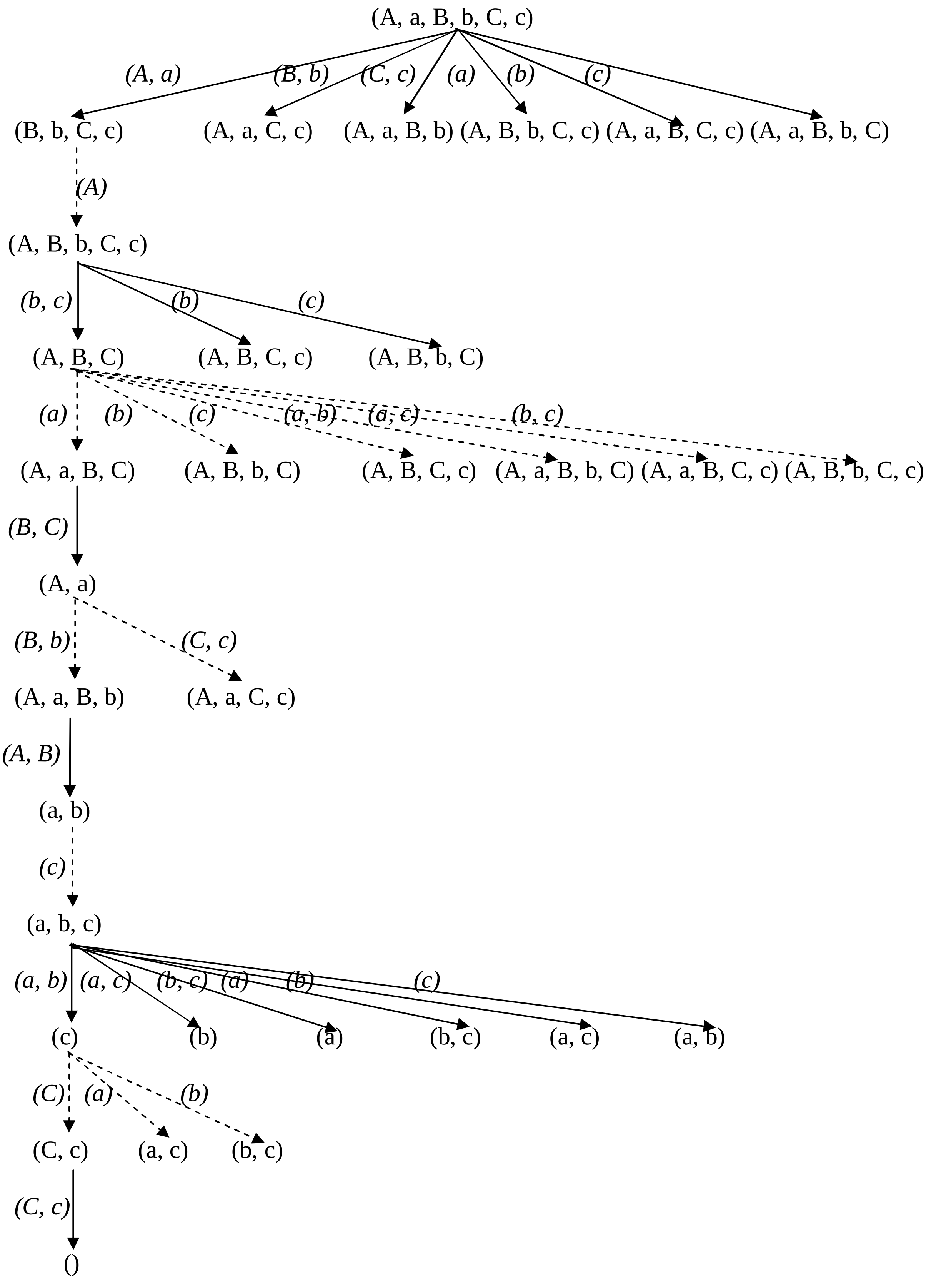
$$Number = \frac{[0.17\times1.0]+[0.83\times1.1]+[0.08\times1.3]+[0.08\times1.4]}{[0.17]+[0.83]+[0.08]+[0.08]} = 1.12 \quad \underline{(1)}$$

Q.3 In the jealous husband problem, there are 3 pairs of husband and wife who need to cross a river. The boat is quite small and at most of 2 people can be on the boat at the same time and at least one traveler is needed for the trip. However each husband is so jealousy that he doesn't allow his wife to be with other guy without him. Formulate this problem by using root node as (A, a, B, b, C, c) when A, B, and C are the husbands of the wives a, b, and c respectively. Each node represents husband(s) and wife(s) who do not cross the river yet. A branch is represented by an arrow with a label of (X, x) or (X) or (x) when the alphabet shows the traveler(s). Going trip is represented by solid arrow and returning trip is represented by dotted arrow.

(a) Find the solution by depth-first search (sort by (1) larger number of travelers in the going trip and by (1) smaller number travelers in the returning trip then by (2) husband traveler, then by (3) alphabetical list of travelers). Draw the tree resulting from depth-first search completely. No looping is allowed in the tree. (25)

**Solution**

(a)




The solution is (A, a, B, b, C, c) → (B, b, C, c) → (A, B, b, C, c) → (A, B, C) → (A, a, B, C) → (A, a) → (A, a, B, b) → (a, b) → (a, b, c) → (c) → (C, c) → ()

Q.4 In classical logic, $(p \Rightarrow q) \Leftrightarrow (\neg p \vee q)$ is always tautology, determine degree of truth of $(p \Rightarrow q) \Leftrightarrow (\neg p \vee q)$ in fuzzy logic using Lukasiewicz's formula. Determine whether it is possible to have 0 degree of truth for $(p \Rightarrow q) \Leftrightarrow (\neg p \vee q)$. (25)

**Solution**

Based on Lukasiewicz's formula

$$(p \Rightarrow q) \Leftrightarrow (\neg p \vee q)$$

$$1 - |min(1, 1 - p + q) - (\max((1 - p), q))|$$

If $p \geq q$ and $1 - p \geq q$

$$1 - |1 - p + q - (1 - p)| = 1 - q$$

If $p \geq q$ and $1 - p < q$

$$1 - |1 - p + q - (q)| = p$$

If $p < q$ and $1 - p \geq q$

$$1 - |1 - (1 - p)| = 1 - p$$

If $p < q$ and $1 - p < q$

$$1 - |1 - (q)| = q$$

It is not possible to have 0 degree of truth from all the 4 conditions.