Midsem Examination AI and Neuro-Fuzzy Theory AT74.05 March 21, 2014

Time: 9:00-11:00 h. **Open Book**
Marks: 100

Attempt all questions.

Q.1 ADALINE network is used to determine parameters; a_0, a_1, a_2, and a_3 of a linear function $f = a_3 z + a_2 y + a_1 x + a_0$ when the input data; *x*, *y*, and *z* and the output data; *f*, are collected and found as the followings.

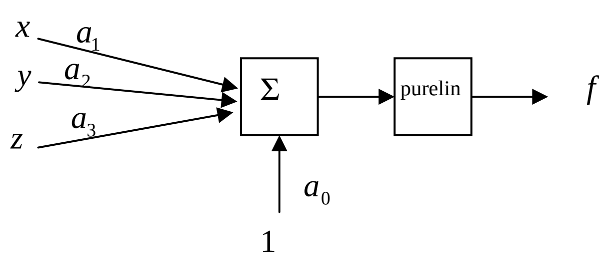
x	1	2	2	3	5
y	1	3	4	5	2
z	1	4	3	10	7
f	50	100	105	165	150

(a) Draw the ADALINE network that can be used to determine the parameters. (4)

(b) If all the data are presented equally, determine the parameters by LMS algorithm. (16)

Solution

(a)



(b)

$$F(x) = E[t^2] - 2x^T E[tz] + x^T E[zz^T]x = c - 2x^T h + x^T R x \quad (1)$$

$$c = \frac{1}{5}\left(50^2 + 100^2 + 105^2 + 165^2 + 150^2\right) = 14650 \quad (2)$$

$$h = \frac{1}{5}\left(50\begin{bmatrix}1\\1\\1\\1\end{bmatrix} + 100\begin{bmatrix}2\\3\\4\\1\end{bmatrix} + 105\begin{bmatrix}2\\4\\3\\1\end{bmatrix} + 165\begin{bmatrix}3\\5\\10\\1\end{bmatrix} + 150\begin{bmatrix}5\\2\\7\\1\end{bmatrix}\right) = \begin{bmatrix}341\\379\\693\\114\end{bmatrix} \quad (3)$$

$$R = \frac{1}{5}\left(\begin{bmatrix}1\\1\\1\\1\end{bmatrix}\begin{bmatrix}1\\1\\1\\1\end{bmatrix}^T + \begin{bmatrix}2\\3\\4\\1\end{bmatrix}\begin{bmatrix}2\\3\\4\\1\end{bmatrix}^T + \begin{bmatrix}2\\4\\3\\1\end{bmatrix}\begin{bmatrix}2\\4\\3\\1\end{bmatrix}^T + \begin{bmatrix}3\\5\\10\\1\end{bmatrix}\begin{bmatrix}3\\5\\10\\1\end{bmatrix}^T + \begin{bmatrix}5\\2\\7\\1\end{bmatrix}\begin{bmatrix}5\\2\\7\\1\end{bmatrix}^T\right) = \begin{bmatrix}8.6 & 8.0 & 16.0 & 2.6\\ 8.0 & 11.0 & 17.8 & 3.0\\ 16.0 & 17.8 & 35.0 & 5.0\\ 2.6 & 3.0 & 5.0 & 1.0\end{bmatrix} \quad (4)$$

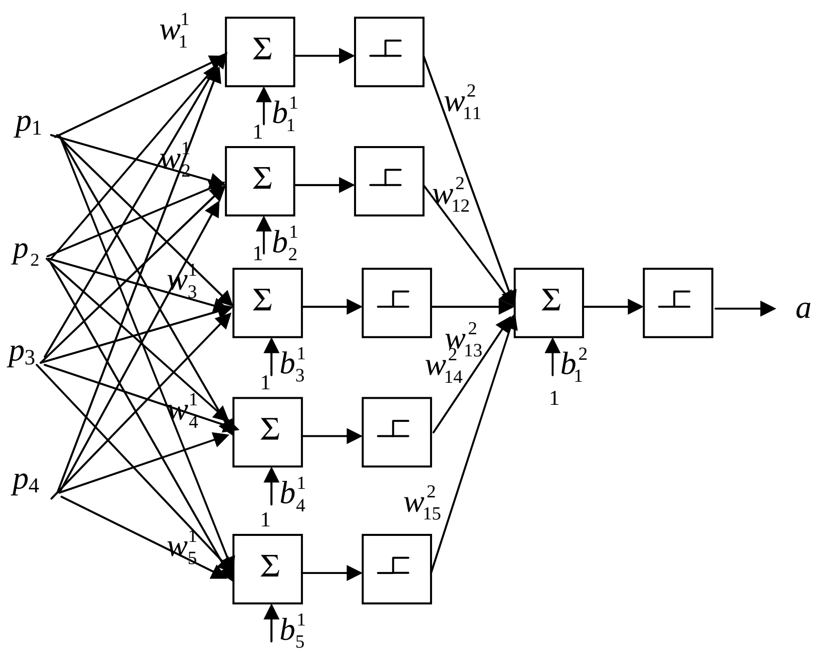
The minimum point is the stationary point of the quadratic function

$$x = \begin{bmatrix}a_1\\a_2\\a_3\\a_0\end{bmatrix} = R^{-1}h = \begin{bmatrix}8.6 & 8.0 & 16.0 & 2.6\\ 8.0 & 11.0 & 17.8 & 3.0\\ 16.0 & 17.8 & 35.0 & 5.0\\ 2.6 & 3.0 & 5.0 & 1.0\end{bmatrix}^{-1}\begin{bmatrix}341\\379\\693\\114\end{bmatrix} = \begin{bmatrix}15\\10\\5\\20\end{bmatrix} \quad (5)$$

Q.2 Design a network with appropriate parameters, which can classify class A occupying inside the polygon which has the corners at the coordinates (10, 0, 0, 0), (0, 12, 0, 0), (0, 0, 18, 0), (0, 0, 0, 15) and (0, 0, 0, 0) from class B which is outside the polygon. (20)

Solution

Multi-Layer Perceptron is selected. The first layer is used to create 5 decision boundaries. The second layer is used to AND all the decision boundaries.



For layer 1

w must point into the shape and it must be perpendicular to decision boundary and edges.

b is determined from equating **n** = 0; and solve for **b**

For the first decision boundary which has the corners at the coordinates (0, 12, 0, 0), (0, 0, 18, 0), (0, 0, 0, 15) and (0, 0, 0, 0),

$$\mathbf{w}_1^1 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (1)$$

$$b_1^1 = 0 \quad (2)$$

For the second decision boundary which has the corners at the coordinates (10, 0, 0, 0), (0, 0, 18, 0), (0, 0, 0, 15) and (0, 0, 0, 0),

$$\mathbf{w}_2^1 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} \quad (3)$$

$$b_2^1 = 0 \quad (4)$$

For the third decision boundary which has the corners at the coordinates (10, 0, 0, 0), (0, 12, 0, 0), (0, 0, 0, 15) and (0, 0, 0, 0),

$$\mathbf{w}_3^1 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \quad (5)$$

$$b_3^1 = 0 \quad (6)$$

For the fourth decision boundary which has the corners at the coordinates (10, 0, 0, 0), (0, 12, 0, 0), (0, 0, 18, 0), and (0, 0, 0, 0),

$$\mathbf{w}_4^1 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \quad (7)$$

$$b_4^1 = 0 \tag{8}$$

For the fifth decision boundary which has the corners at the coordinates (10, 0, 0, 0), (0, 12, 0, 0), (0, 0, 18, 0), and (0, 0, 0, 15),

$$\begin{bmatrix} w_{51}^1 & w_{52}^1 & w_{53}^1 & w_{54}^1 \end{bmatrix} \begin{bmatrix} 10 \\ -12 \\ 0 \\ 0 \end{bmatrix} = 10w_{51}^1 - 12w_{52}^1 = 0 \tag{9}$$

$$\begin{bmatrix} w_{51}^1 & w_{52}^1 & w_{53}^1 & w_{54}^1 \end{bmatrix} \begin{bmatrix} 10 \\ 0 \\ -18 \\ 0 \end{bmatrix} = 10w_{51}^1 - 18w_{53}^1 = 0 \tag{10}$$

$$\begin{bmatrix} w_{51}^1 & w_{52}^1 & w_{53}^1 & w_{54}^1 \end{bmatrix} \begin{bmatrix} 10 \\ 0 \\ 0 \\ -15 \end{bmatrix} = 10w_{51}^1 - 15w_{54}^1 = 0 \tag{11}$$

$$\mathbf{w}_5^1 = \begin{bmatrix} -1 \\ -1/12 \\ -1/18 \\ -1/15 \end{bmatrix} \tag{12}$$

$$n_5^1 = w_{51}^1 p_1 + w_{52}^1 p_2 + w_{53}^1 p_3 + w_{54}^1 p_4 + b_5^1 = 0 \tag{13}$$

at $p_1 = 10$, $p_2 = 0$, $p_3 = 0$and $p_4 = 0$;

$$-1(10) + 0 + 0 + 0 + b_5^1 = 0; b_5^1 = 10 \tag{14}$$

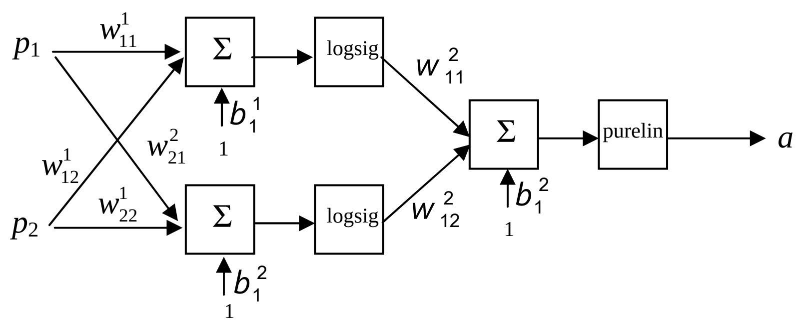
For layer 2

Since this is AND layer, if we select

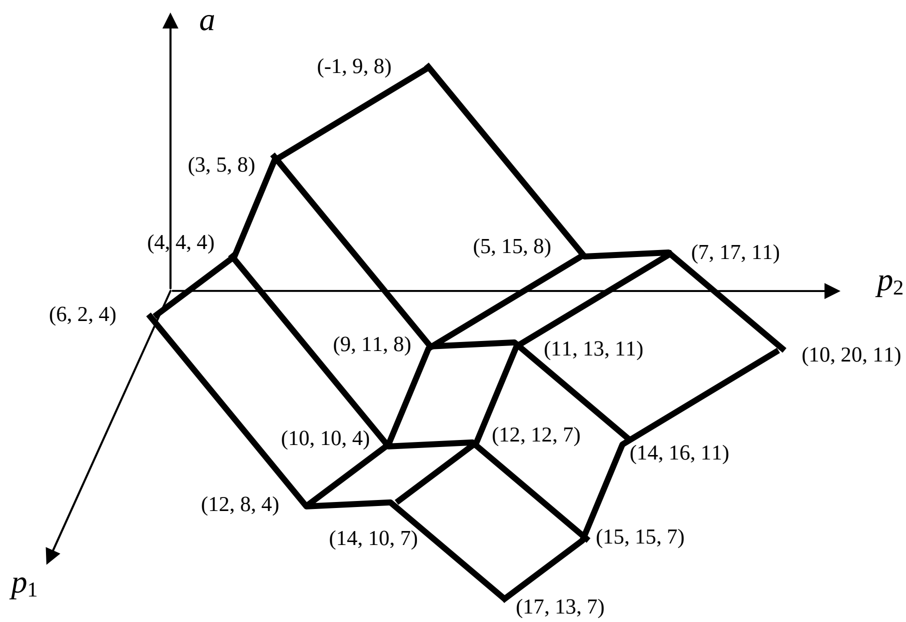
$$w_{11}^2 = w_{12}^2 = w_{13}^2 = w_{14}^2 = w_{15}^2 = 1 \tag{15}$$

$$b_1^2 = -4.5 \tag{16}$$

Q.3 If a 2-2-1 MLP network as shown in the first figure is used to approximate the input-output relation as shown in the second figure, determine one set of the possible weights and biases of the network after convergence. (20)



2-2-1 Network



Input-Output Relation

<u>Solution</u>

The function can be approximated as the superposition of two 2-D log-sigmoid function, the first log-sigmoid has its center following a line $p_2 - p_1 - 1 = 0$ with the output span of 4, the second log-sigmoid has its center following a line $p_2 + p_1 - 22 = 0$ with the output span of 3.

Weight and bias of the first log-sigmoid function, which has its center following a line $p_2 - p_1 - 1 = 0$, are determined from

$$n_1^1 = w_{11}^1 p_1 + w_{12}^1 p_2 + b_1^1 = 0 \quad (1)$$

Thus

$$w_{11}^1 = -1, w_{12}^1 = 1, b_1^1 = -1 \quad \underline{(2)}$$

Weight and bias of the second log-sigmoid function, which has its center following a line $p_2 + p_1 - 22 = 0$, are determined from

$$n_2^1 = w_{21}^1 p_1 + w_{22}^1 p_2 + b_2^1 = 0 \quad (3)$$

Thus

$$w_{21}^1 = 1, w_{22}^1 = 1, b_2^1 = -22 \quad \underline{(4)}$$

Since output span of the first log-sigmoid function is 4, thus

$$w_{11}^2 = 4 \quad \underline{(5)}$$

Since output span of the second log-sigmoid function is 3, thus

$$w_{12}^2 = 3 \quad \underline{(6)}$$

Since the plot starts from the output magnitude of 4, thus

$$b_1^2 = 4 \quad \underline{(7)}$$

Q.4 Consider the below training set when the input is RGB intensity of light and the target is frequency of sound.

$$\left\{p_1 = \begin{bmatrix} 255 \\ 0 \\ 0 \end{bmatrix}, t_1 = [3600]\right\}, \left\{p_2 = \begin{bmatrix} 0 \\ 255 \\ 0 \end{bmatrix}, t_2 = [1200]\right\}, \left\{p_3 = \begin{bmatrix} 0 \\ 0 \\ 255 \end{bmatrix}, t_1 = [2400]\right\}$$

(a) Design the network and its parameters which can convert light to sound of the training set. (15)

(b) Test the result by presenting the input and determine the output from all three inputs in the training set. Then determine the output frequency when the input is $p = \begin{bmatrix} 0 \\ 100 \\ 100 \end{bmatrix}$. (5)

<u>Solution</u>

Linear associator with pseudoinverse rule is applied.

By Pseudoinverse rule,

$$\mathbf{W} = \mathbf{T}\mathbf{P}^{+} \quad (1)$$

Where

$$\mathbf{P}^{+} = (\mathbf{P}^{T}\,\mathbf{P})^{-1}\mathbf{P}^{T} \quad (2)$$

$$\mathbf{T} = \begin{bmatrix} 3600 & 1200 & 2400 \end{bmatrix} \quad (3)$$

$$\mathbf{P} = \begin{bmatrix} 255 & 0 & 0 \\ 0 & 255 & 0 \\ 0 & 0 & 255 \end{bmatrix} \quad (4)$$

$$\mathbf{W} = \mathbf{T}\mathbf{P}^{+} = \begin{bmatrix} 14.1176 & 4.7059 & 9.4118 \end{bmatrix} \quad \underline{(5)}$$

(b)

When apply the first input,

$$\mathbf{a} = \mathbf{W}\mathbf{p}_1 = \begin{bmatrix} 14.1176 & 4.7059 & 9.4118 \end{bmatrix} \begin{bmatrix} 255 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 3600 \end{bmatrix} \quad \underline{(6)}$$

When apply the second input,

$$\mathbf{a} = \mathbf{W}\mathbf{p}_2 = \begin{bmatrix} 14.1176 & 4.7059 & 9.4118 \end{bmatrix} \begin{bmatrix} 0 \\ 255 \\ 0 \end{bmatrix} = \begin{bmatrix} 1200 \end{bmatrix} \quad \underline{(7)}$$

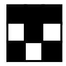
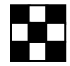
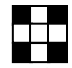
When apply the third input,

$$\mathbf{a} = \mathbf{W}\mathbf{p}_3 = \begin{bmatrix} 14.1176 & 4.7059 & 9.4118 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 255 \end{bmatrix} = \begin{bmatrix} 2400 \end{bmatrix} \quad \underline{(8)}$$

When apply the input $p = \begin{bmatrix} 0 \\ 100 \\ 100 \end{bmatrix}$,

$$\mathbf{a} = \mathbf{W}\mathbf{p} = \begin{bmatrix} 14.1176 & 4.7059 & 9.4118 \end{bmatrix} \begin{bmatrix} 0 \\ 100 \\ 100 \end{bmatrix} = \begin{bmatrix} 1411.8 \end{bmatrix} \quad \underline{(9)}$$

Q.5 To see the influence of sequence of training in Adaptive Resonance Theory, train the network to classify the following input images from the sequence of p_1- p_2- p_3. Then compare the result of training using the sequence of p_3- p_2- p_1. Compare the weight matrices of both sequences.

p_1 p_2 p_3

Use the parameter $\zeta = 1$, $\rho = 0.82$, and number of categories (S^2) = 2. (20)

<u>Solution</u>

$$\mathbf{p}_1 = \begin{bmatrix}1\\1\\1\\0\\1\\0\\1\\0\\1\end{bmatrix}, \quad \mathbf{p}_2 = \begin{bmatrix}1\\0\\1\\0\\1\\0\\1\\0\\1\end{bmatrix}, \quad \mathbf{p}_3 = \begin{bmatrix}1\\0\\1\\0\\0\\0\\1\\0\\1\end{bmatrix} \qquad (1)$$

Each element in $\mathbf{W}^{1:2}$ is equal to $\dfrac{\zeta}{\zeta + S^1 - 1} = \dfrac{1}{1+9-1} = \dfrac{1}{9}$. (2)

Find initial weights

$$\mathbf{W}^{1:2} = \begin{bmatrix}1/9 & 1/9 & 1/9 & 1/9 & 1/9 & 1/9 & 1/9 & 1/9 & 1/9\\ 1/9 & 1/9 & 1/9 & 1/9 & 1/9 & 1/9 & 1/9 & 1/9 & 1/9\end{bmatrix} \qquad (3)$$

$$\mathbf{W}^{2:1} = \begin{bmatrix}1 & 1\\1 & 1\\1 & 1\\1 & 1\\1 & 1\\1 & 1\\1 & 1\\1 & 1\\1 & 1\end{bmatrix} \qquad (4)$$

Train following p_1- p_2- p_3

<u>Train the network with $\mathbf{p}_1$</u>.

1. Compute the Layer 1 response:

$$\mathbf{a}^1 = \mathbf{p}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \\ 1 \\ 0 \\ 1 \\ 0 \\ 1 \end{bmatrix} \tag{5}$$

2. Compute the input to Layer 2:

$$\mathbf{W}^{1:2}\mathbf{a}^1 = \begin{bmatrix} 6/9 \\ 6/9 \end{bmatrix} \tag{6}$$

Since all neurons have the same input, pick the first neuron as the winner.

$$\mathbf{a}^2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \tag{7}$$

3. Compute the L2-L1 expectation:

$$\mathbf{W}^{2:1}\mathbf{a}^2 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} = \mathbf{w}_1^{2:1} \tag{8}$$

4. Adjust the Layer1 output to include the L2-L1 expectation:

$$\mathbf{a}^1 = \mathbf{p}_1 \cap \mathbf{w}_1^{2:1} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \\ 1 \\ 0 \\ 1 \\ 0 \\ 1 \end{bmatrix} \tag{9}$$

5. The Orienting Subsystem determines the degree of match between the expectation and the input pattern.

$$\frac{\|\mathbf{a}^1\|^2}{\|\mathbf{p}_1\|^2} = \frac{6}{6} = 1 > \rho = 0.82 \text{ therefore } a^0 = 0 \text{ (no reset).} \tag{10}$$

6. Since $a^0 = 0$, continue

7. Resonance has occurred, therefore update row 1 of $\mathbf{W}^{1:2}$

$$ {}_{1}\mathbf{w}^{1:2} = \frac{\zeta \mathbf{a}^1}{\zeta + \|\mathbf{a}^1\|^2 - 1} = \begin{bmatrix} 1/6 \\ 1/6 \\ 1/6 \\ 0 \\ 1/6 \\ 0 \\ 1/6 \\ 0 \\ 1/6 \end{bmatrix}, $$

$$ \mathbf{W}^{1:2} = \begin{bmatrix} 1/6 & 1/6 & 1/6 & 0 & 1/6 & 0 & 1/6 & 0 & 1/6 \\ 1/9 & 1/9 & 1/9 & 1/9 & 1/9 & 1/9 & 1/9 & 1/9 & 1/9 \end{bmatrix} \quad (11) $$

8. Update column 1 of $\mathbf{W}^{2:1}$:

$$ \mathbf{w}_1^{2:1} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \\ 1 \\ 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}, \quad \mathbf{W}^{2:1} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 1 & 1 \\ 0 & 1 \\ 1 & 1 \\ 0 & 1 \\ 1 & 1 \\ 0 & 1 \\ 1 & 1 \end{bmatrix} \quad (12) $$

<u>Train the network with $\mathbf{p}_2$.</u>

1. Compute the Layer 1 response:

$$ \mathbf{a}^1 = \mathbf{p}_2 = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \\ 1 \\ 0 \\ 1 \\ 0 \\ 1 \end{bmatrix} \quad (13) $$

2. Compute the input to Layer 2:

$$ \mathbf{W}^{1:2}\mathbf{a}^1 = \begin{bmatrix} 5/6 \\ 5/9 \end{bmatrix} \quad (14) $$

The winner is the first neuron.

$$ \mathbf{a}^2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad (15) $$

3. Compute the L2-L1 expectation:

$$\mathbf{W}^{2:1}\mathbf{a}^2 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \\ 1 \\ 0 \\ 1 \\ 0 \\ 1 \end{bmatrix} = \mathbf{w}_1^{2:1} \tag{16}$$

4. Adjust the Layer1 output to include the L2-L1 expectation:

$$\mathbf{a}^1 = \mathbf{p}_2 \cap \mathbf{w}_1^{2:1} = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \\ 1 \\ 0 \\ 1 \\ 0 \\ 1 \end{bmatrix} \tag{17}$$

5. The Orienting Subsystem determines the degree of match between the expectation and the input pattern.

$$\frac{\left\|\mathbf{a}^1\right\|^2}{\left\|\mathbf{p}_2\right\|^2} = \frac{5}{5} = 1 > \rho = 0.82 \text{ therefore } a^0 = 0 \text{ (no reset).} \tag{18}$$

6. Since $a^0 = 0$, continue

7. Resonance has occurred, therefore update row 1 of $\mathbf{W}^{1:2}$

$${}_1\mathbf{w}^{1:2} = \frac{\zeta\mathbf{a}^1}{\zeta + \left\|\mathbf{a}^1\right\|^2 - 1} = \begin{bmatrix} 1/5 \\ 0 \\ 1/5 \\ 0 \\ 1/5 \\ 0 \\ 1/5 \\ 0 \\ 1/5 \end{bmatrix},$$

$$\mathbf{W}^{1:2} = \begin{bmatrix} 1/5 & 0 & 1/5 & 0 & 1/5 & 0 & 1/5 & 0 & 1/5 \\ 1/9 & 1/9 & 1/9 & 1/9 & 1/9 & 1/9 & 1/9 & 1/9 & 1/9 \end{bmatrix} \tag{19}$$

8. Update column 1 of $\mathbf{W}^{2:1}$:

$$\mathbf{w}_1^{2:1} = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \\ 1 \\ 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}, \quad \mathbf{W}^{2:1} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 1 & 1 \\ 0 & 1 \\ 1 & 1 \\ 0 & 1 \\ 1 & 1 \\ 0 & 1 \\ 1 & 1 \end{bmatrix} \tag{20}$$

Train the network with $\mathbf{p}_3$.

1. Compute the Layer 1 response:

$$\mathbf{a}^1 = \mathbf{p}_3 = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 1 \end{bmatrix} \tag{13}$$

2. Compute the input to Layer 2:

$$\mathbf{W}^{1:2}\mathbf{a}^1 = \begin{bmatrix} 4/5 \\ 4/9 \end{bmatrix} \tag{14}$$

The winner is the first neuron.

$$\mathbf{a}^2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \tag{15}$$

3. Compute the L2-L1 expectation:

$$\mathbf{W}^{2:1}\mathbf{a}^2 = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \\ 1 \\ 0 \\ 1 \\ 0 \\ 1 \end{bmatrix} = \mathbf{w}_1^{2:1} \tag{16}$$

4. Adjust the Layer1 output to include the L2-L1 expectation:

$$\mathbf{a}^1 = \mathbf{p}_3 \cap \mathbf{w}_1^{2:1} = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 1 \end{bmatrix} \tag{17}$$

5. The Orienting Subsystem determines the degree of match between the expectation and the input pattern.

$$\frac{\|\mathbf{a}^1\|^2}{\|\mathbf{p}_2\|^2} = \frac{4}{4} = 1 > \rho = 0.82 \text{ therefore } a^0 = 0 \text{ (no reset).} \tag{18}$$

6. Since $a^0 = 0$, continue

7. Resonance has occurred, therefore update row 1 of $\mathbf{W}^{1:2}$

$$ {}_1\mathbf{w}^{1:2} = \frac{\zeta \mathbf{a}^1}{\zeta + \|\mathbf{a}^1\|^2 - 1} = \begin{bmatrix} 1/4 \\ 0 \\ 1/4 \\ 0 \\ 0 \\ 0 \\ 1/4 \\ 0 \\ 1/4 \end{bmatrix},$$

$$\mathbf{W}^{1:2} = \begin{bmatrix} 1/4 & 0 & 1/4 & 0 & 0 & 0 & 1/4 & 0 & 1/4 \\ 1/9 & 1/9 & 1/9 & 1/9 & 1/9 & 1/9 & 1/9 & 1/9 & 1/9 \end{bmatrix} \tag{19}$$

8. Update column 1 of $\mathbf{W}^{2:1}$:

$$\mathbf{w}_1^{2:1} = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}, \quad \mathbf{W}^{2:1} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 1 & 1 \\ 0 & 1 \\ 0 & 1 \\ 0 & 1 \\ 1 & 1 \\ 0 & 1 \\ 1 & 1 \end{bmatrix} \tag{20}$$

Train following p_3- p_2- p_1

<u>Train the network with $\mathbf{p}_3$.</u>

1. Compute the Layer 1 response:

$$\mathbf{a}^1 = \mathbf{p}_3 = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 1 \end{bmatrix} \tag{21}$$

2. Compute the input to Layer 2:

$$\mathbf{W}^{1:2}\mathbf{a}^1 = \begin{bmatrix} 4/9 \\ 4/9 \end{bmatrix} \tag{22}$$

Since all neurons have the same input, pick the first neuron as the winner.

$$\mathbf{a}^2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \tag{23}$$

3. Compute the L2-L1 expectation:

$$\mathbf{W}^{2:1}\mathbf{a}^2 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} = \mathbf{w}_1^{2:1} \tag{24}$$

4. Adjust the Layer1 output to include the L2-L1 expectation:

$$\mathbf{a}^1 = \mathbf{p}_3 \cap \mathbf{w}_1^{2:1} = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 1 \end{bmatrix} \tag{25}$$

5. The Orienting Subsystem determines the degree of match between the expectation and the input pattern.

$$\frac{\left\|\mathbf{a}^1\right\|^2}{\left\|\mathbf{p}_1\right\|^2} = \frac{4}{4} = 1 > \rho = 0.82 \text{ therefore } a^0 = 0 \text{ (no reset).} \tag{26}$$

6. Since $a^0 = 0$, continue

7. Resonance has occurred, therefore update row 1 of $\mathbf{W}^{1:2}$

$$
{}_1\mathbf{w}^{1:2} = \frac{\zeta \mathbf{a}^1}{\zeta + \|\mathbf{a}^1\|^2 - 1} = \begin{bmatrix} 1/4 \\ 0 \\ 1/4 \\ 0 \\ 0 \\ 0 \\ 1/4 \\ 0 \\ 1/4 \end{bmatrix},
$$

$$
\mathbf{W}^{1:2} = \begin{bmatrix} 1/4 & 0 & 1/4 & 0 & 0 & 0 & 1/4 & 0 & 1/4 \\ 1/9 & 1/9 & 1/9 & 1/9 & 1/9 & 1/9 & 1/9 & 1/9 & 1/9 \end{bmatrix} \tag{27}
$$

8. Update column 1 of $\mathbf{W}^{2:1}$:

$$
\mathbf{w}_1^{2:1} = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}, \quad \mathbf{W}^{2:1} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 1 & 1 \\ 0 & 1 \\ 0 & 1 \\ 0 & 1 \\ 1 & 1 \\ 0 & 1 \\ 1 & 1 \end{bmatrix} \tag{29}
$$

<u>Train the network with $\mathbf{p}_2$</u>.

1. Compute the Layer 1 response:

$$
\mathbf{a}^1 = \mathbf{p}_2 = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \\ 1 \\ 0 \\ 1 \\ 0 \\ 1 \end{bmatrix} \tag{30}
$$

2. Compute the input to Layer 2:

$$
\mathbf{W}^{1:2}\mathbf{a}^1 = \begin{bmatrix} 1 \\ 5/9 \end{bmatrix} \tag{31}
$$

The winner is the first neuron.

$$
\mathbf{a}^2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \tag{32}
$$

3. Compute the L2-L1 expectation:

$$\mathbf{W}^{2:1}\mathbf{a}^2 = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 1 \end{bmatrix} = \mathbf{w}_1^{2:1} \tag{33}$$

4. Adjust the Layer1 output to include the L2-L1 expectation:

$$\mathbf{a}^1 = \mathbf{p}_2 \cap \mathbf{w}_1^{2:1} = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 1 \end{bmatrix} \tag{34}$$

5. The Orienting Subsystem determines the degree of match between the expectation and the input pattern.

$$\frac{\left\|\mathbf{a}^1\right\|^2}{\left\|\mathbf{p}_2\right\|^2} = \frac{4}{5} = 0.8 < \rho = 0.82 \text{ therefore } a^0 = 1 \text{ (reset is sent).} \tag{35}$$

6. Since $a^0 = 1$, disable the winner.

$$\mathbf{a}^2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \tag{36}$$

7. Compute the L2-L1 expectation:

$$\mathbf{W}^{2:1}\mathbf{a}^2 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} = \mathbf{w}_2^{2:1} \tag{37}$$

8. Adjust the Layer1 output to include the L2-L1 expectation:

$$\mathbf{a}^1 = \mathbf{p}_2 \cap \mathbf{w}_1^{2:1} = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \\ 1 \\ 0 \\ 1 \\ 0 \\ 1 \end{bmatrix} \quad (38)$$

9. The Orienting Subsystem determines the degree of match between the expectation and the input pattern.

$$\frac{\|\mathbf{a}^1\|^2}{\|\mathbf{p}_2\|^2} = \frac{5}{5} = 1 > \rho = 0.82 \text{ therefore } a^0 = 0 \text{ (no reset).} \quad (39)$$

10. Since $a^0 = 0$, continue.

11. Resonance has occurred, therefore update row 2 of $\mathbf{W}^{1:2}$

$$_2\mathbf{w}^{1:2} = \frac{\zeta \mathbf{a}^1}{\zeta + \|\mathbf{a}^1\|^2 - 1} = \begin{bmatrix} 1/5 \\ 0 \\ 1/5 \\ 0 \\ 1/5 \\ 0 \\ 1/5 \\ 0 \\ 1/5 \end{bmatrix},$$

$$\mathbf{W}^{1:2} = \begin{bmatrix} 1/4 & 0 & 1/4 & 0 & 0 & 0 & 1/4 & 0 & 1/4 \\ 1/5 & 0 & 1/5 & 0 & 1/5 & 0 & 1/5 & 0 & 1/5 \end{bmatrix} \quad (40)$$

12. Update column 1 of $\mathbf{W}^{2:1}$:

$$\mathbf{w}_2^{2:1} = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \\ 1 \\ 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}, \quad \mathbf{W}^{2:1} = \begin{bmatrix} 1 & 1 \\ 0 & 0 \\ 1 & 1 \\ 0 & 0 \\ 0 & 1 \\ 0 & 0 \\ 1 & 1 \\ 0 & 0 \\ 1 & 1 \end{bmatrix} \quad (41)$$

<u>Train the network with $\mathbf{p}_1$</u>.

1. Compute the Layer 1 response:

$$\mathbf{a}^1 = \mathbf{p}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \\ 1 \\ 0 \\ 1 \\ 0 \\ 1 \end{bmatrix} \tag{42}$$

2. Compute the input to Layer 2:

$$\mathbf{W}^{1:2}\mathbf{a}^1 = \begin{bmatrix} 4/4 \\ 5/5 \end{bmatrix} \tag{43}$$

The winner is the first neuron.

$$\mathbf{a}^2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \tag{44}$$

3. Compute the L2-L1 expectation:

$$\mathbf{W}^{2:1}\mathbf{a}^2 = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 1 \end{bmatrix} = \mathbf{w}_1^{2:1} \tag{45}$$

4. Adjust the Layer1 output to include the L2-L1 expectation:

$$\mathbf{a}^1 = \mathbf{p}_1 \cap \mathbf{w}_1^{2:1} = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 1 \end{bmatrix} \tag{46}$$

5. The Orienting Subsystem determines the degree of match between the expectation and the input pattern.

$$\frac{\left\|\mathbf{a}^1\right\|^2}{\left\|\mathbf{p}_2\right\|^2} = \frac{4}{6} = 0.67 < \rho = 0.82 \text{ therefore } a^0 = 1 \text{ (reset is sent).} \tag{47}$$

6. Since $a^0 = 1$, disable the winner.

$$\mathbf{a}^2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \tag{48}$$

7. Compute the L2-L1 expectation:

$$\mathbf{W}^{2:1}\mathbf{a}^2 = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \\ 1 \\ 0 \\ 1 \\ 0 \\ 1 \end{bmatrix} = \mathbf{w}_2^{2:1} \tag{49}$$

8. Adjust the Layer1 output to include the L2-L1 expectation:

$$\mathbf{a}^1 = \mathbf{p}_1 \cap \mathbf{w}_1^{2:1} = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \\ 1 \\ 0 \\ 1 \\ 0 \\ 1 \end{bmatrix} \tag{50}$$

9. The Orienting Subsystem determines the degree of match between the expectation and the input pattern.

$$\frac{\|\mathbf{a}^1\|^2}{\|\mathbf{p}_2\|^2} = \frac{5}{6} = 0.83 > \rho = 0.82 \text{ therefore } a^0 = 0 \text{ (no reset).} \tag{51}$$

10. Since $a^0 = 0$, continue.

11. Resonance has occurred, therefore update row 2 of $\mathbf{W}^{1:2}$

$$_2\mathbf{w}^{1:2} = \frac{\zeta \mathbf{a}^1}{\zeta + \|\mathbf{a}^1\|^2 - 1} = \begin{bmatrix} 1/5 \\ 0 \\ 1/5 \\ 0 \\ 1/5 \\ 0 \\ 1/5 \\ 0 \\ 1/5 \end{bmatrix},$$

$$\mathbf{W}^{1:2} = \begin{bmatrix} 1/4 & 0 & 1/4 & 0 & 0 & 0 & 1/4 & 0 & 1/4 \\ 1/5 & 0 & 1/5 & 0 & 1/5 & 0 & 1/5 & 0 & 1/5 \end{bmatrix} \tag{52}$$

12. Update column 1 of $\mathbf{W}^{2:1}$:

$$\mathbf{w}_2^{2:1} = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \\ 1 \\ 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}, \quad \mathbf{W}^{2:1} = \begin{bmatrix} 1 & 1 \\ 0 & 0 \\ 1 & 1 \\ 0 & 0 \\ 0 & 1 \\ 0 & 0 \\ 1 & 1 \\ 0 & 0 \\ 1 & 1 \end{bmatrix} \tag{53}$$

The weight matrices of training following p_1- p_2- p_3

$$\mathbf{W}^{1:2} = \begin{bmatrix} 1/4 & 0 & 1/4 & 0 & 0 & 0 & 1/4 & 0 & 1/4 \\ 1/9 & 1/9 & 1/9 & 1/9 & 1/9 & 1/9 & 1/9 & 1/9 & 1/9 \end{bmatrix} \tag{54}$$

$$\mathbf{W}^{2:1} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 1 & 1 \\ 0 & 1 \\ 0 & 1 \\ 0 & 1 \\ 1 & 1 \\ 0 & 1 \\ 1 & 1 \end{bmatrix} \tag{55}$$

The weight matrices of training following p_3- p_2- p_1

$$\mathbf{W}^{1:2} = \begin{bmatrix} 1/4 & 0 & 1/4 & 0 & 0 & 0 & 1/4 & 0 & 1/4 \\ 1/5 & 0 & 1/5 & 0 & 1/5 & 0 & 1/5 & 0 & 1/5 \end{bmatrix} \tag{56}$$

$$\mathbf{W}^{2:1} = \begin{bmatrix} 1 & 1 \\ 0 & 0 \\ 1 & 1 \\ 0 & 0 \\ 0 & 1 \\ 0 & 0 \\ 1 & 1 \\ 0 & 0 \\ 1 & 1 \end{bmatrix} \tag{57}$$