Midsem Examination AI and Neuro-Fuzzy Theory AT74.05 March 6, 2015

Time: 9:00-11:00 h. **Open Book**

Marks: 100

Attempt all questions.

Q.1 ADALINE network is used to determine the parameters; $y(0)$ and a, of free response, $y(t) = y(0)e^{at}$, of a first-order system, $\dot{y} = ay$. The free response data obtained from system identification is obtained as shown in the below table.

$y(t)$	67.67	9.16	1.24	0.17	0.02
t	1	2	3	4	5

(a) Apply logarithmic function to linearize the free response, then draw the ADALINE network that can be used to determine the parameters. (4)

(b) If all the data are presented equally, determine the parameters by LMS algorithm. (16)

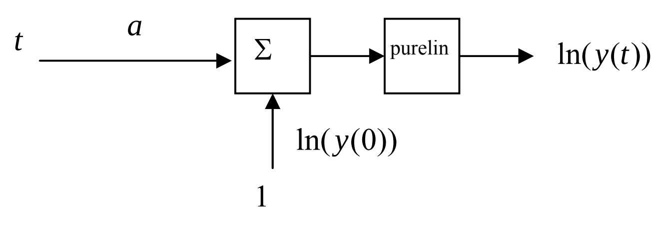
Solution

(a)

$$y(t) = y(0)e^{at}$$

$$\ln(y(t)) = \ln(y(0)) + at$$

$\ln(y(t))$	4.21	2.21	0.21	-1.79	-3.79
t	1	2	3	4	5



(b)

$$F(x) = E[t^2] - 2x^T E[tz] + x^T E[zz^T]x = c - 2x^T h + x^T Rx \quad (1)$$

$$c = \frac{1}{5}\left(4.21^2 + 2.21^2 + 0.21^2 + (-1.79)^2 + (-3.79)^2\right) = 8.04 \quad (2)$$

$$h = \frac{1}{5}\left(4.21\begin{bmatrix}1\\1\end{bmatrix} + 2.21\begin{bmatrix}2\\1\end{bmatrix} + 0.21\begin{bmatrix}3\\1\end{bmatrix} - 1.79\begin{bmatrix}4\\1\end{bmatrix} - 3.79\begin{bmatrix}5\\1\end{bmatrix}\right) = \begin{bmatrix}-3.37\\0.21\end{bmatrix} \quad (3)$$

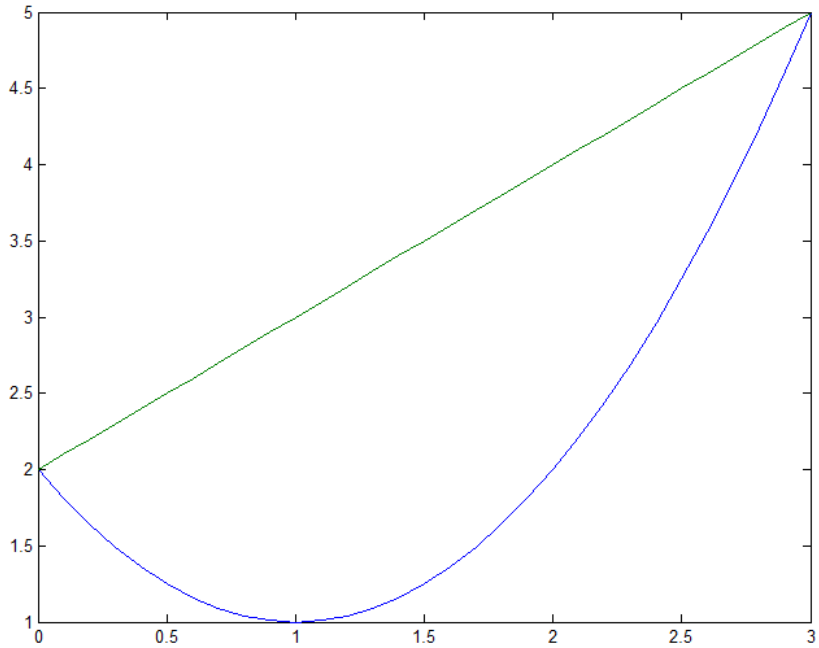
$$R = \frac{1}{5}\left(\begin{bmatrix}1\\1\end{bmatrix}\begin{bmatrix}1\\1\end{bmatrix}^T + \begin{bmatrix}2\\1\end{bmatrix}\begin{bmatrix}2\\1\end{bmatrix}^T + \begin{bmatrix}3\\1\end{bmatrix}\begin{bmatrix}3\\1\end{bmatrix}^T + \begin{bmatrix}4\\1\end{bmatrix}\begin{bmatrix}4\\1\end{bmatrix}^T + \begin{bmatrix}5\\1\end{bmatrix}\begin{bmatrix}5\\1\end{bmatrix}^T\right) = \begin{bmatrix}11 & 3\\3 & 1\end{bmatrix} \quad (4)$$

The minimum point is the stationary point of the quadratic function

$$x = \begin{bmatrix}a\\ \ln(y(0))\end{bmatrix} = R^{-1}h = \begin{bmatrix}11 & 3\\3 & 1\end{bmatrix}^{-1}\begin{bmatrix}-3.37\\0.21\end{bmatrix} = \begin{bmatrix}-2\\6.21\end{bmatrix} \quad (5)$$

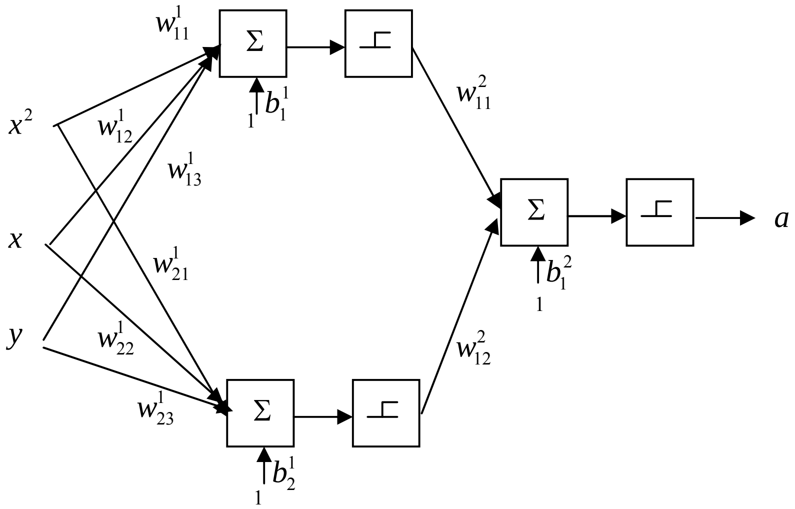
$$\begin{bmatrix}a\\y(0)\end{bmatrix} = \begin{bmatrix}-2\\497.70\end{bmatrix} \quad (6)$$

Q.2 Design a new neural network with appropriate parameters, which can classify class A (output of 1) occupying the area inside the intersection of the graph, $y = x^2 - 2x + 2$, and the graph, $y = x + 2$, from class B (output of 0) which is outside the area as shown in the below figure. The inputs of the network are x, x^2, and y. (20)



Solution

The first layer is used to create 2 decision boundaries. The second layer is used to AND all the decision boundaries.



For layer 1

The first neuron in the first layer is used to create quadratic decision boundary whose output of the upper inside area is group 1.

$$\mathbf{w}_1^1 = [-1 \quad 2 \quad 1] \tag{1}$$

$$b_1^1 = [-2] \tag{2}$$

The second neuron in the first layer is used to create linear decision boundary whose output of the lower area is group 1.

$$\mathbf{w}_2^1 = [0 \quad 1 \quad -1] \tag{3}$$

$$b_2^1 = [2] \tag{4}$$

For layer 2

Since this is AND layer, if we select

$$w_{11}^2 = w_{12}^2 = 1 \tag{5}$$

$$b_1^2 = -1.5 \tag{6}$$

Q.3 Grossberg network is applied to recognize 6 colors of red, green, blue, yellow, cyan, and magenta. The training set consists of $\left\{\begin{bmatrix}255\\0\\0\end{bmatrix}, red\right\}, \left\{\begin{bmatrix}0\\255\\0\end{bmatrix}, green\right\}, \left\{\begin{bmatrix}0\\0\\255\end{bmatrix}, blue\right\},$

$\left\{\begin{bmatrix}255\\255\\0\end{bmatrix}, yellow\right\}, \left\{\begin{bmatrix}0\\255\\255\end{bmatrix}, cyan\right\}, \left\{\begin{bmatrix}255\\0\\255\end{bmatrix}, magenta\right\}.$

(a) Determine the weights of each neuron at the steady-state when ${}^{+}b^1 = 1,\ {}^{-}b^1 = 0.$ (10)

(b) Determine the results of color recognition when the input vectors of $\left\{\begin{bmatrix}255\\192\\203\end{bmatrix}, pink\right\}$ is presented to the network. (10)

Solution

(a) The weights, w_i^2, of each neuron at the steady-state will converge to $n_i^1 = \left(\frac{{}^{+}b^1 P}{1+P}\right)\overline{p_i}$.

$$w_{red}^2 = n_{red}^1 = \left(\frac{1\times 255}{1+255}\right)\begin{bmatrix}\frac{255}{255}\\0\\0\end{bmatrix} = \begin{bmatrix}\frac{255}{256}\\0\\0\end{bmatrix} \tag{1}$$

$$w_{greeen}^2 = n_{green}^1 = \left(\frac{1\times 255}{1+255}\right)\begin{bmatrix}0\\\frac{255}{255}\\0\end{bmatrix} = \begin{bmatrix}0\\\frac{255}{256}\\0\end{bmatrix} \tag{2}$$

$$w_{blue}^2 = n_{blue}^1 = \left(\frac{1\times 255}{1+255}\right)\begin{bmatrix}0\\0\\\frac{255}{255}\end{bmatrix} = \begin{bmatrix}0\\0\\\frac{255}{256}\end{bmatrix} \tag{3}$$

$$w_{yellow}^2 = n_{yellow}^1 = \left(\frac{1\times 510}{1+510}\right)\begin{bmatrix}\frac{255}{510}\\\frac{255}{510}\\0\end{bmatrix} = \begin{bmatrix}\frac{255}{511}\\\frac{255}{511}\\0\end{bmatrix} \tag{4}$$

$$w_{cyan}^2 = n_{cyan}^1 = \left(\frac{1\times 510}{1+510}\right)\begin{bmatrix}0\\\frac{255}{510}\\\frac{255}{510}\end{bmatrix} = \begin{bmatrix}0\\\frac{255}{511}\\\frac{255}{511}\end{bmatrix} \tag{5}$$

$$w_{magenta}^2 = n_{magenta}^1 = \left(\frac{1\times 510}{1+510}\right)\begin{bmatrix}\frac{255}{510}\\0\\\frac{255}{510}\end{bmatrix} = \begin{bmatrix}\frac{255}{511}\\0\\\frac{255}{511}\end{bmatrix} \tag{6}$$

(b)

When $\left\{\begin{bmatrix}255\\192\\203\end{bmatrix}, pink\right\}$ is applied.

$$a_{pink}^{1}=\left(\frac{1\times650}{1+650}\right)\begin{bmatrix}\frac{255}{650}\\\frac{192}{650}\\\frac{203}{650}\end{bmatrix}=\begin{bmatrix}\frac{255}{651}\\\frac{192}{651}\\\frac{203}{651}\end{bmatrix} \tag{7}$$

$$w_{red}^{2}\cdot a_{pink}^{1}=\begin{bmatrix}\frac{255}{256}\\0\\0\end{bmatrix}\cdot\begin{bmatrix}\frac{255}{651}\\\frac{192}{651}\\\frac{203}{651}\end{bmatrix}=0.39 \tag{8}$$

$$w_{greeen}^{2}\cdot a_{pink}^{1}=\begin{bmatrix}0\\\frac{255}{256}\\0\end{bmatrix}\cdot\begin{bmatrix}\frac{255}{651}\\\frac{192}{651}\\\frac{203}{651}\end{bmatrix}=0.29 \tag{9}$$

$$w_{blue}^{2}\cdot a_{pink}^{1}=\begin{bmatrix}0\\0\\\frac{255}{256}\end{bmatrix}\cdot\begin{bmatrix}\frac{255}{651}\\\frac{192}{651}\\\frac{203}{651}\end{bmatrix}=0.31 \tag{10}$$

$$w_{yellow}^{2}\cdot a_{pink}^{1}=\begin{bmatrix}\frac{255}{511}\\\frac{255}{511}\\0\end{bmatrix}\cdot\begin{bmatrix}\frac{255}{651}\\\frac{192}{651}\\\frac{203}{651}\end{bmatrix}=0.34 \tag{11}$$

$$w_{cyan}^{2}\cdot a_{pink}^{1}=\begin{bmatrix}0\\\frac{255}{511}\\\frac{255}{511}\end{bmatrix}\cdot\begin{bmatrix}\frac{255}{651}\\\frac{192}{651}\\\frac{203}{651}\end{bmatrix}=0.30 \tag{12}$$

$$w_{magenta}^{2}\cdot a_{pink}^{1}=\begin{bmatrix}\frac{255}{511}\\0\\\frac{255}{511}\end{bmatrix}\cdot\begin{bmatrix}\frac{255}{651}\\\frac{192}{651}\\\frac{203}{651}\end{bmatrix}=0.35 \tag{13}$$

Since the correlation of pink color and red color is the highest, then the input pink color is recognized as red color.

Q.4 Pseudo-inverse rule for linear associator is applicable when the number of rows in the input vector is not less than the number of data in the training set.

(a) Determine the weight matrix by pseudo-inverse rule when the training set consists of

$\left\{p_1 = \begin{bmatrix}1\\2\\3\end{bmatrix}, t_1 = [10]\right\}, \left\{p_2 = \begin{bmatrix}4\\5\\6\end{bmatrix}, t_2 = [20]\right\}$, determine the output when each input is presented. (10)

(b) Is it possible to determine the weight matrix by pseudo-inverse rule when the training set consists of

$\left\{p_1 = \begin{bmatrix}1\\2\\3\end{bmatrix}, t_1 = [10]\right\}, \left\{p_2 = \begin{bmatrix}4\\5\\6\end{bmatrix}, t_2 = [20]\right\}, \left\{p_3 = \begin{bmatrix}7\\8\\9\end{bmatrix}, t_1 = [30]\right\}, \left\{p_4 = \begin{bmatrix}10\\11\\12\end{bmatrix}, t_1 = [40]\right\}$? Explain the reason. (10)

<u>Solution</u>

By Pseudoinverse rule,

$$\mathbf{W} = \mathbf{TP}^{+} \quad (1)$$

Where

$$\mathbf{P}^{+} = (\mathbf{P}^{T}\mathbf{P})^{-1}\mathbf{P}^{T} \quad (2)$$

(a)

$$\mathbf{T} = \begin{bmatrix}10 & 20\end{bmatrix} \quad (3)$$

$$\mathbf{P} = \begin{bmatrix}1 & 4\\2 & 5\\3 & 6\end{bmatrix} \quad (4)$$

$$\mathbf{W} = \mathbf{TP}^{+} = \begin{bmatrix}-0.56 & 1.11 & 2.78\end{bmatrix} \quad \underline{(5)}$$

$$\mathbf{W}p_1 = \begin{bmatrix}-0.56 & 1.11 & 2.78\end{bmatrix}\begin{bmatrix}1\\2\\3\end{bmatrix} = 10 \quad \underline{(6)}$$

$$\mathbf{W}p_1 = \begin{bmatrix}-0.56 & 1.11 & 2.78\end{bmatrix}\begin{bmatrix}4\\5\\6\end{bmatrix} = 20 \quad \underline{(7)}$$

(b)

$$\mathbf{T} = \begin{bmatrix}10 & 20 & 30 & 40\end{bmatrix} \quad (8)$$

$$\mathbf{P} = \begin{bmatrix} 1 & 4 & 7 & 10 \\ 2 & 5 & 8 & 11 \\ 3 & 6 & 9 & 12 \end{bmatrix} \tag{9}$$

$$\left|\mathbf{P}^{\mathrm{T}}\mathbf{P}\right| = \begin{vmatrix} 14 & 32 & 50 & 68 \\ 32 & 77 & 122 & 167 \\ 50 & 122 & 194 & 266 \\ 68 & 167 & 266 & 365 \end{vmatrix} = 0 \tag{10}$$

Since matrix $\mathbf{P}^T\mathbf{P}$ is a singular matrix, there is no inverse of this matrix. Thus, pseudo-inverse matrix cannot be determined. It is not possible to determine the weight matrix by pseudo-inverse rule.

Q.5 1-3-1 multi-layer perceptron network is applied to approximate a polynomial function, $y = \frac{x^3}{3} - \frac{3x^2}{2} + 2x + 5$ when $0 \le x \le 3$. Assume log-sigmoid function is applied in the first layer and pure-linear function is applied in the second layer. Determine possible weights and biases of the network. (20)

Solution

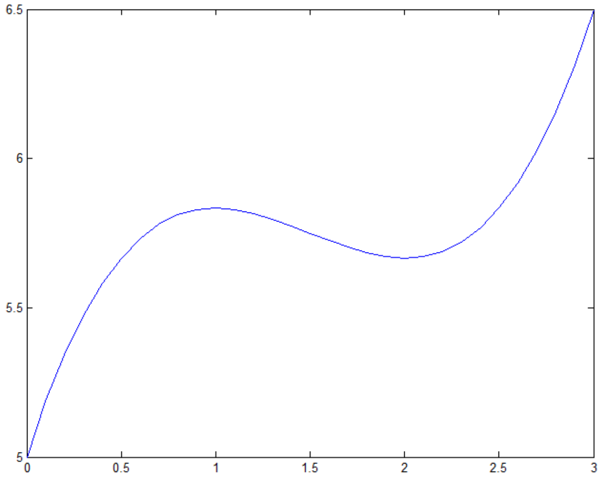
$$y = \frac{x^3}{3} - \frac{3x^2}{2} + 2x + 5 \tag{1}$$

Stationary points are determined.

$$\frac{d}{dx}y = x^2 - 3x + 2 = (x-1)(x-2) = 0 \tag{2}$$

Zero second derivative points are determined.

$$\frac{d^2}{dx^2}y = 2x - 3 = \left(x - \frac{3}{2}\right) = 0 \tag{3}$$



3 log-sigmoid functions in the first layer should have the center at $x = 0, 1.5,$ and 3.

For the log-sigmoid with the center at $x = 0$,

$$w_1^1 = 1, b_1^1 = 0 \tag{4}$$

For the log-sigmoid with the center at $x = 1.5$,

$$w_2^1 = 1, b_1^1 = -1.5 \tag{5}$$

For the log-sigmoid with the center at $x = 3$,

$$w_3^1 = 1, b_1^1 = -3 \tag{6}$$

Determine ranges of the log-sigmoid functions and the bias.

$$\frac{0^3}{3} - \frac{3\times 0^2}{2} + 2 \times 0 + 5 = 5 \tag{7}$$

$$\frac{1^3}{3} - \frac{3\times 1^2}{2} + 2 \times 1 + 5 = 5.83 \tag{8}$$

$$\frac{2^3}{3} - \frac{3\times 2^2}{2} + 2 \times 2 + 5 = 5.67 \tag{9}$$

$$\frac{3^3}{3} - \frac{3\times 3^2}{2} + 2 \times 3 + 5 = 6.5 \tag{10}$$

Thus,

$$w_1^2 = 1.66, w_2^2 = -0.16, w_3^2 = 1.66, b_1^2 = 4.17 \tag{11}$$