Final Examination AI and Neuro-Fuzzy Theory AT74.05 November 28, 2023

Time: 13:00-15:00 h. Open Book

Marks: 100

Attempt all questions.

Q.1 An uncertain quadratic equation which is expressed by

$$Ax^2 + Bx + C = 0$$

When A is about 1, B is about -13 and C is about 40 and the membership functions of these fuzzy numbers follow

$$A(x) = \begin{cases} 1 - |x-1| & ; 0 \le x \le 2 \\ 0 & ; otherwise \end{cases}$$

$$B(x) = \begin{cases} 1 - |x+13| & ; -14 \le x \le -12 \\ 0 & ; otherwise \end{cases}$$

$$C(x) = \begin{cases} 1 - |x-40| & ; 39 \le x \le 41 \\ 0 & ; otherwise \end{cases}$$

Determine the membership function of both solutions, $S_1(x)$ and $S_2(x)$, of the quadratic equation when their ranges of the alpha-cut operation are real numbers and the range of the alpha values.

(30)

<u>Solution</u>

$$S = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

$$^{\alpha}(A) = [\alpha \quad 2-\alpha]$$

$$^{\alpha}(B) = [\alpha - 14 \quad -12-\alpha]$$

$$^{\alpha}(C) = [\alpha + 39 \quad 41-\alpha]$$

$$^{\alpha}(2A) = [2\alpha \quad 4-2\alpha]$$

$$^{\alpha}(-B) = [12+\alpha \quad 14-\alpha]$$

$$^{\alpha}(B^2) = [\alpha^2 + 24\alpha + 144 \quad \alpha^2 - 28\alpha + 196]$$

$$^{\alpha}(AC) = [\alpha^2 + 39\alpha \quad \alpha^2 - 43\alpha + 82]$$

$$^{\alpha}(4AC) = [4\alpha^2 + 156\alpha \quad 4\alpha^2 - 172\alpha + 328]$$

$$^{\alpha}(B^2 - 4AC) = [-3\alpha^2 + 196\alpha - 184 \quad -3\alpha^2 - 184\alpha + 196]$$

$$-3\alpha^2 + 196\alpha - 184 > 0$$

$$\alpha > 0.95$$

$$^{\alpha}(\sqrt{B^2 - 4AC}) = \left[\sqrt{-3\alpha^2 + 196\alpha - 184} \quad \sqrt{-3\alpha^2 - 184\alpha + 196}\right]$$

$$^{\alpha}(-B + \sqrt{B^2 - 4AC})$$

$$= \left[12 + \alpha + \sqrt{-3\alpha^2 + 196\alpha - 184} \quad 14 - \alpha + \sqrt{-3\alpha^2 - 184\alpha + 196}\right]$$

$$^{\alpha}(-B - \sqrt{B^2 - 4AC})$$

$$= \left[12 + \alpha - \sqrt{-3\alpha^2 - 184\alpha + 196} \quad 14 - \alpha - \sqrt{-3\alpha^2 + 196\alpha - 184}\right]$$

$$^{\alpha}\left(\frac{-B + \sqrt{B^2 - 4AC}}{2A}\right) = {}^{\alpha}(S_1) = [SL1 \quad SR1]$$

$$= \left[\frac{12 + \alpha + \sqrt{-3\alpha^2 + 196\alpha - 184}}{4 - 2\alpha} \quad \frac{14 - \alpha + \sqrt{-3\alpha^2 - 184\alpha + 196}}{2\alpha}\right]$$

$$^{\alpha}\left(\frac{-B - \sqrt{B^2 - 4AC}}{2A}\right) = {}^{\alpha}(S_2) = [SL2 \quad SR2]$$

$$= \left[\frac{12 + \alpha - \sqrt{-3\alpha^2 - 184\alpha + 196}}{4 - 2\alpha} \quad \frac{14 - \alpha - \sqrt{-3\alpha^2 + 196\alpha - 184}}{2\alpha}\right]$$

$$SL1 = \frac{12+\alpha+\sqrt{-3\alpha^2+196\alpha-184}}{4-2\alpha} = x \text{ when } 0.95 < \alpha < 1; 6.22 < x < 8$$

$$\alpha^2[4x^2 + 8] - \alpha[16x^2 + 132] + [16x^2 + 232] = 0$$

$$\alpha = \frac{[16x^2 + 132] - \sqrt{[16x^2 + 132]^2 - 4[4x^2 + 8][16x^2 + 232]}}{2[4x^2 + 8]}$$

$$SR1 = \frac{14-\alpha+\sqrt{-3\alpha^2-184\alpha+196}}{2\alpha} = x \text{ when } 0.95 < \alpha < 1; 8 < x < 9.13$$

$$\alpha^2[4x^2 + 4x + 4] - \alpha[56x - 156] = 0$$

$$\alpha = \frac{56x - 156}{4x^2 + 4x + 4}$$

$$SL2 = \frac{12+\alpha-\sqrt{-3\alpha^2-184\alpha+196}}{4-2\alpha} \text{ when } 0.95 < \alpha < 1; 4.12 < x < 5$$

$$\alpha^2[4x^2+4x+5]-\alpha[16x^2-40x-24]+[16x^2-96x+144]=0$$

$$\alpha=\frac{[16x^2-40x-24]+\sqrt{[16x^2-40x-24]^2-4[4x^2+4x+5][16x^2-96x+144]}}{2[4x^2+4x+5]}$$

$$SR2=\frac{14-\alpha-\sqrt{-3\alpha^2+196\alpha-184}}{2\alpha}=x \text{ when } 0.95<\alpha<1; 5<x<6.81$$

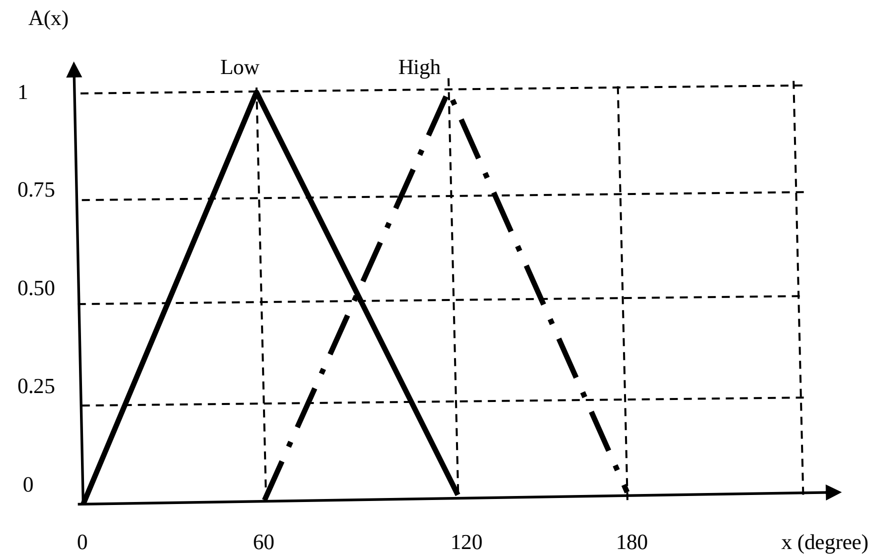
$$\alpha^2[4x^2+4x+4]-\alpha[56x+224]+[380]=0$$

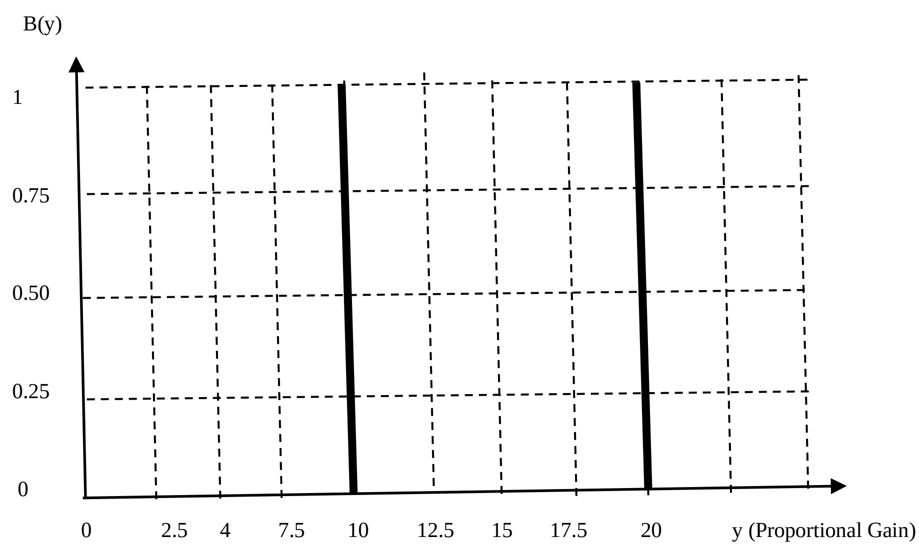
$$\alpha=\frac{[56x+224]+\sqrt{[56x+224]^2-4[4x^2+4x+4][380]}}{2[4x^2+4x+4]}$$

$$S_1(x)=\begin{cases}\dfrac{[16x^2+132]-\sqrt{[16x^2+132]^2-4[4x^2+8][16x^2+232]}}{2[4x^2+8]} & ; 6.22<x<8\\ \dfrac{56x-156}{4x^2+4x+4} & ; 8<x<9.13\\ 0 & ; otherwise\end{cases}$$

$$S_2(x)=\begin{cases}\dfrac{[16x^2-40x-24]+\sqrt{[16x^2-40x-24]^2-4[4x^2+4x+5][16x^2-96x+144]}}{2[4x^2+4x+5]} & ; 4.12<x<5\\ \dfrac{[56x+224]+\sqrt{[56x+224]^2-4[4x^2+4x+4][380]}}{2[4x^2+4x+4]} & ; 5<x<6.81\\ 0 & ; otherwise\end{cases}$$

Q.2 A Sugeno-type fuzzy inference system is used to determine a proportional gain of an adaptive servo motor. If the input is the position error of the motor which is classified into Low and High as shown in the membership functions, A(x), the output is the singleton type proportional gain and is designed to be 10 for High position error and 20 for Low position error as shown in the output membership functions, B(y). Determine the formula of the proportional gain as a function of the position error. (20)





Solution

When the 0 < position error < 60,

$$Proportional\ Gain = \frac{20\ (A(x))}{(A(x))} = 20$$

When the 60 < position error < 120,

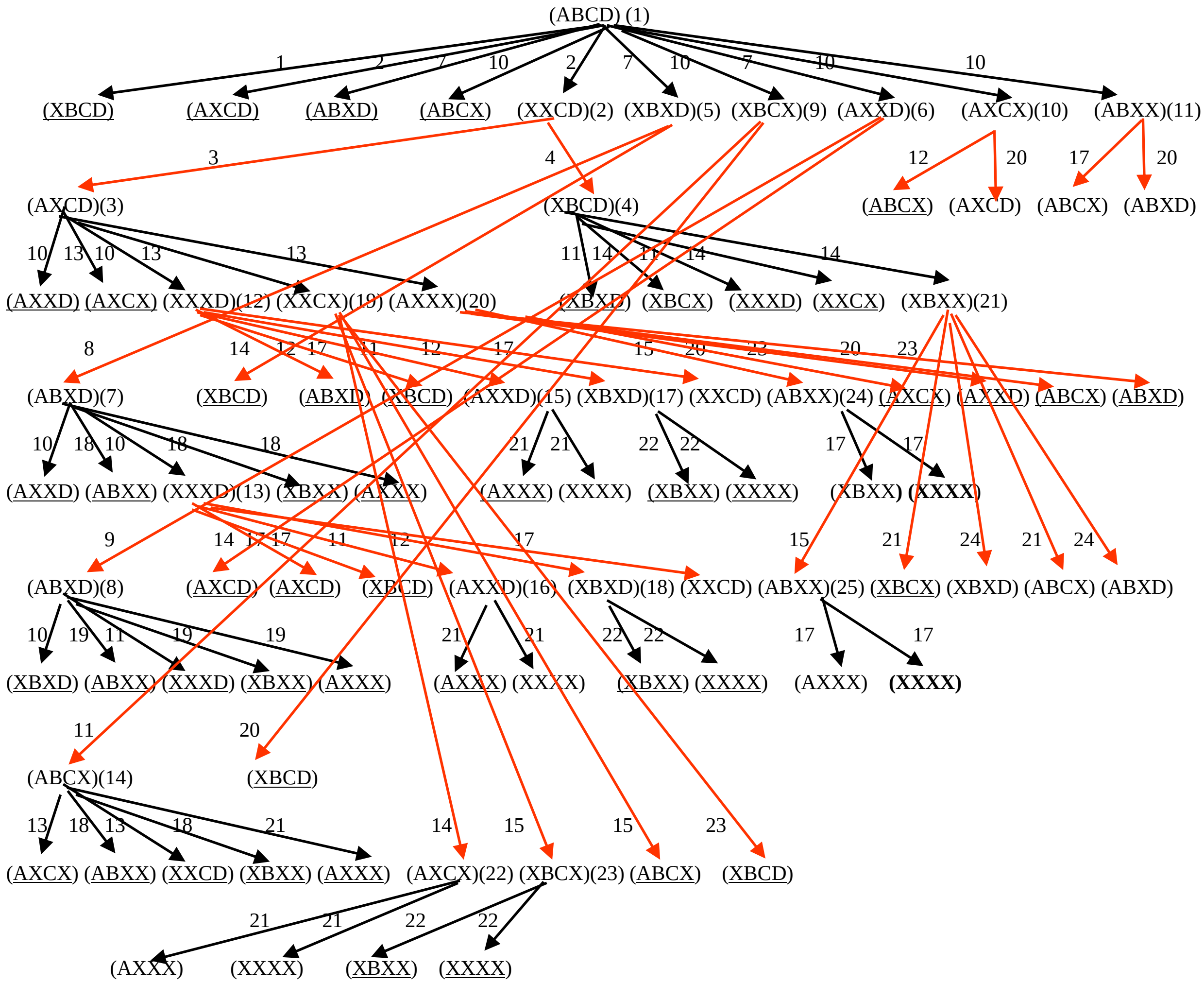
$$Proportional\ Gain = \frac{20\ \left(\frac{-\mathrm{x}}{60} + 2\right) + 10\left(\frac{\mathrm{x}}{60} - 1\right)}{\left(\frac{-\mathrm{x}}{60} + 2\right) + \left(\frac{\mathrm{x}}{60} - 1\right)} = \frac{-\mathrm{x}}{6} + 30$$

When the 120 < position error < 180,

$$Proportional\ Gain = \frac{10\ (A(x))}{(A(x))} = 10$$

Q.3 Four men, A, B, C, and D, are walking cross a rickety bridge at night. They have only one torch and the bridge is too dangerous to cross without a torch. The bridge can support only two men at a time. A, B, C, and D takes 1, 2, 7, and 10 minutes to cross the bridge respectively. If two men cross the bridge, time of the longer man is taken. After crossing the bridge, the torch has to be taken back by at least a man. Determine the shortest time needed for all the men to cross the bridge by dynamic programming method using accumulated time as the objective function and all the solutions that achieve this shortest time. Draw the tree and node opening order. The root node is represented by (ABCD) which means all the men don't cross the bridge yet and the objective is to arrive the node (XXXX) which means all the men already crossed the bridge. (30)

Solution



The shortest time is 17 seconds.

The solutions are

(ABCD)->(XXCD)->(AXCD)->(AXXX)->(ABXX)->(XXXX)

(ABCD)->(XXCD)->(XBCD)->(XBXX)->(ABXX)->(XXXX)

Q.4 PSO is used to determine the solutions of a quadratic function (20)

$$x^2 + y^2 - 100x - 30y + 2700 = 0$$

Show how to determine two more generations of the particles based on the following setting:

(a) Particle format = (x, y)

(b) Fitness function = $f = e^{-\left|\frac{x^2+y^2-100x-30y+2700}{10000}\right|}$

(c) Population = 4

(d) Initial particles = {(0, 0), (100, 20), (10, 80), (70, 90)}

(e) Initial velocities = ={(0, 0), (0, 0), (0, 0), (0, 0)}

(f) Velocity of each particle follows $v(k+1) = wv(k) + c_1r_1\big(P(k) - x(k)\big) + c_2r_2\big(G(k) - x(k)\big)$ when $w = 0.2, c_1 = 0.5, c_2 = 0.3, r_1 = 1 - f, r_2 = f$

(g) Position of each particle is updated using $x(k+1) = x(k) + v(k+1)$

<u>Solution</u>

Generation 0:

$v(0) =${(0, 0), (0, 0), (0, 0), (0, 0)}

$x(0) =$ {(0, 0), (100, 20), (10, 80), (70, 90)}

$$f(0) = \{e^{-\left|\frac{0^2+0^2-100(0)-30(0)+2700}{10000}\right|}, e^{-\left|\frac{100^2+20^2-100(100)-30(20)+2700}{10000}\right|}, e^{-\left|\frac{10^2+80^2-100(10)-30(80)+2700}{10000}\right|},$$

$$e^{-\left|\frac{70^2+90^2-100(70)-30(90)+2700}{10000}\right|}\} = \left\{e^{-|0.27|}, e^{-|0.25|}, e^{-|0.58|}, e^{-|0.60|}\right\}$$

= {0.76, 0.78, 0.56, 0.55}

$P(0) =${(0, 0), (100, 20), (10, 80), (70, 90)}

$G(0) =$(100, 20)

Generation 1:

$$v(1) = \{0.2(0,0) + 0.5(1 - 0.76)\big((0,0) - (0,0)\big) + 0.3(0.76)((100,20) - (0,0)),$$

$$0.2(0,0) + 0.5(1 - 0.78)\big((100,20) - (100,20)\big) + 0.3(0.78)((100,20) - (100,20)),$$

$$0.2(0,0) + 0.5(1 - 0.56)\big((10,80) - (10,80)\big) + 0.3(0.56)((100,20) - (10,80)),$$

$$0.2(0,0) + 0.5(1 - 0.55)\big((70,90) - (70,90)\big) + 0.3(0.55)((100,20) - (70,90))\}$$

$$= \{(22.80,4.56), (0,0), (15.12,-10.08), (4.95,-11.55)\}$$

$$x(1) = \{(0 + 22.80, 0 + 4.56), (100 + 0, 20 + 0), (10 + 15.12, 80 - 10.08),$$

$$(70 + 4.95, 90 - 11.55)\} = \{(22.80,4.56), (100,20), (25.12,69.92), (74.95,78.45)\}$$

$$f(1) = \{e^{-\left|\frac{22.80^2+4.56^2-100(22.80)-30(4.56)+2700}{10000}\right|}, e^{-\left|\frac{100^2+20^2-100(100)-30(20)+2700}{10000}\right|},$$

$$e^{-\left|\frac{25.12^2+69.92^2-100(25.12)-30(69.92)+2700}{10000}\right|}, e^{-\left|\frac{74.95^2+78.45^2-100(74.95)-30(78.45)+2700}{10000}\right|}\}$$

$= \left\{e^{-|0.08|}, e^{-|0.25|}, e^{-|0.36|}, e^{-|0.46|}\right\}$ = {0.92, 0.78, 0.70, 0.63}

$P(1) =${(22.80, 4.56), (100, 20), (25.12, 69.92), (74.95, 78.45)}

$G(1) =$(22.80, 4.56)

Generation 2:

$$v(2) = \{0.2(22.80,4.56) + 0.5(1-0.92)\big((22.80,4.56) - (22.80,4.56)\big) + 0.3(0.92)((22.80,4.56) - (22.80,4.56)),$$
$$0.2(0,0) + 0.5(1-0.78)\big((100,20) - (100,20)\big) + 0.3(0.78)((22.80,4.56) - (100,20)),$$
$$0.2(15.12,-10.08) + 0.5(1-0.70)\big((25.12,69.92) - (25.12,69.92)\big) + 0.3(0.70)((22.80,4.56) - (25.12,69.92)),$$
$$0.2(4.95,-11.55) + 0.5(1-0.63)\big((74.95,78.45) - (74.95,78.45)\big) + 0.3(0.63)((22.80,4.56) - (74.95,78.45))\}$$
$$= \{(4.56,0.91), (-18.06,-3.61), (2.54,-15.74), (-8.87,-16.28)\}$$
$$x(2) = \{(22.80+4.56, 4.56+0.91), (100-18.06, 20-3.61),$$
$$(25.12+2.54, 69.92-15.74), (74.95-8.87, 78.45-16.28)\}$$
. $$= \{(27.36,5.47), (81.94,16.39), (27.66,54.18), (66.08,62.17)\}$$