

Final Examination AI and Neuro-Fuzzy Theory AT07.24 April 30, 2019

Time: 9:00-11:00 h.

Open Book

Marks: 100

Attempt all questions.

Q.1 Membership functions of two fuzzy numbers; about 5, $A(x)$, and about -4, $B(x)$, are given as follows

$$A(x) = \begin{cases} \frac{x}{4} - \frac{1}{4} & ; 1 < x \leq 5 \\ -\frac{x}{4} + \frac{9}{4} & ; 5 < x \leq 9 \\ 0 & ; otherwise \end{cases}$$

$$B(x) = \begin{cases} \frac{x}{2} + 3 & ; -6 < x \leq -4 \\ -\frac{x}{2} - 1 & ; -4 < x \leq -2 \\ 0 & ; otherwise \end{cases}$$

(a) Determine membership function of $A^2 - B^2$. (10)

(b) Determine membership function of $(A - B)(A + B)$. (10)

(c) Is the relation, $A^2 - B^2 = (A - B)(A + B)$, valid on fuzzy numbers? (10)

Solution

(a)

$$\alpha(A) = [4\alpha + 1 \quad 9 - 4\alpha]$$

$$\alpha(B) = [2\alpha - 6 \quad -2 - 2\alpha]$$

$$\alpha(A^2) = [4\alpha + 1 \quad 9 - 4\alpha] \cdot [4\alpha + 1 \quad 9 - 4\alpha] = [16\alpha^2 + 8\alpha + 1 \quad 16\alpha^2 - 72\alpha + 81]$$

$$\alpha(B^2) = [2\alpha - 6 \quad -2 - 2\alpha] \cdot [2\alpha - 6 \quad -2 - 2\alpha] = [4\alpha^2 + 8\alpha + 4 \quad 4\alpha^2 - 24\alpha + 36]$$

$$\begin{aligned} \alpha(A^2) - \alpha(B^2) &= [16\alpha^2 + 8\alpha + 1 \quad 16\alpha^2 - 72\alpha + 81] - [4\alpha^2 + 8\alpha + 4 \quad 4\alpha^2 - 24\alpha + 36] \\ &= [12\alpha^2 + 32\alpha - 35 \quad 12\alpha^2 - 80\alpha + 77] \end{aligned}$$

$$A^2 - B^2 = \begin{cases} \frac{\sqrt{2704 + 48x}}{24} - \frac{4}{3} & ; -35 < x \leq 9 \\ \frac{10}{3} - \frac{\sqrt{2704 + 48x}}{24} & ; 9 < x \leq 77 \\ 0 & ; otherwise \end{cases}$$

(b)

$$\alpha(A) = [4\alpha + 1 \quad 9 - 4\alpha]$$

$$\alpha(B) = [2\alpha - 6 \quad -2 - 2\alpha]$$

$$\alpha(A) - \alpha(B) = [4\alpha + 1 \quad 9 - 4\alpha] - [2\alpha - 6 \quad -2 - 2\alpha] = [6\alpha + 3 \quad 15 - 6\alpha]$$

$$\alpha(A) + \alpha(B) = [4\alpha + 1 \quad 9 - 4\alpha] + [2\alpha - 6 \quad -2 - 2\alpha] = [6\alpha - 5 \quad 7 - 6\alpha]$$

$$(\alpha(A) - \alpha(B))(\alpha(A) + \alpha(B)) = [6\alpha + 3 \quad 15 - 6\alpha] \cdot [6\alpha - 5 \quad 7 - 6\alpha]$$

$$= \begin{cases} [-36\alpha^2 + 120\alpha - 75 \quad 36\alpha^2 - 132\alpha + 105] & ; 0 \leq \alpha < 5/6 \\ [36\alpha^2 - 12\alpha - 15 \quad 36\alpha^2 - 132\alpha + 105] & ; 5/6 \leq \alpha \leq 1 \end{cases}$$

$$(A - B)(A + B) = \begin{cases} \frac{10}{6} - \frac{\sqrt{3600 - 144x}}{72} & , -75 < x \leq 0 \\ \frac{1}{6} + \frac{\sqrt{2304 + 144x}}{72} & , 0 < x \leq 9 \\ \frac{11}{6} - \frac{\sqrt{2304 + 144x}}{72} & , 9 < x \leq 105 \\ 0 & , otherwise \end{cases}$$

(c) The relation, $A^2 - B^2 = (A - B)(A + B)$, is not valid on fuzzy numbers.

Q.2 Fuzzy logic controller (FLC) is applied to control water quality of a fish pond. The input of FLC consists of (1) dissolved oxygen (DO) and (2) pH level of the fish pond, and the output are turbine speed (v) and water circulation flow rate (f).

DO, x , in mg/l unit is categorized as Low (L), Medium (M), and High (H) with the following membership functions.

$$L(x) = e^{-(x-5)^2/2}$$

$$M(x) = e^{-(x-10)^2/2}$$

$$H(x) = e^{-(x-15)^2/2}$$

pH level, y , is categorized as Acid (A), Neutral (N), and Base (B) with the following membership functions.

$$A(y) = e^{-(x-5)^2}$$

$$N(y) = e^{-(x-7)^2}$$

$$B(y) = e^{-(x-9)^2}$$

Turbine speed, w , in rpm unit is categorized as Very Low (VL), Low (L), Medium (M), High (H) and Very High (VH) with the following membership functions.

$$VL(w) = \max\left(1 - \frac{|w - 40|}{5}, 0\right)$$

$$L(w) = \max\left(1 - \frac{|w - 50|}{5}, 0\right)$$

$$M(w) = \max\left(1 - \frac{|w - 60|}{5}, 0\right)$$

$$H(w) = \max\left(1 - \frac{|w - 70|}{5}, 0\right)$$

$$VH(w) = \max\left(1 - \frac{|w - 80|}{5}, 0\right)$$

Water circulation flowrate, f , in m^3/s unit has singleton-type membership function.

Fuzzy inference rule to determine turbine speed is given below.

DO \ PH	A	N	B
L	VH	M	VH
M	H	L	H
H	M	VL	M

Fuzzy inference rule to determine water circulation flowrate is given below.

DO \ PH	A	N	B
L	2.0	0	1.5
M	1.5	0	1.0

H	1.0	0	0.5
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Determine turbine speed and water flow rate when DO is measured at 8 mg/l and pH level is measured at 6.5. (30)

Solution

(a) Degree of membership in the fuzzy inference rule is determined.

For turbine speed

PH (6.5) \ DO (8 mg/l)	A (0.11)	N (0.78)	B (0.00)
L (0.01)	VH (0.01)	M (0.01)	VH (0.00)
M (0.14)	H (0.11)	L (0.14)	H (0.00)
H (0.00)	M (0.00)	VL (0.00)	M (0.00)

Defuzzification is applied to determine the turbine speed,

$$w = \frac{[50 \times 0.5 \times 0.14 \times (20 - 2 \times 5 \times 0.14)] + [60 \times 0.5 \times 0.01 \times (20 - 2 \times 5 \times 0.01)] + [70 \times 0.5 \times 0.11 \times (20 - 2 \times 5 \times 0.11)] + [80 \times 0.5 \times 0.01 \times (20 - 2 \times 5 \times 0.01)]}{[0.5 \times 0.14 \times (20 - 2 \times 5 \times 0.14)] + [0.5 \times 0.01 \times (20 - 2 \times 5 \times 0.01)] + [0.5 \times 0.11 \times (20 - 2 \times 5 \times 0.11)] + [0.5 \times 0.01 \times (20 - 2 \times 5 \times 0.01)]}$$

$$w = 59.75rpm$$

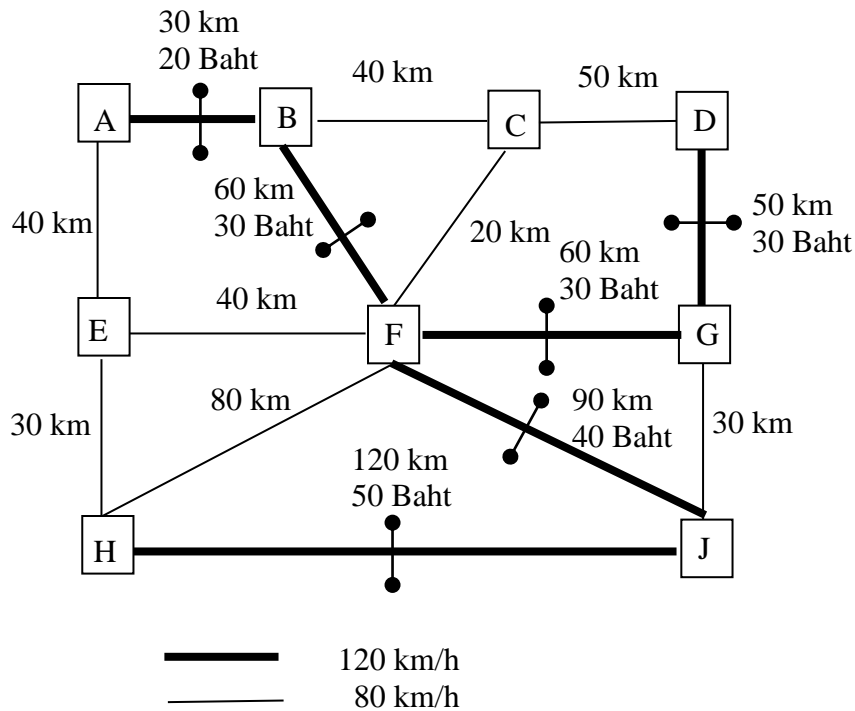
For water circulation flowrate

PH (6.5) \ DO (8 mg/l)	A (0.11)	N (0.78)	B (0.00)
L (0.01)	2.0 (0.01)	0 (0.01)	1.5 (0.00)
M (0.14)	1.5 (0.11)	0 (0.14)	1.0 (0.00)
H (0.00)	1.0 (0.00)	0 (0.00)	0.5 (0.00)

Defuzzification is applied to determine the circulation flowrate,

$$f = \frac{[0 \times 0.14] + [1.5 \times 0.11] + [2 \times 0.01]}{[0.14] + [0.11] + [0.01]} = 0.71 m^3/s$$

Q.3 AI is used in a car navigation system. Consider the map below showing road distances between cities (in km), with speed limit (in km/h), and toll fee (in Baht).

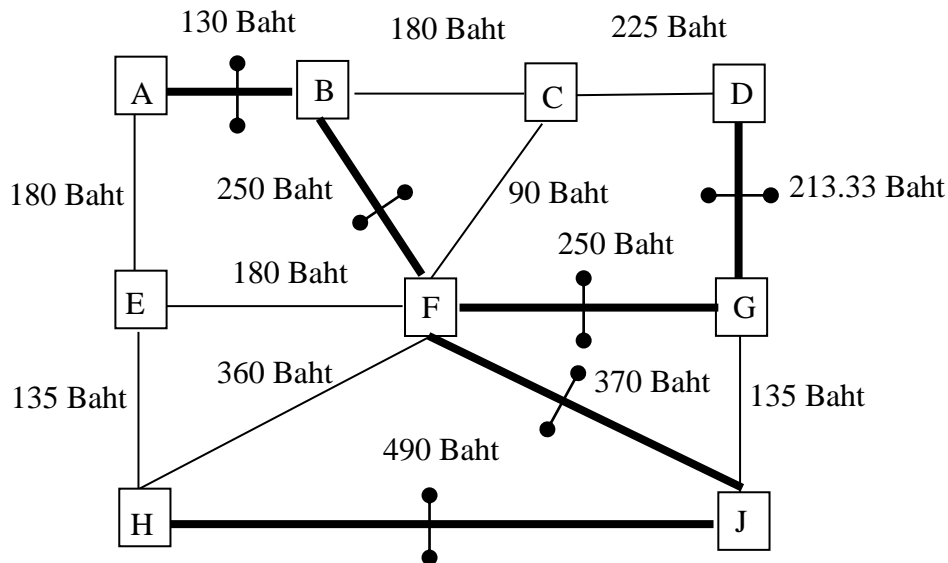


Determine the best route from city A to city J according to the cost function defined by

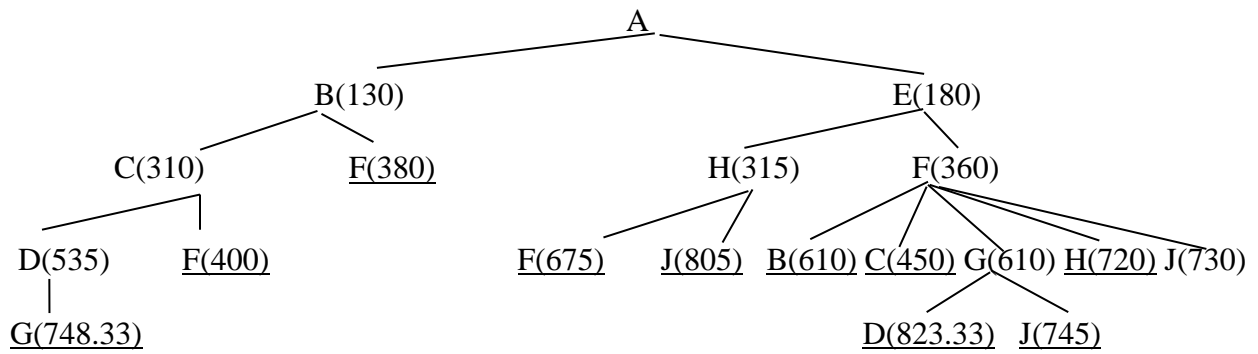
Cost Function = [Distance (km) x 2 (Baht)] + [Travelling Time (hour) x 200 (Baht)] + [Toll Fee (Baht, if any)] by dynamic programming. Show search tree, order of opened nodes, route, and total cost. (30)

Solution

Determine costs between cities.



By dynamic programming,



Order of the opened nodes: A, B(130), E(180), C(310), H(315), F(360), D(535), G(610)

The route is A-> E-> F-> J

Total cost is 730 Baht

Q.4 If Genetic Algorithm (GA) is applied to determine the unknown coefficients of the relation between input, x , and output, y , of a system, expressed by $y = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$. Explain in detail the step to apply GA with examples. (10)