Final Examination AI and Neuro-Fuzzy Theory AT07.24 December 4, 2021

Time: 9:00-11:00 h. Open Book

Marks: 100

Attempt all questions.

Q.1 Consider a logarithmic relation $\log\left(\frac{A}{B}\right) = \log(A) - \log(B)$. If A is a fuzzy number of about 100 and expressed by

$$A(x) = \begin{cases} 1 - \dfrac{|x-100|}{5} & ; 95 \le x \le 105 \\ 0 & ; otherwise \end{cases}$$

and B is a fuzzy number of about 10 and expressed by

$$B(x) = \begin{cases} 1 - \dfrac{|x-10|}{0.5} & ; 9.5 \le x \le 10.5 \\ 0 & ; otherwise \end{cases}$$

Determine whether the membership function of $\log\left(\frac{A}{B}\right)$, is the same as the membership function of $\log(A) - \log(B)$ or not. (25)

Solution

$$^{\alpha}(A) = [95 + 5\alpha \quad 105 - 5\alpha]$$

$$^{\alpha}(B) = [9.5 + 0.5\alpha \quad 10.5 - 0.5\alpha]$$

$$^{\alpha}\left(\frac{A}{B}\right) = \left[\frac{95 + 5\alpha}{10.5 - 0.5\alpha} \quad \frac{105 - 5\alpha}{9.5 + 0.5\alpha}\right]$$

$$^{\alpha}\log\left(\frac{A}{B}\right) = \left[\log\left(\frac{95 + 5\alpha}{10.5 - 0.5\alpha}\right) \quad \log\left(\frac{105 - 5\alpha}{9.5 + 0.5\alpha}\right)\right]$$

$$\log\left(\frac{A}{B}\right) = \begin{cases} \dfrac{10.5 \times 10^{x} - 95}{0.5 \times 10^{x} + 5} & ; 0.96 < x \le 1 \\ \dfrac{105 - 9.5 \times 10^{x}}{0.5 \times 10^{x} + 5} & ; 1 < x \le 1.04 \\ 0 & ; otherwise \end{cases}$$

$$^{\alpha}(\log(A)) = [\log(95 + 5\alpha) \quad \log(105 - 5\alpha)]$$

$$^{\alpha}(\log(B)) = [\log(9.5 + 0.5\alpha) \quad \log(10.5 - 0.5\alpha)]$$

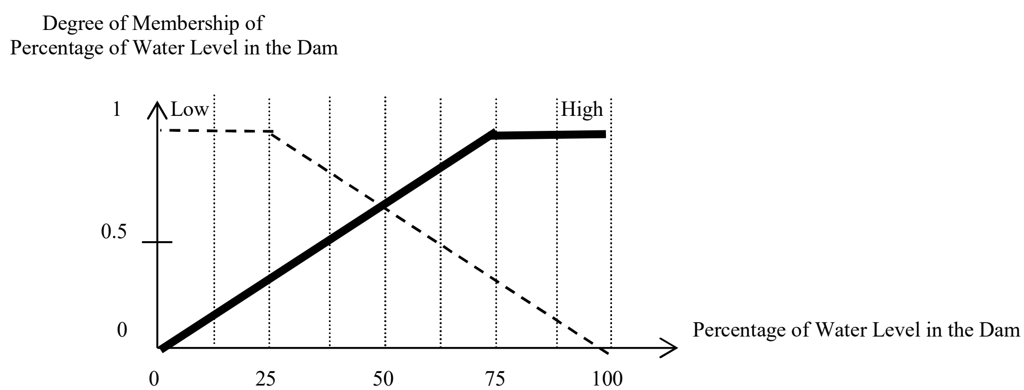
$$^{\alpha}(\log(A) - \log(B)) = [\log(95 + 5\alpha) - \log(10.5 - 0.5\alpha) \quad \log(105 - 5\alpha) - \log(9.5 + 0.5\alpha)]$$

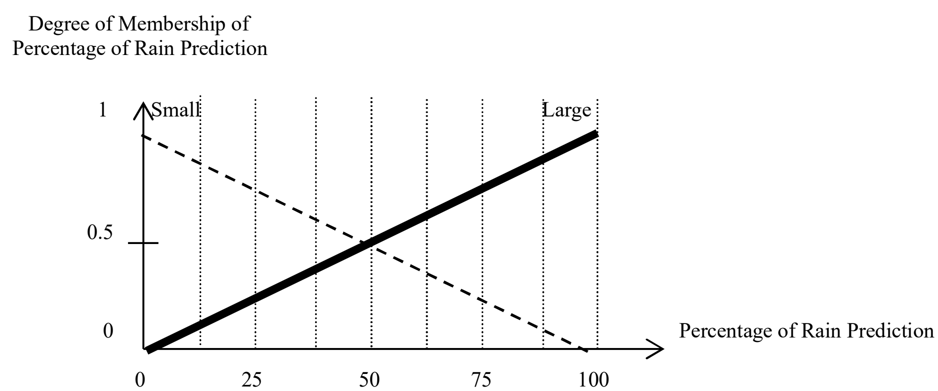
$$\log(A) - \log(B) = \begin{cases} \dfrac{10.5 \times 10^x - 95}{0.5 \times 10^x + 5} & ; 0.96 < x \leq 1 \\ \dfrac{105 - 9.5 \times 10^x}{0.5 \times 10^x + 5} & ; 1 < x \leq 1.04 \\ 0 & ; otherwise \end{cases}$$

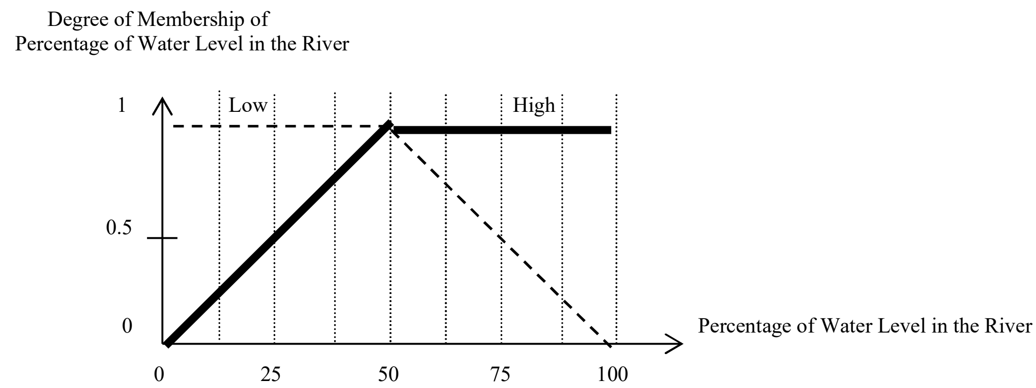
The membership function of $\log\left(\frac{A}{B}\right)$, is the same as the membership function of $\log(A) - \log(B)$.

Q.2 Fuzzy logic controller (FLC) is applied to control the percentage of gate opening of a dam. If the three inputs consist of 1. percentage of water level in the dam (Low, High), 2. percentage of rain prediction (Small, Large), and 3. percentage of water level of the river after the dam (Low, High). If the output of the controller is the percentage of gate opening and represented by singleton type membership functions (0%, 25%, 50%, 75%, 100%).

Membership functions of all the three inputs are shown below.







Fuzzy Inference Rule is given below.

Water Level in Dam	Rain Prediction	Water Level in River	Gate Opening
Low	Small	Low	50
Low	Small	High	25
Low	Large	Low	25
Low	Large	High	0
High	Small	Low	100
High	Small	High	75
High	Large	Low	75
High	Large	High	50

Determine the amount of gate opening at the instant when the percentage of water level in the dam is 80%, the percentage of rain prediction is 30%, and the percentage of water level of the river after the dam is 50%. (25)

Solution

Water Level in Dam 80%	Rain Prediction 30%	Water Level in River 50%	Gate Opening
Low (0.27)	Small (0.7)	Low (1)	50 (0.27)

Low (0.27)	Small (0.7)	High (1)	25 (0.27)
Low (0.27)	Large (0.3)	Low (1)	25 (0.27)
Low (0.27)	Large (0.3)	High (1)	0 (0.27)
High (1)	Small (0.7)	Low (1)	100 (0.7)
High (1)	Small (0.7)	High (1)	75 (0.7)
High (1)	Large (0.3)	Low (1)	75 (0.3)
High (1)	Large (0.3)	High (1)	50 (0.3)

Based on the highest degree of membership of the output,

100 (0.7), 75(0.7), 50(0.3), 25(0.27), 0(0.27)

Thus,

$$Gate\ Opening = \frac{100\ (0.7) +\ 75(0.7) + 50(0.3) +\ 25(0.27) +\ 0(0.27)}{(0.7 + 0.7 + 0.3 + 0.27 + 0.27)} = 64.40\%$$

Q.3 Consider the water jug problem. There are 2 jugs with the capacities of 4 L and 3 L respectively. Assume there is unlimited of supply water from a tap to the jug and assume that we are allowed to pour water unlimited amount to the ground. If there is no device to measure the amount of water which means we can fill water from the tap to make the jug full from the initial amount and we can also pour the remaining water to the ground. We can pour water from one jug to the other jug until the receiving jug is full or until there is no water left in the giving jug. Use a tuple of (*x, y*) to represent the amount of water *x* L in the 4-L jug and the amount of water *y* L in the 3-L jug. The initial state is (0, 0) and the desired final state is (2, 0).

Determine the steps to have 2 L of water inside the 4-L jug by

(a) Hill-climbing search

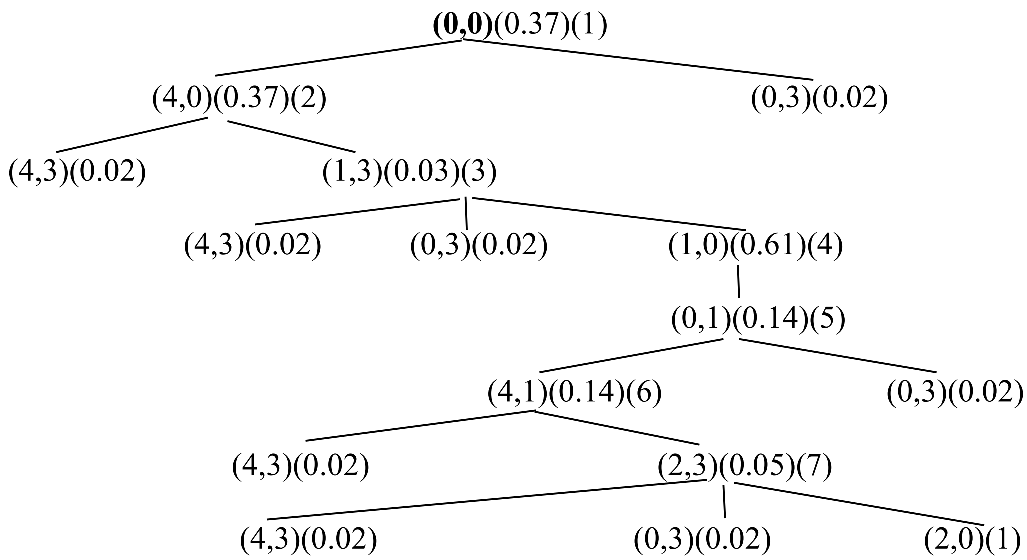
(b) Best-first search

Apply the function of $f(x,y) = e^{-\left|\frac{x-2}{2}\right|-|y|}$ as the heuristic function in both (a) and (b). This function generates the maximum value of 1 when the state is (2, 0), and generates less value when the state deviates from the state (2, 0) with the importance of the deviation of x from 2 is 2 times higher than the deviation of y from 0.

Draw the tree, determine the heuristic function value of all the opened nodes, and label the order of node opening. (25)

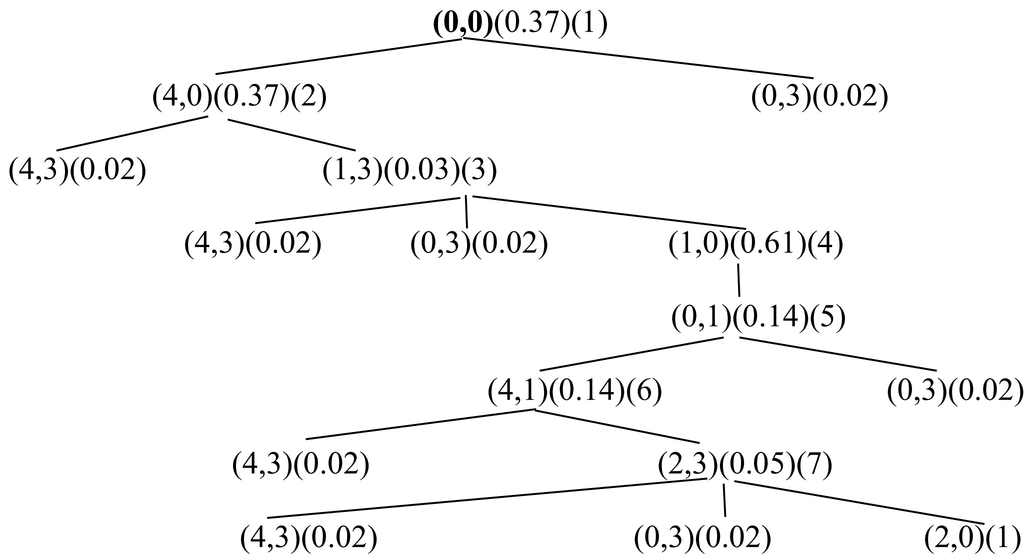
Solution

(a) Hill-climbing search



Solution: (0,0)->(4,0)->(1,3)->(1,0)->(0,1)->(4,1)->(2,3)->(2,0)

(b) Best-first search,



Solution: (0,0)->(4,0)->(1,3)->(1,0)->(0,1)->(4,1)->(2,3)->(2,0)

Q.4 Make the truth table using Lukasiewicz's formula of the 3-value logic as expressed by

$$\overline{A \wedge (A \vee B)} \leftrightarrow \bar{A}$$

When $\wedge$ represents AND, $\vee$ represents OR, $\bar{A}$ represents NOT of A, $\rightarrow$ represents IF THEN, and $\leftrightarrow$ represents IF AND ONLY IF. (25)

Solution

A	B	$A \vee B$	$A \wedge (A \vee B)$	$\overline{A \wedge (A \vee B)}$	$\bar{A}$	$\overline{A \wedge (A \vee B)} \leftrightarrow \bar{A}$
0	0	0	0	1	1	1
0	0.5	0.5	0	1	1	1
0	1	1	0	1	1	1
0.5	0	0.5	0.5	0.5	0.5	1
0.5	0.5	0.5	0.5	0.5	0.5	1
0.5	1	1	0.5	0.5	0.5	1
1	0	1	1	0	0	1
1	0.5	1	1	0	0	1
1	1	1	1	0	0	1