

Time: 13:00-15:00 h.

Open Book

Marks: 100

Attempt all questions.

Q.1 Log-sigmoid function is expressed by

$$y(n) = \frac{1}{1 + e^{-n}}$$

If n is a triangular shape fuzzy number with membership function expressed by

$$n(x) = \begin{cases} 1 - \frac{|x - n|}{s} & ; n - s < x \leq n + s \\ 0 & ; otherwise \end{cases}$$

(a) Determine membership function of Log-sigmoid function, $y(x)$. (20)

(b) If $n = 0, s = 2$, roughly draw membership function of $y(x)$, and determine $^{0+}y(x)$. (10)

Solution

(a) Rewrite the membership function of $n(x)$,

$$n(x) = \begin{cases} \frac{x + s - n}{s} & ; n - s < x \\ \frac{-x + s + n}{s} & ; x \leq n + s \\ 0 & ; otherwise \end{cases}$$

$$^{\alpha}(n) = [s(\alpha - 1) + n \quad s(1 - \alpha) + n]$$

$$^{\alpha}(-n) = [s(\alpha - 1) - n \quad s(1 - \alpha) - n]$$

$$^{\alpha}(e^{-n}) = [e^{s(\alpha-1)-n} \quad e^{s(1-\alpha)-n}]$$

$$^{\alpha}(1 + e^{-n}) = [1 + e^{s(\alpha-1)-n} \quad 1 + e^{s(1-\alpha)-n}]$$

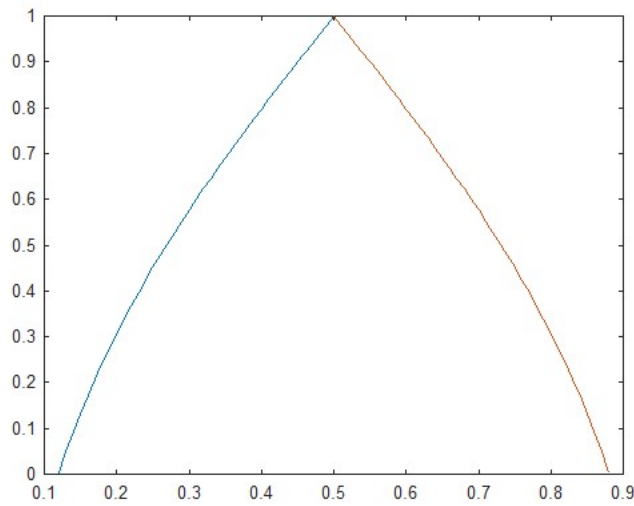
$$^{\alpha}\left(\frac{1}{1 + e^{-n}}\right) = \left[\frac{1}{1 + e^{s(1-\alpha)-n}} \quad \frac{1}{1 + e^{s(\alpha-1)-n}}\right]$$

$$y(x) = \begin{cases} 1 - \frac{n + \ln\left(\frac{1}{x} - 1\right)}{s} & ; \frac{1}{1 + e^{s-n}} \leq x < \frac{1}{2} \\ 1 + \frac{n + \ln\left(\frac{1}{x} - 1\right)}{s} & ; \frac{1}{2} \leq x < \frac{1}{1 + e^{-s-n}} \\ 0 & ; \text{otherwise} \end{cases}$$

(b) $n = 0, s = 2,$

$$y(x) = \begin{cases} 1 - \frac{\ln\left(\frac{1}{x} - 1\right)}{2} & ; \frac{1}{1 + e^2} \leq x < \frac{1}{2} \\ 1 + \frac{\ln\left(\frac{1}{x} - 1\right)}{2} & ; \frac{1}{2} \leq x < \frac{1}{1 + e^{-2}} \\ 0 & ; \text{otherwise} \end{cases}$$

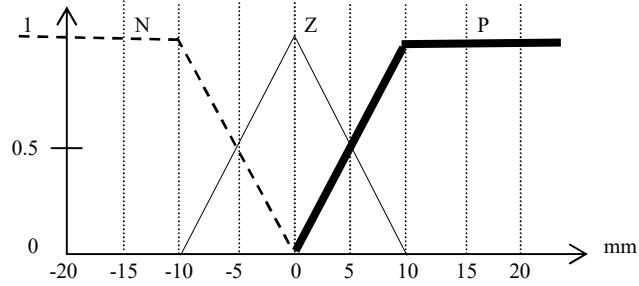
$${}^{0+}y(x) = \left(\frac{1}{1 + e^2} \quad \frac{1}{1 + e^{-2}} \right) = (0.12, 0.88)$$



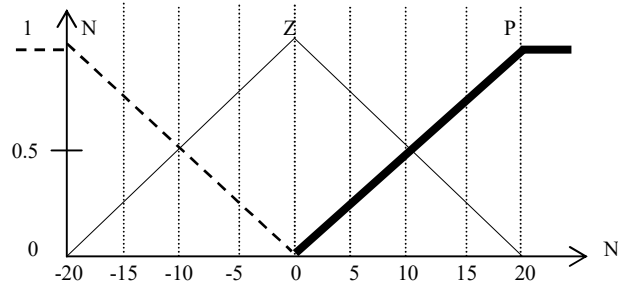
Q.2 Sugeno-type fuzzy logic controller (FLC) is applied in haptics control of a haptics gripper. The input of FLC consists of (1) position error, p , (desired gripper position - actual gripper position measured from an encoder) and (2) force error, f , (desired gripping force - actual gripping force measured from a load cell), and the output is voltage, v , to drive the motor of the haptics device.

Position error and force error are categorized as Negative (N), Zero (Z), Positive (P) and the output motor voltage is singleton-type membership function. The membership functions of position error and force error are shown below.

Degree of Membership of Position Error



Degree of Membership of Force error



Fuzzy inference rule to determine output voltage (Volts) is given below.

Force Error \ Position Error	N	Z	P
N	-24	-3	6
Z	-12	0	12
P	-6	3	24

Determine the required motor voltage of the haptics device when the desired gripper position is set at 60 mm and the desired gripping force is at 40 N.

(a) If the actual gripper position is measured at 54 mm and the actual gripping force is measured at 25 N. (10)

(b) If the actual gripper position is measured at 45 mm and the actual gripping force is measured at 0 N. (10)

(c) If the actual gripper position is measured at 65 mm and the actual gripping force is measured at 44 N. (10)

Solution

(a) Degree of membership in the fuzzy inference rule is determined.

Force Error Position Error (6mm) (15 N)	N (0)	Z (0.25)	P (0.75)
N (0)	-24 (0)	-3 (0)	6 (0)
Z (0.4)	-12 (0)	0 (0.25)	12 (0.4)
P (0.6)	-6 (0)	3 (0.25)	24 (0.6)

Defuzzification is applied to determine the motor voltage,

$$v = \frac{[0 \times 0.25] + [3 \times 0.25] + [12 \times 0.4] + [24 \times 0.6]}{[0.25] + [0.25] + [0.4] + [0.6]} = 13.3 \text{ Volts}$$

(b) Degree of membership in the fuzzy inference rule is determined.

Force Error Position Error (15mm) (40 N)	N (0)	Z (0)	P (1)
N (0)	-24 (0)	-3 (0)	6 (0)
Z (0)	-12 (0)	0 (0)	12 (0)
P (1)	-6 (0)	3 (0)	24 (1)

Defuzzification is applied to determine the motor voltage,

$$v = \frac{[24 \times 1]}{[1]} = 24 \text{ Volts}$$

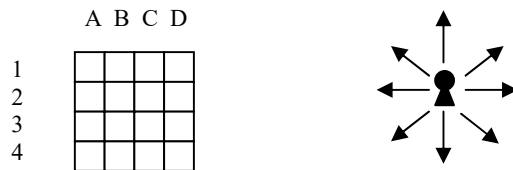
(c) Degree of membership in the fuzzy inference rule is determined.

Force Error Position Error (-5mm) (-4 N)	N (0.2)	Z (0.8)	P (0)
N (0.5)	-24 (0.2)	-3 (0.5)	6 (0)
Z (0.5)	-12 (0.2)	0 (0.5)	12 (0)
P (0)	-6 (0)	3 (0)	24 (0)

Defuzzification is applied to determine the motor voltage,

$$v = \frac{[-24 \times 0.2] + [-12 \times 0.2] + [-3 \times 0.5] + [0 \times 0.5]}{[0.2] + [0.2] + [0.5] + [0.5]} = -6.21 \text{ Volts}$$

Q.3 Depth-first search and breadth-first search are applied to determine solution of 4-queen problem. Four queens must be placed in a 4x4 chessboard from the first row until the fourth row safely. Each queen can move in horizontal, vertical, and diagonal directions, thus, placing a new queen in these directions of the previously placed queens is prohibited as shown below.



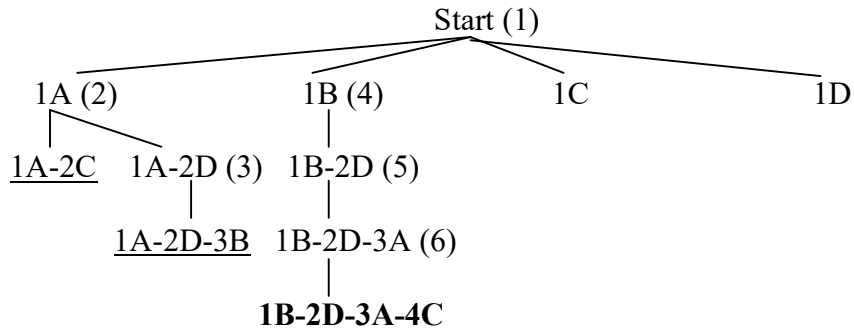
If the board configuration of 1A-2C means the first queen is placed at 1A and the second queen is placed at 2C for instance. Label order of node opening from Start with label 1 and sort the children nodes following alphabetic list.

(a) Show search tree and solution from depth-fist search. (15)

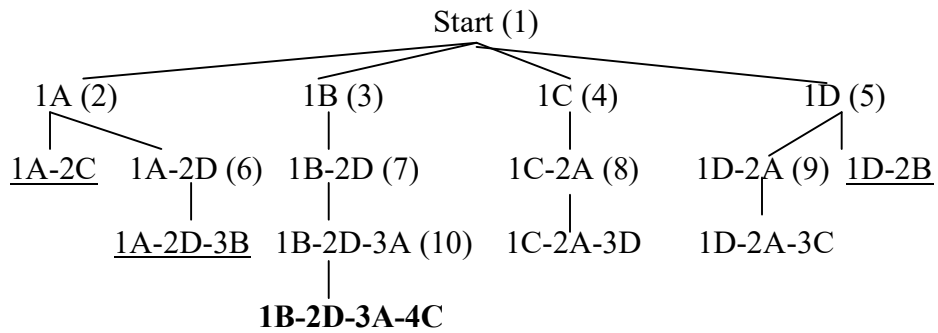
(b) Show search tree and solution from breadth-first search. (15)

Solution

(a) Depth-first search



(b) Breadth-first search



Q.4 Apply either fuzzy or artificial intelligence to a new application. Explain in detail how you design the system. (10)