Time: 9:00-11:00 h.   Open Book
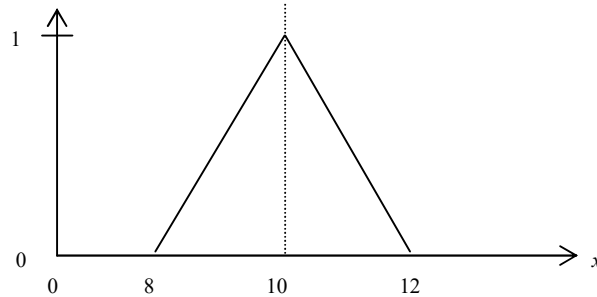Marks: 100

Attempt all questions.

Q.1   From the relation

$$(A + B)^2 = A^2 + 2AB + B^2$$

If A is about 10 and B is about 15 and their membership functions are given below, please determine membership functions of the results by using $(A + B)^2$ and $A^2 + 2AB + B^2$, then compare the results whether both membership functions are the same or not.   (35)

A(x), Degree of Membership of
      about 10

B(x), Degree of Membership of
      about 15
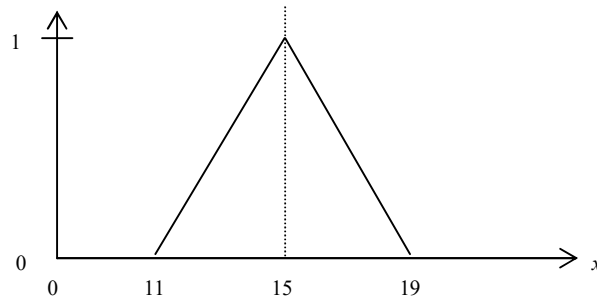
Solution

When $8 \leq x \leq 10$,
$$A(x) = 0.5x - 4 \tag{1}$$

When $10 \leq x \leq 12$,
$$A(x) = -0.5x + 6 \tag{2}$$

$$^{\alpha}A = [2\alpha + 8, 12 - 2\alpha] \tag{3}$$

When $11 \leq x \leq 15$,

$$B(x) = 0.25x - 2.75 \tag{4}$$

When $15 \leq x \leq 19$,

$$B(x) = -0.25x + 4.75 \tag{5}$$

$$^{\alpha}B = [4\alpha + 11, 19 - 4\alpha] \tag{6}$$

$$^{\alpha}(A + B) = [6\alpha + 19 \quad 31 - 6\alpha] \tag{7}$$

$$^{\alpha}(A + B)^2 = [36\alpha^2 + 228\alpha + 361 \quad 36\alpha^2 - 372\alpha + 961] \tag{8}$$

$$36\alpha^2 + 228\alpha + 361 = x \; ; 361 \leq x \leq 625 \tag{9}$$

$$\alpha = \frac{-228 + \sqrt{51984 - 144(361 - x)}}{72} \; ; 361 \leq x \leq 625 \tag{10}$$

$$36\alpha^2 - 372\alpha + 961 = x \; ; 625 \leq x \leq 961 \tag{11}$$

$$\alpha = \frac{372 - \sqrt{138384 - 14\;(961 - x)}}{72} \; ; 625 \leq x \leq 961 \tag{12}$$

$$(A + B)^2 = \begin{cases} \frac{-228 + \sqrt{51984 - 144(361 - x)}}{72} & 361 \leq x \leq 625 \\ \frac{372 - \sqrt{138384 - 144(961 - x)}}{72} & 625 \leq x \leq 961 \\ 0 & otherwise \end{cases} \tag{13}$$

$$^{\alpha}A^2 = [4\alpha^2 + 32\alpha + 64, 4\alpha^2 - 48\alpha + 144] \tag{14}$$

$$^{\alpha}B^2 = [16\alpha^2 + 88\alpha + 121, 16\alpha^2 - 152\alpha + 361] \tag{15}$$

$$^{\alpha}2AB = [16\alpha^2 + 108\alpha + 176, 16\alpha^2 - 172\alpha + 456] \tag{16}$$

$$^{\alpha}A^2 + 2AB + B^2 = [36\alpha^2 + 228\alpha + 361, 36\alpha^2 - 372\alpha + 961] \tag{17}$$

$$A^2 + 2AB + B^2 = \begin{cases} \frac{-228 + \sqrt{51984 - 144(361 -)}}{72} & 361 \leq x \leq 625 \\ \frac{372 - \sqrt{138384 - 144(961 - x)}}{72} & 625 \leq x \leq 961 \\ 0 & otherwise \end{cases} \tag{18}$$

The membership functions are the same.

Q.2　Fuzzy logic controller (FLC) is applied to swing a moving-cart type inverted pendulum from the bottom position up to the upright position as shown in the below figure. The input of

FLC consists of pendulum angular position, θ, and pendulum angular velocity, $\dot{\theta}$, and the output is voltage to drive the cart motor, V.
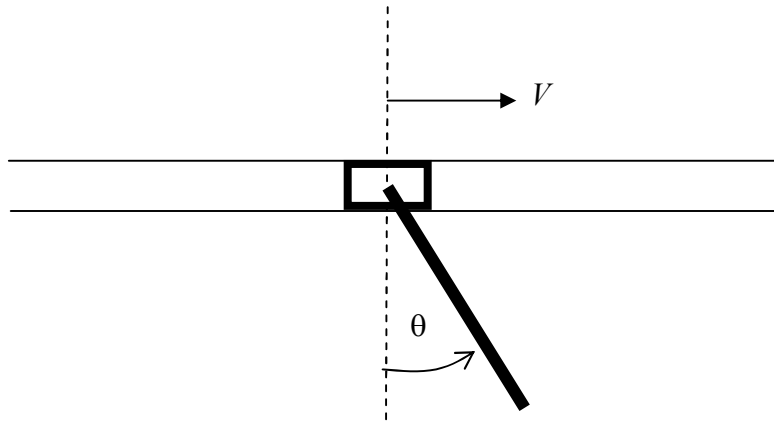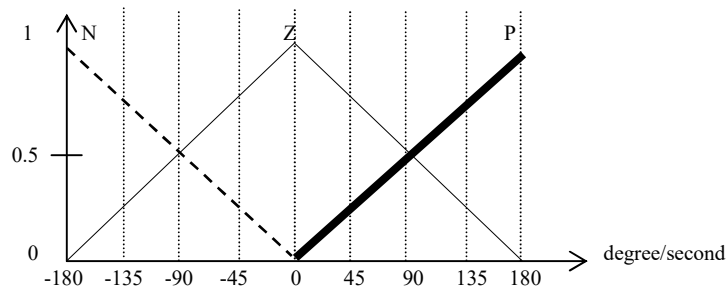

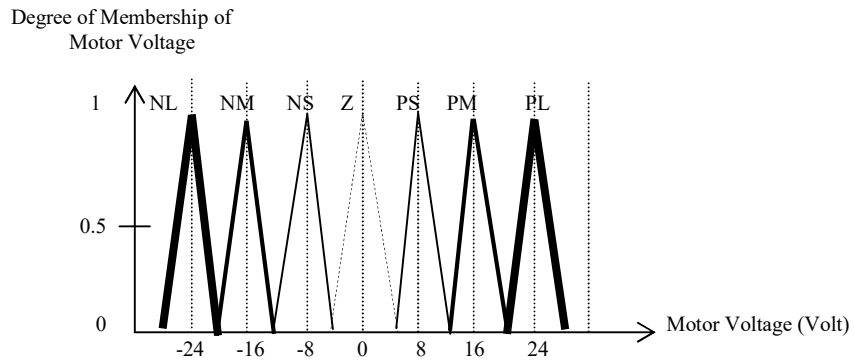Pendulum angular position and velocity are categorized as Negative (N), Zero (Z), Positive (P) and the output motor voltage is categorized as Negative Large (NL), Negative Medium (NM), Negative Small (NS), Zero (Z), Positive Small (PS), Positive Medium (PM), Positive Large (PL). All the membership functions are shown below.

Degree of Membership of Pendulum Angular Position
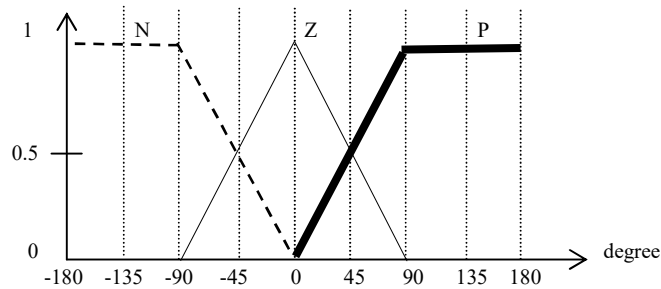
Degree of Membership of Pendulum Angular Velocity

Degree of Membership of
Motor Voltage

1 — NL   NM   NS   Z   PS   PM   PL

0.5 —

0 —

-24   -16   -8   0   8   16   24   Motor Voltage (Volt)

Fuzzy Inference Rule is given below.

Angular Velocity / Angular Position	N	Z	P
N	NS	PS	PL
Z	NM	Z	PM
P	NL	NS	PS

Determine the required motor voltage at the instant when the pendulum angular position is at 30 degree the pendulum angular velocity is at -18 degree/second by center of gravity defuzzification method.   (35)

Solution

Fuzzy Inference Rule is given below.

Angular Position (30) / Angular Velocity (-18)	N (0.10)	Z (0.90)	P (0.00)
N (0.00)	NS (0.00)	PS (0.00)	PL (0.00)
Z (0.67)	NM (0.10)	Z (0.67)	PM (0.00)
P (0.33)	NL (0.10)	NS (0.33)	PS (0.00)
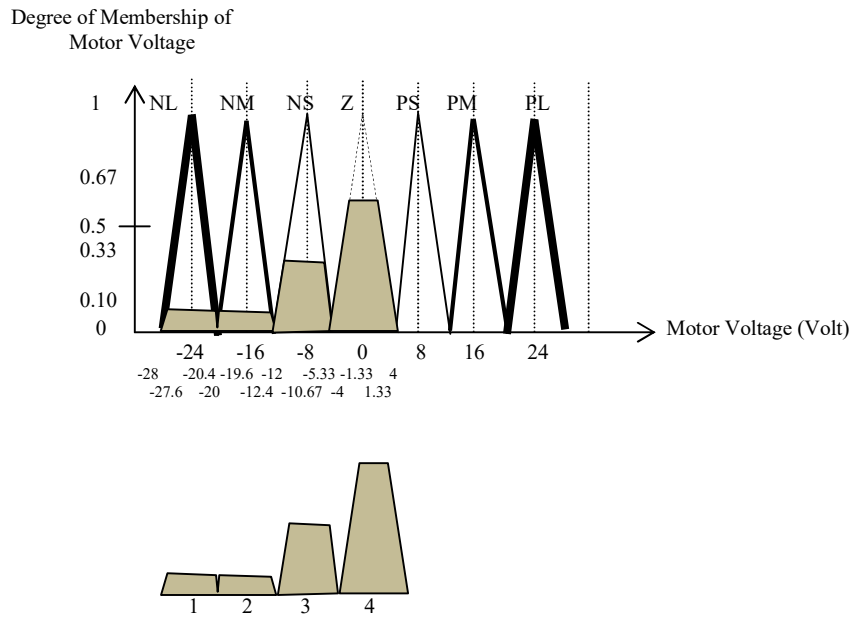
Defuzzification by center of gravity method,

$$V = \frac{[0.5\times15.2\times0.1\times(-24)]+[0.5\times15.2\times0.1\times(-1\ )]+[0.5\times13.33\times0.33\times(-8)]+[0.5\times10.67\times0.67\times(0)]}{[0.5\times15.2\times0.1]+[0.5\times15.2\times0.1]+[0.5\times13.33\times0.33]+[0.5\times10.67\times0.67]} = -6.58\,V$$

Q.3   A monkey, a dog, a cat, a bird, and a worm stay together in an old room. If all animals have to move to the new room however the walkway is so small that at most of 2 animals can move at the same time. After going to the new room, at least one animal has to come back to the old room to notify that the walkway is ready. Furthermore, the monkey cannot leave the pair of the dog and the cat or the pair of the cat and the bird or the pair of the bird and the worm to stay together without the monkey otherwise the dog will bite the cat or the cat will bite the bird or the bird will bite the worm. Determine the successful pattern of motion by hill climbing method using the following priority: (1) more number of animals travelling to the new room or less number of animals travelling back to the old room and (2) priority list: monkey > dog > cat > bird > worm. Each state is represented by the animals who stay on the old room, thus the initial state is represented by MDCBW. When a node is opened, all children nodes must be listed in the semantic tree.   (30)

Solution

5

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                                    MDCBW
        MCW          MCB        MCBW       MDBW      MDCW       MDCB

    MDCW      MCBW

  CW            DW

  MCBW

  MCB

  MDCB

      DB          MCB        MDB

  MDB         MDBW

    B             D

    DB

    -
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