**Final Examination   AI and Neuro-Fuzzy Theory AT07.24   May 9, 2014**
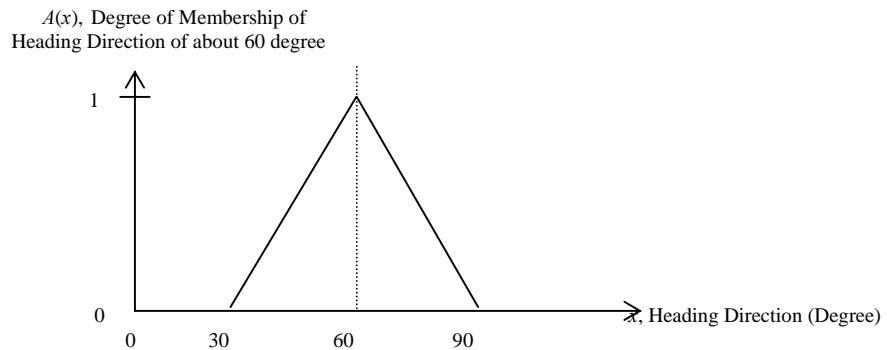
Time: 10:00-12:00 h.   Open Book
Marks: 100

Attempt all questions.

Q.1 A compass is used to measure heading direction of a vehicle respect to north pole. If the compass indicates heading direction of about 60 degree which is expressed by the following fuzzy number, $A(x)$.

A(x), Degree of Membership of
Heading Direction of about 60 degree

Determine the membership function of cosine of about 60 degree, $B(y)$, where alpha-cut of $B(y)$ is always the same as cosine of alpha-cut of $A(x)$. Then roughly plot the membership function of $B(y)$. (25)

Solution

When $30 \leq x \leq 60$,

$$A(x) = \alpha = \frac{x}{30} - 1 \tag{1}$$

When $60 \leq x \leq 90$,

$$A(x) = \alpha = -\frac{x}{30} + 3 \tag{2}$$

$$^\alpha A = [30(\alpha + 1), 30(3 - \alpha)] \tag{3}$$

$$^\alpha B = \cos\left(^\alpha A\right) = [\cos(30(3 - \alpha)), \cos(30(\alpha + 1))] \tag{4}$$
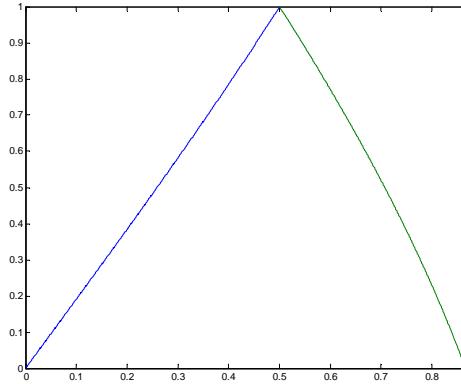
$$^\alpha B = \cos\left(^\alpha A\right) = [\sin(30\alpha), \cos(30(\alpha + 1))] \tag{5}$$

When $0 \leq y \leq 0.5$,
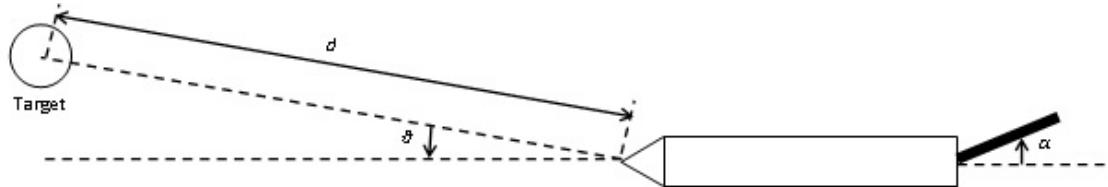
$$\alpha = \frac{\sin^{-1}(y)}{30} \tag{6}$$

When $0.5 \leq y \leq 0.866$
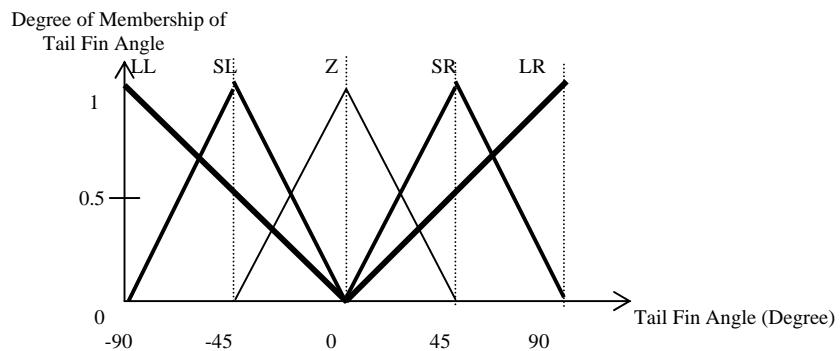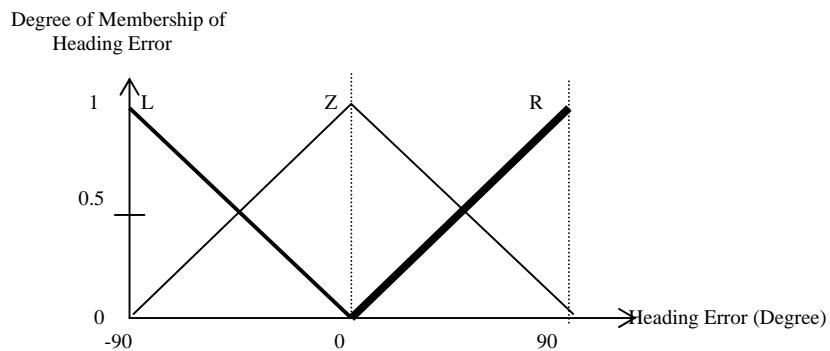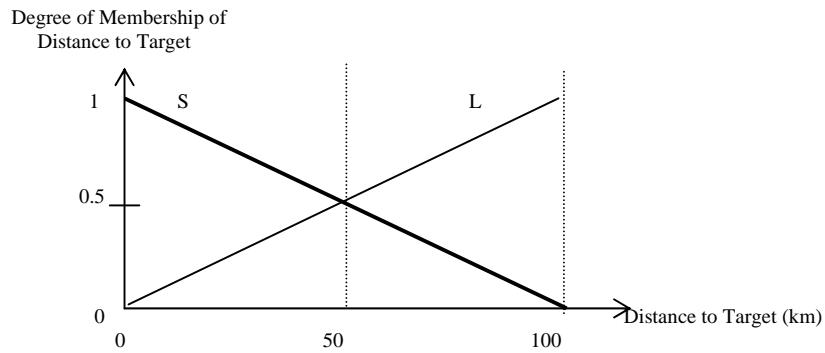
$$\alpha = \frac{\cos^{-1}(y)}{30} - 1 \tag{7}$$

Q.2 Fuzzy is applied to control tail fin angle of a missile as shown in the below figure. The inputs of the fuzzy controller are distance to target, d, and missile heading error, θ, while the output of the fuzzy controller is the tail fin angle, α.

Distance to target is categorized as Short (S), and Long (L). Missile heading error is categorized as Left (L), Zero (Z), and Right (R). Tail fin angle is categorized as Large Right (LR), Small Right (SR), Zero (Z), Small Left (SL), and Large Left (LL). All the membership functions are shown below.

Degree of Membership of Distance to Target

Degree of Membership of Heading Error

Degree of Membership of Tail Fin Angle

Fuzzy Inference Rule is given below.

Heading Error / Distance to Target	L	Z	R
S	LR	Z	LL
L	SR	Z	SL
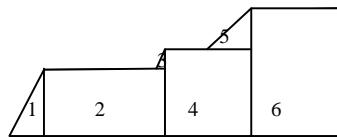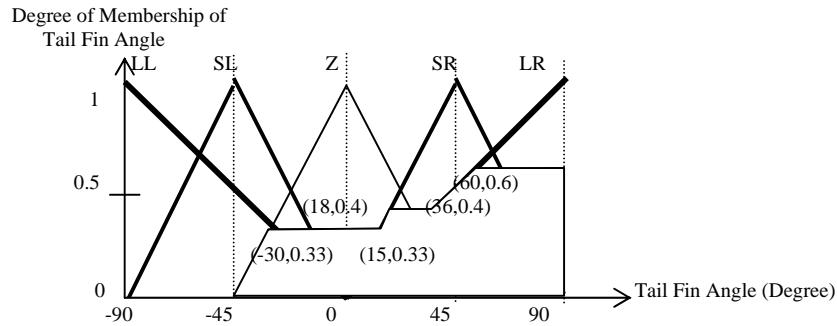
Determine tail fin angle of the missile when the distance to target is 40 km and the heading error is -60 degree. (25)

Solution

3

Heading Error / Distance to Target	L(-60) = 0.67	Z(-60) = 0.33	R(-60) = 0
S(40) = 0.6	LR(0.6)	Z(0.33)	LL(0)
L(40) = 0.4	SR(0.4)	Z(0.33)	SL(0)
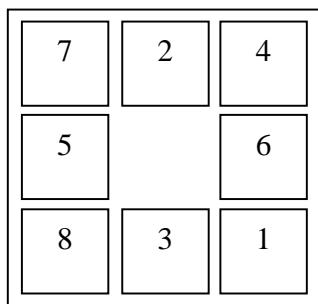
Degree of Membership of Tail Fin Angle

α
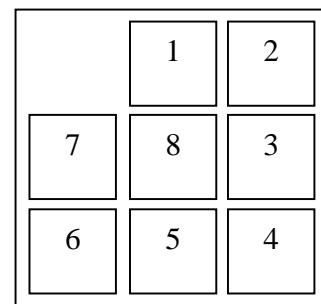
$$= \frac{[0.5 \times 15 \times 0.33 \times (-35)] + [48 \times 0.33 \times (-6)] + [0.5 \times 3 \times 0.07 \times (17)] + [42 \times 0.4 \times (39)] + [0.5 \times 24 \times 0.2 \times (52)] + [30 \times 0.6 \times (75)]}{[0.5 \times 15 \times 0.33] + [48 \times 0.33] + [0.5 \times 3 \times 0.07] + [42 \times 0.4] + [0.5 \times 24 \times 0.2] + [30 \times 0.6]}$$

$= 35.06$ (1)

Q.3   In 8-puzzle problem, a start state and a goal state are given as shown in the below figure.
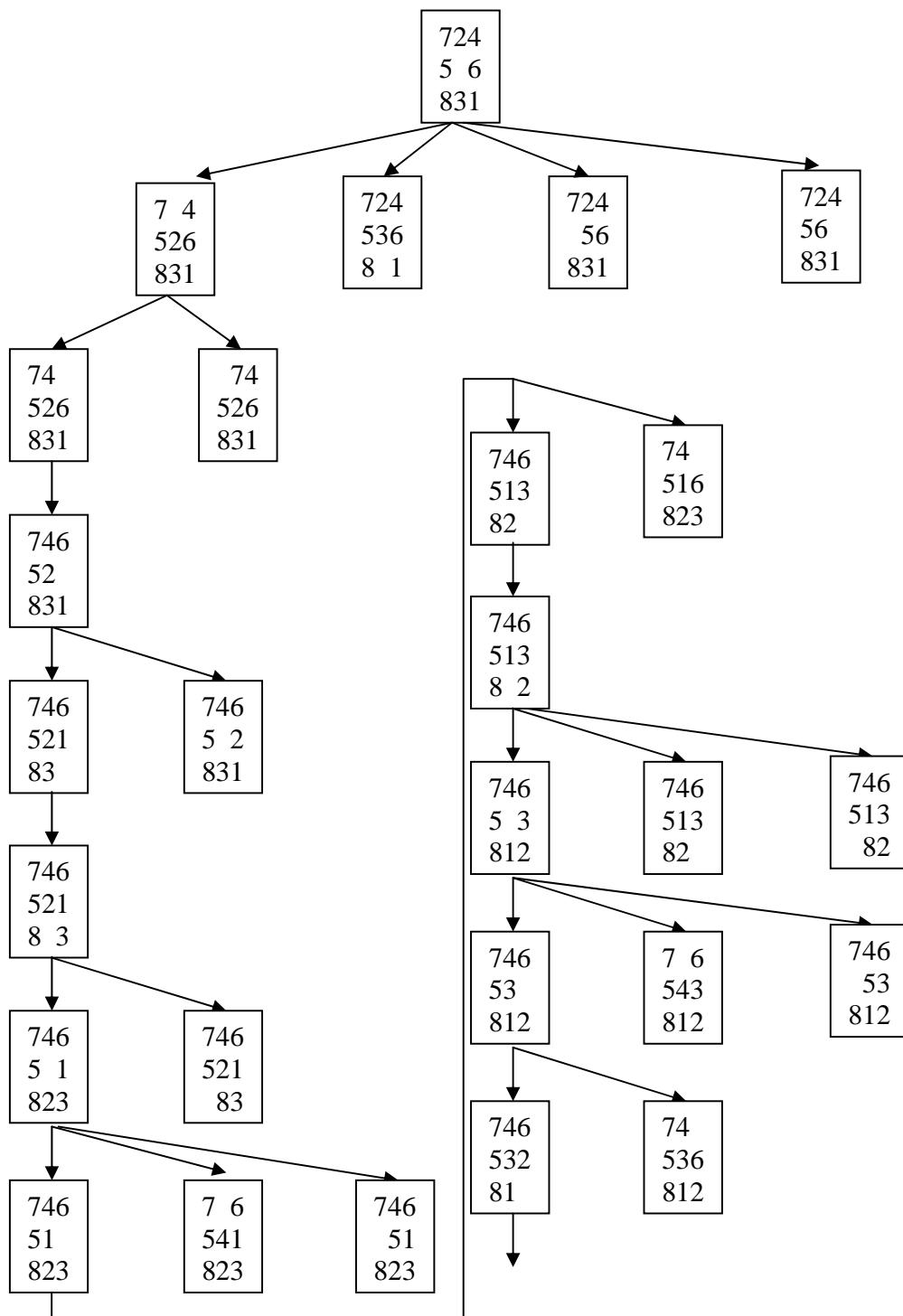
Start State   Goal State
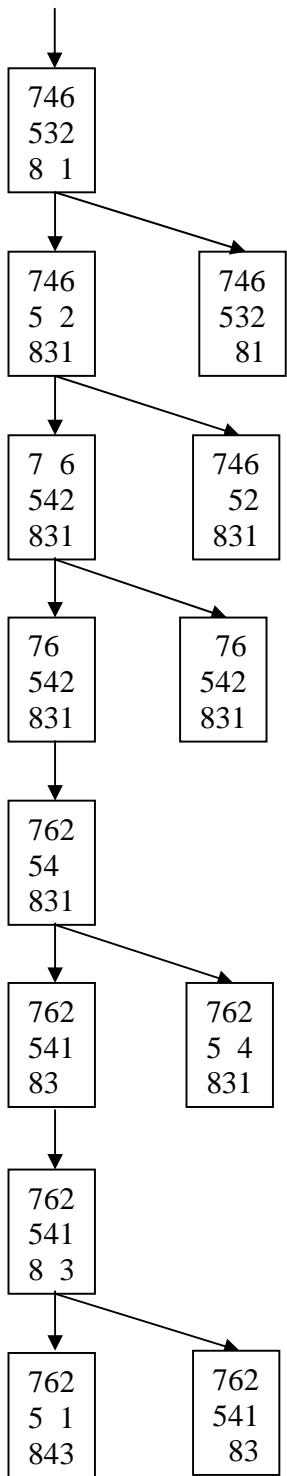
The possible moves are either up, down, left, and right of the numbers to the empty space.

4

(a) Show the result of search tree by depth-first search sort, by moving of smaller numerical order until the depth of 20. No looping is allowed in the tree.   (25)

Solution

(a)

```
┌─────┐
│ 746 │
│ 532 │
│ 8  1│
└─────┘
   │    ╲
   │     ╲
   ▼      ▼
┌─────┐  ┌─────┐
│ 746 │  │ 746 │
│ 5  2│  │ 532 │
│ 831 │  │  81 │
└─────┘  └─────┘
   │    ╲
   ▼      ▼
┌─────┐  ┌─────┐
│ 7  6│  │ 746 │
│ 542 │  │  52 │
│ 831 │  │ 831 │
└─────┘  └─────┘
   │    ╲
   ▼      ▼
┌─────┐  ┌─────┐
│ 76  │  │ 76  │
│ 542 │  │ 542 │
│ 831 │  │ 831 │
└─────┘  └─────┘
   │
   ▼
┌─────┐
│ 762 │
│ 54  │
│ 831 │
└─────┘
   │    ╲
   ▼      ▼
┌─────┐  ┌─────┐
│ 762 │  │ 762 │
│ 541 │  │ 5  4│
│ 83  │  │ 831 │
└─────┘  └─────┘
   │
   ▼
┌─────┐
│ 762 │
│ 541 │
│ 8  3│
└─────┘
   │    ╲
   ▼      ▼
┌─────┐  ┌─────┐
│ 762 │  │ 762 │
│ 5  1│  │ 541 │
│ 843 │  │  83 │
└─────┘  └─────┘
```

(b) Show the result of search tree by hill-climbing search, sort by (1) shorter summation of distance between empty space-1-2-3-4-5-6-7-8-empty space (2) shorter distance of empty space to the corner top-left position, (3) shorter distance of 2 to the corner top-right position until the depth of 10. No looping is allowed in the tree. Show the distance(s) used in sorting beside the node. (25)

<u>Solution</u>

```
                724
                5 6
                831   17.77
     /          |        \            \
  724         724         724          7 4
  536          56          56          526
  8 1  15.12   831 16.36,1 831 16.36,2.24  831 17.59
   /    \
 724    724
 536    536
  81 15.89  81  16.36
   |
 724
 53
 816  17.37
   |        \
 724        72
 5 3        534
 816 16.13  816 18.13
   |    \        \
 724   724       7 4
  53   513       523
 816   8 6       816
 14.49 14.71     16.54
   |    \
 724   24
 853   753
  16   816
 13.07 14.71
   |
```

```
        │
        ▼
    ┌───────┐
    │ 724   │
    │ 853   │
    │ 1  6  │
    └───────┘ 13.72
        │         ╲
        ▼          ╲
    ┌───────┐    ┌───────┐
    │ 724   │    │ 724   │
    │ 8  3  │    │ 853   │
    │ 156   │    │ 16    │
    └───────┘ 14.13  └───────┘ 14.54
        │    ╲        ╲
        ▼     ▼         ▼
    ┌───────┐ ┌───────┐ ┌───────┐
    │ 724   │ │ 7  4  │ │ 724   │
    │ 83    │ │ 823   │ │ 83    │
    │ 156   │ │ 156   │ │ 156   │
    └───────┘ └───────┘ └───────┘ 15.95
  14.13,1,1   14.13,1,1.41
        │    ╲
        ▼     ▼
    ┌───────┐ ┌───────┐
    │ 724   │ │  24   │
    │ 183   │ │ 783   │
    │ 56    │ │ 156   │
    └───────┘ └───────┘
      13.72      14.54
```