Final Examination AI and Neuro-Fuzzy Theory AT07.24 May 4, 2016

Time: 9:00-11:00 h. Open Book

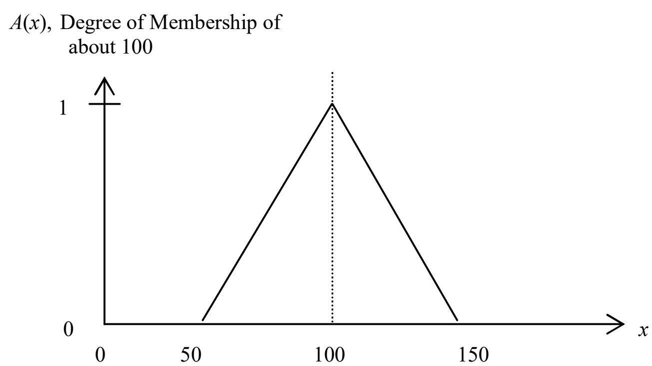
Marks: 100

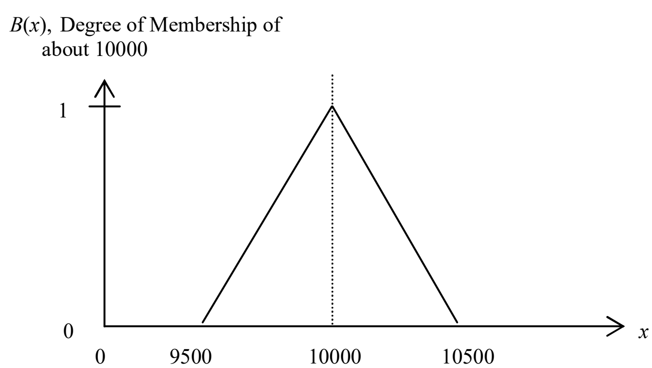
Attempt all questions.

Q.1 Logarithm is used to linearize multiplication operation as expressed by

$$\log(AB) = \log(A) + \log(B).$$

If membership functions of A, about 100, and B, about 10000, are given below. Determine membership functions of the fuzzy numbers $AB, \log(A), \log(B), \log(AB)$. (40)





Solution

When $50 \leq x \leq 100$,

$$A(x) = 0.02x - 1 \tag{1}$$

When $100 \leq x \leq 150$,

$$A(x) = -0.02x + 3 \tag{2}$$

$$^{\alpha}A = [50\alpha + 50, 150 - 50\alpha] \tag{3}$$

When $9500 \leq x \leq 10000$,

$$B(x) = 0.002x - 19 \tag{4}$$

When $10000 \leq x \leq 10500$,

$$B(x) = -0.002x + 21 \tag{5}$$

$$^{\alpha}B = [500\alpha + 9500, 10500 - 500\alpha] \tag{6}$$

$$^{\alpha}(AB) = [25000\alpha^2 + 500000\alpha + 475000 \quad 25000\alpha^2 - 600000\alpha + 1575000] \tag{7}$$

$$25000\alpha^2 + 500000\alpha + 475000 = x;\ 475000 \leq x \leq 1000000 \tag{8}$$

$$\alpha = \frac{-500000 + \sqrt{500000^2 - 100000(475000 - x)}}{50000};\ 475000 \leq x \leq 1000000 \tag{9}$$

$$25000\alpha^2 - 600000\alpha + 1575000 = x;\ 1000000 \leq x \leq 1575000 \tag{10}$$

$$\alpha = \frac{-600000 + \sqrt{600000^2 - 1000 \quad (1575000 - x)}}{50000};\ 1000000 \leq x \leq 1575000 \tag{11}$$

$$AB = \begin{cases} \frac{-500000 + \sqrt{500000^2 - 100000(475000 - x)}}{50000} & 475000 \leq x \leq 1000000 \\ \frac{-600000 + \sqrt{600000^2 - 100000(1575000 - x)}}{50000} & 1000000 \leq x \leq 1575000 \\ 0 & otherwise \end{cases} \tag{12}$$

$$^{\alpha}\log(A) = [\log(50\alpha + 50), \log(150 - 50\alpha)] \tag{13}$$

$$\log(50\alpha + 50) = x;\ 1.70 \leq x \leq 2 \tag{14}$$

$$\alpha = \frac{10^x - 50}{50};\ 1.70 \leq x \leq 2 \tag{15}$$

$$\log(150 - 50\alpha) = x;\ 2 \leq x \leq 2.18 \tag{16}$$

$$\alpha = \frac{150 - 10^x}{50};\ 2 \leq x \leq 2.18 \tag{17}$$

$$\log(A) = \begin{cases} \frac{10^x - 50}{50} & 1.70 \leq x \leq 2 \\ \frac{150 - 10^x}{50} & 2 \leq x \leq 2.18 \\ 0 & otherwise \end{cases} \tag{18}$$

$$^{\alpha}\log(B) = [\log(500\alpha + 9500), \log(10500 - 500\alpha)] \tag{19}$$

$$\log(500\alpha + 9500) = x;\ 3.98 \leq x \leq 4 \tag{20}$$

$$\alpha = \frac{10^x - 95}{500};\ 3.98 \leq x \leq 4 \tag{21}$$

$$\log(10500 - 500\alpha) = x;\ 4 \leq x \leq 4.02 \tag{22}$$

$$\alpha = \frac{10500-10^{x}}{500}; 4 \le x \le 4.02 \tag{23}$$

$$\log(A) = \begin{cases} \frac{10^{x}-95}{500} & 3.98 \le x \le 4 \\ \frac{10500-10^{x}}{500} & 4 \le x \le 4.02 \\ 0 & otherwise \end{cases} \tag{24}$$

$$^{\alpha}(\log(A) + \log(B)) = [\log(50\alpha + 50), \log(150 - 50\alpha)] + [\log(500\alpha + 9500), \log(10500 - 500\alpha)] \tag{25}$$

$$^{\alpha}(\log(A) + \log(B)) = [\log(50\alpha + 50) + \log(500\alpha + 9500), \log(150 - 50\alpha) + \log(10500 - 500\alpha)] \tag{26}$$

$$^{\alpha}(\log(AB)) = [\log(25000\alpha^{2} + 500000\alpha + 475000), \log(25000\alpha^{2} - 600000\alpha + 1575000)] \tag{27}$$

$$25000\alpha^{2} + 500000\alpha + 475000 = 10^{x}; 5.68 \le x \le 6 \tag{28}$$

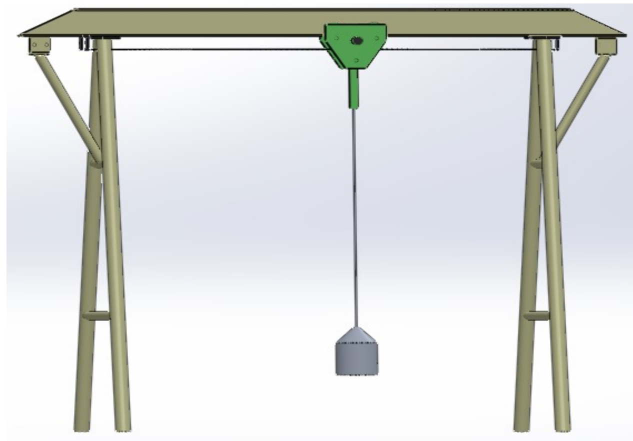
$$\alpha = \frac{-500000+\sqrt{500000^{2}-100000(475000-1\ \ ^{x})}}{50000}; 5.68 \le x \le 6 \tag{29}$$

$$25000\alpha^{2} - 600000\alpha + 1575000 = 10^{x}; 6 \le x \le 6.20 \tag{30}$$

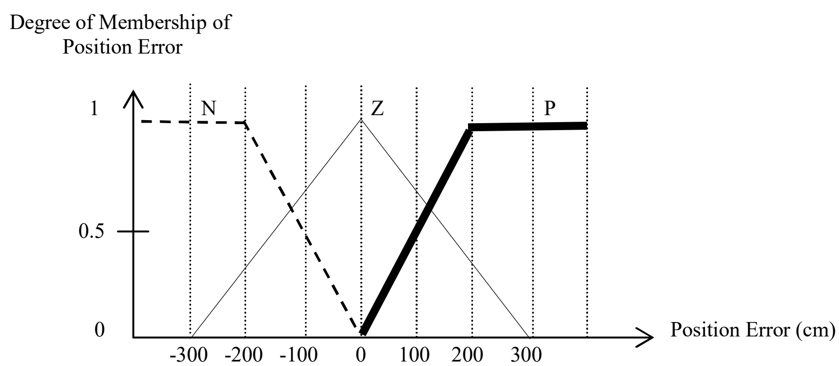
$$\alpha = \frac{-600000+\sqrt{600000^{2}-100000(1575000-10^{x})}}{50000}; 6 \le x \le 6.20 \tag{31}$$

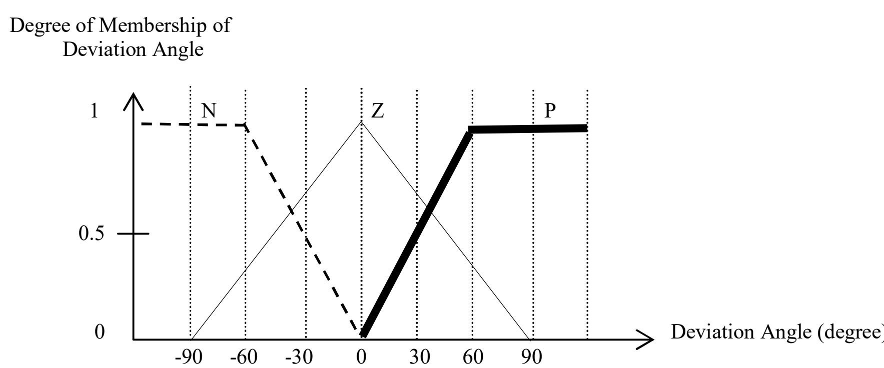
$$\log(AB) = \begin{cases} \frac{-500000+\sqrt{500000^{2}-1000\quad(475000-1\ \ ^{x})}}{50000} & 5.68 \le x \le 6 \\ \frac{-600000+\sqrt{600000^{2}-100000(1575000-10^{x})}}{50000} & 6 \le x \le 6.20 \\ 0 & otherwise \end{cases} \tag{32}$$

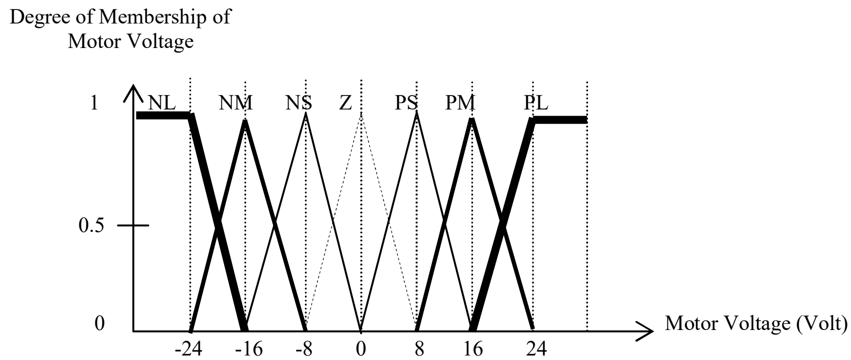
Q.2 Fuzzy logic controller (FLC) is applied to control position and to suppress vibration of an overhead crane as shown in the below figure. The input of FLC consists of position error and deviation angle of crane cable from vertical axis and the output is voltage to drive the crane motor.



Position error and deviation angle are categorized as Negative (N), Zero (Z), Positive (P) and the output motor voltage is categorized as Negative Large (NL), Negative Medium (NM), Negative Small (NS), Zero (Z), Positive Small (PS), Positive Medium (PM), Positive Large (PL). All the membership functions are shown below.







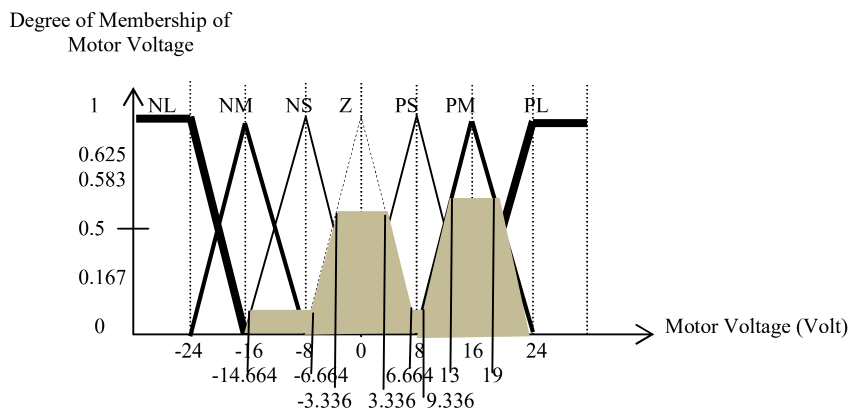
Fuzzy Inference Rule is given below.

Position Error \ Deviation Angle	N	Z	P
N	NL	NM	NS
Z	NS	Z	PS
P	PS	PM	PL

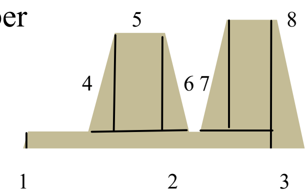
Determine the required motor voltage if the position error is detected 125 cm and the deviation angle is detected -10 degree by center of gravity defuzzification method. (30)

<u>Solution</u>

Position Error (125) \ Deviation Angle (-10)	N (0.167)	Z (0.889)	P (0)
N (0)	NL (0)	NM (0)	NS (0)
Z (0.583)	NS (0.167)	Z (0.583)	PS (0)
P (0.625)	PS (0.167)	PM (0.625)	PL (0)



Area Number



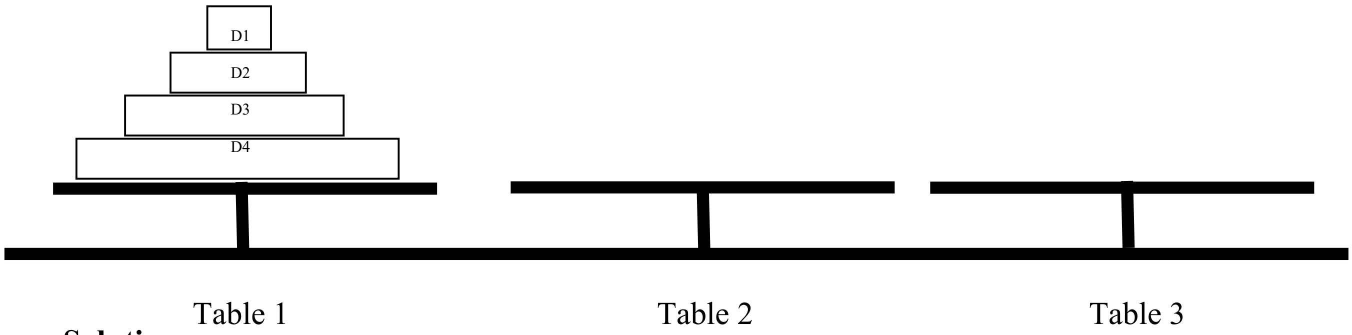
Defuzzification by center of gravity method,

$$V = \frac{[0.5\times1.336\times0.167\times(-15.109)]+[33.664\times0.167\times(2.168)]+[0.5\times5\times0.625\times(20.667)]+[0.5\times3.328\times0.416\times(-4.445)]}{[0.5\times1.336\times0.167]+[33.664\times0.167]+[0.5\times5\times0.625]+[0.5\times3.328\times0.416]+[6.672\times0.416]+[0.5\times\times3.328\times0.416]+[0.5\times3.664\times0.458]+[6\times0.458]}$$

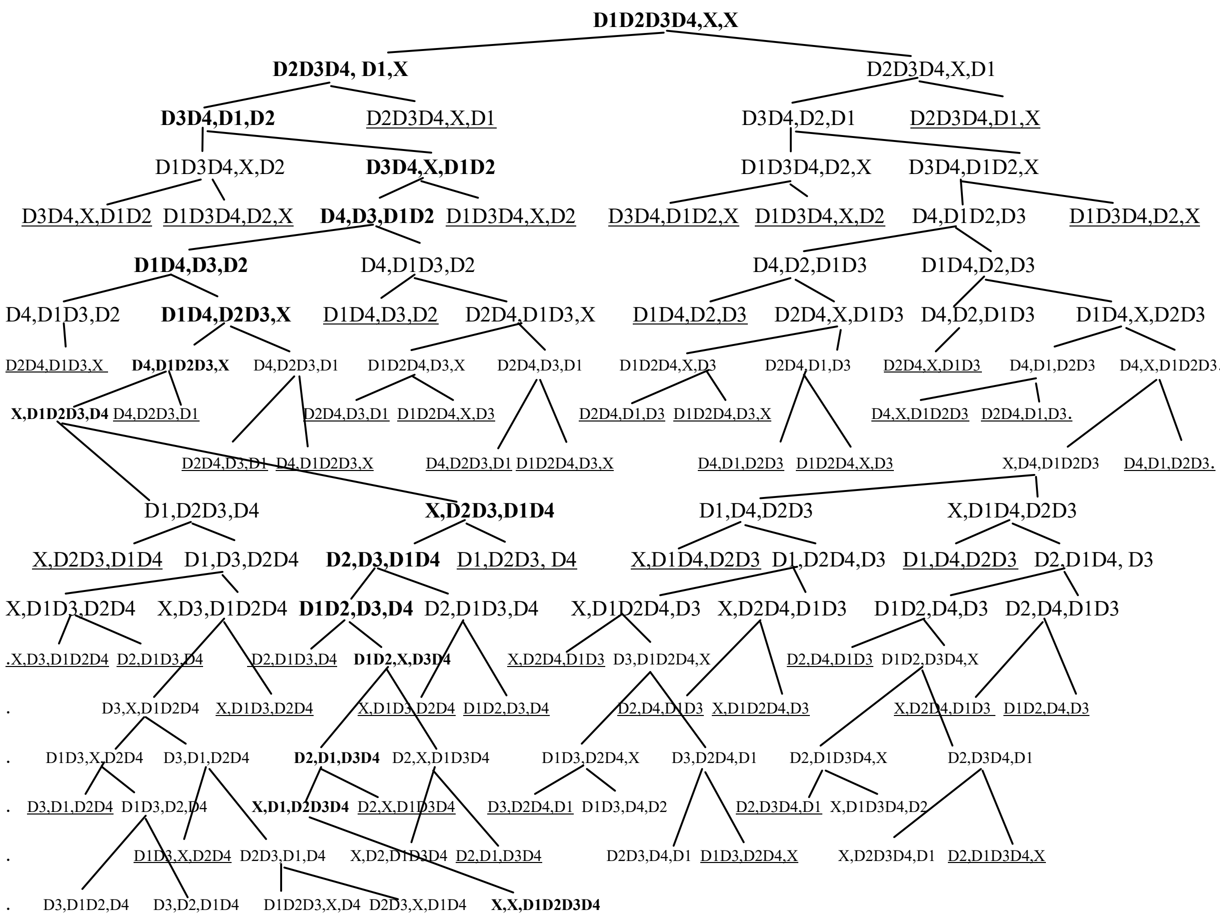
$$+\frac{[6.672\times0.416\times(0)]+[0.5\times\times3.328\times0.416\times(4.445)]+[0.5\times3.664\times0.458\times(11.779)]+[6\times0.458\times(16)]}{[0.5\times1.336\times0.167]+[33.664\times0.167]+[0.5\times5\times0.625]+[0.5\times3.328\times0.416]+[6.672\times0.416]+[0.5\times\times3.328\times0.416]+[0.5\times3.664\times0.458]+[6\times0.458]}$$

$$V = 6.425\, V$$

Q.3 In Tower of Hanoi problems, 4 circular cylinders with different diameter (D1 < D2 < D3 < D4) are firstly located on table 1 of the three tables as shown in the below figure from side view. If only one cylinder at a time can be moved from one table to another table and only smaller cylinder can be placed on top of bigger cylinder. Determine the steps in order to place again all 4 cylinders on table 3 using dynamic programming method when the objective function is the number of moves. Use symbol D1D2D3D4, X, X to represent the initial state. If D1 is moved to table 2, the state becomes, D2D3D4, D1, X. (30)



Solution



The steps follow:

D1D2D3D4,X,X -> D2D3D4,D1,X -> D3D4,D1,D2 -> D3D4,X,D1D2 -> D4,D3,D1D2 -> D1D4,D3,D2 -> D1D4,D2D3,X -> D4,D1D2D3,X -> X,D1D2D3,D4 -> X,D2D3,D1D4 -> D2,D3,D1D4 -> D1D2,D3,D4 -> D1D2,X,D3D4 -> D2,D1,D3D4 -> X,D1,D2D3D4 -> X,X,D1D2D3D4