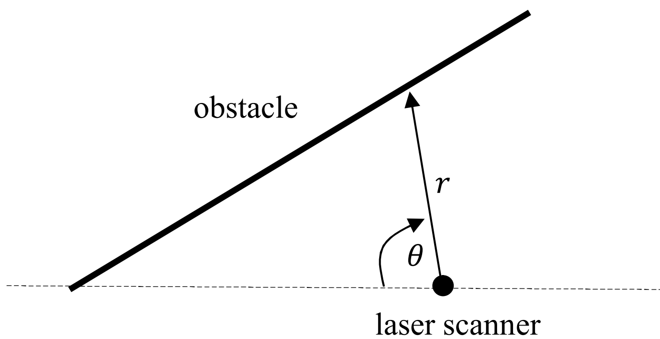
Midsem Examination AI and Neuro-Fuzzy Theory AT74.05 March 2, 2018

Time: 9:00-11:00 h. **Open Book**
Marks: 100

Attempt all questions.

Q.1 A laser scanner is used to create map of an unknown environment. The outputs from the laser scanner are range in cm, r, and direction in degree, θ, of the obstacle respect to the laser scanner. $\theta = 90°$ is the direction of right in front of the laser scanner and clockwise direction is positive direction.



Assume a linear wall locates in the unknown environment and data from the laser scanner is collected and shown in the below table.

θ	0	30	60	90
r	50	56.12	92.03	1000

ADALINE network is used to determine the cartesian parameters, (m, c), of the linear wall according to $y = mx + c$. Cartesian coordinate of the laser scanner location is set at $(x, y) = (0,0)$.

(a) Convert the data from (r, θ) to Cartesian coordinate (x, y). (10)

(b) Draw the ADALINE network that can be used to determine the parameters. (5)

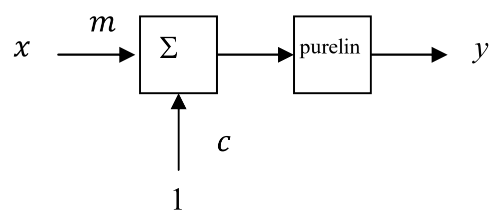
(c) Determine all the parameters of the linear wall by LMS algorithm. (10)

<u>Solution</u>

(a)

x	-50	-48.60	-46.02	0
y	0	28.06	79.70	1000

(b)



(b)

$$F(x) = E[t^2] - 2x^T E[tz] + x^T E[zz^T]x = c - 2x^T h + x^T R x \quad (1)$$

$$c = \tfrac{1}{4}(0^2 + 28.06^2 + 79.70^2 + 1000^2) = 251784.9 \quad (2)$$

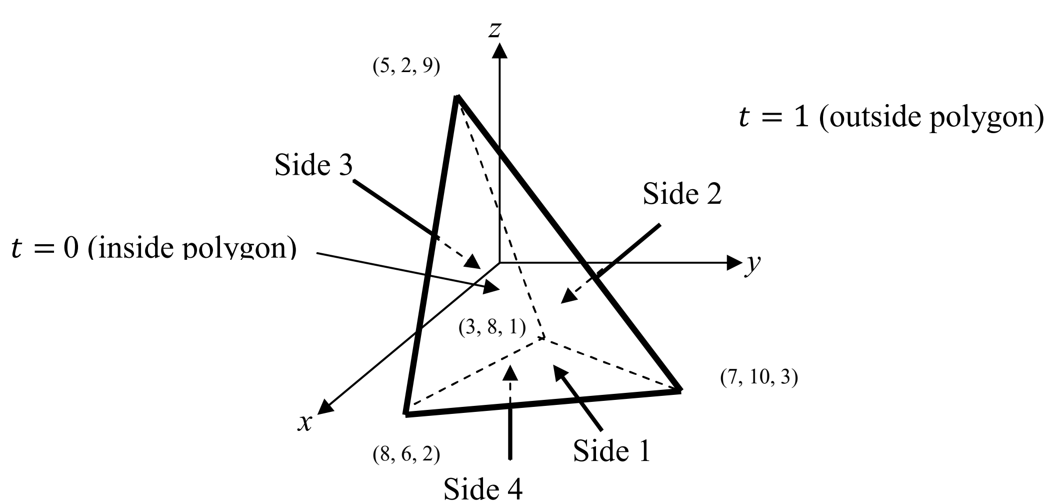
$$h = \tfrac{1}{4}\left(0\begin{bmatrix}-50\\1\end{bmatrix} + 28.06\begin{bmatrix}-48.60\\1\end{bmatrix} + 79.70\begin{bmatrix}-46.02\\1\end{bmatrix} + 1000\begin{bmatrix}0\\1\end{bmatrix}\right) = \begin{bmatrix}-1257.9\\276.9\end{bmatrix} \quad (3)$$

$$R = \tfrac{1}{4}\left(\begin{bmatrix}-50\\1\end{bmatrix}\begin{bmatrix}-50\\1\end{bmatrix}^T + \begin{bmatrix}-48.60\\1\end{bmatrix}\begin{bmatrix}-48.60\\1\end{bmatrix}^T + \begin{bmatrix}-46.02\\1\end{bmatrix}\begin{bmatrix}-46.02\\1\end{bmatrix}^T + \begin{bmatrix}0\\1\end{bmatrix}\begin{bmatrix}0\\1\end{bmatrix}^T\right) = \begin{bmatrix}1745.0 & -36.2\\-36.2 & 1\end{bmatrix} \quad (4)$$

The minimum point is the stationary point of the quadratic function.

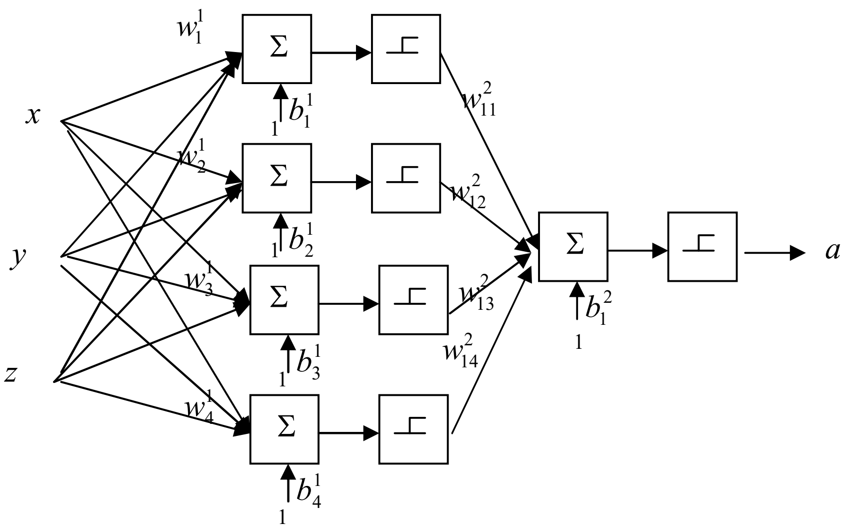
$$\begin{bmatrix}m\\c\end{bmatrix} = \begin{bmatrix}1745.0 & -36.2\\-36.2 & 1\end{bmatrix}^{-1}\begin{bmatrix}-1257.9\\276.9\end{bmatrix} = \begin{bmatrix}20\\1000\end{bmatrix} \quad (5)$$

Q.2 Design a two-layer perceptron network with appropriate parameters, which generates output of 0 when the input vector is inside the polygon and generates output of 1 when the input vector is outside the polygon. Use integer for all coordinates of weight vectors. (25)



Solution

Two-Layer Perceptron is selected. The first layer is used to create decision boundaries 1, 2, 3, and 4 when the weight vectors point outward from the polygon. The second layer is used to OR decision boundaries 1, 2, 3, and 4.



For layer 1

w must point outward from the polygon and it must be perpendicular to decision boundary and edges.

b is determined from equating **n** = 0; and solve for **b**

For the first decision boundary,

$$\begin{bmatrix} w_{11}^1 & w_{12}^1 & w_{13}^1 \end{bmatrix} \begin{bmatrix} -1 \\ 4 \\ 1 \end{bmatrix} = -w_{11}^1 + 4w_{12}^1 + w_{13}^1 = 0 \quad (1)$$

$$\begin{bmatrix} w_{11}^1 & w_{12}^1 & w_{13}^1 \end{bmatrix} \begin{bmatrix} -2 \\ -8 \\ 6 \end{bmatrix} = -2w_{11}^1 - 8w_{12}^1 + 6w_{13}^1 = 0 \quad (2)$$

$$\mathbf{w}_1^1 = \begin{bmatrix} 8 \\ 1 \\ 4 \end{bmatrix} \quad (3)$$

$$n_1^1 = w_{11}^1 x + w_{12}^1 y + w_{13}^1 z + b_1^1 = 0 \quad (4)$$

at $x = 8$, $y = 6$, and $z = 2$;

$$8(8)+1(6)+4(2)+b_1^1=0; b_1^1=-78 \quad \underline{(5)}$$

<u>For the second decision boundary</u>,

$$\begin{bmatrix} w_{21}^1 & w_{22}^1 & w_{23}^1 \end{bmatrix}\begin{bmatrix} -2 \\ -8 \\ 6 \end{bmatrix} = -2w_{21}^1 - 8w_{22}^1 + 6w_{23}^1 = 0 \quad (6)$$

$$\begin{bmatrix} w_{21}^1 & w_{22}^1 & w_{23}^1 \end{bmatrix}\begin{bmatrix} 2 \\ -6 \\ 8 \end{bmatrix} = 2w_{21}^1 - 6w_{22}^1 + 8w_{23}^1 = 0 \quad (7)$$

$$\mathbf{w}_2^1 = \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix} \quad \underline{(8)}$$

$$n_2^1 = w_{21}^1 x + w_{22}^1 y + w_{23}^1 z + b_2^1 = 0 \quad (9)$$

at $x = 7$, $y = 10$, and $z = 3$

$$7(-1)+10(1)+3(1)+b_2^1=0; b_2^1=-6 \quad \underline{(10)}$$

<u>For the third decision boundary</u>,

$$\begin{bmatrix} w_{31}^1 & w_{32}^1 & w_{33}^1 \end{bmatrix}\begin{bmatrix} 2 \\ -6 \\ 8 \end{bmatrix} = 2w_{31}^1 - 6w_{32}^1 + 8w_{33}^1 = 0 \quad (11)$$

$$\begin{bmatrix} w_{31}^1 & w_{32}^1 & w_{33}^1 \end{bmatrix}\begin{bmatrix} 5 \\ -2 \\ 1 \end{bmatrix} = 5w_{31}^1 - 2w_{32}^1 + 1w_{33}^1 = 0 \quad (12)$$

$$\mathbf{w}_3^1 = \begin{bmatrix} -5 \\ -19 \\ -13 \end{bmatrix} \quad \underline{(13)}$$

at $x = 3$, $y = 8$, and $z = 1$

$$3(-5)+8(-19)+1(-13)+b_3^1=0; b_3^1=180 \quad \underline{(14)}$$

<u>For the fourth decision boundary</u>,

$$\begin{bmatrix} w_{41}^1 & w_{42}^1 & w_{43}^1 \end{bmatrix} \begin{bmatrix} -1 \\ 4 \\ 1 \end{bmatrix} = -w_{41}^1 + 4w_{42}^1 + w_{43}^1 = 0 \quad (15)$$

$$\begin{bmatrix} w_{41}^1 & w_{42}^1 & w_{43}^1 \end{bmatrix} \begin{bmatrix} 4 \\ 2 \\ 2 \end{bmatrix} = 4w_{41}^1 + 2w_{42}^1 + 2w_{43}^1 = 0 \quad (16)$$

$$\mathbf{w}_4^1 = \begin{bmatrix} 1 \\ 1 \\ -3 \end{bmatrix} \quad (17)$$

at $x = 8$, $y = 6$, and $z = 2$

$$1(8) + 1(6) - 3(2) + b_4^1 = 0; b_4^1 = -8 \quad (18)$$

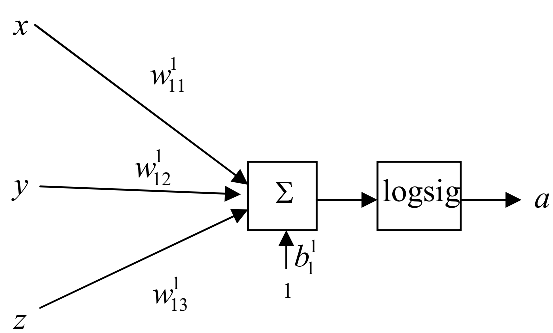
For layer 2

Since this is OR layer, if we select

$$w_{11}^2 = w_{12}^2 = w_{13}^2 = w_{14}^2 = 1 \quad (19)$$

$$b_1^2 = -0.5 \quad (20)$$

Q.3 Consider a 3-1 neural network as shown below.



Train this network by SDBP using both training sets for one round with learning rate, $\alpha = 0.5$. The training sets are given as

$$\left\{ p_1 = \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix}, t_1 = [0] \right\}, \left\{ p_2 = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}, t_1 = [1] \right\}$$

The initial weights and bias are as follows. $w_{11}^1 = 0.1, w_{12}^1 = -0.2, w_{13}^1 = 0.3, b_1^1 = -0.4.$ (25)

Solution

Determine derivative of the transfer function,

$$f^1 = (1+e^{-n})^{-1}, \dot{f}^1 = (1-a^1)(a^1) \quad (1)$$

Present p_1,

$$n_1^1 = \begin{bmatrix}0.1 & -0.2 & 0.3\end{bmatrix}\begin{bmatrix}1\\-1\\-1\end{bmatrix} + \begin{bmatrix}-0.4\end{bmatrix} = -0.4, a_1^1 = (1+e^{-n_1^1})^{-1} = (1+e^{0.4})^{-1} = 0.4013 \quad (2)$$

$$e = t - a = 0 - (0.4013) = -0.4013 \quad (3)$$

$$s^1 = -2\dot{f}^1 e = -2(1-0.4013)(0.4013)(-0.4013) = 0.1928 \quad (4)$$

$$w^1(1) = w^1(0) - \alpha s^1 (a^0)^T = \begin{bmatrix}0.1 & -0.2 & 0.3\end{bmatrix} - 0.5\begin{bmatrix}0.1928\end{bmatrix}\begin{bmatrix}1 & -1 & -1\end{bmatrix} = \begin{bmatrix}0.0036 & -0.1036 & 0.3964\end{bmatrix} \quad (5)$$

$$b^1(1) = b^1(0) - \alpha s^1 = \begin{bmatrix}-0.4\end{bmatrix} - 0.5\begin{bmatrix}0.1928\end{bmatrix} = \begin{bmatrix}-0.4964\end{bmatrix} \quad (6)$$

Present p_2,

$$n_1^1 = \begin{bmatrix}0.0036 & -0.1036 & 0.3964\end{bmatrix}\begin{bmatrix}1\\1\\-1\end{bmatrix} + \begin{bmatrix}-0.4964\end{bmatrix} = -0.9928, a_1^1 = (1+e^{-n_1^1})^{-1} = (1+e^{0.9928})^{-1} = 0.2704 \quad (7)$$

$$e = t - a = 1 - (0.2704) = 0.7296 \quad (8)$$

$$s^1 = -2\dot{f}^1 e = -2(1-0.2704)(0.2704)(0.7296) = \text{-}0.2879 \quad (9)$$

$$w^1(2) = w^1(1) - \alpha s^1 (a^0)^T = \begin{bmatrix}0.0036 & -0.1036 & 0.3964\end{bmatrix} - 0.5\begin{bmatrix}\text{-}0.2879\end{bmatrix}\begin{bmatrix}1 & 1 & -1\end{bmatrix} = \begin{bmatrix}0.1475 & 0.0403 & 0.2525\end{bmatrix} \quad (10)$$

$$b^1(2) = b^1(1) - \alpha s^1 = \begin{bmatrix}-0.4964\end{bmatrix} - 0.5\begin{bmatrix}-0.2879\end{bmatrix} = \begin{bmatrix}\text{-}0.3525\end{bmatrix} \quad (11)$$

Q.4 LVQ network is used to recognize 3 patterns according to the following training sets.

$$\left\{p_1 = \begin{bmatrix}10\\10\\10\end{bmatrix}, t_1 = \begin{bmatrix}1\\0\\0\end{bmatrix}\right\}, \left\{p_2 = \begin{bmatrix}10.5\\9.8\\9.6\end{bmatrix}, t_2 = \begin{bmatrix}1\\0\\0\end{bmatrix}\right\}, \left\{p_3 = \begin{bmatrix}9.7\\10.3\\10.2\end{bmatrix}, t_3 = \begin{bmatrix}1\\0\\0\end{bmatrix}\right\},$$

$$\left\{p_4 = \begin{bmatrix}5\\-15\\0\end{bmatrix}, t_4 = \begin{bmatrix}0\\1\\0\end{bmatrix}\right\}, \left\{p_5 = \begin{bmatrix}4.7\\-14.8\\0.3\end{bmatrix}, t_5 = \begin{bmatrix}0\\1\\0\end{bmatrix}\right\}, \left\{p_6 = \begin{bmatrix}-20\\-20\\-20\end{bmatrix}, t_6 = \begin{bmatrix}1\\0\\0\end{bmatrix}\right\},$$

$$\left\{p_7 = \begin{bmatrix} -19.6 \\ -20.2 \\ -20.3 \end{bmatrix}, t_7 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}\right\}, \left\{p_8 = \begin{bmatrix} -20.3 \\ -19.5 \\ -19.1 \end{bmatrix}, t_8 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}\right\}, \left\{p_9 = \begin{bmatrix} -5 \\ 20 \\ -15 \end{bmatrix}, t_9 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}\right\},$$

$$\left\{p_{10} = \begin{bmatrix} -4.8 \\ 19.7 \\ -15.4 \end{bmatrix}, t_{10} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}\right\}, \left\{p_{11} = \begin{bmatrix} 15 \\ 15 \\ 15 \end{bmatrix}, t_{11} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}\right\}, \left\{p_{12} = \begin{bmatrix} 15.4 \\ 14.7 \\ 14.9 \end{bmatrix}, t_{12} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}\right\}$$

If a very small learning rate, α, is applied, determine the weights of the first and the second layer of the LVQ network. (25)

Solution

From the training sets, there are 5 sub classes.

In the first layer of LVQ, the trained weights vector will be at the center of each sub class.

$$w_1 = \frac{1}{3}\begin{bmatrix} 10+10.5+9.7 \\ 10+9.8+10.3 \\ 10+9.6+10.2 \end{bmatrix} = \begin{bmatrix} 10.07 \\ 10.03 \\ 9.93 \end{bmatrix} \quad (1)$$

$$w_2 = \frac{1}{2}\begin{bmatrix} 5+4.7 \\ -15-14.8 \\ 0+0.3 \end{bmatrix} = \begin{bmatrix} 4.85 \\ -14.90 \\ 0.15 \end{bmatrix} \quad (2)$$

$$w_3 = \frac{1}{3}\begin{bmatrix} -20-19.6-20.3 \\ -20-20.2-19.5 \\ -20-20.3-19.1 \end{bmatrix} = \begin{bmatrix} -19.97 \\ -19.90 \\ -19.80 \end{bmatrix} \quad (3)$$

$$w_4 = \frac{1}{2}\begin{bmatrix} -5-4.8 \\ 20+19.7 \\ -15-15.4 \end{bmatrix} = \begin{bmatrix} -4.90 \\ 19.85 \\ -15.20 \end{bmatrix} \quad (4)$$

$$w_5 = \frac{1}{2}\begin{bmatrix} 15+15.4 \\ 15+14.7 \\ 15+14.9 \end{bmatrix} = \begin{bmatrix} 15.20 \\ 14.85 \\ 14.95 \end{bmatrix} \quad (5)$$

The weight matrix of the first layer is thus

$$W^1 = \begin{bmatrix} 10.07 & 4.85 & -19.97 & -4.90 & 15.20 \\ 10.03 & -14.90 & -19.90 & 19.85 & 14.85 \\ 9.93 & 0.15 & -19.80 & -15.20 & 14.95 \end{bmatrix} \quad (6)$$

The weight matrix of the second layer combines and recognizes subclasses 1 and 3 as class 1, subclass 2 as class 2 and sub classes 4 and 5 as class 3 respectively.

$$W^2 = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} \tag{7}$$