

Time: 9:00-11:00 h.

Open Book

Marks: 100

Attempt all questions.

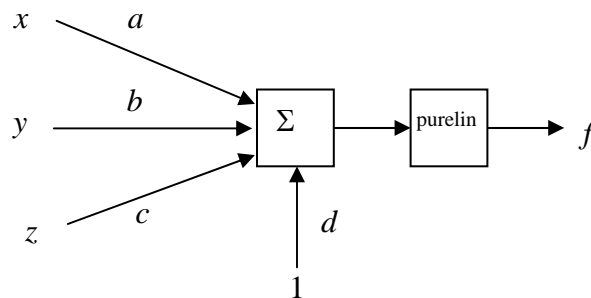
Q.1 ADALINE network is used to identify mathematic model of a system which has the relation in the form of $f = ax + by + cz + d$; when f denotes the output, while x, y, z denotes the three inputs. Determine the parameters (a, b, c, d) of the relation when the data are collected and found as shown in the below table.

x	0	1	1	1	1	1	1	1	1	1
y	0	1	1	1	2	2	2	3	3	3
z	0	1	2	3	1	2	3	1	2	3
f	8	15	19	23	17	21	25	19	23	27

- (a) Draw the network when the inputs are x, y, z , and the output is f . (5)
- (b) If all the data are presented equally, determine the parameters by LMS algorithm. (10)
- (c) Determine the parameters after three iterations sequentially by Widrow-Hoff learning algorithm, assume that all the parameters are 0 at the beginning and learning rate of 0.05 is used. (10)

Solution

(a)



(b)

$$F(x) = E[t^2] - 2x^T E[tz] + x^T E[zz^T]x \tag{1}$$

$$E[t^2] = \frac{1}{10} (8^2 + 15^2 + 19^2 + 23^2 + 17^2 + 21^2 + 25^2 + 19^2 + 23^2 + 27^2) = 415.3 \tag{2}$$

$$E[tz] = \frac{1}{10} \left(8 \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} + 15 \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} + 19 \begin{bmatrix} 1 \\ 1 \\ 2 \\ 1 \end{bmatrix} + 23 \begin{bmatrix} 1 \\ 1 \\ 3 \\ 1 \end{bmatrix} + 17 \begin{bmatrix} 1 \\ 2 \\ 1 \\ 1 \end{bmatrix} + 21 \begin{bmatrix} 1 \\ 2 \\ 2 \\ 1 \end{bmatrix} + 25 \begin{bmatrix} 1 \\ 2 \\ 3 \\ 1 \end{bmatrix} + 19 \begin{bmatrix} 1 \\ 3 \\ 1 \\ 1 \end{bmatrix} + 23 \begin{bmatrix} 1 \\ 3 \\ 2 \\ 1 \end{bmatrix} + 27 \begin{bmatrix} 1 \\ 3 \\ 3 \\ 1 \end{bmatrix} \right) = \begin{bmatrix} 18.9 \\ 39 \\ 40.2 \\ 19.7 \end{bmatrix} \quad (3)$$

$$E[zz^T] = \frac{1}{10} \left(\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 1 \end{bmatrix}^T + \begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 1 & 1 \\ 1 & 1 \end{bmatrix}^T + \begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 2 & 2 \\ 1 & 1 \end{bmatrix}^T + \begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 3 & 3 \\ 1 & 1 \end{bmatrix}^T + \begin{bmatrix} 1 & 1 \\ 2 & 2 \\ 1 & 1 \\ 1 & 1 \end{bmatrix}^T + \begin{bmatrix} 1 & 1 \\ 2 & 2 \\ 2 & 2 \\ 1 & 1 \end{bmatrix}^T + \begin{bmatrix} 1 & 1 \\ 2 & 2 \\ 3 & 3 \\ 1 & 1 \end{bmatrix}^T + \begin{bmatrix} 1 & 1 \\ 3 & 3 \\ 1 & 1 \\ 1 & 1 \end{bmatrix}^T + \begin{bmatrix} 1 & 1 \\ 3 & 3 \\ 2 & 2 \\ 1 & 1 \end{bmatrix}^T + \begin{bmatrix} 1 & 1 \\ 3 & 3 \\ 3 & 3 \\ 1 & 1 \end{bmatrix}^T \right) = \begin{bmatrix} 0.9 & 1.8 & 1.8 & 0.9 \\ 1.8 & 4.2 & 3.6 & 1.8 \\ 1.8 & 3.6 & 4.2 & 1.8 \\ 0.9 & 1.8 & 1.8 & 1.0 \end{bmatrix} \quad (4)$$

The minimum point is the stationary point of the quadratic function

$$x = \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = E[zz^T]^{-1} E[tz] = \begin{bmatrix} 0.9 & 1.8 & 1.8 & 0.9 \\ 1.8 & 4.2 & 3.6 & 1.8 \\ 1.8 & 3.6 & 4.2 & 1.8 \\ 0.9 & 1.8 & 1.8 & 1.0 \end{bmatrix}^{-1} \begin{bmatrix} 18.9 \\ 39 \\ 40.2 \\ 19.7 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 4 \\ 8 \end{bmatrix} \quad (5)$$

(c)

Present (0, 0, 0, 8),

$$f(1) = 0(0) + 0(0) + 0(0) + 0 = 0; e(1) = 8 - 0 = 8 \quad (6)$$

$$\begin{bmatrix} a(1) \\ b(1) \\ c(1) \\ d(1) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + 2(0.05)(8) \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0.8 \end{bmatrix} \quad (7)$$

Present (1, 1, 1, 15),

$$f(2) = 0(1) + 0(1) + 0(1) + 0.8 = 0.8; e(2) = 15 - 0.8 = 14.2 \quad (8)$$

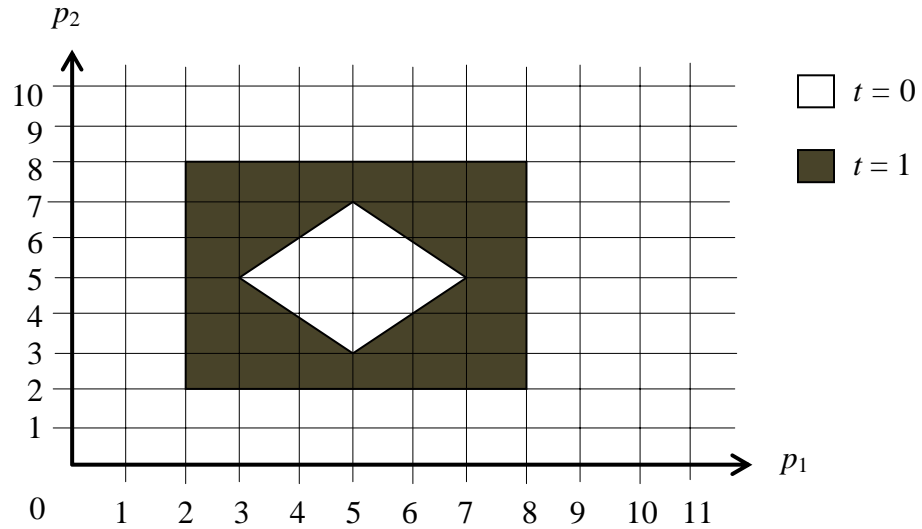
$$\begin{bmatrix} a(2) \\ b(2) \\ c(2) \\ d(2) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0.8 \end{bmatrix} + 2(0.05)(14.2) \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1.42 \\ 1.42 \\ 1.42 \\ 2.22 \end{bmatrix} \quad (9)$$

Present (1, 1, 2, 19),

$$f(3) = 1.42(1) + 1.42(1) + 1.42(2) + 2.22 = 6.48; e(3) = 19 - 6.48 = 12.52 \quad (10)$$

$$\begin{bmatrix} a(3) \\ b(3) \\ c(3) \\ d(3) \end{bmatrix} = \begin{bmatrix} 1.42 \\ 1.42 \\ 1.42 \\ 2.22 \end{bmatrix} + 2(0.05)(12.52) \begin{bmatrix} 1 \\ 1 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 2.67 \\ 2.67 \\ 3.92 \\ 3.47 \end{bmatrix} \quad (11)$$

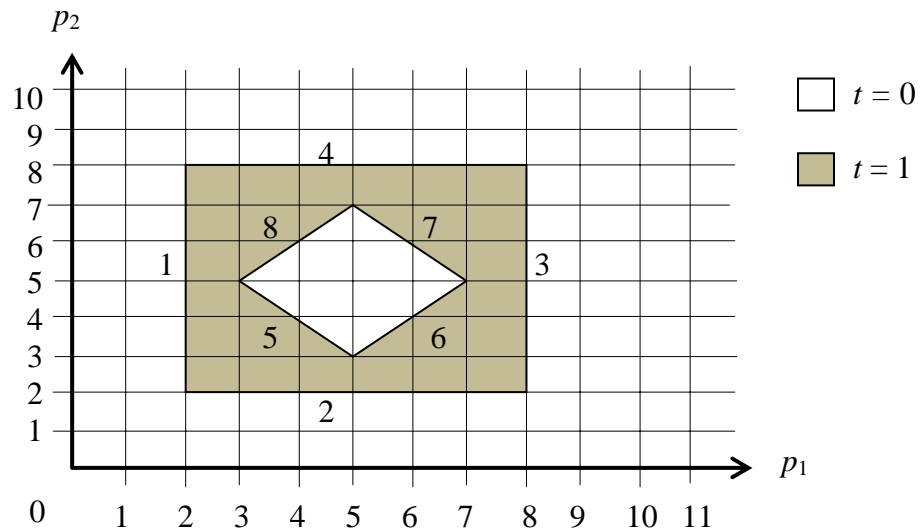
Q.2 A well known relation of set is $A - B = A \cap \bar{B}$. Design a neural network that applies this relation to recognize patterns in class 1 and class 0 as shown in the below vector space. (25)



Solution

Multi-Layer Perceptron is used. Define set A is in the shaded rectangular shape area, define set B is in the white diamond shape area. The first layer will create decision boundaries of set A and set B. The second layer will AND the decision boundaries of set A and also for set B. Then only set B will be NOTed before ANDing with set A.

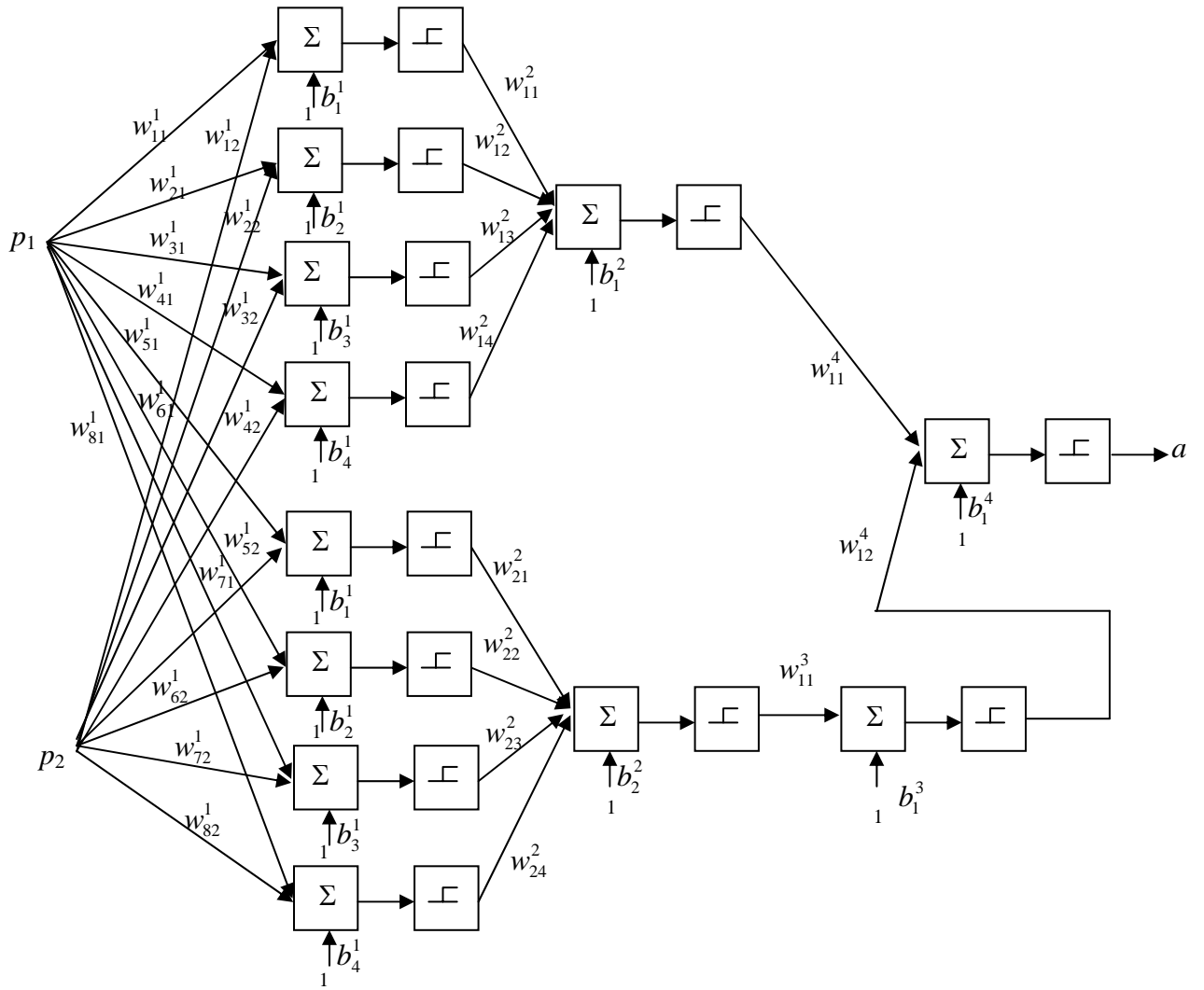
Label the decision boundaries as shown below.



Decision Boundary

AND

NOT (only polygon B) AND



For layer 1

Since \mathbf{w} should point into the polygon and should be perpendicular to decision boundary. Thus, select

$$\mathbf{w}_1^1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad (1)$$

$$\mathbf{w}_2^1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad (2)$$

$$\mathbf{w}_3^1 = \begin{bmatrix} -1 \\ 0 \end{bmatrix} \quad (3)$$

$$\mathbf{w}_4^1 = \begin{bmatrix} 0 \\ -1 \end{bmatrix} \quad (4)$$

$$\mathbf{w}_5^1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad (5)$$

$$\mathbf{w}_6^1 = \begin{bmatrix} -1 \\ 1 \end{bmatrix} \quad (6)$$

$$\mathbf{w}_7^1 = \begin{bmatrix} -1 \\ -1 \end{bmatrix} \quad (7)$$

$$\mathbf{w}_8^1 = \begin{bmatrix} 1 \\ -1 \end{bmatrix} \quad (8)$$

\mathbf{b} is determined from equating $\mathbf{n} = 0$; and solve for \mathbf{b}

Decision boundary 1,

$$n_1^1 = w_{11}^1 p_1 + w_{12}^1 p_2 + b_1^1 = 0 \quad (9)$$

at $p_1 = 2, p_2 = 2$;

$$1(2) + 0(2) + b_1^1 = 0; b_1^1 = -2 \quad (10)$$

Decision boundary 2,

$$n_2^1 = w_{21}^1 p_1 + w_{22}^1 p_2 + b_2^1 = 0 \quad (11)$$

at $p_1 = 2, p_2 = 2$;

$$0(2) + 1(2) + b_2^1 = 0; b_2^1 = -2 \quad (12)$$

Decision boundary 3,

$$n_3^1 = w_{31}^1 p_1 + w_{32}^1 p_2 + b_3^1 = 0 \quad (13)$$

at $p_1 = 8, p_2 = 8$;

$$-1(8) + 0(8) + b_3^1 = 0; b_3^1 = 8 \quad (14)$$

Decision boundary 4,

$$n_4^1 = w_{41}^1 p_1 + w_{42}^1 p_2 + b_4^1 = 0 \quad (15)$$

at $p_1 = 8, p_2 = 8$;

$$0(8) - 1(8) + b_4^1 = 0; b_4^1 = 8 \quad (16)$$

Decision boundary 5,

$$n_5^1 = w_{51}^1 p_1 + w_{52}^1 p_2 + b_5^1 = 0 \quad (17)$$

at $p_1 = 5, p_2 = 3$;

$$1(5) + 1(3) + b_5^1 = 0; b_5^1 = -8 \quad (18)$$

Decision boundary 6,

$$n_6^1 = w_{61}^1 p_1 + w_{62}^1 p_2 + b_6^1 = 0 \quad (19)$$

at $p_1 = 5, p_2 = 3$;

$$-1(5) + 1(3) + b_6^1 = 0; b_6^1 = 2 \quad (20)$$

Decision boundary 7,

$$n_7^1 = w_{71}^1 p_1 + w_{72}^1 p_2 + b_7^1 = 0 \quad (21)$$

at $p_1 = 5, p_2 = 7$;

$$-1(5) - 1(7) + b_7^1 = 0; b_7^1 = 12 \quad (22)$$

Decision boundary 8,

$$n_8^1 = w_{81}^1 p_1 + w_{82}^1 p_2 + b_8^1 = 0 \quad (23)$$

at $p_1 = 5, p_2 = 7$;

$$1(5) - 1(7) + b_8^1 = 0; b_8^1 = 2 \quad (24)$$

For layer 2

Since this is AND layer, if we select

$$w_{11}^2 = w_{12}^2 = w_{13}^2 = w_{14}^2 = 1 \quad (25)$$

$$w_{21}^2 = w_{22}^2 = w_{23}^2 = w_{24}^2 = 1 \quad (26)$$

\mathbf{b} must be selected such that only when all inputs are 1s, output is 1. Thus, select

$$b_1^2 = b_2^2 = -3.5 \quad (27)$$

For layer 3

Since this is NOT layer for set B, if we select

$$w_{11}^3 = -1 \quad (28)$$

b must be selected such that when the input is 1, output is 0. Thus, select

$$b_1^3 = 0.5 \quad (29)$$

For layer 4

Since this is AND layer, if we select

$$w_{11}^4 = w_{12}^4 = 1 \quad (30)$$

b must be selected such that only when all inputs are 1s, output is 1. Thus, select

$$b_1^4 = -1.5 \quad (31)$$

Q.3 A 1-4-2 network is used to model the following relation between input and output.

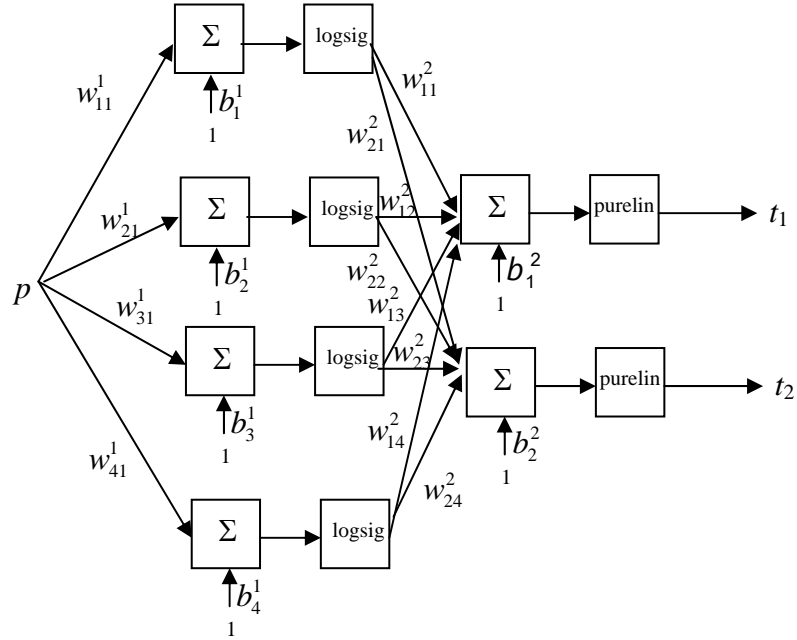
$$\left\{ p_1 = 0, t_1 = \begin{bmatrix} 0 \\ 4 \end{bmatrix} \right\}, \left\{ p_2 = 1, t_2 = \begin{bmatrix} 5 \\ 0 \end{bmatrix} \right\}, \left\{ p_3 = 2, t_3 = \begin{bmatrix} 10 \\ 4 \end{bmatrix} \right\}$$

Levenberg-Marquard method is used to train the network with the following initial weights and biases.

$$w_{11}^1 = -0.1, w_{21}^1 = 0.2, w_{31}^1 = 0.1, w_{41}^1 = -0.2, b_1^1 = 0.1, b_2^1 = 0.2, b_3^1 = 0.3, b_4^1 = 0.4, \\ w_{11}^2 = 0.1, w_{12}^2 = -0.2, w_{13}^2 = 0.3, w_{14}^2 = -0.4, b_1^2 = 0.1, w_{21}^2 = 0.4, w_{22}^2 = -0.3, w_{23}^2 = -0.2, w_{24}^2 = 0.1, b_2^2 = 0.1$$

Assume that log-sigmoid function is applied in the first layer and linear function is applied in the second layer, determine the summation of squared error of the current set of weights and biases and determine the Maquard sensitivities of the first layer and second layer of all the inputs during the first iteration. (25)

Solution



Determine derivative of transfer function,

$$f^1 = (1 + e^{-n})^{-1}, \dot{f}^1 = (1 - a^1)(a^1), f^2 = n, \dot{f}^2 = 1 \quad (1)$$

Present p_1 ,

$$p = p_1 = 0, n^1 = w^1 p + b^1 = \begin{bmatrix} -0.1 \\ 0.2 \\ 0.1 \\ -0.2 \end{bmatrix} 0 + \begin{bmatrix} 0.1 \\ 0.2 \\ 0.3 \\ 0.4 \end{bmatrix} = \begin{bmatrix} 0.1 \\ 0.2 \\ 0.3 \\ 0.4 \end{bmatrix}, a^1 = \begin{bmatrix} (1 + e^{-n_1})^{-1} \\ (1 + e^{-n_2})^{-1} \\ (1 + e^{-n_3})^{-1} \\ (1 + e^{-n_4})^{-1} \end{bmatrix} = \begin{bmatrix} (1 + e^{-0.1})^{-1} \\ (1 + e^{-0.2})^{-1} \\ (1 + e^{-0.3})^{-1} \\ (1 + e^{-0.4})^{-1} \end{bmatrix} = \begin{bmatrix} 0.53 \\ 0.55 \\ 0.57 \\ 0.60 \end{bmatrix} \quad (2)$$

$$n^2 = w^2 a^1 + b^2 = \begin{bmatrix} 0.1 & -0.2 & 0.3 & -0.4 \\ 0.4 & -0.3 & -0.2 & 0.1 \end{bmatrix} \begin{bmatrix} 0.53 \\ 0.55 \\ 0.57 \\ 0.60 \end{bmatrix} + \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix} = \begin{bmatrix} -0.03 \\ 0.09 \end{bmatrix}, a^2 = a = n^2 = \begin{bmatrix} -0.03 \\ 0.09 \end{bmatrix} \quad (3)$$

$$e = e_1 = t - a = \begin{bmatrix} 0 \\ 4 \end{bmatrix} - \begin{bmatrix} -0.03 \\ 0.09 \end{bmatrix} = \begin{bmatrix} 0.03 \\ 3.91 \end{bmatrix} \quad (4)$$

$$\tilde{S}^2 = -\dot{f}^2 = \begin{bmatrix} -1 \\ -1 \end{bmatrix}, \tilde{S}^1 = \dot{f}^1 (w^2)^T \tilde{S}^2 = \begin{bmatrix} (1-0.53)0.53 & 0 & 0 & 0 \\ 0 & (1-0.55)0.55 & 0 & 0 \\ 0 & 0 & (1-0.57)0.57 & 0 \\ 0 & 0 & 0 & (1-0.60)0.60 \end{bmatrix} \begin{bmatrix} 0.1 & 0.4 \\ -0.2 & -0.3 \\ 0.3 & -0.2 \\ -0.4 & 0.1 \end{bmatrix} \begin{bmatrix} -1 \\ -1 \end{bmatrix} = \begin{bmatrix} -0.12 \\ 0.12 \\ -0.02 \\ 0.07 \end{bmatrix} \quad (5)$$

Present p_2 ,

$$p = p_2 = 1, n^1 = w^1 p + b^1 = \begin{bmatrix} -0.1 \\ 0.2 \\ 0.1 \\ -0.2 \end{bmatrix} 1 + \begin{bmatrix} 0.1 \\ 0.2 \\ 0.3 \\ 0.4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0.4 \\ 0.4 \\ 0.2 \end{bmatrix}, a^1 = \begin{bmatrix} (1+e^{-n_1}^{-1})^{-1} \\ (1+e^{-n_2}^{-1})^{-1} \\ (1+e^{-n_3}^{-1})^{-1} \\ (1+e^{-n_4}^{-1})^{-1} \end{bmatrix} = \begin{bmatrix} (1+e^0)^{-1} \\ (1+e^{-0.4})^{-1} \\ (1+e^{-0.4})^{-1} \\ (1+e^{-0.2})^{-1} \end{bmatrix} = \begin{bmatrix} 0.50 \\ 0.60 \\ 0.60 \\ 0.55 \end{bmatrix} \quad (6)$$

$$n^2 = w^2 a^1 + b^2 = \begin{bmatrix} 0.1 & -0.2 & 0.3 & -0.4 \\ 0.4 & -0.3 & -0.2 & 0.1 \end{bmatrix} \begin{bmatrix} 0.50 \\ 0.60 \\ 0.60 \\ 0.55 \end{bmatrix} + \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix} = \begin{bmatrix} -0.03 \\ 0.09 \end{bmatrix}, a^2 = a = n^2 = \begin{bmatrix} -0.03 \\ 0.09 \end{bmatrix} \quad (7)$$

$$e = e_2 = t - a = \begin{bmatrix} 5 \\ 0 \end{bmatrix} - \begin{bmatrix} -0.03 \\ 0.09 \end{bmatrix} = \begin{bmatrix} 5.03 \\ -0.09 \end{bmatrix} \quad (8)$$

$$\tilde{S}^2 = -\dot{f}^2 = \begin{bmatrix} -1 \\ -1 \end{bmatrix}, \tilde{S}^1 = \dot{f}^1 (w^2)^T \tilde{S}^2 = \begin{bmatrix} (1-0.50)0.50 & 0 & 0 & 0 \\ 0 & (1-0.60)0.60 & 0 & 0 \\ 0 & 0 & (1-0.60)0.60 & 0 \\ 0 & 0 & 0 & (1-0.55)0.55 \end{bmatrix} \begin{bmatrix} 0.1 & 0.4 \\ -0.2 & -0.3 \\ 0.3 & -0.2 \\ -0.4 & 0.1 \end{bmatrix} \begin{bmatrix} -1 \\ -1 \end{bmatrix} = \begin{bmatrix} -0.13 \\ 0.12 \\ -0.02 \\ 0.07 \end{bmatrix} \quad (9)$$

Present p_3 ,

$$p = p_3 = 2, n^1 = w^1 p + b^1 = \begin{bmatrix} -0.1 \\ 0.2 \\ 0.1 \\ -0.2 \end{bmatrix} 2 + \begin{bmatrix} 0.1 \\ 0.2 \\ 0.3 \\ 0.4 \end{bmatrix} = \begin{bmatrix} -0.1 \\ 0.6 \\ 0.5 \\ 0 \end{bmatrix}, a^1 = \begin{bmatrix} (1+e^{-n_1}^{-1})^{-1} \\ (1+e^{-n_2}^{-1})^{-1} \\ (1+e^{-n_3}^{-1})^{-1} \\ (1+e^{-n_4}^{-1})^{-1} \end{bmatrix} = \begin{bmatrix} (1+e^{0.1})^{-1} \\ (1+e^{-0.6})^{-1} \\ (1+e^{-0.5})^{-1} \\ (1+e^0)^{-1} \end{bmatrix} = \begin{bmatrix} 0.48 \\ 0.65 \\ 0.62 \\ 0.50 \end{bmatrix} \quad (10)$$

$$n^2 = w^2 a^1 + b^2 = \begin{bmatrix} 0.1 & -0.2 & 0.3 & -0.4 \\ 0.4 & -0.3 & -0.2 & 0.1 \end{bmatrix} \begin{bmatrix} 0.48 \\ 0.65 \\ 0.62 \\ 0.50 \end{bmatrix} + \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix} = \begin{bmatrix} 0.00 \\ 0.02 \end{bmatrix}, a^2 = a = n^2 = \begin{bmatrix} 0.00 \\ 0.02 \end{bmatrix} \quad (11)$$

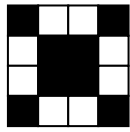
$$e = e_3 = t - a = \begin{bmatrix} 10 \\ 4 \end{bmatrix} - \begin{bmatrix} 0.00 \\ 0.02 \end{bmatrix} = \begin{bmatrix} 10 \\ 3.98 \end{bmatrix} \quad (12)$$

$$\tilde{S}^2 = -\dot{f}^2 = \begin{bmatrix} -1 \\ -1 \end{bmatrix}, \tilde{S}^1 = \dot{f}^1 (w^2)^T \tilde{S}^2 = \begin{bmatrix} (1-0.48)0.48 & 0 & 0 & 0 \\ 0 & (1-0.65)0.65 & 0 & 0 \\ 0 & 0 & (1-0.62)0.62 & 0 \\ 0 & 0 & 0 & (1-0.50)0.50 \end{bmatrix} \begin{bmatrix} 0.1 & 0.4 \\ -0.2 & -0.3 \\ 0.3 & -0.2 \\ -0.4 & 0.1 \end{bmatrix} \begin{bmatrix} -1 \\ -1 \end{bmatrix} = \begin{bmatrix} -0.13 \\ 0.11 \\ -0.02 \\ 0.08 \end{bmatrix} \quad (13)$$

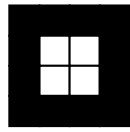
Determine summation of error square,

$$F(x) = \sum e^2 = 0.03^2 + 3.91^2 + 5.03^2 + (-0.09)^2 + 10^2 + 3.98^2 = 156.44 \quad (14)$$

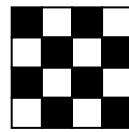
Q.4 If Hebb's rule is used to determine weight of a Hopfield network, having 3 patterns as shown below in the ± 1 binary number system.



Pattern 1

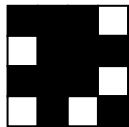


Pattern 2

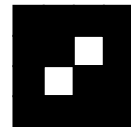


Pattern 3

Determine the output after the second iteration of each input when the two inputs as shown below are presented each at a time to the network. Assume symmetrical hard limit function is used in the network. (25)



Input 1



Input 2

Solution

Patterns are represented by

$$\mathbf{p}_1 = \begin{bmatrix} 1 \\ -1 \\ -1 \\ 1 \\ -1 \\ 1 \\ 1 \\ -1 \\ -1 \\ 1 \\ 1 \\ -1 \\ 1 \\ -1 \\ -1 \\ 1 \end{bmatrix}, \quad \mathbf{p}_2 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ -1 \\ -1 \\ 1 \\ 1 \\ -1 \\ -1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \quad \mathbf{p}_3 = \begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \\ -1 \\ 1 \\ -1 \\ 1 \\ -1 \\ 1 \\ -1 \\ -1 \\ 1 \\ 1 \\ -1 \\ 1 \end{bmatrix} \tag{1}$$

By Hebb's rule, the weight become

$$\mathbf{W} = \sum_{q=1}^3 (\mathbf{p}_q \mathbf{p}_q^T) = \begin{pmatrix} 1 & 1 & 1 \\ -1 & -1 & -1 \\ -1 & -1 & 1 \\ 1 & 1 & 1 \\ -1 & -1 & 1 \\ 1 & 1 & -1 \\ 1 & 1 & -1 \\ -1 & -1 & 1 \\ -1 & -1 & -1 \\ 1 & 1 & 1 \\ 1 & 1 & -1 \\ -1 & -1 & 1 \\ 1 & 1 & 1 \\ -1 & -1 & -1 \\ -1 & -1 & 1 \\ 1 & 1 & 1 \\ -1 & -1 & -1 \\ 1 & 1 & 1 \end{pmatrix} + \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \\ -1 & -1 & -1 \\ -1 & -1 & -1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \\ -1 & -1 & -1 \\ -1 & -1 & -1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \\ -1 & -1 & -1 \\ -1 & -1 & -1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + \begin{pmatrix} 1 & 1 & 1 \\ -1 & -1 & -1 \\ 1 & 1 & 1 \\ -1 & -1 & -1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 3 & -1 & 1 & 1 & -1 & 1 & -1 & 1 & 1 & -1 & 1 & -1 & 1 & 1 & -1 & 3 \\ -1 & 3 & 1 & 1 & 3 & -3 & -1 & 1 & 1 & -1 & -3 & 3 & 1 & 1 & 3 & -1 \\ 1 & 1 & 3 & -1 & 1 & -1 & -3 & 3 & 3 & -3 & -1 & 1 & -1 & 3 & 1 & 1 \\ 1 & 1 & -1 & 3 & 1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 & 3 & -1 & 1 & 1 \\ -1 & 3 & 1 & 1 & 3 & -3 & -1 & 1 & 1 & -1 & -3 & 3 & 1 & 1 & 3 & -1 \\ 1 & -3 & -1 & -1 & -3 & 3 & 1 & -1 & -1 & 1 & 3 & -3 & -1 & -1 & -3 & 1 \\ -1 & -1 & -3 & 1 & -1 & 1 & 3 & -3 & -3 & 3 & 1 & -1 & 1 & -3 & -1 & -1 \\ 1 & 1 & 3 & -1 & 1 & -1 & -3 & 3 & 3 & -3 & -1 & 1 & -1 & 3 & 1 & 1 \\ 1 & 1 & 3 & -1 & 1 & -1 & -3 & 3 & 3 & -3 & -1 & 1 & -1 & 3 & 1 & 1 \\ -1 & -1 & -3 & 1 & -1 & 1 & 3 & -3 & -3 & 3 & 1 & -1 & 1 & -3 & -1 & -1 \\ 1 & -3 & -1 & -1 & -3 & 3 & 1 & -1 & -1 & 1 & 3 & -3 & -1 & -1 & -3 & 1 \\ -1 & 3 & 1 & 1 & 3 & -3 & -1 & 1 & 1 & -1 & -3 & 3 & 1 & 1 & 3 & -1 \\ 1 & 1 & -1 & 3 & 1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 & 3 & -1 & 1 & 1 \\ 1 & 1 & 3 & -1 & 1 & -1 & -3 & 3 & 3 & -3 & -1 & 1 & -1 & 3 & 1 & 1 \\ -1 & 3 & 1 & 1 & 3 & -3 & -1 & 1 & 1 & -1 & -3 & 3 & 1 & 1 & 3 & -1 \\ 3 & -1 & 1 & 1 & -1 & 1 & -1 & 1 & 1 & -1 & 1 & -1 & 1 & 1 & -1 & 3 \end{pmatrix} \quad (2)$$

the bias is zero

$$\mathbf{b} = \mathbf{0} \quad (3)$$

The first input is represented by

$$\mathbf{p} = \begin{pmatrix} 1 \\ 1 \\ 1 \\ -1 \\ -1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ -1 \\ -1 \\ 1 \\ -1 \\ 1 \end{pmatrix} \quad (4)$$

Since

$$\mathbf{p} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ -1 \\ 1 \\ 1 \\ -1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \tag{8}$$

Since

$$\mathbf{a}(0) = \mathbf{p} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ -1 \\ 1 \\ 1 \\ -1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \tag{9}$$

