

Time: 9:00-11:00 h.

Open Book

Marks: 100

Attempt all questions.

Q.1 Single Layer Perceptron network is applied to recognize 2 patterns. Determine the necessary parameters by Perceptron learning rule when the following training sets are provided.

$$\left\{ p_1 = \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix}, t_1 = [1] \right\}, \left\{ p_2 = \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix}, t_2 = [0] \right\}, \left\{ p_3 = \begin{bmatrix} 1 \\ 4 \\ 1 \end{bmatrix}, t_3 = [1] \right\},$$

$$\left\{ p_4 = \begin{bmatrix} -1 \\ 2 \\ -1 \end{bmatrix}, t_4 = [0] \right\}, \left\{ p_5 = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}, t_5 = [1] \right\}, \left\{ p_6 = \begin{bmatrix} -1 \\ -1 \\ 0 \end{bmatrix}, t_6 = [0] \right\}$$

Apply initial weight matrix of  $W(0) = [0 \ 0 \ 0]$  and initial bias of  $b(0) = [0]$  and the training sets are presented sequentially until convergence of the parameters. (20)

**Solution**

Present  $\left\{ p_1 = \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix}, t_1 = [1] \right\}$

$$a(1) = \text{hardlim} \left( [0 \ 0 \ 0] \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix} + [0] \right) = 1 = t_1$$

$$W(1) = [0 \ 0 \ 0], b(1) = [0]$$

Present  $\left\{ p_2 = \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix}, t_2 = [0] \right\}$

$$a(2) = \text{hardlim} \left( [0 \ 0 \ 0] \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix} + [0] \right) = 1 \neq t_2$$

$$W(2) = [0 \ 0 \ 0] - [1 \ -1 \ -1] = [-1 \ 1 \ 1], b(2) = [0] - [1] = [-1]$$

Present  $\left\{ p_3 = \begin{bmatrix} 1 \\ 4 \\ 1 \end{bmatrix}, t_3 = [1] \right\}$

$$a(3) = \text{hardlim} \left( [-1 \ 1 \ 1] \begin{bmatrix} 1 \\ 4 \\ 1 \end{bmatrix} + [-1] \right) = 1 = t_3$$

$$W(3) = [-1 \ 1 \ 1], b(3) = [-1]$$

Present  $\left\{ p_4 = \begin{bmatrix} -1 \\ 2 \\ -1 \end{bmatrix}, t_4 = [0] \right\}$

$$a(4) = \text{hardlim} \left( [-1 \ 1 \ 1] \begin{bmatrix} -1 \\ 2 \\ -1 \end{bmatrix} + [-1] \right) = 1 \neq t_5$$

$$W(4) = [-1 \ 1 \ 1] - [-1 \ 2 \ -1] = [0 \ -1 \ 0], b(4) = [-1] - [1] = [-2]$$

Present  $\left\{ p_5 = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}, t_5 = [1] \right\}$

$$a(5) = \text{hardlim} \left( [0 \ -1 \ 0] \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} + [-2] \right) = 0 \neq t_5$$

$$W(5) = [0 \quad -1 \quad 0] + [1 \quad 1 \quad 2] = [1 \quad 0 \quad 2], b(5) = [-2] + [1] = [-1]$$

Present  $\left\{ p_6 = \begin{bmatrix} -1 \\ -1 \\ 0 \end{bmatrix}, t_6 = [0] \right\}$

$$a(6) = \text{hardlim} \left( [1 \quad 0 \quad 2] \begin{bmatrix} -1 \\ -1 \\ 0 \end{bmatrix} + [-1] \right) = 0 = t_6$$

$$W(6) = [1 \quad 0 \quad 2], b(6) = [-1]$$

Present  $\left\{ p_1 = \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix}, t_1 = [1] \right\}$

$$a(7) = \text{hardlim} \left( [1 \quad 0 \quad 2] \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix} + [-1] \right) = 1 = t_1$$

$$W(7) = [1 \quad 0 \quad 2], b(7) = [-1]$$

Present  $\left\{ p_2 = \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix}, t_2 = [0] \right\}$

$$a(8) = \text{hardlim} \left( [1 \quad 0 \quad 2] \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix} + [-1] \right) = 0 = t_2$$

$$W(8) = [1 \quad 0 \quad 2], b(8) = [-1]$$

Present  $\left\{ p_3 = \begin{bmatrix} 1 \\ 4 \\ 1 \end{bmatrix}, t_3 = [1] \right\}$

$$a(9) = \text{hardlim} \left( [1 \quad 0 \quad 2] \begin{bmatrix} 1 \\ 4 \\ 1 \end{bmatrix} + [-1] \right) = 1 = t_3$$

$$W(9) = [1 \quad 0 \quad 2], b(9) = [-1]$$

Present  $\left\{ p_4 = \begin{bmatrix} -1 \\ 2 \\ -1 \end{bmatrix}, t_4 = [0] \right\}$

$$a(10) = \text{hardlim} \left( [1 \quad 0 \quad 2] \begin{bmatrix} -1 \\ 2 \\ -1 \end{bmatrix} + [-1] \right) = 0 = t_4$$

$$W(10) = [1 \quad 0 \quad 2], b(10) = [-1]$$

Present  $\left\{ p_5 = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}, t_5 = [1] \right\}$

$$a(11) = \text{hardlim} \left( [1 \quad 0 \quad 2] \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} + [-1] \right) = 1 = t_5$$

$$W(11) = [1 \quad 0 \quad 2], b(11) = [-1]$$

Present  $\left\{ p_6 = \begin{bmatrix} -1 \\ -1 \\ 0 \end{bmatrix}, t_6 = [0] \right\}$

$$a(12) = \text{hardlim} \left( [1 \quad 0 \quad 2] \begin{bmatrix} -1 \\ -1 \\ 0 \end{bmatrix} + [-1] \right) = 0 = t_6$$

$$W(12) = [1 \quad 0 \quad 2], b(12) = [-1]$$

Thus the required parameters are

$$W = [1 \quad 0 \quad 2], b = [-1]$$

Q.2 If ADALINE Network with 3 inputs and a bias is applied in Q.1, determine the optimal parameters by LMS algorithm. Determine the mean square value of the error. (20)

**Solution**

$$F(x) = E[t^2] - 2x^T E[tz] + x^T E[zz^T]x = c - 2x^T h + x^T R x$$

$$c = \frac{1}{6}(1^2 + 0^2 + 1^2 + 0^2 + 1^2 + 0^2) = \frac{1}{2}$$

$$h = \frac{1}{6} \left( [1] \begin{bmatrix} 3 \\ 1 \\ 1 \\ 1 \end{bmatrix} + [0] \begin{bmatrix} 1 \\ -1 \\ -1 \\ 1 \end{bmatrix} + [1] \begin{bmatrix} 1 \\ 4 \\ 1 \\ 1 \end{bmatrix} + [0] \begin{bmatrix} -1 \\ 2 \\ -1 \\ 1 \end{bmatrix} + [1] \begin{bmatrix} 1 \\ 1 \\ 2 \\ 1 \end{bmatrix} + [0] \begin{bmatrix} -1 \\ 0 \\ -1 \\ 1 \end{bmatrix} \right) = \frac{1}{6} \begin{bmatrix} 5 \\ 6 \\ 4 \\ 3 \end{bmatrix}$$

$$R = \frac{1}{6} \left( \begin{bmatrix} 3 \\ 1 \\ 1 \\ 1 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \\ 1 \\ 1 \end{bmatrix}^T + \begin{bmatrix} 1 \\ -1 \\ -1 \\ 1 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ -1 \\ 1 \end{bmatrix}^T + \begin{bmatrix} 1 \\ 4 \\ 1 \\ 1 \end{bmatrix} \begin{bmatrix} 1 \\ 4 \\ 1 \\ 1 \end{bmatrix}^T + \begin{bmatrix} -1 \\ 2 \\ -1 \\ 1 \end{bmatrix} \begin{bmatrix} -1 \\ 2 \\ -1 \\ 1 \end{bmatrix}^T + \begin{bmatrix} 1 \\ 1 \\ 2 \\ 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 2 \\ 1 \end{bmatrix}^T + \begin{bmatrix} -1 \\ 0 \\ -1 \\ 1 \end{bmatrix} \begin{bmatrix} -1 \\ 0 \\ -1 \\ 1 \end{bmatrix}^T \right)$$

$$= \frac{1}{6} \begin{bmatrix} 14 & 6 & 6 & 4 \\ 6 & 24 & 6 & 6 \\ 6 & 6 & 8 & 2 \\ 4 & 6 & 2 & 6 \end{bmatrix}$$

The minimum point is the stationary point of the quadratic function.

$$\begin{bmatrix} w_{11} \\ w_{12} \\ w_{13} \\ b_1 \end{bmatrix} = R^{-1}h = \begin{bmatrix} 14 & 6 & 6 & 4 \\ 6 & 24 & 6 & 6 \\ 6 & 6 & 8 & 2 \\ 4 & 6 & 2 & 6 \end{bmatrix}^{-1} \begin{bmatrix} 5 \\ 6 \\ 4 \\ 3 \end{bmatrix} = \begin{bmatrix} 0.14 \\ 0.09 \\ 0.27 \\ 0.23 \end{bmatrix}$$

Present  $\left\{ p_1 = \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix}, t_1 = [1] \right\}$

$$a_1 = \text{purelin} \left( [0.14 \quad 0.09 \quad 0.27] \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix} + [0.23] \right) = 1.01$$

$$e_1 = t_1 - a_1 = 1 - 1.01 = -0.01$$

Present  $\left\{ p_2 = \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix}, t_2 = [0] \right\}$

$$a_2 = \text{purelin} \left( [0.14 \quad 0.09 \quad 0.27] \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix} + [0.23] \right) = 0.01$$

$$e_2 = t_2 - a_2 = 0 - 0.01 = -0.01$$

Present  $\left\{ p_3 = \begin{bmatrix} 1 \\ 4 \\ 1 \end{bmatrix}, t_3 = [1] \right\}$

$$a_3 = \text{purelin} \left( [0.14 \quad 0.09 \quad 0.27] \begin{bmatrix} 1 \\ 4 \\ 1 \end{bmatrix} + [0.23] \right) = 1$$

$$e_3 = t_3 - a_3 = 1 - 1 = 0$$

Present  $\left\{ p_4 = \begin{bmatrix} -1 \\ 2 \\ -1 \end{bmatrix}, t_4 = [0] \right\}$

$$a_4 = \text{purelin} \left( [0.14 \quad 0.09 \quad 0.27] \begin{bmatrix} -1 \\ 2 \\ -1 \end{bmatrix} + [0.23] \right) = 0$$

$$e_4 = t_4 - a_4 = 0 - 0 = 0$$

Present  $\left\{ p_5 = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}, t_5 = [1] \right\}$

$$a_5 = \text{purelin} \left( [0.14 \quad 0.09 \quad 0.27] \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} + [0.23] \right) = 1$$

$$e_5 = t_5 - a_5 = 1 - 1 = 0$$

Present  $\left\{ p_6 = \begin{bmatrix} -1 \\ -1 \\ 0 \end{bmatrix}, t_6 = [0] \right\}$

$$a_6 = \text{purelin} \left( [0.14 \quad 0.09 \quad 0.27] \begin{bmatrix} -1 \\ -1 \\ 0 \end{bmatrix} + [0.23] \right) = 0$$

$$e_6 = t_6 - a_6 = 0 - 0 = 0$$

Thus mean square error becomes,

$$E[e^2] = \frac{1}{6} [(-0.01)^2 + (-0.01)^2 + 0^2 + 0^2 + 0^2 + 0^2] = 0.00$$

Q.3 Consider the vector space shown below. When the input vector is in the polygon shaded area, the

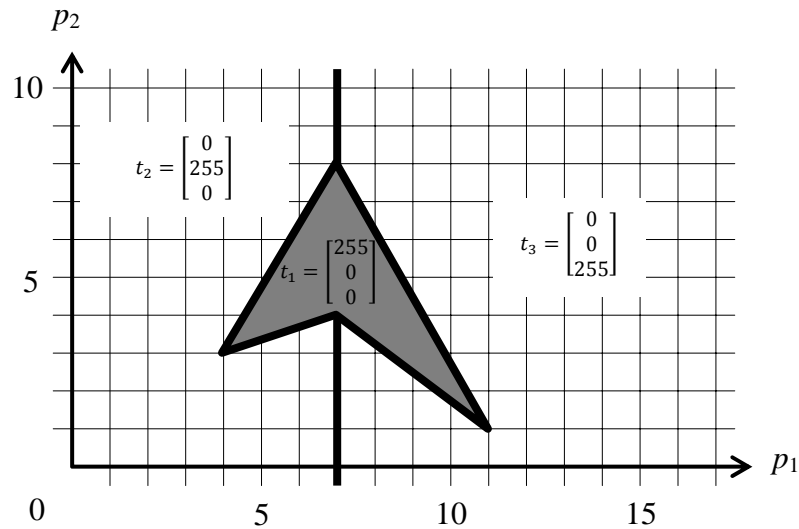
target vector is  $t_1 = \begin{bmatrix} 255 \\ 100 \\ 50 \end{bmatrix}$ . When the input vector is outside the polygon and on the left side, the

target vector is  $t_2 = \begin{bmatrix} 200 \\ 255 \\ 100 \end{bmatrix}$ . When the input vector is outside the polygon and on the right side, the

target vector is  $t_3 = \begin{bmatrix} 0 \\ 100 \\ 255 \end{bmatrix}$ . Design the neural network and draw its architecture then determine all

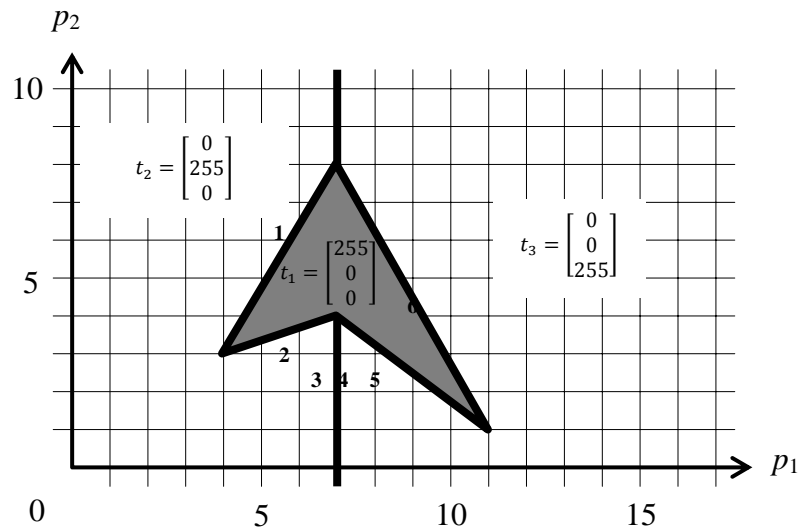
the necessary parameters that can associate the input vectors with the output vectors correctly.

(30)



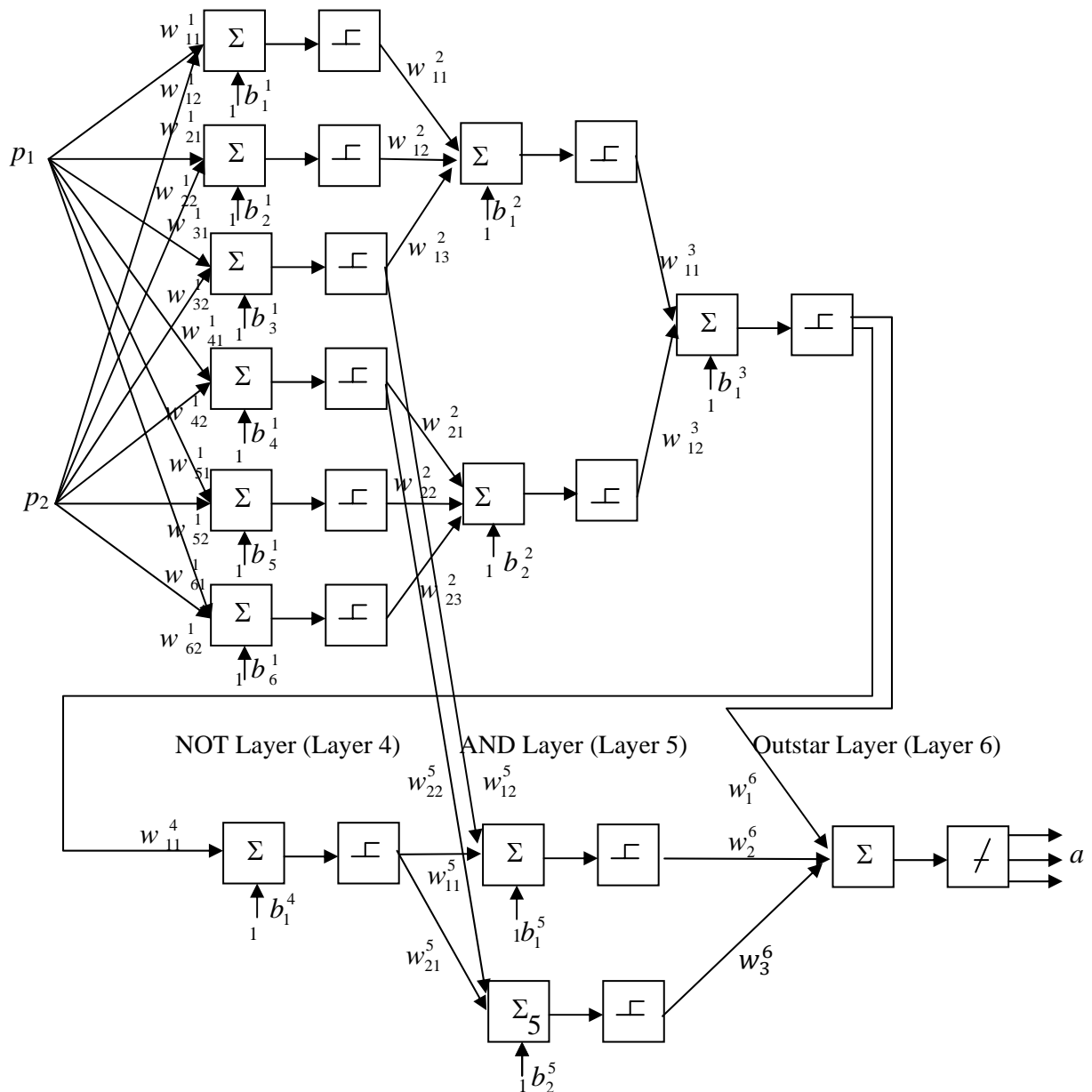
**Solution**

(a) Each decision boundary is numbered as shown below.



Combination of Multi-Layer Perceptron with outstar network can be selected.

Decision Boundary Layer (Layer 1)    AND Layer (Layer 2)    OR Layer (Layer 3)



## For layer 1

Since  $\mathbf{w}$  should point into the area and it should be perpendicular to decision boundary. Thus, select

$$\mathbf{w}_1^1 = \begin{bmatrix} 5 \\ -3 \end{bmatrix} \quad (1)$$

$$\mathbf{w}_2^1 = \begin{bmatrix} -1 \\ 3 \end{bmatrix} \quad (2)$$

$$\mathbf{w}_3^1 = \begin{bmatrix} -1 \\ 0 \end{bmatrix} \quad (3)$$

$$\mathbf{w}_4^1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad (4)$$

$$\mathbf{w}_5^1 = \begin{bmatrix} 3 \\ 4 \end{bmatrix} \quad (5)$$

$$\mathbf{w}_6^1 = \begin{bmatrix} -7 \\ -4 \end{bmatrix} \quad (6)$$

$\mathbf{b}$  is determined from equating  $\mathbf{n} = 0$ ; and solve for  $\mathbf{b}$

For decision boundary 1,

$$n_1^1 = w_{11}^1 p_1 + w_{12}^1 p_2 + b_1^1 = 0 \quad (7)$$

at  $p_1 = 4, p_2 = 3$ ;

$$5(4) - 3(3) + b_1^1 = 0; b_1^1 = -11 \quad (8)$$

For decision boundary 2,

$$n_2^1 = w_{21}^1 p_1 + w_{22}^1 p_2 + b_2^1 = 0 \quad (9)$$

at  $p_1 = 4, p_2 = 3$ ;

$$-1(4) + 3(3) + b_2^1 = 0; b_2^1 = -5 \quad (10)$$

For decision boundary 3,

$$n_3^1 = w_{31}^1 p_1 + w_{32}^1 p_2 + b_3^1 = 0 \quad (11)$$

at  $p_1 = 7, p_2 = 0$ ;

$$-1(7) + 0(0) + b_3^1 = 0; b_3^1 = 7 \quad (12)$$

For decision boundary 4,

$$n_4^1 = w_{41}^1 p_1 + w_{42}^1 p_2 + b_4^1 = 0 \quad (13)$$

at  $p_1 = 7, p_2 = 0$ ;

$$1(7) + 0(0) + b_4^1 = 0; b_4^1 = -7 \quad (14)$$

For decision boundary 5,

$$n_5^1 = w_{51}^1 p_1 + w_{52}^1 p_2 + b_5^1 = 0 \quad (15)$$

at  $p_1 = 11, p_2 = 1$ ;

$$3(11) + 4(1) + b_5^1 = 0; b_5^1 = -37 \quad (16)$$

For decision boundary 6,

$$n_6^1 = w_{61}^1 p_1 + w_{62}^1 p_2 + b_6^1 = 0 \quad (17)$$

at  $p_1 = 11, p_2 = 1$ ;

$$-7(11) - 4(1) + b_6^1 = 0; b_6^1 = 81 \quad (18)$$

## For layer 2

Since this is AND layer, if we select

$$w_{11}^2 = w_{12}^2 = w_{13}^2 = 1 \quad (19)$$

$$w_{21}^2 = w_{22}^2 = w_{23}^2 = 1 \quad (20)$$

$\mathbf{b}$  must be selected such that only when all inputs are 1s, output is 1. Thus, select

$$b_1^2 = b_2^2 = -2.5 \quad (21)$$

**For layer 3**

Since this is OR layer, if we select

$$w_{11}^3 = w_{12}^3 = 1 \quad (22)$$

**b** must be selected such that even one input is 1, output is 1. Thus, select

$$b_1^3 = -0.5 \quad (23)$$

**For layer 4**

Since this is NOT layer,

$$w_{11}^4 = -1 \quad (24)$$

**b** must be selected such that when the input is 1, output is 0. Thus, select

$$b_1^4 = 0.5 \quad (25)$$

**For layer 5**

Since this is AND layer, if we select

$$w_{11}^5 = w_{12}^5 = 1 \quad (26)$$

$$w_{21}^5 = w_{22}^5 = 1 \quad (27)$$

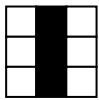
**b** must be selected such that only when all inputs are 1s, output is 1. Thus, select

$$b_1^5 = b_2^5 = -1.5 \quad (28)$$

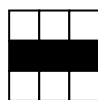
**For layer 6, Outstar network is selected.**

$$w^6 = [t_1 \quad t_2 \quad t_3] = \begin{bmatrix} 255 & 200 & 0 \\ 100 & 255 & 100 \\ 50 & 100 & 255 \end{bmatrix} \quad (29)$$

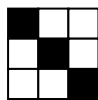
Q.4 Determine weights of a Hopfield network by Hebb rule, W-QI rule (when W is the weight by Hebb rule, Q is number of patterns, and I is identity matrix), and pseudo inverse rule of the patterns shown below in the  $\pm 1$  binary number system (1 for black pixel, -1 for white pixel and row by row numbering), assume the bias is selected to be 0 matrix.



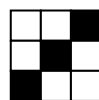
Pattern 1



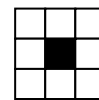
Pattern 2



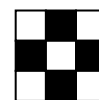
Pattern 3



Pattern 4

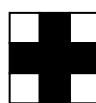


Pattern 5



Pattern 6

In order to compare performance of weights, determine output of the iterations until convergence to one of the pattern of the weights by all the rules when an input shown below is presented to the network. Assume symmetrical hard limit function is used in the network. (20)



Input

**Solution**

Patterns represent by

$$\mathbf{p}_1 = \begin{bmatrix} -1 \\ 1 \\ -1 \\ -1 \\ 1 \\ -1 \\ -1 \\ 1 \\ -1 \end{bmatrix}, \mathbf{p}_2 = \begin{bmatrix} -1 \\ -1 \\ -1 \\ 1 \\ 1 \\ -1 \\ -1 \\ -1 \\ -1 \end{bmatrix}, \mathbf{p}_3 = \begin{bmatrix} 1 \\ -1 \\ -1 \\ -1 \\ -1 \\ -1 \\ -1 \\ -1 \\ 1 \end{bmatrix}, \mathbf{p}_4 = \begin{bmatrix} -1 \\ -1 \\ 1 \\ -1 \\ -1 \\ 1 \\ -1 \\ -1 \\ -1 \end{bmatrix}, \mathbf{p}_5 = \begin{bmatrix} -1 \\ -1 \\ -1 \\ -1 \\ -1 \\ -1 \\ -1 \\ -1 \\ -1 \end{bmatrix}, \mathbf{p}_6 = \begin{bmatrix} -1 \\ 1 \\ -1 \\ 1 \\ -1 \\ 1 \\ -1 \\ 1 \\ -1 \end{bmatrix} \quad (1)$$

By Hebb rule, the weight becomes

$$\mathbf{W}_{Hebb} = \sum_{q=1}^6 (\mathbf{p}_q \mathbf{p}_q^T) = \begin{pmatrix} \begin{bmatrix} -1 \\ 1 \\ -1 \\ -1 \\ 1 \\ -1 \\ -1 \\ 1 \\ -1 \end{bmatrix} \begin{bmatrix} -1 & 1 \end{bmatrix}^T + \begin{bmatrix} -1 \\ -1 \\ -1 \\ 1 \\ 1 \\ -1 \\ -1 \\ -1 \\ -1 \end{bmatrix} \begin{bmatrix} -1 & -1 \end{bmatrix}^T + \begin{bmatrix} 1 \\ -1 \\ -1 \\ -1 \\ -1 \\ -1 \\ -1 \\ -1 \\ 1 \end{bmatrix} \begin{bmatrix} 1 & -1 \end{bmatrix}^T + \begin{bmatrix} -1 \\ -1 \\ 1 \\ -1 \\ -1 \\ 1 \\ -1 \\ -1 \\ -1 \end{bmatrix} \begin{bmatrix} -1 & -1 \end{bmatrix}^T + \begin{bmatrix} -1 \\ -1 \\ -1 \\ -1 \\ -1 \\ -1 \\ -1 \\ -1 \\ -1 \end{bmatrix} \begin{bmatrix} -1 & -1 \end{bmatrix}^T + \begin{bmatrix} -1 \\ 1 \\ -1 \\ 1 \\ -1 \\ 1 \\ -1 \\ 1 \\ -1 \end{bmatrix} \begin{bmatrix} -1 & 1 \end{bmatrix}^T \end{pmatrix}$$
$$\mathbf{W}_{Hebb} = \begin{bmatrix} 6 & 0 & 2 & 0 & -2 & 0 & 2 & 0 & 6 \\ 0 & 6 & 0 & 2 & -4 & 2 & 0 & 6 & 0 \\ 2 & 0 & 6 & 0 & -2 & 0 & 6 & 0 & 2 \\ 0 & 2 & 0 & 6 & -4 & 6 & 0 & 2 & 0 \\ -2 & -4 & -2 & -4 & 6 & -4 & -2 & -4 & -2 \\ 0 & 2 & 0 & 6 & -4 & 6 & 0 & 2 & 0 \\ 2 & 0 & 6 & 0 & -2 & 0 & 6 & 0 & 2 \\ 0 & 6 & 0 & 2 & -4 & 2 & 0 & 6 & 0 \\ 6 & 0 & 2 & 0 & -2 & 0 & 2 & 0 & 6 \end{bmatrix} \quad (2)$$

By W-QI rule, the weight becomes

$$\mathbf{W}_{w-QI} = \begin{bmatrix} 0 & 0 & 2 & 0 & -2 & 0 & 2 & 0 & 6 \\ 0 & 0 & 0 & 2 & -4 & 2 & 0 & 6 & 0 \\ 2 & 0 & 0 & 0 & -2 & 0 & 6 & 0 & 2 \\ 0 & 2 & 0 & 0 & -4 & 6 & 0 & 2 & 0 \\ -2 & -4 & -2 & -4 & 0 & -4 & -2 & -4 & -2 \\ 0 & 2 & 0 & 6 & -4 & 0 & 0 & 2 & 0 \\ 2 & 0 & 6 & 0 & -2 & 0 & 0 & 0 & 2 \\ 0 & 6 & 0 & 2 & -4 & 2 & 0 & 0 & 0 \\ 6 & 0 & 2 & 0 & -2 & 0 & 2 & 0 & 0 \end{bmatrix} \quad (3)$$

By Pseudo Inverse rule,



$$W = TP^+ = T(P^T P)^{-1} P^T = \begin{bmatrix} -1 & -1 & 1 & -1 & -1 & -1 \\ 1 & -1 & -1 & -1 & -1 & 1 \\ -1 & -1 & -1 & 1 & -1 & -1 \\ -1 & 1 & -1 & -1 & -1 & 1 \\ 1 & 1 & 1 & 1 & 1 & -1 \\ -1 & 1 & -1 & -1 & -1 & 1 \\ -1 & -1 & -1 & 1 & -1 & -1 \\ 1 & -1 & -1 & -1 & -1 & 1 \\ -1 & -1 & 1 & -1 & -1 & -1 \end{bmatrix} \begin{bmatrix} -1 & -1 & 1 & -1 & -1 & -1 \\ 1 & -1 & -1 & -1 & -1 & 1 \\ -1 & -1 & -1 & 1 & -1 & -1 \\ -1 & 1 & -1 & -1 & -1 & 1 \\ 1 & 1 & 1 & 1 & 1 & -1 \\ -1 & 1 & -1 & -1 & -1 & 1 \\ -1 & -1 & -1 & 1 & -1 & -1 \\ 1 & -1 & -1 & -1 & -1 & 1 \\ -1 & -1 & 1 & -1 & -1 & -1 \end{bmatrix}^{-1} \begin{bmatrix} -1 & -1 & 1 & -1 & -1 & -1 \\ 1 & -1 & -1 & -1 & -1 & 1 \\ -1 & -1 & -1 & 1 & -1 & -1 \\ -1 & 1 & -1 & -1 & -1 & 1 \\ 1 & 1 & 1 & 1 & 1 & -1 \\ -1 & 1 & -1 & -1 & -1 & 1 \\ -1 & -1 & -1 & 1 & -1 & -1 \\ 1 & -1 & -1 & -1 & -1 & 1 \\ -1 & -1 & 1 & -1 & -1 & -1 \end{bmatrix}^{-T}$$

$$W = \begin{bmatrix} 0.56 & 0 & 0.19 & 0 & -0.06 & 0 & 0.19 & 0 & 0.56 \\ 0.06 & 0.25 & 0.06 & -0.25 & 0.19 & -0.25 & 0.06 & 0.25 & 0.06 \\ 0 & -0.19 & 0.63 & 0.06 & 0 & 0.06 & 0.63 & -0.19 & 0 \\ 0.25 & 0.06 & -0.13 & 0.56 & -0.13 & 0.56 & -0.13 & 0.06 & 0.25 \\ -0.38 & 0.06 & 0 & 0.06 & 1 & 0.06 & 0 & 0.06 & -0.38 \\ 0.25 & 0.06 & -0.13 & 0.56 & -0.13 & 0.56 & -0.13 & 0.06 & 0.25 \\ 0 & -0.19 & 0.63 & 0.06 & 0 & 0.06 & 0.63 & -0.19 & 0 \\ 0.06 & 0.25 & 0.06 & -0.25 & 0.19 & -0.25 & 0.06 & 0.25 & 0.06 \\ 0.56 & 0 & 0.19 & 0 & -0.06 & 0 & 0.19 & 0 & 0.56 \end{bmatrix} \quad (4)$$

The input represents by

$$\mathbf{p} = \begin{bmatrix} -1 \\ 1 \\ -1 \\ 1 \\ 1 \\ 1 \\ -1 \\ 1 \\ -1 \end{bmatrix} \quad (5)$$

Since

$$\mathbf{a}(0) = \mathbf{p} = \begin{bmatrix} -1 \\ 1 \\ -1 \\ 1 \\ 1 \\ 1 \\ -1 \\ 1 \\ -1 \end{bmatrix} \quad (6)$$

By Hebb rule,

$$\mathbf{a}(1) = \text{hardlims}(\mathbf{W}\mathbf{a}(0)) = \text{hardlims} \left( \begin{bmatrix} 6 & 0 & 2 & 0 & -2 & 0 & 2 & 0 & 6 \\ 0 & 6 & 0 & 2 & -4 & 2 & 0 & 6 & 0 \\ 2 & 0 & 6 & 0 & -2 & 0 & 6 & 0 & 2 \\ 0 & 2 & 0 & 6 & -4 & 6 & 0 & 2 & 0 \\ -2 & -4 & -2 & -4 & 6 & -4 & -2 & -4 & -2 \\ 0 & 2 & 0 & 6 & -4 & 6 & 0 & 2 & 0 \\ 2 & 0 & 6 & 0 & -2 & 0 & 6 & 0 & 2 \\ 0 & 6 & 0 & 2 & -4 & 2 & 0 & 6 & 0 \\ 6 & 0 & 2 & 0 & -2 & 0 & 2 & 0 & 6 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \\ -1 \\ 1 \\ 1 \\ -1 \\ 1 \\ -1 \\ -1 \end{bmatrix} \right) = \text{hardlims} \begin{bmatrix} -18 \\ 12 \\ -18 \\ 12 \\ -2 \\ 12 \\ -18 \\ 12 \\ -18 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \\ -1 \\ 1 \\ -1 \\ 1 \\ -1 \\ 1 \\ -1 \end{bmatrix} = \mathbf{p}_6 \quad (7)$$

By W-QI rule,

$$\mathbf{a}(1) = \text{hardlims}(\mathbf{W}\mathbf{a}(0)) = \text{hardlims} \left( \begin{bmatrix} 0 & 0 & 2 & 0 & -2 & 0 & 2 & 0 & 6 \\ 0 & 0 & 0 & 2 & -4 & 2 & 0 & 6 & 0 \\ 2 & 0 & 0 & 0 & -2 & 0 & 6 & 0 & 2 \\ 0 & 2 & 0 & 0 & -4 & 6 & 0 & 2 & 0 \\ -2 & -4 & -2 & -4 & 0 & -4 & -2 & -4 & -2 \\ 0 & 2 & 0 & 6 & -4 & 0 & 0 & 2 & 0 \\ 2 & 0 & 6 & 0 & -2 & 0 & 0 & 0 & 2 \\ 0 & 6 & 0 & 2 & -4 & 2 & 0 & 0 & 0 \\ 6 & 0 & 2 & 0 & -2 & 0 & 2 & 0 & 0 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \\ -1 \\ 1 \\ 1 \\ -1 \\ 1 \\ -1 \\ -1 \end{bmatrix} \right) = \text{hardlims} \begin{bmatrix} -12 \\ 6 \\ -12 \\ 6 \\ -8 \\ 6 \\ -12 \\ 6 \\ -12 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \\ -1 \\ 1 \\ -1 \\ 1 \\ -1 \\ 1 \\ -1 \end{bmatrix} = \mathbf{p}_6 \quad (8)$$

By pseudoinverse rule,

$$\mathbf{a}(1) = \text{hardlims}(\mathbf{W}\mathbf{a}(0)) = \text{hardlims} \left( \begin{bmatrix} 0.56 & 0 & 0.19 & 0 & -0.06 & 0 & 0.19 & 0 & 0.56 \\ 0.06 & 0.25 & 0.06 & -0.25 & 0.19 & -0.25 & 0.06 & 0.25 & 0.06 \\ 0 & -0.19 & 0.63 & 0.06 & 0 & 0.06 & 0.63 & -0.19 & 0 \\ 0.25 & 0.06 & -0.13 & 0.56 & -0.13 & 0.56 & -0.13 & 0.06 & 0.25 \\ -0.38 & 0.06 & 0 & 0.06 & 1 & 0.06 & 0 & 0.06 & -0.38 \\ 0.25 & 0.06 & -0.13 & 0.56 & -0.13 & 0.56 & -0.13 & 0.06 & 0.25 \\ 0 & -0.19 & 0.63 & 0.06 & 0 & 0.06 & 0.63 & -0.19 & 0 \\ 0.06 & 0.25 & 0.06 & -0.25 & 0.19 & -0.25 & 0.06 & 0.25 & 0.06 \\ 0.56 & 0 & 0.19 & 0 & -0.06 & 0 & 0.19 & 0 & 0.56 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \\ -1 \\ 1 \\ 1 \\ -1 \\ 1 \\ -1 \\ -1 \end{bmatrix} \right) = \text{hardlims} \begin{bmatrix} -1.56 \\ -0.06 \\ -1.50 \\ 0.88 \\ 2 \\ 0.88 \\ -1.50 \\ -0.06 \\ -1.56 \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \\ -1 \\ 1 \\ 1 \\ 1 \\ -1 \\ -1 \\ -1 \end{bmatrix} = \mathbf{p}_2 \quad (9)$$

Q.5 Discuss how to apply neural network to your research work. What kind of network is appropriate and how to prepare the training set? (10)