

Time: 9:00-11:00 h.

Open Book

Marks: 100

Attempt all questions.

Q.1 ADALINE network is used to determine the parameters of a polynomial function. If the relation between input,  $x$ , and output,  $y$ , from a system is collected and shown in the below table.

$x$	0	1	2	3	4
$y$	2	5	1	8	3

If there is a requirement that no error is allowed in the polynomial model.

(a) Draw the ADALINE network that can be used to determine the parameters. (10)

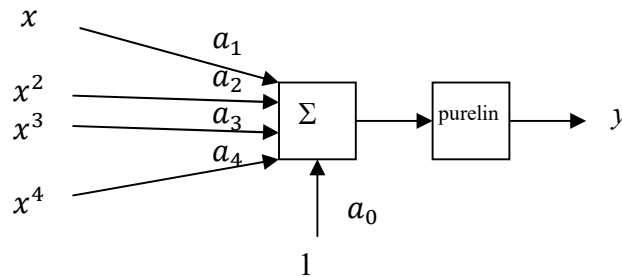
(b) Determine all parameters of the polynomial function by LMS algorithm. (10)

**Solution**

There are 5 data. There will be no error if the polynomial has degree 4 as expressed by

$$y = a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4$$

(a)



(b)

$$F(x) = E[t^2] - 2x^T E[tz] + x^T E[zz^T]x = c - 2x^T h + x^T R x \tag{1}$$

$$c = \frac{1}{5}(2^2 + 5^2 + 1^2 + 8^2 + 3^2) = 20.6 \tag{2}$$

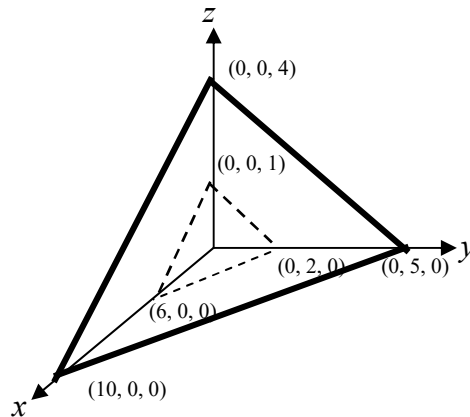
$$h = \frac{1}{5} \left( 2 \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + 5 \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} + 1 \begin{bmatrix} 1 \\ 2 \\ 4 \\ 8 \\ 16 \end{bmatrix} + 8 \begin{bmatrix} 1 \\ 3 \\ 9 \\ 27 \\ 81 \end{bmatrix} + 3 \begin{bmatrix} 1 \\ 4 \\ 16 \\ 64 \\ 256 \end{bmatrix} \right) = \begin{bmatrix} 3.8 \\ 8.6 \\ 25.8 \\ 84.2 \\ 287.4 \end{bmatrix} \tag{3}$$

$$R = \frac{1}{5} \left( \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}^T + \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}^T + \begin{bmatrix} 1 \\ 2 \\ 4 \\ 8 \\ 16 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 4 \\ 8 \\ 16 \end{bmatrix}^T + \begin{bmatrix} 1 \\ 3 \\ 9 \\ 27 \\ 81 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \\ 9 \\ 27 \\ 81 \end{bmatrix}^T + \begin{bmatrix} 1 \\ 4 \\ 16 \\ 64 \\ 256 \end{bmatrix} \begin{bmatrix} 1 \\ 4 \\ 16 \\ 64 \\ 256 \end{bmatrix}^T \right) = \begin{bmatrix} 1 & 2 & 6 & 20 & 70.8 \\ 2 & 6 & 20 & 70.8 & 260 \\ 6 & 20 & 70.8 & 260 & 978 \\ 20 & 70.8 & 260 & 978 & 3740 \\ 70.8 & 260 & 978 & 3740 & 14470.8 \end{bmatrix} \tag{4}$$

The minimum point is the stationary point of the quadratic function.

$$\begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 6 & 20 & 70.8 \\ 2 & 6 & 20 & 70.8 & 260 \\ 6 & 20 & 70.8 & 260 & 978 \\ 20 & 70.8 & 260 & 978 & 3740 \\ 70.8 & 260 & 978 & 3740 & 14470.8 \end{bmatrix}^{-1} \begin{bmatrix} 3.8 \\ 8.6 \\ 25.8 \\ 84.2 \\ 287.4 \end{bmatrix} = \begin{bmatrix} 2 \\ 22.75 \\ -31.2917 \\ 13.25 \\ -1.7083 \end{bmatrix} \quad (5)$$

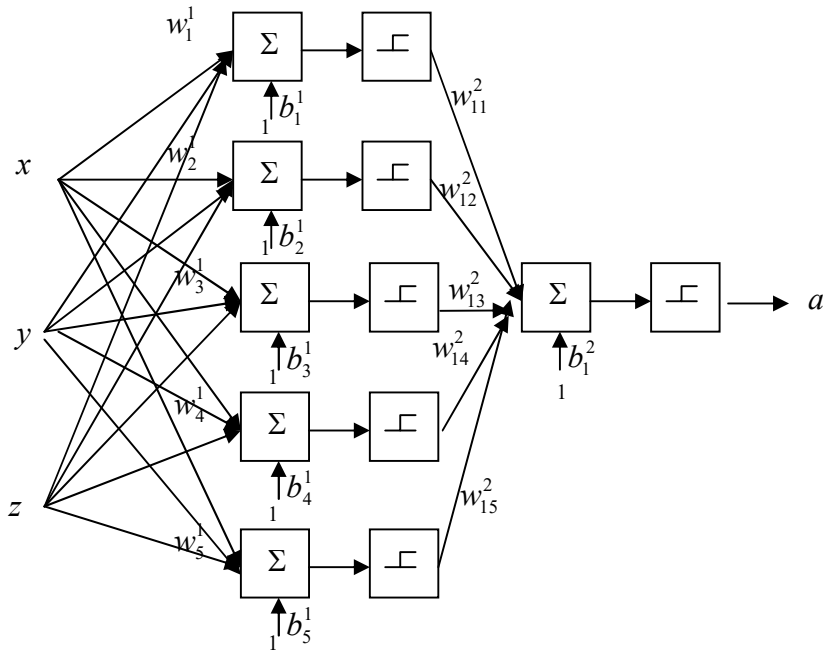
**Q.2** Design a network with appropriate parameters, which can classify class A occupying the polygon shape with 5 sides as shown in the below figure from class B which is outside. Side 1 of the polygon has 3 corners at (10,0,0), (0,5,0) and (0,0,4). Side 2 has 3 corners at (6,0,0), (0,2,0) and (0,0,1). Side 3 represents by  $x = 0$ , side 4 represents by  $y = 0$ , and side 5 represents by  $z = 0$ . Use integer for all coordinates of weight vectors. (20)



**Solution**

Two-Layer Perceptron is selected. The first layer is used to create decision boundaries 1, 2, 3, 4, and 5.

The second layer is used to AND decision boundaries 1, 2, 3, 4, and 5.



**For layer 1**

**w** must point into the shape and it must be perpendicular to decision boundary and edges.

**b** is determined from equating  $\mathbf{n} = 0$ ; and solve for **b**

For the first decision boundary,

$$\begin{bmatrix} w_{11}^1 & w_{12}^1 & w_{13}^1 \end{bmatrix} \begin{bmatrix} 10 \\ -5 \\ 0 \end{bmatrix} = 10w_{11}^1 - 5w_{12}^1 = 0 \quad (1)$$

$$\begin{bmatrix} w_{11}^1 & w_{12}^1 & w_{13}^1 \end{bmatrix} \begin{bmatrix} 0 \\ 5 \\ -4 \end{bmatrix} = 5w_{12}^1 - 4w_{13}^1 = 0 \quad (2)$$

$$\mathbf{w}_1^1 = \begin{bmatrix} -2 \\ -4 \\ -5 \end{bmatrix} \quad (3)$$

$$n_1^1 = w_{11}^1 x + w_{12}^1 y + w_{13}^1 z + b_1^1 = 0 \quad (4)$$

at  $x = 10, y = 0,$  and  $z = 0$ ;

$$-2(10) + 0 + 0 + b_1^1 = 0; b_1^1 = 20 \quad (5)$$

For the second decision boundary,

$$\begin{bmatrix} w_{21}^1 & w_{22}^1 & w_{23}^1 \end{bmatrix} \begin{bmatrix} 6 \\ -2 \\ 0 \end{bmatrix} = 6w_{21}^1 - 2w_{22}^1 = 0 \quad (6)$$

$$\begin{bmatrix} w_{21}^1 & w_{22}^1 & w_{23}^1 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ -1 \end{bmatrix} = 2w_{22}^1 - 1w_{23}^1 = 0 \quad (7)$$

$$\mathbf{w}_2^1 = \begin{bmatrix} 1 \\ 3 \\ 6 \end{bmatrix} \quad (8)$$

$$n_2^1 = w_{21}^1 x + w_{22}^1 y + w_{23}^1 z + b_2^1 = 0 \quad (9)$$

at  $x = 6, y = 0,$  and  $z = 0$

$$6(1) + 0 + 0 + b_2^1 = 0; b_2^1 = -6 \quad (10)$$

For the third decision boundary,

$$\mathbf{w}_3^1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad (11)$$

$$b_3^1 = 0 \quad (12)$$

For the fourth decision boundary,

$$\mathbf{w}_4^1 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \quad (13)$$

$$b_4^1 = 0 \quad (14)$$

For the fifth decision boundary,

$$\mathbf{w}_5^1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad (15)$$

$$b_5^1 = 0 \quad (16)$$

## For layer 2

Since this is AND layer, if we select

$$w_{11}^2 = w_{12}^2 = w_{13}^2 = w_{14}^2 = w_{15}^2 = 1 \quad (17)$$

$$b_1^2 = -4.5 \quad (18)$$

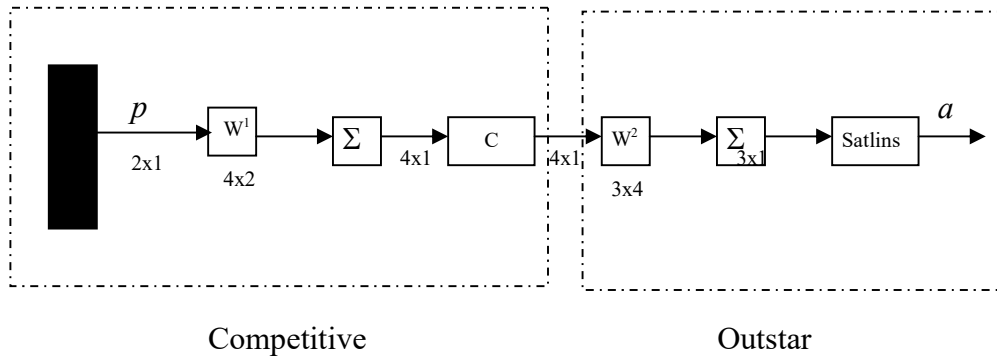
**Q.3** Voltage levels from two analog sensors are used to generate colors from RGB LED. If the relations between voltage levels and RGB intensities are desired as shown below.

$$\left\{ p_1 = \begin{bmatrix} 5 \\ 5 \end{bmatrix}, t_1 = \begin{bmatrix} 255 \\ 0 \\ 0 \end{bmatrix} \right\}, \left\{ p_2 = \begin{bmatrix} -5 \\ -5 \end{bmatrix}, t_2 = \begin{bmatrix} 0 \\ 255 \\ 0 \end{bmatrix} \right\}, \left\{ p_3 = \begin{bmatrix} 5 \\ -5 \end{bmatrix}, t_3 = \begin{bmatrix} 0 \\ 0 \\ 255 \end{bmatrix} \right\}, \left\{ p_4 = \begin{bmatrix} -5 \\ 5 \end{bmatrix}, t_4 = \begin{bmatrix} 255 \\ 255 \\ 255 \end{bmatrix} \right\}$$

Design the neural network that can perform this relations correctly. Then determine the colors if the input is represented by  $p = \begin{bmatrix} 0.1 \\ -0.2 \end{bmatrix}$ . (20)

### Solution

Combination of competitive and outstar is selected here.



Parameters of instar network are designed as followings.

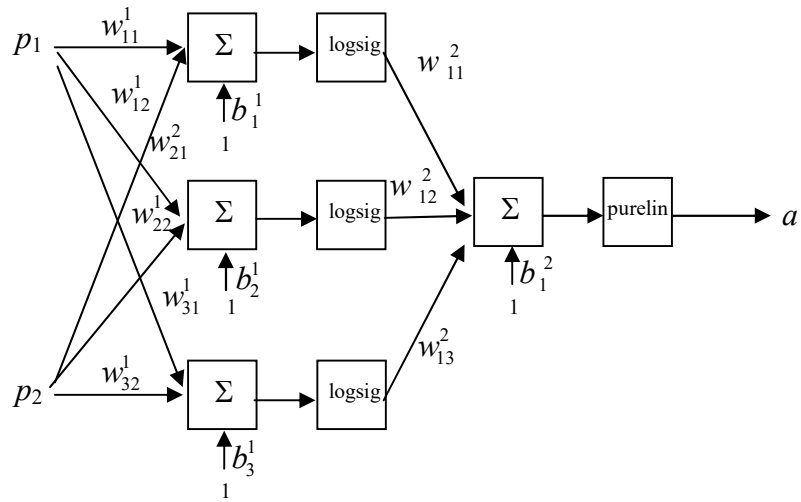
$$W^1 = \begin{bmatrix} p_1^T \\ p_2^T \\ p_3^T \\ p_4^T \end{bmatrix}$$

Parameters of outstar network are designed as followings.

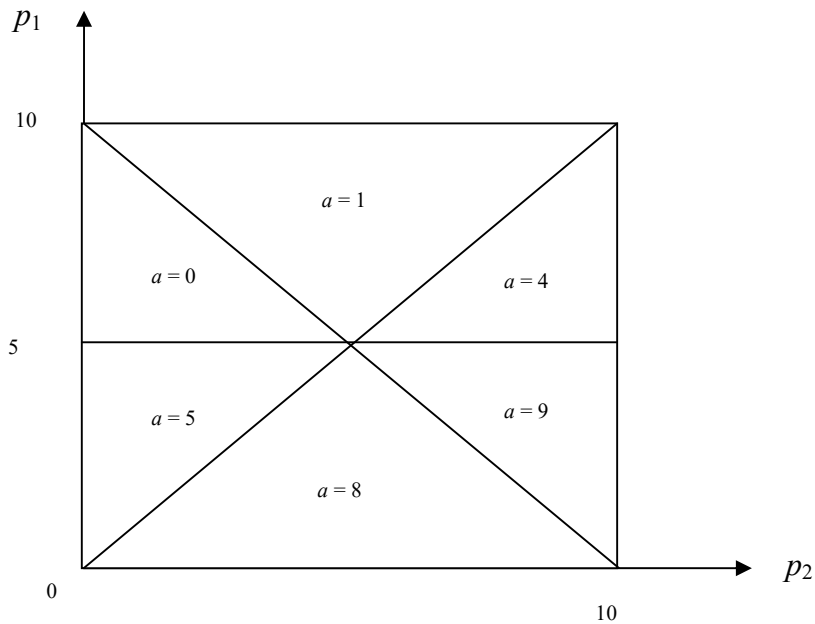
$$W^2 = [t_1 \quad t_2 \quad t_3 \quad t_4]$$

When  $p = \begin{bmatrix} 0.1 \\ -0.2 \end{bmatrix}$  is presented, neuron number 3 wins the competition, thus  $a = \begin{bmatrix} 0 \\ 0 \\ 255 \end{bmatrix}$ .

**Q.4** If a 2-3-1 MLP network as shown in the first figure is used to approximate the input-output relation as shown in the second figure, determine one set of the possible weights and biases of the network after convergence. (20)

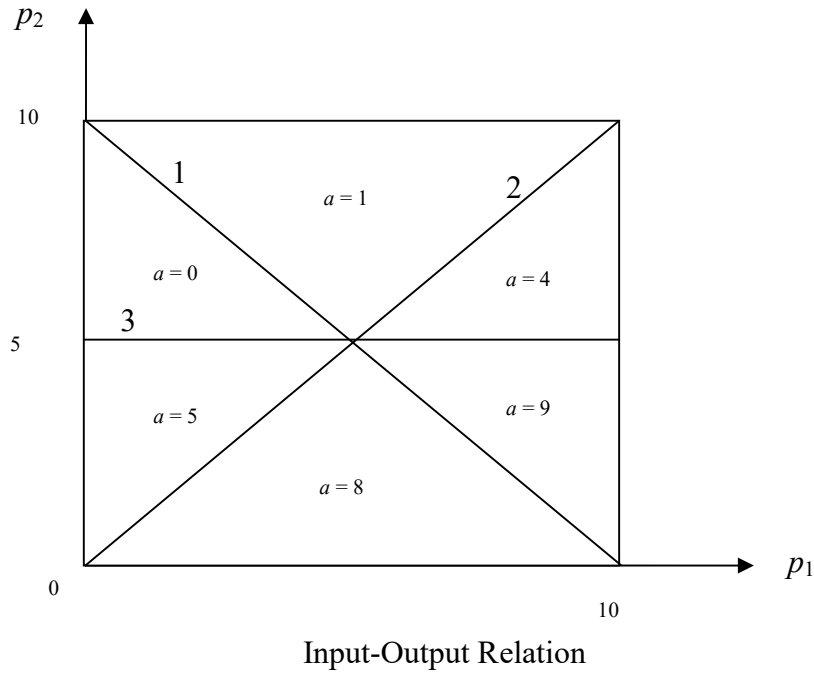


2-3-1 Network



Input-Output Relation

**Solution**



The function can be approximated as the superposition of three 2-D log-sigmoid function. The centers of log-sigmoid are shown by straight lines labeling with 1, 2, and 3 respectively. The first log-sigmoid has its center following a line  $p_2 + p_1 - 10 = 0$  with the output span of 1. The second log-sigmoid has its center following a line  $p_1 - p_2 = 0$  with the output span of 3. The third log-sigmoid has its center following a line  $-p_2 + 5 = 0$  with the output span of 5.

Weight and bias of the first log-sigmoid function, which has its center following a line  $p_2 + p_1 - 10 = 0$ , are determined from

$$n_1^1 = w_{11}^1 p_1 + w_{12}^1 p_2 + b_1^1 = 0 \quad (1)$$

Thus

$$w_{11}^1 = 1, w_{12}^1 = 1, b_1^1 = -10 \quad (2)$$

Weight and bias of the second log-sigmoid function, which has its center following a line  $p_1 - p_2 = 0$ , are determined from

$$n_2^1 = w_{21}^1 p_1 + w_{22}^1 p_2 + b_2^1 = 0 \quad (3)$$

Thus

$$w_{21}^1 = 1, w_{22}^1 = -1, b_2^1 = 0 \quad (4)$$

Weight and bias of the third log-sigmoid function, which has its center following a line  $-p_2 + 5 = 0$ , are determined from

$$n_3^1 = w_{31}^1 p_1 + w_{32}^1 p_2 + b_3^1 = 0 \quad (5)$$

Thus

$$w_{31}^1 = 0, w_{32}^1 = -1, b_3^1 = 5 \quad (6)$$

Since output span of the first log-sigmoid function is 1, thus

$$w_{11}^2 = 1 \quad (7)$$

Since output span of the second log-sigmoid function is 3, thus

$$w_{12}^2 = 3 \quad (8)$$

Since output span of the second log-sigmoid function is 5, thus

$$w_{13}^2 = 5 \quad (9)$$

Since the plot starts from the output magnitude of 0, thus

$$b_1^2 = 0 \quad (10)$$

**Q.5** Discuss how to apply neural network to your research work. What kind of network is appropriate and how to prepare the training set? (20)