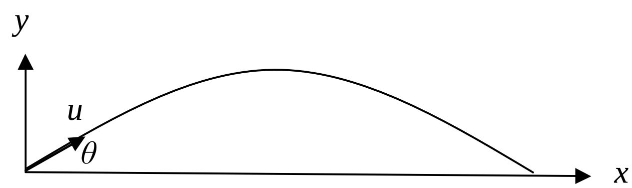
Midsem Examination **AI and Neuro-Fuzzy Theory AT74.05** **March 15, 2013**

Time: 9:00-11:00 h. **Open Book**
Marks: 100

Attempt all questions.

Q.1 ADALINE network is used to determine the initial speed, u, and its angle from ground, θ, of a ball kicked by a soccer player as shown in the below figure.



Assume the ball trajectory follows a projectile motion with no drag force and the gravitational acceleration is 10 m/s^2. The ball coordinates are collected and shown in the table.

x (m)	0	20	40	60	80
y (m)	0	15	20	15	0

(a) Draw the ADALINE network which can be applied to determine the initial speed and the angle, then determine the inputs and output of the network. (10)

(b) If all the data are presented equally, determine the parameters of the ADALINE by LMS algorithm. Then determine the initial speed and the angle. (15)

Solution

Based on projectile motion,

$$U_x = u\cos(\theta) \tag{1}$$

$$U_y = u\sin(\theta) \tag{2}$$

$$x = U_x t \tag{3}$$

$$y = U_y t - \frac{gt^2}{2} \tag{4}$$

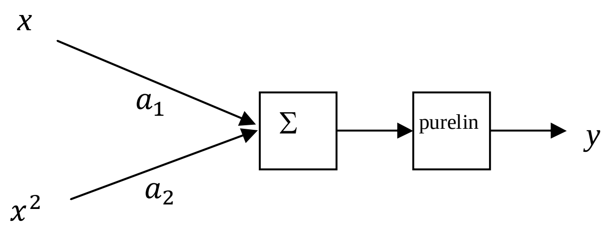
From (3),

$$t = \frac{x}{U_x} \tag{5}$$

Substitute (5) into (4),

$$y = \frac{U_y x}{U_x} - \frac{g x^2}{2U_x^2} = a_1 x + a_2 x^2 \tag{6}$$

(a)



(b)

$$F(x) = E[t^2] - 2x^T E[tz] + x^T E[zz^T]x \tag{7}$$

$$E[t^2] = \frac{1}{5}\left(0^2 + 15^2 + 20^2 + 15^2 + 0^2\right) = 170 \tag{8}$$

$$E[tz] = \frac{1}{5}\left(0\begin{bmatrix}0\\0\end{bmatrix} + 15\begin{bmatrix}20\\400\end{bmatrix} + 20\begin{bmatrix}40\\1600\end{bmatrix} + 15\begin{bmatrix}60\\3600\end{bmatrix} + 0\begin{bmatrix}80\\6400\end{bmatrix}\right) = \begin{bmatrix}400\\18400\end{bmatrix} \tag{9}$$

$$E[zz^T] = \frac{1}{5}\left(\begin{bmatrix}0\\0\end{bmatrix}\begin{bmatrix}0\\0\end{bmatrix}^T + \begin{bmatrix}20\\400\end{bmatrix}\begin{bmatrix}20\\400\end{bmatrix}^T + \begin{bmatrix}40\\1600\end{bmatrix}\begin{bmatrix}40\\1600\end{bmatrix}^T + \begin{bmatrix}60\\3600\end{bmatrix}\begin{bmatrix}60\\3600\end{bmatrix}^T + \begin{bmatrix}80\\6400\end{bmatrix}\begin{bmatrix}80\\6400\end{bmatrix}^T\right) = \begin{bmatrix}2400 & 160000\\160000 & 11328000\end{bmatrix} \tag{10}$$

The minimum point is the stationary point of the quadratic function

$$x = \begin{bmatrix}a_1\\a_2\end{bmatrix} = E[zz^T]^{-1}E[tz] = \begin{bmatrix}2400 & 160000\\160000 & 11328000\end{bmatrix}^{-1}\begin{bmatrix}400\\18400\end{bmatrix} = \begin{bmatrix}1\\-0.0125\end{bmatrix} \tag{11}$$

$$a_2 = -0.0125 = -\frac{g}{2U_x^2} = -\frac{5}{U_x^2} \tag{12}$$

$$U_x = 20\ \text{m/s} \tag{13}$$

$$a_1 = 1 = \frac{U_y}{U_x} = \frac{U_y}{20} \tag{14}$$

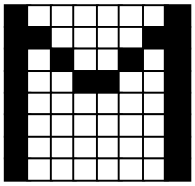
$$U_y = 20\ \text{m/s} \tag{15}$$

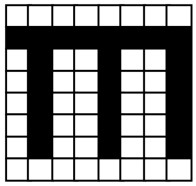
$$\tan(\theta) = \frac{U_y}{U_x} = 1 \tag{16}$$

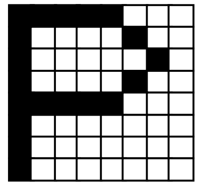
$$\theta = 45^\circ \tag{17}$$

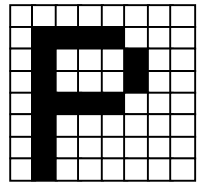
$$u = \sqrt{U_x^2 + U_y^2} = 20\sqrt{2} = 28.28 \text{ m/s} \qquad (18)$$

Q.2 Design the network and its parameters which can recognize correctly the following 2 patterns.

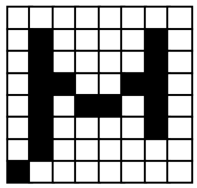

Pattern 1


Pattern 1


Pattern 2


Pattern 2

Determine whether the following input is recognized as pattern 1 or pattern 2 based on your design.


Input

(25)

Solution

Since each pattern consists of the vectors which are not very similar, LVQ is selected. Black pixel is represented by 1, white pixel can be represented by either 0 or -1 (0 is used in this case since the function in the first layer of LVQ is the negative norm of the different between the pattern vector and the input vector).

$w_1{}^T$ = [1 0 0 0 0 0 0 1, 1 1 0 0 0 0 1 1, 1 0 1 0 0 1 0 1, 1 0 0 1 1 0 0 1, 1 0 0 0 0 0 0 1, 1 0 0 0 0 0 0 1, 1 0 0 0 0 0 0 1, 1 0 0 0 0 0 0 1] (1)

$w_2{}^T$ = [0 0 0 0 0 0 0 0, 1 1 1 1 1 1 1 1, 0 1 0 0 1 0 0 1, 0 1 0 0 1 0 0 1, 0 1 0 0 1 0 0 1, 0 1 0 0 1 0 0 1, 0 1 0 0 1 0 0 1, 0 0 0 0 0 0 0 0] (2)

$w_3{}^T$ = [1 1 1 1 1 0 0 0, 1 0 0 0 0 1 0 0, 1 0 0 0 0 0 1 0, 1 0 0 0 0 1 0 0, 1 1 1 1 1 0 0 0, 1 0 0 0 0 0 0 0, 1 0 0 0 0 0 0 0, 1 0 0 0 0 0 0 0] (3)

$w_4{}^T$ = [0 0 0 0 0 0 0 0, 0 1 1 1 1 0 0 0, 0 1 0 0 0 1 0 0, 0 1 0 0 0 1 0 0, 0 1 1 1 1 0 0 0, 0 1 0 0 0 0 0 0, 0 1 0 0 0 0 0 0, 0 1 0 0 0 0 0 0] (4)

$$W^1 = \begin{bmatrix} w_1^T \\ w_2^T \\ w_3^T \\ w_4^T \end{bmatrix} \qquad (5)$$

$$W^2 = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} \quad (6)$$

The input,

p^T = [0 0 0 0 0 0 0 0, 0 1 0 0 0 0 1 0, 0 1 0 0 0 0 1 0, 0 1 1 0 0 1 1 0, 0 1 0 1 1 0 1 0, 0 1 0 0 0 0 1 0, 0 1 0 0 0 0 0 0, 1 0 0 0 0 0 0 0] (7)

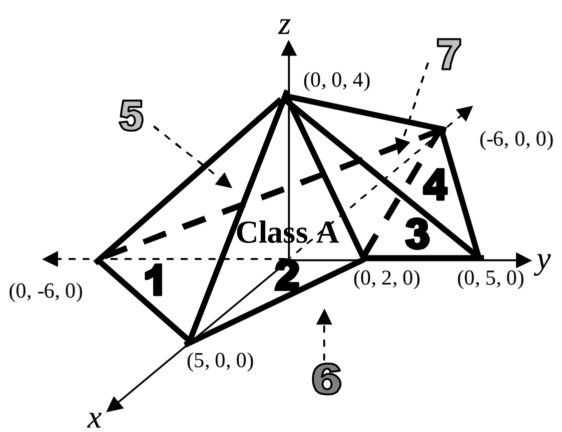
The output from the first layer,

$$a^1 = compet\begin{bmatrix} -|w_1 - p| \\ -|w_2 - p| \\ -|w_3 - p| \\ -|w_4 - p| \end{bmatrix} = compet\begin{bmatrix} -\sqrt{32} \\ -\sqrt{23} \\ -\sqrt{23} \\ -\sqrt{13} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \quad (8)$$

The output from the second layer,

$$a^2 = purelin\left[\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}\right] = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad (9)$$

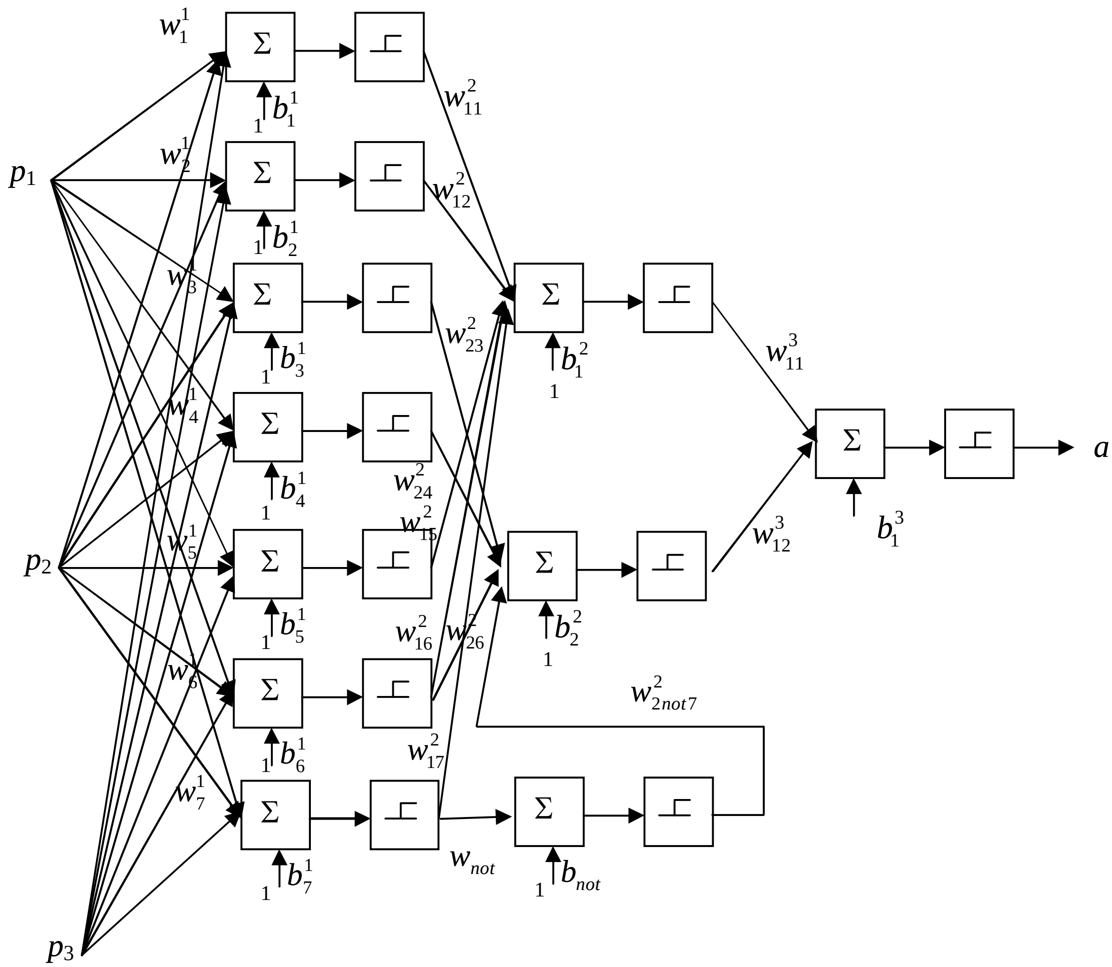
Q.3 Design a network with appropriate parameters, which can classify class A occupying the shape as shown in the below figure from class B which is outside. (25)



Class B

<u>**Solution**</u>

Multi-Layer Perceptron is selected. The first layer is used to create decision boundaries 1, 2, 3, 4, 5, 6, and 7. The second layer is used to AND decision boundaries 1, 2, 5, 6, and 7 and to AND decision boundaries 3, 4, 6, and (NOT of 7). The third layer is used to OR the 2 polygons obtained from the second layer.



For layer 1

w must point into the shape and it must be perpendicular to decision boundary and edges.

b is determined from equating **n** = 0; and solve for **b**

For the first decision boundary,

$$\begin{bmatrix} w_{11}^1 & w_{12}^1 & w_{13}^1 \end{bmatrix} \begin{bmatrix} 5 \\ 6 \\ 0 \end{bmatrix} = 5w_{11}^1 + 6w_{12}^1 = 0 \tag{1}$$

$$\begin{bmatrix} w_{11}^1 & w_{12}^1 & w_{13}^1 \end{bmatrix} \begin{bmatrix} 5 \\ 0 \\ -4 \end{bmatrix} = 5w_{11}^1 - 4w_{13}^1 = 0 \tag{2}$$

$$\mathbf{w}_1^1 = \begin{bmatrix} -1 \\ 5/6 \\ -5/4 \end{bmatrix} \tag{3}$$

$$n_1^1 = w_{11}^1 p_1 + w_{12}^1 p_2 + w_{13}^1 p_3 + b_1^1 = 0 \tag{4}$$

at $p_1 = 5$, $p_2 = 0$, and $p_3 = 0$;

$$-1(5) + 0 + 0 + b_1^1 = 0; b_1^1 = 5 \tag{5}$$

For the second decision boundary,

$$\begin{bmatrix} w_{21}^1 & w_{22}^1 & w_{23}^1 \end{bmatrix} \begin{bmatrix} 5 \\ -2 \\ 0 \end{bmatrix} = 5w_{21}^1 - 2w_{22}^1 = 0 \tag{6}$$

$$\begin{bmatrix} w_{21}^1 & w_{22}^1 & w_{23}^1 \end{bmatrix} \begin{bmatrix} 5 \\ 0 \\ -4 \end{bmatrix} = 5w_{21}^1 - 4w_{23}^1 = 0 \tag{7}$$

$$\mathbf{w}_2^1 = \begin{bmatrix} -1 \\ -5/2 \\ -5/4 \end{bmatrix} \tag{8}$$

$$n_2^1 = w_{21}^1 p_1 + w_{22}^1 p_2 + w_{23}^1 p_3 + b_2^1 = 0 \tag{9}$$

at $p_1 = 5$, $p_2 = 0$, and $p_3 = 0$;

$$-1(5) + 0 + 0 + b_2^1 = 0; b_2^1 = 5 \tag{10}$$

For the third decision boundary,

$$\begin{bmatrix} w_{31}^1 & w_{32}^1 & w_{33}^1 \end{bmatrix} \begin{bmatrix} 0 \\ 3 \\ 0 \end{bmatrix} = 3w_{32}^1 = 0 \tag{11}$$

$$\begin{bmatrix} w_{31}^1 & w_{32}^1 & w_{33}^1 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ -4 \end{bmatrix} = 2w_{32}^1 - 4w_{33}^1 = 0 \tag{12}$$

$$\mathbf{w}_3^1 = \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix} \tag{13}$$

$$n_3^1 = w_{31}^1 p_1 + w_{32}^1 p_2 + w_{33}^1 p_3 + b_3^1 = 0 \tag{14}$$

at $p_1 = 0$, $p_2 = 0$, and $p_3 = 4$;

$$0+0+0+b_3^1=0; b_3^1=0 \tag{15}$$

For the fourth decision boundary,

$$\begin{bmatrix} w_{41}^1 & w_{42}^1 & w_{43}^1 \end{bmatrix} \begin{bmatrix} 6 \\ 5 \\ 0 \end{bmatrix} = 6w_{41}^1 + 5w_{42}^1 = 0 \tag{16}$$

$$\begin{bmatrix} w_{41}^1 & w_{42}^1 & w_{43}^1 \end{bmatrix} \begin{bmatrix} 0 \\ 5 \\ -4 \end{bmatrix} = 5w_{42}^1 - 4w_{43}^1 = 0 \tag{17}$$

$$\mathbf{w}_4^1 = \begin{bmatrix} 5/6 \\ -1 \\ -5/4 \end{bmatrix} \tag{18}$$

$$n_4^1 = w_{41}^1 p_1 + w_{42}^1 p_2 + w_{43}^1 p_3 + b_4^1 = 0 \tag{19}$$

at $p_1 = 0$, $p_2 = 0$, and $p_3 = 4$;

$$0+0+(-5/4)4+b_4^1=0; b_4^1=5 \tag{20}$$

For the fifth decision boundary,

$$\begin{bmatrix} w_{51}^1 & w_{52}^1 & w_{53}^1 \end{bmatrix} \begin{bmatrix} 6 \\ -6 \\ 0 \end{bmatrix} = 6w_{51}^1 - 6w_{52}^1 = 0 \tag{21}$$

$$\begin{bmatrix} w_{51}^1 & w_{52}^1 & w_{53}^1 \end{bmatrix} \begin{bmatrix} 0 \\ -6 \\ -4 \end{bmatrix} = -6w_{52}^1 - 4w_{53}^1 = 0 \tag{22}$$

$$\mathbf{w}_5^1 = \begin{bmatrix} 1 \\ 1 \\ -6/4 \end{bmatrix} \tag{23}$$

$$n_5^1 = w_{51}^1 p_1 + w_{52}^1 p_2 + w_{53}^1 p_3 + b_5^1 = 0 \tag{24}$$

at $p_1 = 0$, $p_2 = 0$, and $p_3 = 4$;

$$0+0+(-6/4)4+b_5^1=0; b_5^1=6 \tag{25}$$

For the sixth decision boundary,

$$\mathbf{w}_6^1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad (26)$$

$$n_6^1 = w_{61}^1 p_1 + w_{62}^1 p_2 + w_{63}^1 p_3 + b_6^1 = 0 \quad (27)$$

at $p_1 = 0$, $p_2 = 0$, and $p_3 = 0$;

$$0 + 0 + 0 + b_6^1 = 0; b_6^1 = 0 \quad (28)$$

For the seventh decision boundary,

$$\begin{bmatrix} w_{71}^1 & w_{72}^1 & w_{73}^1 \end{bmatrix} \begin{bmatrix} 6 \\ 2 \\ 0 \end{bmatrix} = 6w_{71}^1 + 2w_{72}^1 = 0 \quad (29)$$

$$\begin{bmatrix} w_{71}^1 & w_{72}^1 & w_{73}^1 \end{bmatrix} \begin{bmatrix} -6 \\ 0 \\ -4 \end{bmatrix} = -6w_{71}^1 - 4w_{73}^1 = 0 \quad (30)$$

$$\mathbf{w}_7^1 = \begin{bmatrix} 1 \\ -3 \\ -6/4 \end{bmatrix} \quad (31)$$

$$n_7^1 = w_{71}^1 p_1 + w_{72}^1 p_2 + w_{73}^1 p_3 + b_7^1 = 0 \quad (32)$$

at $p_1 = 0$, $p_2 = 0$, and $p_3 = 4$;

$$0 + 0 + (-6/4)4 + b_7^1 = 0; b_7^1 = 6 \quad (33)$$

NOT operation is required for the second polygon.

$$w_{not} = -1 \quad (34)$$

$$b_{not} = 0.5 \quad (35)$$

For layer 2

Since this is AND layer, if we select

$$w_{11}^2 = w_{12}^2 = w_{15}^2 = w_{16}^2 = w_{17}^2 = 1 \quad (36)$$

$$b_1^2 = -4.5 \tag{37}$$

$$w_{23}^2 = w_{24}^2 = w_{26}^2 = w_{2not7}^2 = 1 \tag{38}$$

$$b_2^2 = -3.5 \tag{39}$$

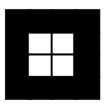
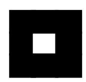
For layer 3

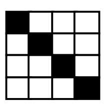
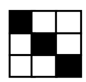
Since this is OR layer, if we select

$$w_{11}^3 = w_{12}^3 = 1 \tag{40}$$

$$b_1^3 = -0.5 \tag{41}$$

Q.4 Design the network and its parameters which can change resolution of the two images from 4x4 to 3x3 as shown below. Then test the result by presenting the input and determine the output from both patterns. (25)

p_1 =  , t_1 = 

p_2 =  , t_2 = 

<u>Solution</u>

Both Linear Associator with Pseudo-inverse rule and Symmetrical Hard Limit Associator with Hebb's rule are applicable. Select Symmetrical Hard Limit Associator with Hebb's rule.

Inputs and targets are represented by

$$\left\{ \mathbf{p}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ -1 \\ -1 \\ 1 \\ 1 \\ -1 \\ -1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \mathrm{t}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ -1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \right\}, \left\{ \mathbf{p}_2 = \begin{bmatrix} 1 \\ -1 \\ -1 \\ -1 \\ -1 \\ 1 \\ -1 \\ -1 \\ -1 \\ -1 \\ 1 \\ -1 \\ -1 \\ -1 \\ -1 \\ 1 \end{bmatrix}, \mathrm{t}_2 = \begin{bmatrix} 1 \\ -1 \\ -1 \\ -1 \\ 1 \\ -1 \\ -1 \\ -1 \\ 1 \end{bmatrix} \right\} \tag{1}$$

By Hebb's rule, the weight become

$$\mathbf{W} = \sum_{q=1}^{2} (\mathrm{t}_q \mathbf{p}_q^T) = \left(\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ -1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ -1 \\ -1 \\ 1 \\ 1 \\ -1 \\ -1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}^T + \begin{bmatrix} 1 \\ -1 \\ -1 \\ -1 \\ 1 \\ -1 \\ -1 \\ -1 \\ 1 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ -1 \\ -1 \\ -1 \\ 1 \\ -1 \\ -1 \\ -1 \\ -1 \\ 1 \\ -1 \\ -1 \\ -1 \\ -1 \\ 1 \end{bmatrix}^T \right) = \begin{bmatrix} 2 & 0 & 0 & 0 & 0 & 0 & -2 & 0 & 0 & -2 & 0 & 0 & 0 & 0 & 0 & 2 \\ 0 & 2 & 2 & 2 & 2 & -2 & 0 & 2 & 2 & 0 & -2 & 2 & 2 & 2 & 2 & 0 \\ 0 & 2 & 2 & 2 & 2 & -2 & 0 & 2 & 2 & 0 & -2 & 2 & 2 & 2 & 2 & 0 \\ 0 & 2 & 2 & 2 & 2 & -2 & 0 & 2 & 2 & 0 & -2 & 2 & 2 & 2 & 2 & 0 \\ 0 & -2 & -2 & -2 & -2 & 2 & 0 & -2 & -2 & 0 & 2 & -2 & -2 & -2 & -2 & 0 \\ 0 & 2 & 2 & 2 & 2 & -2 & 0 & 2 & 2 & 0 & -2 & 2 & 2 & 2 & 2 & 0 \\ 0 & 2 & 2 & 2 & 2 & -2 & 0 & 2 & 2 & 0 & -2 & 2 & 2 & 2 & 2 & 0 \\ 0 & 2 & 2 & 2 & 2 & -2 & 0 & 2 & 2 & 0 & -2 & 2 & 2 & 2 & 2 & 0 \\ 2 & 0 & 0 & 0 & 0 & 0 & -2 & 0 & 0 & -2 & 0 & 0 & 0 & 0 & 0 & 2 \end{bmatrix} \tag{2}$$

The bias is zero

$$\mathbf{b} = \mathbf{0} \tag{3}$$

Present the first input,

$$\mathbf{p}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ -1 \\ -1 \\ 1 \\ 1 \\ -1 \\ -1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \tag{4}$$

$$\mathbf{a} = \text{hardlims}(\text{W}\,\text{p} + \text{b}) = \text{hardlims}\begin{bmatrix} 8 \\ 24 \\ 24 \\ 24 \\ -24 \\ 24 \\ 24 \\ 24 \\ 8 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ -1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} = \text{t}_1 \tag{5}$$

Present the second input,

$$\left\{ \mathbf{p}_2 = \begin{bmatrix} 1 \\ -1 \\ -1 \\ -1 \\ -1 \\ 1 \\ -1 \\ -1 \\ -1 \\ -1 \\ 1 \\ -1 \\ -1 \\ -1 \\ -1 \\ 1 \end{bmatrix}, \right\} \tag{6}$$

$$\mathbf{a} = \text{hardlims}(\text{W}\,\text{p} + \text{b}) = \text{hardlims}\begin{bmatrix} 8 \\ -24 \\ -24 \\ -24 \\ 24 \\ -24 \\ -24 \\ -24 \\ 8 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ -1 \\ -1 \\ 1 \\ -1 \\ -1 \\ -1 \\ 1 \end{bmatrix} = \text{t}_2 \tag{7}$$