Final Examination AI and Neuro-Fuzzy Theory AT74.05 November 29, 2022

Time: 9:00-11:00 h. Open Book

Marks: 100

Attempt all questions.

Q.1 A rocket moving vertically at a constant acceleration from the base locating at about the sea level at the time about 9:00 h. This rocket passes the first check point locating at the distance about 10,000 m above the sea level at the time about 9:30 h. If the fuzzy numbers of the location of the base ($S_1(x)$), the launching time ($T_1(y)$), the location of the first check point ($S_2(x)$), the time at the first check point ($T_2(y)$) are given as

$$S_1(x) = \begin{cases} 1 - \dfrac{x}{5} & ; 0\, m \le x \le 5\, m \\ 0 & ; otherwise \end{cases}$$

$$S_2(x) = \begin{cases} 1 - \dfrac{|x - 10000|}{100} & ; 9900\, m \le x \le 10100\, m \\ 0 & ; otherwise \end{cases}$$

$$T_1(y) = \begin{cases} 1 - \dfrac{|y - 9{:}00|}{0{:}02} & ; 8{:}58\, h \le y \le 9{:}02\, h \\ 0 & ; otherwise \end{cases}$$

$$T_2(y) = \begin{cases} 1 - \dfrac{|y - 9{:}30|}{0{:}02} & ; 9{:}28\, h \le y \le 9{:}32\, h \\ 0 & ; otherwise \end{cases}$$

Determine the membership function of the acceleration ($A(z)$) when z has the unit of m/s^2. (25)

<u>Solution</u>

$$a = \frac{2s}{t^2}$$

$$^{\alpha}(S_1) = [0 \quad 5 - 5\alpha]$$

$$^{\alpha}(S_2) = [9{,}900 + 100\alpha \quad 10{,}100 - 100\alpha]$$

$$^{\alpha}(T_1) = [8{:}58 + 0{:}02\alpha \quad 9{:}02 - 0{:}02\alpha]$$

$$^{\alpha}(T_2) = [9{:}28 + 0{:}02\alpha \quad 9{:}32 - 0{:}02\alpha]$$

$$^{\alpha}(s) = {}^{\alpha}(S_2 - S_1) = {}^{\alpha}(S_2) - {}^{\alpha}(S_1) = [9{,}900 + 100\alpha \quad 10{,}100 - 100\alpha] - [0 \quad 5 - 5\alpha]$$

$$^{\alpha}(s) = [9{,}895 + 105\alpha \quad 10{,}100 - 100\alpha]$$

$$^{\alpha}(2s) = [19{,}790 + 210\alpha \quad 20{,}200 - 200\alpha]$$

$$^{\alpha}(t) = {}^{\alpha}(T_2 - T_1) = {}^{\alpha}(T_2) - {}^{\alpha}(T_1)$$

$$^{\alpha}(t) = [9{:}28 + 0{:}02\alpha \quad 9{:}32 - 0{:}02\alpha] - [8{:}58 + 0{:}02\alpha \quad 9{:}02 - 0{:}02\alpha]$$

$$^{\alpha}(t) = [0{:}26 + 0{:}04\alpha \quad 0{:}34 - 0{:}04\alpha]$$

$$^{\alpha}(t) = [1{,}560 + 240\alpha \quad 2{,}040 - 240\alpha]$$

$$^{\alpha}(t^2) = [1{,}560 + 240\alpha \quad 2{,}040 - 240\alpha][1{,}560 + 240\alpha \quad 2{,}040 - 240\alpha]$$

$$^{\alpha}(t^2) = [57{,}600\alpha^2 + 748{,}800\alpha + 2{,}433{,}600 \quad 57{,}600\alpha^2 - 979{,}200\alpha + 4{,}161{,}600]$$

$$^{\alpha}(\frac{2s}{t^2}) = [19{,}790 + 210\alpha \quad 20{,}200 - 200\alpha]$$

$$/[57{,}600\alpha^2 + 748{,}800\alpha + 2{,}433{,}600 \quad 57{,}600\alpha^2 - 979{,}200\alpha + 4{,}161{,}600]$$

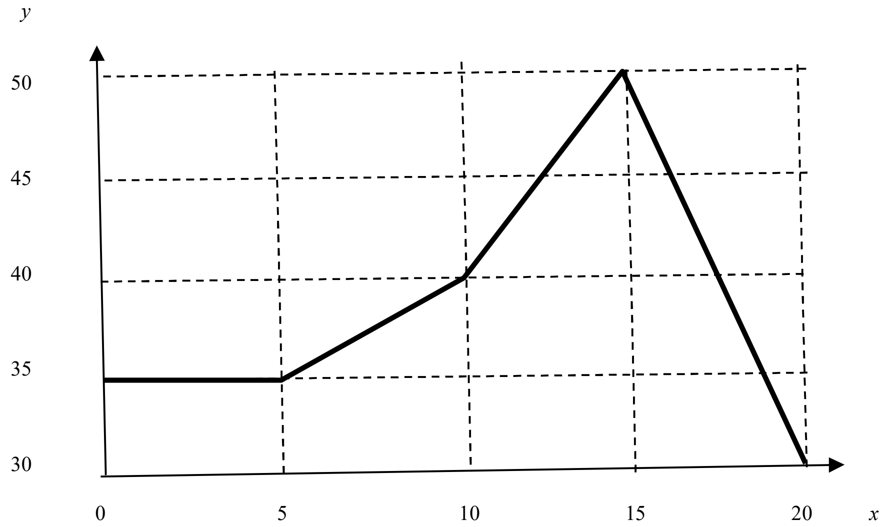
$$^{\alpha}(\frac{2s}{t^2}) = [19{,}790 + 210\alpha \quad 20{,}200 - 200\alpha]$$

$$\times \left[\frac{1}{57{,}600\alpha^2 - 979{,}200\alpha + 4{,}161{,}600} \quad \frac{1}{57{,}600\alpha^2 + 748{,}800\alpha + 2{,}433{,}600}\right]$$

$$^{\alpha}(A) = {}^{\alpha}(\frac{2s}{t^2}) = \left[\frac{19{,}790 + 210\alpha}{57{,}600\alpha^2 - 979{,}200\alpha + 4{,}161{,}600} \quad \frac{20{,}200 - 200\alpha}{57{,}600\alpha^2 + 748{,}800\alpha + 2{,}433{,}600}\right]$$

$$A(z) = \begin{cases} 8.5 + \frac{0.0018}{z} - \sqrt{\frac{0.3746}{z} + \frac{0.00000332}{z^2}} & ; 0.0048 m/s^2 < z \leq 0.0062\, m/s^2 \\ -6.5 - \frac{0.0017}{z} + \sqrt{\frac{0.3733}{z} + \frac{0.00000301}{z^2}} & ; 0.0062 m/s^2 < z \leq 0.0083 m/s^2 \\ 0 & ; otherwise \end{cases}$$

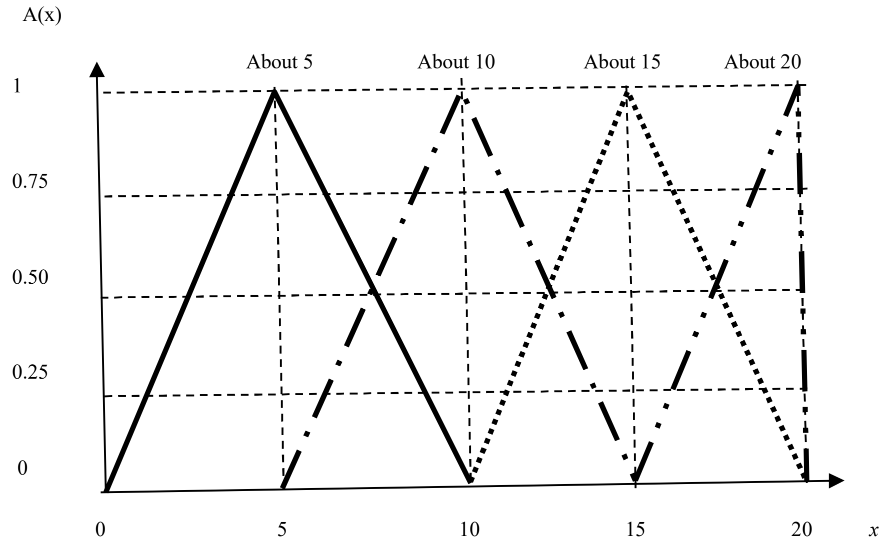
Q.2 ANFIS applies Sugeno-type fuzzy inference system to create a relationship between the input, x, and the output, y, of a system. If the collected input, output data is shown below.



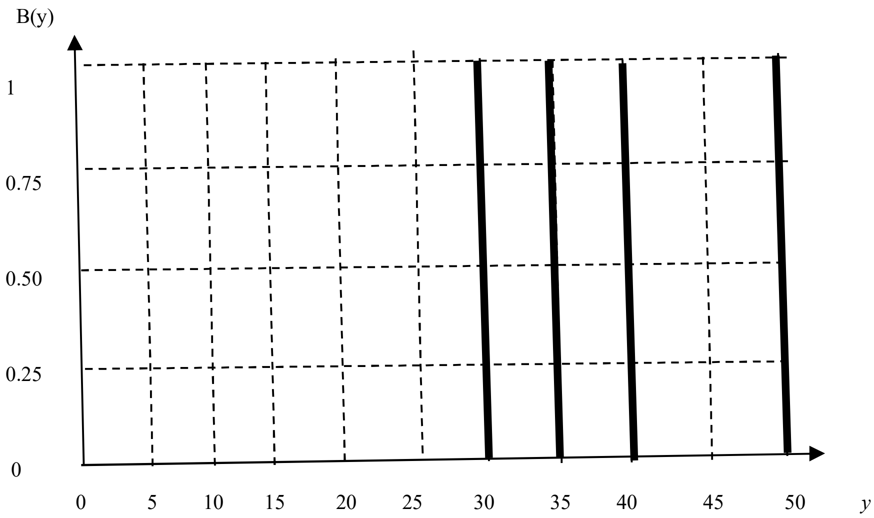
If 4 triangular-shape membership functions (defined by point a, b, c) are used for the input and 4 constant membership functions (defined by the value) are used for the output. Determine all the possible input and output membership functions along with the fuzzy inference rules that can create this relationship. (25)

<u>Solution</u>

Since graph changes its slope at x of 5, 10, 15, and 20. These points are used as point b in the triangular-shape membership functions of the input of about 5, about 10, about 15 and about 20.



Since graph changes its slope at y of 30, 35, 40, and 50. These points are used as the value in the constant membership functions of the output.



Fuzzy inference system is designed as

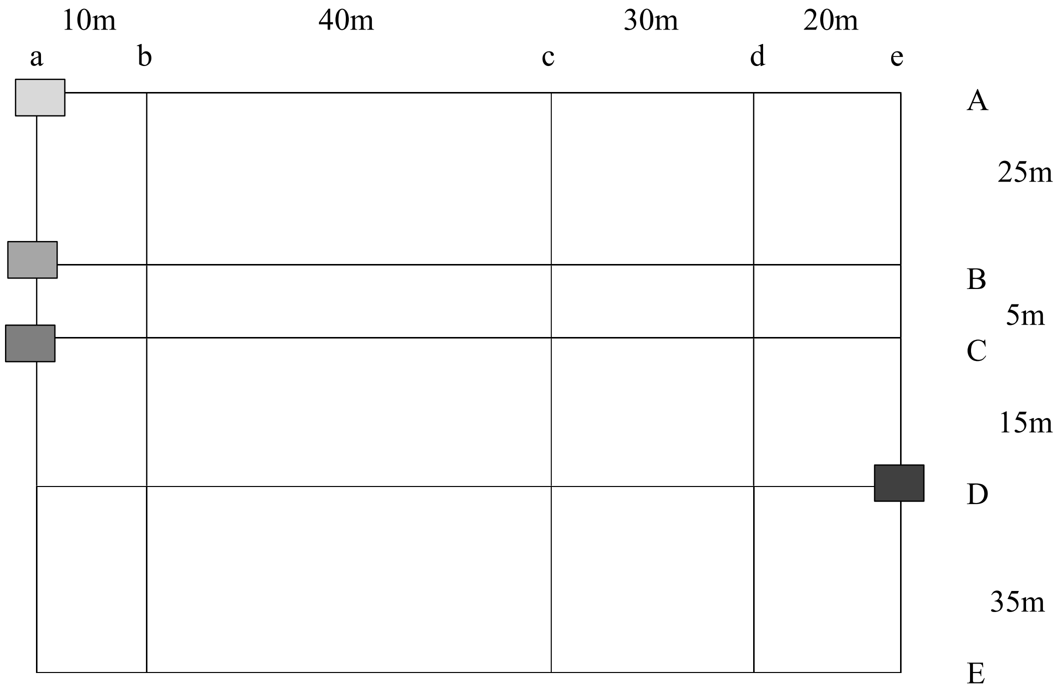
If x is about 5, then y is 35

If x is about 10, then y is 40

If x is about 15, then y is 50

If x is about 20, then y is 30

Q.3 AI is used to navigate multiple robots path planning. Four robots have to move from the starting coordinates of Aa, Ba, Ca, and De to the destination coordinates of Ee, De, Ce, and Ba respectively as shown in map below. Assume the road is small and can accommodate only at most of one robot at a time.



Determine the paths of all the four robots by hill climbing method using the accumulated distance of each robot as the objective functions. This objective function has the highest priority. The robots move from the coordinates to the next coordinates at the same time at each level in the search tree. The motion priority of the robots is designed as robot 1 > robot 2 > robot 3 > robot 4 to avoid crashing. A robot must keep moving from one node to the connected node and is not allowed to stop except at the destination where it cannot move anymore. Looping to the previous visited coordinate is not allowed. No need to show all the children nodes since the branching factor is about 3^4 which is too many, show only the nodes that are opened. The node in the search tree is represented by the coordinates of all the four robots respectively; for example, the root node is represented by (Aa, Ba, Ca, De). Show the accumulated distances of all the robots near the link. Show also the order of node opening. (25)

<u>Solution</u>

Hill-climbing search

(Aa, Ba, Ca, De) (1)

(10, 5, 10, 15)

(Ab, Ca, Cb, Ce) (2)

(35, 15, 25, 20)

(Bb, Cb, Db, Be) (3)

(40, 30, 35, 40)

(Cb, Db, Da, Bd) (4)

(50, 40, 70, 45)

(Ca, Da, Ea, Cd) (5)

(55, 75, 80, 60) (55, 75, 80, 60)

(<u>Ba</u>, Ea, Eb, Dd) (Da, Ea, Eb, Dd) (6)

(65, 85, 120, 90)

(Db, Eb, Ec, Dc) (7)

(100, 125, 150, 105)

(Eb, Ec, Ed, Cc) (8)

(110, 155, 175, 110) (140, 155, 175, 110)

(Ea, Ed, Ee, Bc) (Ec, Ed, Ee, Bc) (9)

(170, 175, 210, 135)

(Ed, Ee, De, Ac) (10)

(190, 210, 225, 165) (190, 210, 225, 175)

(**<u>Ee</u>**, **<u>De</u>**, **<u>Ce</u>**, Ad) (11) **<u>Ee</u>**, **<u>De</u>**, **<u>Ce</u>**, Ab) (12)

(190, 210, 225, 185)   (190, 210, 225, 185)

(**Ee**,, **De**, **Ce**, Ae)   (**Ee**, **De**, **Ce**, Aa) (13)

(190, 210, 225, 210)

(**Ee**, **De**, **Ce**, **Ba**)

Q.4 Make the truth table using Lukasiewicz's formula of the 3-value logic as expressed by

$$(A \leftrightarrow B) \leftrightarrow \big((A \to B) \wedge (B \to A)\big)$$

When $\wedge$ represents AND, $\vee$ represents OR, $\bar{A}$ represents NOT of A, $\to$ represents IF THEN, and $\leftrightarrow$ represents IF AND ONLY IF. (25)

Solution

A	B	$A \leftrightarrow B$	$(A \leftrightarrow B) \leftrightarrow ((A \to B) \wedge (B \to A))$	$A \to B$	$(A \to B) \wedge (B \to A)$	$B \to A$
0	0	1	1	1	1	1
0	0.5	0.5	1	1	0.5	0.5
0	1	0	1	1	0	0
0.5	0	0.5	1	0.5	0.5	1
0.5	0.5	1	1	1	1	1
0.5	1	0.5	1	1	0.5	0.5
1	0	0	1	0	0	1
1	0.5	0.5	1	0.5	0.5	1
1	1	1	1	1	1	1