Final Examination AI and Neuro-Fuzzy Theory AT07.24 November 27, 2020

Time: 9:00-11:00 h. Open Book
Marks: 100

Attempt all questions.

Q.1 Consider a linear equation $y = mx + c$. If the slope is a fuzzy number of about 2 and expressed by

$$m(z) = \begin{cases} 1 - \dfrac{|z-2|}{0.5} & ; 1.5 \le z \le 2.5 \\ 0 & ; otherwise \end{cases}$$

and the y-intercept is a fuzzy number of about 8 and expressed by

$$c(z) = \begin{cases} 1 - \dfrac{|z-8|}{2} & ; 6 \le z \le 10 \\ 0 & ; otherwise \end{cases}$$

Determine degree of membership of the result, y (about 18), when x is a fuzzy number of about 5 and expressed by

$$x(z) = \begin{cases} 1 - \dfrac{|z-5|}{1} & ; 4 \le z \le 6 \\ 0 & ; otherwise \end{cases}$$

(25)

Solution

$${}^{\alpha}(m) = [1.5 + 0.5\alpha \quad 2.5 - 0.5\alpha]$$

$${}^{\alpha}(c) = [6 + 2\alpha \quad 10 - 2\alpha]$$

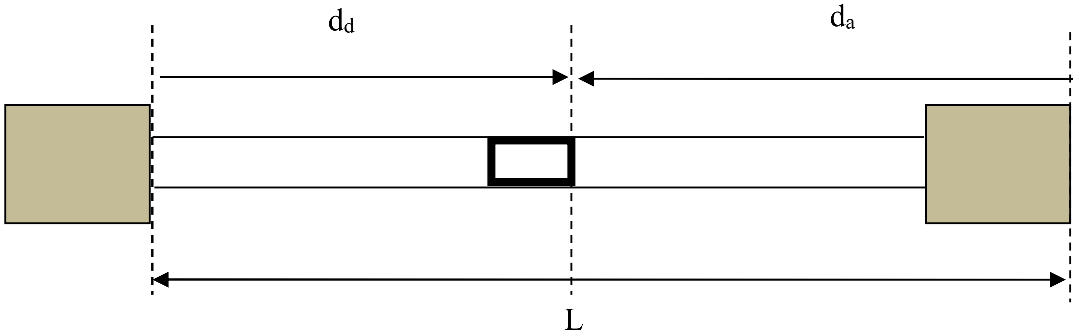
$${}^{\alpha}(x) = [4 + \alpha \quad 6 - \alpha]$$

$${}^{\alpha}(mx) = [0.5\alpha^2 + 3.5\alpha + 6 \quad 0.5\alpha^2 - 5.5\alpha + 15]$$

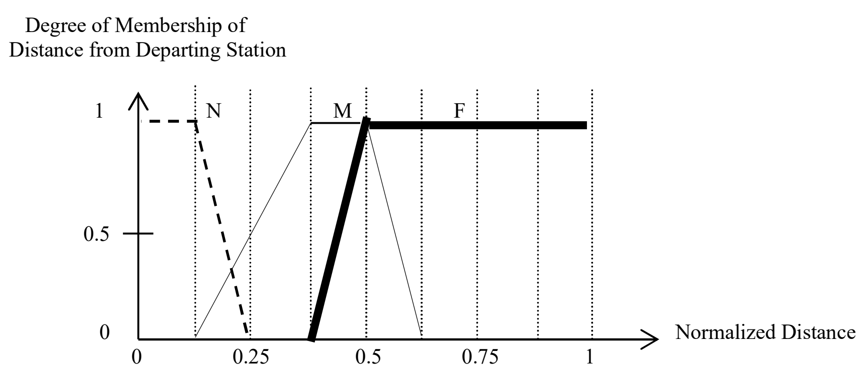
$${}^{\alpha}(mx + c) = [0.5\alpha^2 + 5.5\alpha + 12 \quad 0.5\alpha^2 - 7.5\alpha + 25]$$

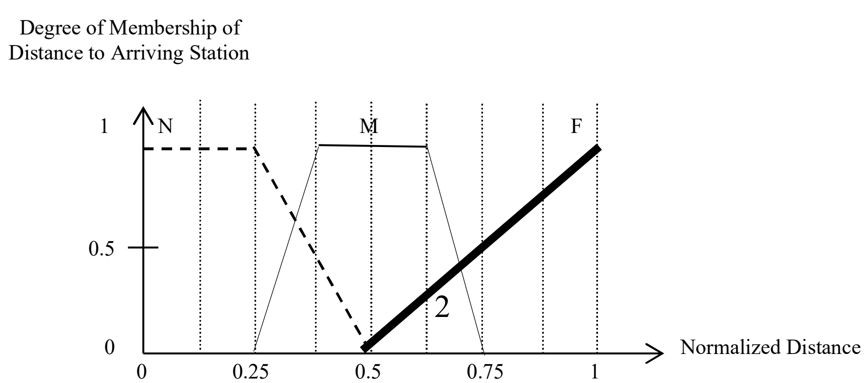
$$y = \begin{cases} -5.5 + \sqrt{30.25 - 24 + 2y} & ; 12 < y \le 18 \\ 7.5 - \sqrt{56.25 - 50 + 2y} & ; 18 < y \le 25 \\ 0 & ; otherwise \end{cases}$$

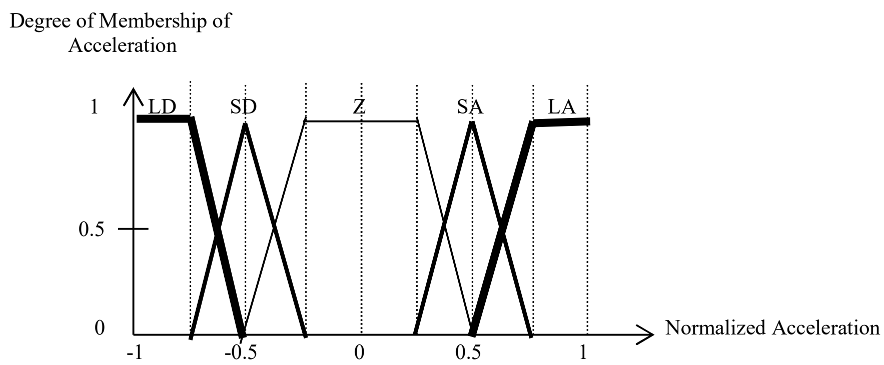
Q.2 Fuzzy logic controller (FLC) is applied to control an automatic train to run from a departing station to an arriving station as shown below. If the inputs of the controller are the normalized distance from the departing station, (d_d/L), and the normalized distance to the arriving station, (d_a/L), and the output of the controller is the train acceleration.



The normalized distance from the departing station and the normalized distance to the arriving station are categorized as Near (N), Medium (M), Far (F) and the output normalized acceleration is categorized as Large Deceleration (LD), Small Deceleration (SD), Zero (Z), Small Acceleration (SA), Large Acceleration (LA). All the membership functions are shown below.







Fuzzy Inference Rule is given below.

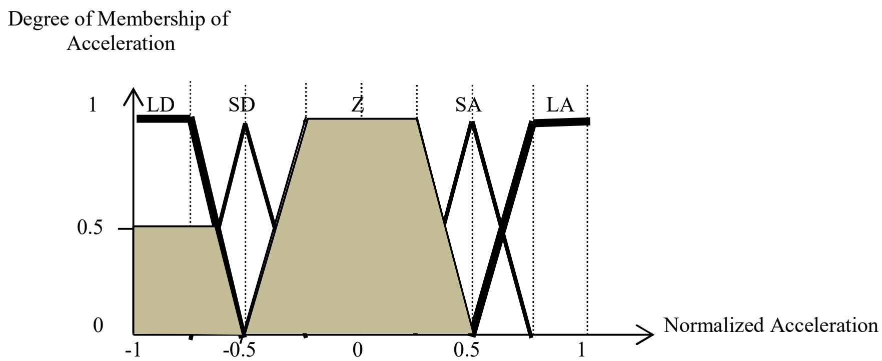
Distance to Arriving Station / Distance from Departing Station	F	M	N
N	SA	Z	SD
M	LA	Z	LD
F	LA	Z	LD

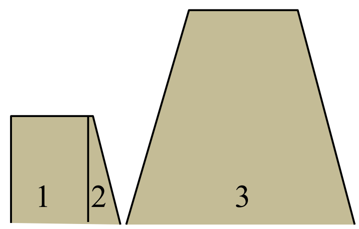
Determine the required acceleration at the instant when the train is at 1250 m from the departing station and the distance between the station is 2000 m by using center of gravity defuzzification. Show all the steps in determining the required acceleration clearly. (25)

<u>Solution</u>

When the train is at 1250 m from the departing station and the distance between the station is 2000 m, $\frac{d_d}{L} = 0.625, \frac{d_a}{L} = 0.375.$

Distance to Arriving Station / Distance from Departing Station	F (0)	M (1)	N (0.5)
N (0)	SA (0)	Z (0)	SD (0)
M (0)	LA (0)	Z (0)	LD (0)
F (1)	LA (0)	Z (1)	LD (0.5)

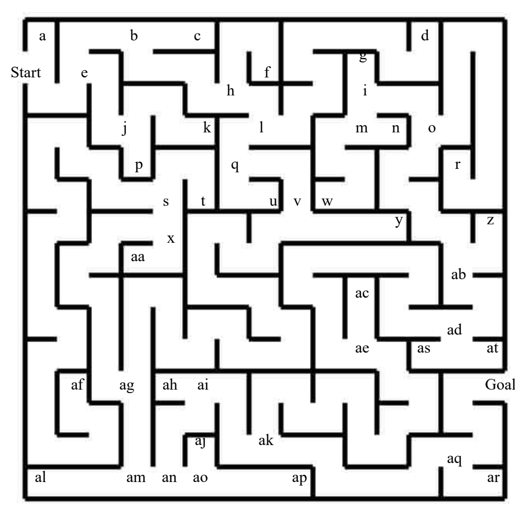




Defuzzification by center of gravity method,

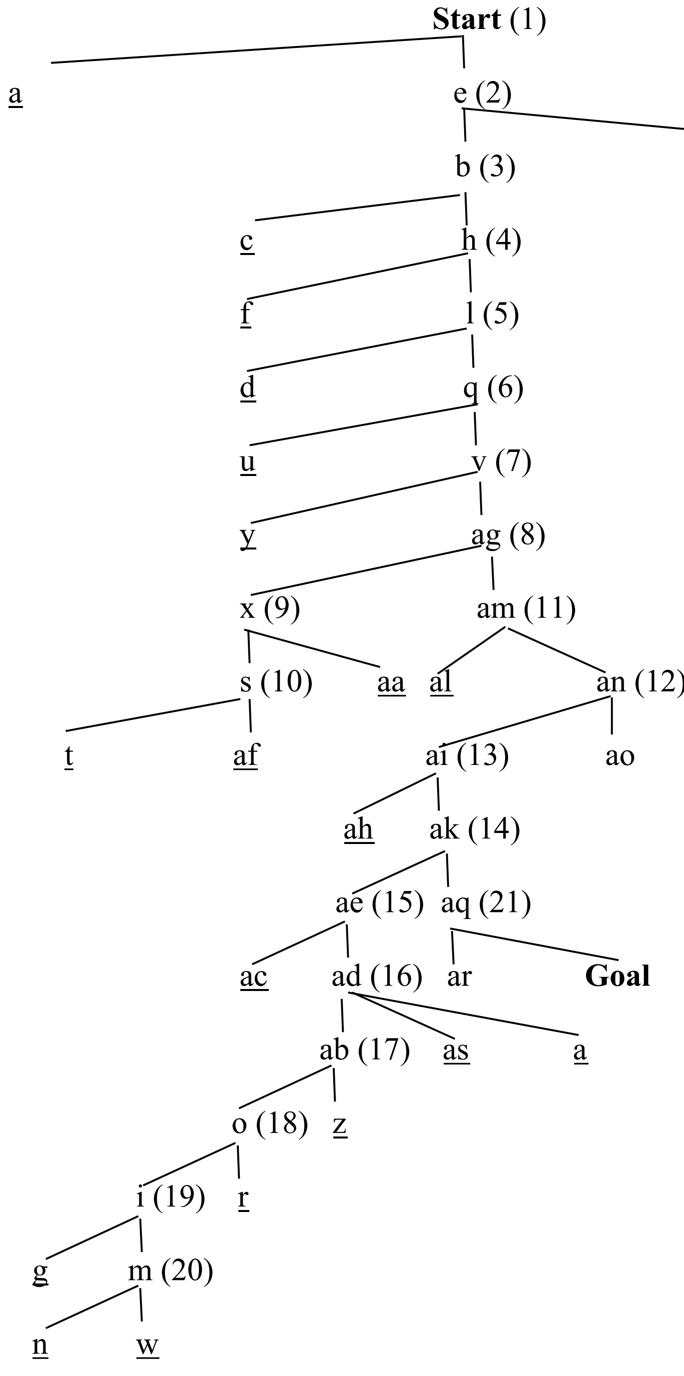
$$Norm\ Acc = \frac{[0.5\times 0.375\times(-0.8125)]+[0.5\times 0.5\times 0.125\times(-0.5833)]+[0.5\times 1\times 1.5\times(0)]}{[0.5\times 0.375]+[0.5\times 0.5\times 0.125]+[0.5\times 1\times 1.5]} = -0.176$$

Q.3 Consider a maze as shown in the below figure. Nodes shown by alphabetic symbols (Start, Goal, a, b, .. aa, ab, ..) are defined at the start, the goal, the junctions, and the dead ends. Determine the path from Start to Goal by Depth-first search (use alphabetic order for sorting). Please show the trees and label the order of node opening. (25)



Solution

Depth-first search (use alphabetic order for sorting)



Solution: Start-e-b-h-l-q-v-ag-am-an-ai-ak-aq-Goal

Q.4 Make the truth table using Lukasiewicz's formula of the 3-value logic as expressed by

$$[(A \wedge B) \rightarrow C] \leftrightarrow [\bar{A} \vee \bar{B} \vee C]$$

When $\wedge$ represents AND, $\vee$ represents OR, $\bar{A}$ represents NOT of A, $\rightarrow$ represents IF THEN, and $\leftrightarrow$ represents IF AND ONLY IF. (25)

Solution

A	B	C	$(A \wedge B)$	$[(A \wedge B) \rightarrow C]$	$\leftrightarrow$	$\bar{A}$	$\bar{B}$	$[\bar{A} \vee \bar{B} \vee C]$
0	0	0	0	1	**1**	1	1	1
0	0	0.5	0	1	**1**	1	1	1
0	0	1	0	1	**1**	1	1	1
0	0.5	0	0	1	**1**	1	0.5	1
0	0.5	0.5	0	1	**1**	1	0.5	1
0	0.5	1	0	1	**1**	1	0.5	1
0	1	0	0	1	**1**	1	0	1
0	1	0.5	0	1	**1**	1	0	1
0	1	1	0	1	**1**	1	0	1
0.5	0	0	0	1	**1**	0.5	1	1
0.5	0	0.5	0	1	**1**	0.5	1	1
0.5	0	1	0	1	**1**	0.5	1	1
0.5	0.5	0	0	1	**0.5**	0.5	0.5	0.5
0.5	0.5	0.5	0	1	**0.5**	0.5	0.5	0.5
0.5	0.5	1	0	1	**1**	0.5	0.5	1
0.5	1	0	0.5	0.5	**1**	0.5	0	0.5
0.5	1	0.5	0.5	1	**0.5**	0.5	0	0.5
0.5	1	1	0.5	1	**1**	0.5	0	1
1	0	0	0	1	**1**	0	1	1
1	0	0.5	0	1	**1**	0	1	1
1	0	1	0	1	**1**	0	1	1
1	0.5	0	0.5	0.5	**1**	0	0.5	0.5
1	0.5	0.5	0.5	1	**0.5**	0	0.5	0.5
1	0.5	1	0.5	1	**1**	0	0.5	1
1	1	0	1	0	**1**	0	0	0
1	1	0.5	1	0.5	**1**	0	0	0.5
1	1	1	1	1	**1**	0	0	1