

Time: 9:00-11:00 h.
Marks: 100

Open Book

Attempt all questions.

Q.1 ADALINE network is applied to determine number of gas molecules (n) in a closed flexible container. The relation between pressure (P), volume (V), number of gas molecules (n), and Temperature (T) follows the ideal gas law relation

$$PV = nKT$$

When K is Boltzmann constant ($1.38 \times 10^{-23} \text{ J/K}$).

(a) Differentiate pressure (dP) and express the result as a linear function of differentiation of temperature (dT) and volume (dV) of ideal gas in the closed flexible container. (10)

Assume the experiment data is recorded as shown in the below table.

dP (Pa)	0	4000	6000	10000
dT (°K)	0	10	20	30
dV (m ³)	0	0.1	0.4	0.5

(b) Draw the ADALINE network that can be used to determine the parameters in (a). (5)

(c) Determine the number of gas molecules by LMS algorithm when the data above are obtained at the reference volume of the ideal gas at 1 m³. (10)

Solution

(a)

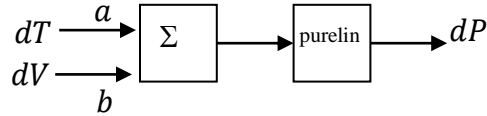
$$PV = nKT$$

$$P = \frac{nKT}{V}$$

$$dP = \frac{nK}{V_0} dT - \frac{nKT_0}{V_0^2} dV$$

$$dP = adT + bdV$$

(b)



(c)

$$F(x) = E[t^2] - 2x^T E[tz] + x^T E[zz^T]x = c - 2x^T h + x^T R x \quad (1)$$

$$c = \frac{1}{4}(0^2 + 4000^2 + 6000^2 + 10000^2) = 38000000 \quad (2)$$

$$h = \frac{1}{4}\left(0 \begin{bmatrix} 0 \\ 0 \end{bmatrix} + 4000 \begin{bmatrix} 10 \\ 0.1 \end{bmatrix} + 6000 \begin{bmatrix} 20 \\ 0.4 \end{bmatrix} + 10000 \begin{bmatrix} 30 \\ 0.5 \end{bmatrix}\right) = \begin{bmatrix} 115000 \\ 1950 \end{bmatrix} \quad (3)$$

$$R = \frac{1}{4}\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \end{bmatrix}^T + \begin{bmatrix} 10 \\ 0.1 \end{bmatrix}\begin{bmatrix} 10 \\ 0.1 \end{bmatrix}^T + \begin{bmatrix} 20 \\ 0.4 \end{bmatrix}\begin{bmatrix} 20 \\ 0.4 \end{bmatrix}^T + \begin{bmatrix} 30 \\ 0.5 \end{bmatrix}\begin{bmatrix} 30 \\ 0.5 \end{bmatrix}^T\right) = \begin{bmatrix} 350 & 6 \\ 6 & 0.105 \end{bmatrix} \quad (4)$$

The minimum point is the stationary point of the quadratic function.

$$\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 350 & 6 \\ 6 & 0.105 \end{bmatrix}^{-1} \begin{bmatrix} 115000 \\ 1950 \end{bmatrix} = \begin{bmatrix} 500 \\ -10000 \end{bmatrix} \quad (5)$$

$$\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} \frac{nK}{V_0} \\ -\frac{nKT_0}{V_0^2} \end{bmatrix} = \begin{bmatrix} 500 \\ -10000 \end{bmatrix} \quad (6)$$

$$\frac{b}{a} = -\frac{T_0}{V_0} = -20 \quad (7)$$

Since $V_0 = 1 \text{ m}^3$, thus $V_0 = 20 \text{ K}$.

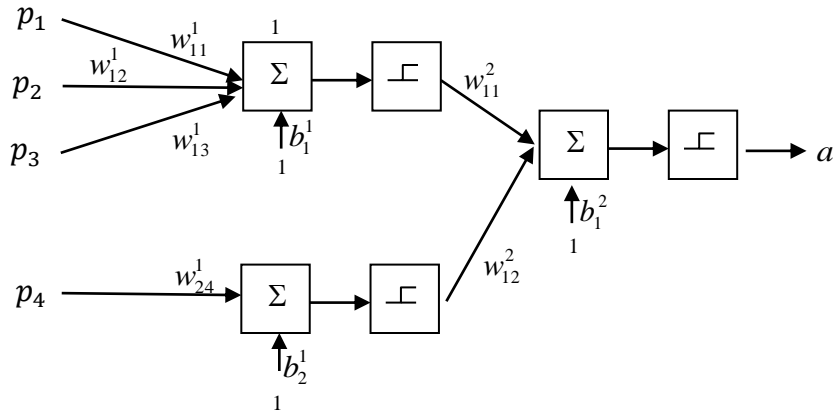
Substitute $K = 1.38 \times 10^{-23} \text{ J/K}$,

$$\frac{nK}{V_0} = \frac{n1.38 \times 10^{-23}}{1} = 500 \quad (8)$$

$$n = 3.62 \times 10^{25} \text{ molecules} \quad (9)$$

Q.2 Multi-layer perceptron (MLP) is used to implement the logic $(p_1 \wedge p_2 \wedge p_3) \vee \bar{p}_4$. When \wedge represents AND, \vee represents OR, \bar{p}_4 represents NOT of p_4 . Design the network that can implement the required logic, determine all the weights and biases of the network. (25)

Solution



For layer 1

The first neuron in the first layer performs AND operation with three inputs and one output.

Select

$$w_{11}^1 = 1 \text{ and } w_{12}^1 = 1 \text{ and } w_{13}^1 = 1 \text{ and } b_1^1 = -2.5$$

The second neuron in the first layer perform NOT operation with one input and one output.

Select

$$w_{24}^1 = -1 \text{ and } b_2^1 = 0.5$$

For layer 2

The neuron in the second layer performs OR operation with two inputs and one output.

Select

$$w_{11}^2 = 1 \text{ and } w_{12}^2 = 1 \text{ and } b_1^2 = -0.5$$

Q.3 Consider the training sets below.

$$\left\{ p_1 = \begin{bmatrix} -11 \\ 9 \\ 22 \end{bmatrix}, t_1 = \begin{bmatrix} 10 \\ 20 \\ 30 \\ 40 \\ 50 \end{bmatrix} \right\}, \left\{ p_2 = \begin{bmatrix} -5 \\ 6 \\ 9 \end{bmatrix}, t_2 = \begin{bmatrix} 25 \\ 24 \\ 23 \\ 22 \\ 21 \end{bmatrix} \right\}, \left\{ p_3 = \begin{bmatrix} 13 \\ 7 \\ 9 \end{bmatrix}, t_3 = \begin{bmatrix} 10 \\ 20 \\ 30 \\ 40 \\ 50 \end{bmatrix} \right\},$$

$$\left\{ p_4 = \begin{bmatrix} -9 \\ 11 \\ 18 \end{bmatrix}, t_4 = \begin{bmatrix} 10 \\ 20 \\ 30 \\ 40 \\ 50 \end{bmatrix} \right\}, \left\{ p_5 = \begin{bmatrix} -5 \\ 4 \\ 11 \end{bmatrix}, t_5 = \begin{bmatrix} 25 \\ 24 \\ 23 \\ 22 \\ 21 \end{bmatrix} \right\}, \left\{ p_6 = \begin{bmatrix} 17 \\ 3 \\ 11 \end{bmatrix}, t_6 = \begin{bmatrix} 10 \\ 20 \\ 30 \\ 40 \\ 50 \end{bmatrix} \right\},$$

$$\left\{ p_7 = \begin{bmatrix} 12 \\ 1 \\ -35 \end{bmatrix}, t_7 = \begin{bmatrix} 25 \\ 24 \\ 23 \\ 22 \\ 21 \end{bmatrix} \right\}, \left\{ p_8 = \begin{bmatrix} -1 \\ 2 \\ -10 \end{bmatrix}, t_8 = \begin{bmatrix} 10 \\ 20 \\ 30 \\ 40 \\ 50 \end{bmatrix} \right\}, \left\{ p_9 = \begin{bmatrix} -6 \\ -7 \\ 0 \end{bmatrix}, t_9 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \right\},$$

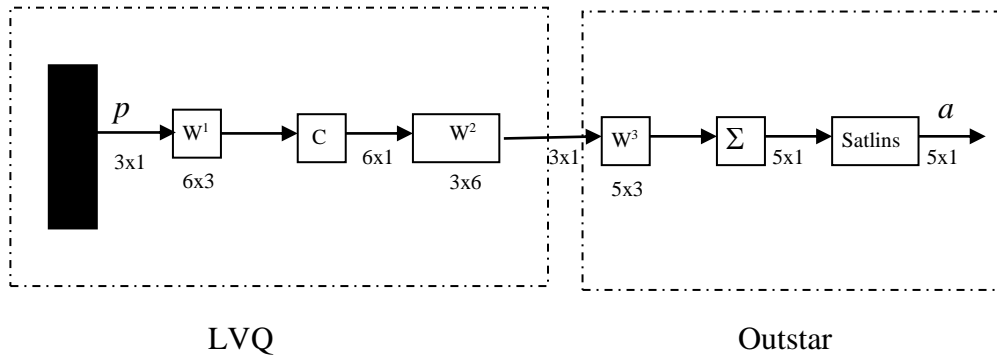
$$\left\{ p_{10} = \begin{bmatrix} 1 \\ -2 \\ -10 \end{bmatrix}, t_{10} = \begin{bmatrix} 10 \\ 20 \\ 30 \\ 40 \\ 50 \end{bmatrix} \right\}, \left\{ p_{11} = \begin{bmatrix} -4 \\ -3 \\ 0 \end{bmatrix}, t_{11} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \right\}, \left\{ p_{12} = \begin{bmatrix} 8 \\ -1 \\ -45 \end{bmatrix}, t_{12} = \begin{bmatrix} 25 \\ 24 \\ 23 \\ 22 \\ 21 \end{bmatrix} \right\}$$

Design the network which can generate the output correctly for all the input in the training set. (25)

Solution

The combination of LVQ with Outstar can be applied.

LVQ is used for input pattern recognition and Outstar network is used to recall the output pattern.



For LVQ

From the training sets, there are 3 classes and 6 sub classes. Subclass 1 consists of training set numbers 1 and 4, subclass 2 consists of training set numbers 2 and 5, subclass 3 consists of training set numbers 3 and 6, subclass 4 consists of training set numbers 8 and 10, subclass 5 consists of training set numbers 7 and 12, subclass 6 consists of training set numbers 9 and 11.

In the first layer of LVQ, the trained weights vector will be at the center of each subclass.

$$w_1^1 = \frac{1}{2} \begin{bmatrix} -11 - 9 \\ 9 + 11 \\ 22 + 18 \end{bmatrix} = \begin{bmatrix} -10 \\ 10 \\ 20 \end{bmatrix} \quad (1)$$

$$w_2^1 = \frac{1}{2} \begin{bmatrix} -5 - 5 \\ 6 + 4 \\ 9 + 11 \end{bmatrix} = \begin{bmatrix} -5 \\ 5 \\ 10 \end{bmatrix} \quad (2)$$

$$w_3^1 = \frac{1}{2} \begin{bmatrix} 13 + 17 \\ 7 + 3 \\ 9 + 11 \end{bmatrix} = \begin{bmatrix} 15 \\ 5 \\ 10 \end{bmatrix} \quad (3)$$

$$w_4^1 = \frac{1}{2} \begin{bmatrix} -1 + 1 \\ 2 - 2 \\ -10 - 10 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -10 \end{bmatrix} \quad (4)$$

$$w_5^1 = \frac{1}{2} \begin{bmatrix} 12 + 8 \\ 1 - 1 \\ -35 - 45 \end{bmatrix} = \begin{bmatrix} 10 \\ 0 \\ -40 \end{bmatrix} \quad (5)$$

$$w_6^1 = \frac{1}{2} \begin{bmatrix} -6 - 4 \\ -7 - 3 \\ 0 + 0 \end{bmatrix} = \begin{bmatrix} -5 \\ -5 \\ 0 \end{bmatrix} \quad (6)$$

The weight matrix of the first layer is thus

$$W^1 = \begin{bmatrix} -10 & 10 & 20 \\ -5 & 5 & 10 \\ 15 & 5 & 10 \\ 0 & 0 & -10 \\ 10 & 0 & -40 \\ -5 & -5 & 0 \end{bmatrix} \quad (7)$$

The weight matrix of the second layer combines and recognizes subclasses as class. Class 1 consists of subclasses 1, 3 and 4. Class 2 consists of subclasses 2 and 5. Class 3 consists of subclass 6. respectively.

$$W^2 = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (8)$$

Parameters of Outstar network are designed as followings.

$$W^3 = \begin{bmatrix} 10 & 25 & 1 \\ 20 & 24 & 1 \\ 30 & 23 & 1 \\ 40 & 22 & 1 \\ 50 & 21 & 1 \end{bmatrix} \quad (9)$$

Q.4 1-5-1 network is used to approximate a function. The function in the first layer is log-sigmoid function. The function in the second layer is pure linear function. Roughly plot graph of the relation between input and output when the following weights and biases are used. (25)

$$W^1 = \begin{bmatrix} 1 \\ 5 \\ 8 \\ 10 \\ 12 \end{bmatrix}, b^1 = \begin{bmatrix} 20 \\ 0 \\ -80 \\ -300 \\ -600 \end{bmatrix}, W^2 = [25 \quad -10 \quad -20 \quad 40 \quad -50], b^2 = [15]$$

Solution

Centers of the 5 log-sigmoid function in the first layer are determined.

$$n_1^1 = w_{11}^1 p + b_1^1 = 1p + 20 = 0; p = -20$$

$$n_2^1 = w_{21}^1 p + b_2^1 = 5p + 0 = 0; p = 0$$

$$n_3^1 = w_{31}^1 p + b_3^1 = 8p - 80 = 0; p = 10$$

$$n_4^1 = w_{41}^1 p + b_4^1 = 10p - 300 = 0; p = 30$$

$$n_5^1 = w_{51}^1 p + b_5^1 = 12p - 600 = 0; p = 50$$

Weights of the second layers represent span ranges of the log sigmoid functions while bias of the second layer represents the starting value of the output. Thus, graph of the relation can be roughly plotted as shown below.

