Final Examination AI and Neuro-Fuzzy Theory AT07.24 April 29, 2020

Time: 13:00-15:00 h. Open Book
Marks: 100

Attempt all questions.

Q.1 AIT student ID number is expressed by 12ABCD. If A, B, C, and D are fuzzy numbers with triangular shape membership function having the center at its value and the span, s, of 0.5 as expressed by the following membership function for fuzzy number A. (Membership functions for fuzzy numbers B, C, and D are obtained by substation of A in the below equation by B, C, or D accordingly)

$$A(x) = \begin{cases} 1 - \dfrac{|x - A|}{s} & ; A - s \leq x \leq A + s \\ 0 & ; otherwise \end{cases}$$

Substitute A, B, C, and D by the numbers of your AIT student ID number. Then determine membership function of the result of $(A - B) \times (C + D)$. (40)

Solution

$$^{\alpha}(A) = [A - 0.5 + 0.5\alpha \quad A + 0.5 - 0.5\alpha]$$

$$^{\alpha}(B) = [B - 0.5 + 0.5\alpha \quad B + 0.5 - 0.5\alpha]$$

$$^{\alpha}(C) = [C - 0.5 + 0.5\alpha \quad C + 0.5 - 0.5\alpha]$$

$$^{\alpha}(D) = [D - 0.5 + 0.5\alpha \quad D + 0.5 - 0.5\alpha]$$

$$^{\alpha}(A - B) = [A - B - 1 + \alpha \quad A - B + 1 - \alpha]$$

$$^{\alpha}(C + D) = [C + D - 1 + \alpha \quad C + D + 1 - \alpha]$$

If $A > B$,

$$^{\alpha}[(A - B) \times (C + D)]$$

$$= [(A - B - 1 + \alpha)(C + D - 1 + \alpha) \quad (A - B + 1 - \alpha)(C + D + 1 - \alpha)]$$

$$(A-B)\times(C+D)=M\times N$$

$$=\begin{cases}\dfrac{2-(M+N)+\sqrt{(M+N)^2-4(MN-x)}}{2} & ; MN-(M+N)+1<x\le MN\\ \dfrac{2+(M+N)-\sqrt{(M+N)^2-4(MN-x)}}{2} & ; MN<x\le MN+(M+N)+1\\ 0 & ; otherwise\end{cases}$$

If $A<B$,

$$^{\alpha}[(A-B)\times(C+D)]$$

$$=[(A-B-1+\alpha)(C+D+1-\alpha)\quad(A-B+1-\alpha)(C+D-1+\alpha)]$$

$$(A-B)\times(C+D)=M\times N$$

$$=\begin{cases}\dfrac{2+(N-M)-\sqrt{(N-M)^2+4(MN-x)}}{2} & ; MN+(M-N)-1<x\le MN\\ \dfrac{2+(M-N)-\sqrt{(M-N)^2+4(MN-x)}}{2} & ; MN<x\le MN+(N-M)-1\\ 0 & ; otherwise\end{cases}$$

If $A=B$,

$$^{\alpha}[(A-B)\times(C+D)]$$

$$=[(A-B-1+\alpha)(C+D+1-\alpha)\quad(A-B+1-\alpha)(C+D+1-\alpha)]$$

$$(A-B)\times(C+D)=M\times N=\begin{cases}\frac{2+(N-M)-\sqrt{(N-M)^2+4(MN-x)}}{2} & ; MN+(M-N)-1<x\le MN\\ \frac{2+(M+N)-\sqrt{(M+N)^2-4(MN-x)}}{2} & ; MN<x\le MN+(M+N)+1\\ 0 & ; otherwise\end{cases}$$

To show the concept, I use my AIT ID number (6347),

$$(6-3)\times(4+7)=3\times 11=\begin{cases}\dfrac{-12+\sqrt{(14)^2-4(33-x)}}{2} & ; 20<x\le 33\\ \dfrac{16-\sqrt{(14)^2-4(33-x)}}{2} & ; 33<x\le 48\\ 0 & ; otherwise\end{cases}$$

Q.2 Particle Swarm Optimization (PSO) is applied to determine the minimum point of a polynomial function $f(x)=1x^5-2x^4+Ax^3-Bx^2+Cx-D$ in the range $-999\le x\le 999$, when A, B, C, and D are the numbers obtained from your AIT student ID number as expressed by 12ABCD. Show all the particle positions after 2 generations by applying the following conditions to PSO. (30)

- Population size = 5.
- Particle position is represented by $x_i = (x_{i1})$. Particle velocity is represented by $v_i = (v_{i1})$.
- The initial particles are obtained by evoking 5 random numbers with uniform distribution. (Use [rand] command in matlab or [RND] key from calculator.net/scientific-calculator.html) Limit the number to the first three digits after dot. If the evoked random number is $0.mnpxxx$, the particle initial position locates at $\pm mnp$. If p is odd number, use negative value for the particle initial position. If p is even number, use positive value for the particle initial position.
- All the particle initial velocities are 0.
- The objective function is the function $f(x) = 1x^5 - 2x^4 + Ax^3 - Bx^2 + Cx - D$, when smaller value is better.
- The particle velocity is calculated from $v_i(k+1) = 0.4v_i(k) + 0.3r_1(P_i(k) - x_i(k)) + 0.3r_2(G(k) - x_i(k))$. When r_1 and r_2 are random numbers in between 0.000 and 0.999 obtained by evoking random numbers similar to the random numbers obtained in the initial particles but use only positive value. The velocity is limited to the range $-999 \leq v_i \leq 999$ and rounded off to the nearest integer.
- The particle position is updated by using $x_i(k+1) = x_i(k) + v_i(k+1)$ and also limited to the range $-999 \leq x_i \leq 999$.

Solution

To show the concept, I use my AIT ID number,

$$f(x) = 1x^5 - 2x^4 + 6x^3 - 3x^2 + 4x - 7$$

Generation 0,

Particle Positions are initialized at

$$x_1(0) = 632, x_2(0) = -97, x_3(0) = 278, x_4(0) = 546, x_5(0) = -957$$

Objective function values are determined

$$f(x_1) = 1.0 \times 10^{14}, f(x_2) = -8.8 \times 10^{9}, f(x_3) = 1.6 \times 10^{12},$$

$$f(x_4) = 4.8 \times 10^{13}, f(x_5) = -8.0 \times 10^{14}$$

$$P_1(0) = 632, P_2(0) = -97, P_3(0) = 278, P_4(0) = 546, P_5(0) = -957, G(0) = -957$$

Generation 1,

$$v_1(1) = 0.4(0) + 0.3(0.965)(632 - 632) + 0.3(0.158)(-957 - 632) = -75$$

$$v_2(1) = 0.4(0) + 0.3(0.970)(-97 + 97) + 0.3(0.957)(-957 + 97) = -247$$

$$v_3(1) = 0.4(0) + 0.3(0.485)(278 - 278) + 0.3(0.800)(-957 - 278) = -333$$

$$v_4(1) = 0.4(0) + 0.3(0.142)(546 - 546) + 0.3(0.422)(-957 - 546) = -190$$

$$v_5(1) = 0.4(0) + 0.3(0.916)(-957 + 957) + 0.3(0.792)(-957 + 957) = 0$$

$$x_1(1) = 632 - 75 = 557$$

$$x_2(1) = -97 - 247 = -344$$

$$x_3(1) = 278 - 333 = -55$$

$$x_4(1) = 546 - 190 = 356$$

$$x_5(1) = -957 + 0 = -957$$

$$f(x_1) = 5.3 \times 10^{13}, f(x_2) = -4.8 \times 10^{12}, f(x_3) = -5.2 \times 10^{8},$$

$$f(x_4) = 5.7 \times 10^{12}, f(x_5) = -8.0 \times 10^{14}$$

$$P_1(1) = 557, P_2(0) = -344, P_3(0) = -55, P_4(0) = 356, P_5(0) = -957, G(0) = -957$$

Generation 2,

$$v_1(2) = 0.4(-75) + 0.3(0.960)(557 - 557) + 0.3(0.656)(-957 - 557) = -328$$

$$v_2(2) = 0.4(-247) + 0.3(0.036)(-344 + 344) + 0.3(0.849)(-957 + 344) = -255$$

$$v_3(2) = 0.4(-333) + 0.3(0.934)(-55 + 55) + 0.3(0.679)(-957 + 55) = -317$$

$$v_4(2) = 0.4(-190) + 0.3(0.758)(356 - 356) + 0.3(0.743)(-957 - 356) = -369$$

$$v_5(2) = 0.4(0) + 0.3(0.392)(-957 + 957) + 0.3(0.656)(-957 + 957) = 0$$

$$x_1(2) = 557 - 328 = 229$$

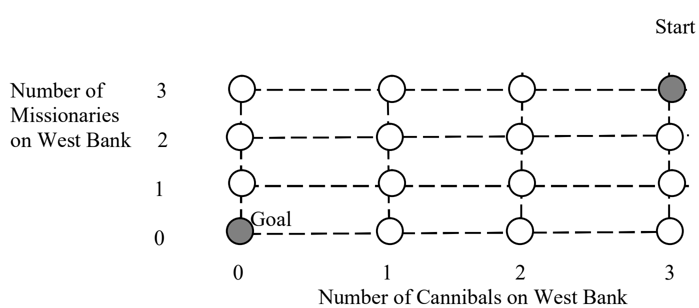
$$x_2(2) = -344 - 255 = -599$$

$$x_3(2) = -55 - 317 = -372$$

$$x_4(2) = 356 - 369 = -13$$

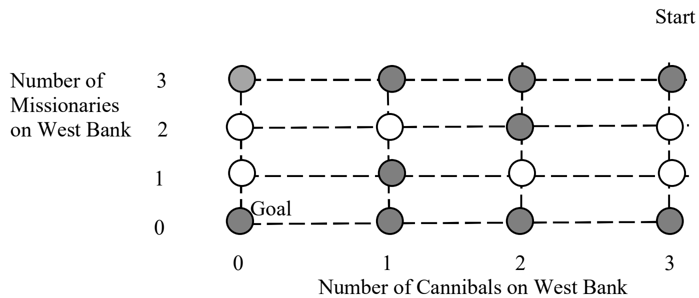
$$x_5(2) = -957 + 0 = -957$$

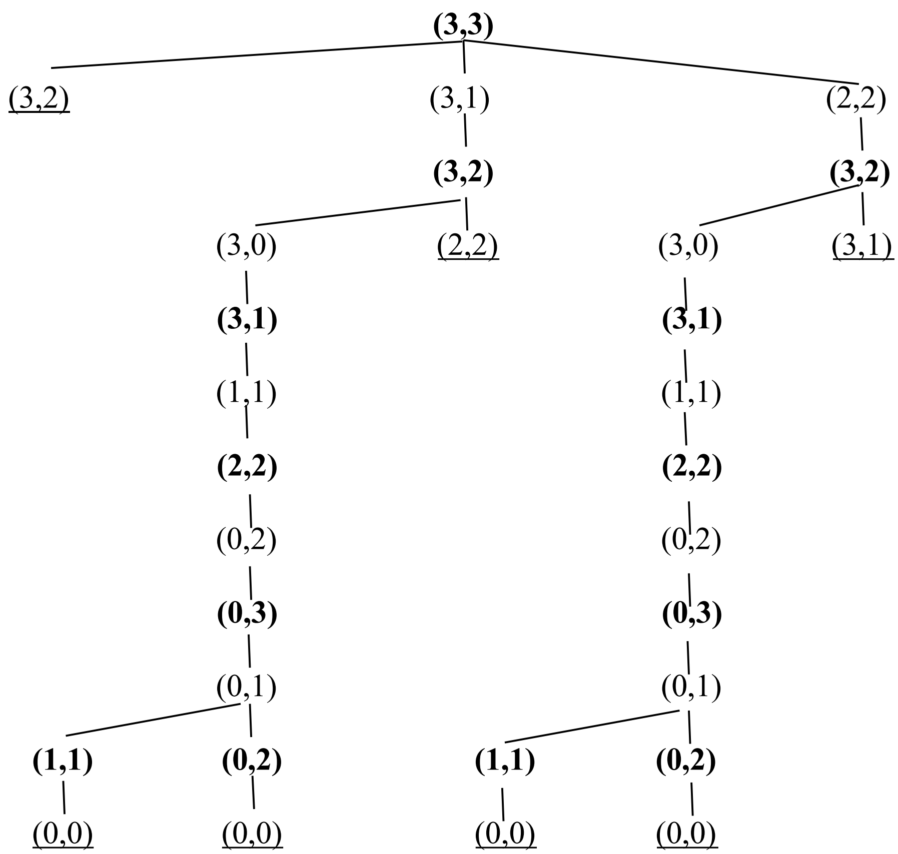
Q.3 Consider the missionaries-cannibals problem from the class exercise represented in the state space representation as shown in the figure below when (*m*, *n*) shows *m* missionaries and *n* cannibals on the west bank of a river. Initially, there are 3 missionaries and 3 cannibals on the west bank of the river. A small boat can carry at most of 2 persons. If the number of cannibals is larger than the number of missionaries either on the west bank or on the east bank of the river, the missionaries will be eaten by the cannibals. The boat without passenger cannot move. Draw the whole search tree and show the step of all solutions. Use regular font (*m*, *n*) for the result of west-east trip and use bold font ***(m, n)*** for the result of east-west trip. (30)



Solution

Label the safe nodes,





Solution 1: **(3,3)** - (3,1) - **(3,2)** - (3,0) - **(3,1)** - (1,1) - **(2,2)** - (0,2) - **(0,3)** - (0,1) - **(1,1)** - (0,0)

Solution 2: **(3,3)** - (3,1) - **(3,2)** - (3,0) - **(3,1)** - (1,1) - **(2,2)** - (0,2) - **(0,3)** - (0,1) - **(0,2)** - (0,0)

Solution 3: **(3,3)** - (2,2) - **(3,2)** - (3,0) - **(3,1)** - (1,1) - **(2,2)** - (0,2) - **(0,3)** - (0,1) - **(1,1)** - (0,0)

Solution 4: **(3,3)** - (2,2) - **(3,2)** - (3,0) - **(3,1)** - (1,1) - **(2,2)** - (0,2) - **(0,3)** - (0,1) - **(0,2)** - (0,0)