

Time: 9:00-11:00 h.

Marks: 100

Open Book

Attempt all questions.

Q.1 ADALINE network is applied to determine a relation between the input, x , and the output, y , of a system as expressed by

$$y = \frac{ax^2 + bx + c}{x}$$

(a) Draw the network architecture. (5)

(b) Determine the parameters; a , b , and c in the relation that minimizes the mean square error when the data with noises are collected and expressed as shown in the below table. (20)

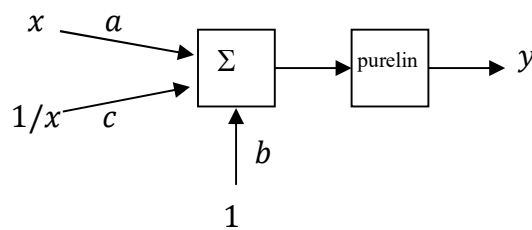
x	1	2	4	8
y	13	16	22	43

Solution

(a)

Rewrite the relation

$$y = \frac{ax^2 + bx + c}{x} = ax + b + \frac{c}{x}$$



(b)

$$F(x) = E[t^2] - 2x^T E[tz] + x^T E[zz^T]x = c - 2x^T h + x^T R x \quad (1)$$

$$c = \frac{1}{4}(13^2 + 16^2 + 22^2 + 43^2) = 689.5 \quad (2)$$

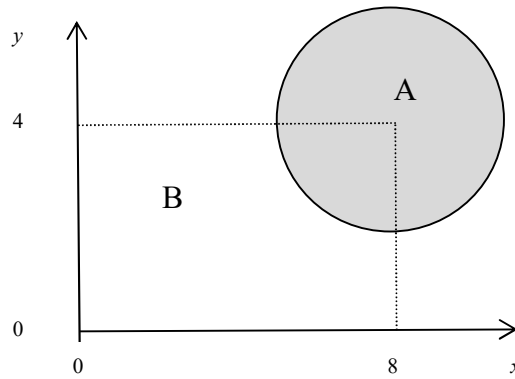
$$h = \frac{1}{4} \left(13 \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + 16 \begin{bmatrix} 2 \\ 1/2 \\ 1 \end{bmatrix} + 22 \begin{bmatrix} 4 \\ 1/4 \\ 1 \end{bmatrix} + 43 \begin{bmatrix} 8 \\ 1/8 \\ 1 \end{bmatrix} \right) = \begin{bmatrix} 119.25 \\ 7.97 \\ 23.5 \end{bmatrix} \quad (3)$$

$$R = \frac{1}{4} \left(\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}^T + \begin{bmatrix} 2 \\ 1/2 \\ 1 \end{bmatrix} \begin{bmatrix} 2 \\ 1/2 \\ 1 \end{bmatrix}^T + \begin{bmatrix} 4 \\ 1/4 \\ 1 \end{bmatrix} \begin{bmatrix} 4 \\ 1/4 \\ 1 \end{bmatrix}^T + \begin{bmatrix} 8 \\ 1/8 \\ 1 \end{bmatrix} \begin{bmatrix} 8 \\ 1/8 \\ 1 \end{bmatrix}^T \right) = \begin{bmatrix} 21.25 & 1 & 3.75 \\ 1 & 0.33 & 0.47 \\ 3.75 & 0.47 & 1 \end{bmatrix} \quad (4)$$

The parameters are determined.

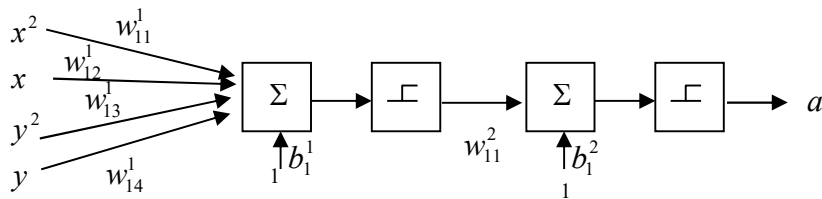
$$\begin{bmatrix} a \\ c \\ b \end{bmatrix} = \begin{bmatrix} 21.25 & 1 & 3.75 \\ 1 & 0.33 & 0.47 \\ 3.75 & 0.47 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 119.25 \\ 7.97 \\ 23.5 \end{bmatrix} = \begin{bmatrix} 5.09 \\ 7.25 \\ 1 \end{bmatrix} \quad (5)$$

Q.2 Design a new neural network with appropriate parameters, which can classify class A (output of 1) occupying the area inside the circle, $(x - 8)^2 + (y - 4)^2 = 2$, from class B (output of 0) which is outside the circle as shown in the below figure. The inputs of the network are x, x^2, y , and y^2 . (25)



Solution

The first layer is used to create a circle decision boundary with output 1 outside the circle. The second layer is used to NOT the output from layer 1.



For layer 1

The first neuron in the first layer is used to create a circle decision boundary whose the output 1 is outside the circle.

$$(x - 8)^2 + (y - 4)^2 = 2 \quad (1)$$

$$x^2 - 16x + y^2 - 8y = -76 \quad (2)$$

$$w^1 = [1 \quad -16 \quad 1 \quad -8] \quad (3)$$

$$b^1 = -76 \quad (4)$$

For layer 2

Since this is NOT layer, if we select

$$w^2 = [-1] \quad (5)$$

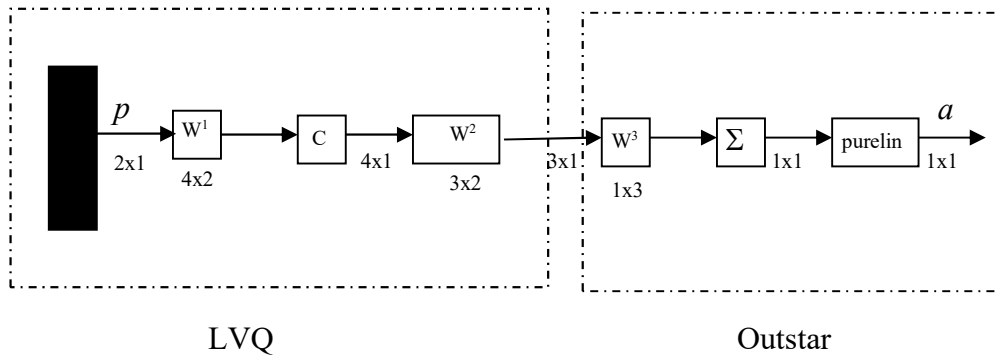
$$b^2 = 0.5 \quad (6)$$

Q.3 Design a network that can associate the following relations. $\{p_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, t_1 = [1]\}, \{p_2 = \begin{bmatrix} 3 \\ 3 \end{bmatrix}, t_2 = [2]\}, \{p_3 = \begin{bmatrix} 4 \\ 2 \end{bmatrix}, t_3 = [3]\}, \{p_4 = \begin{bmatrix} 5 \\ 4 \end{bmatrix}, t_4 = [1]\}$ (25)

Solution

The combination of LVQ with Outstar (replace satlins to purelin function) can be applied.

LVQ is used for input pattern recognition and Outstar network is used to recall the output pattern.



For LVQ

From the training sets, there are 3 classes and 4 sub classes.

In the first layer of LVQ, the weights vector will be at the center of each subclass.

$$w_1^1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad (1)$$

$$w_2^1 = \begin{bmatrix} 3 \\ 3 \end{bmatrix} \quad (2)$$

$$w_3^1 = \begin{bmatrix} 4 \\ 2 \end{bmatrix} \quad (3)$$

$$w_4^1 = \begin{bmatrix} 5 \\ 4 \end{bmatrix} \quad (4)$$

The weight matrix of the first layer is thus

$$W^1 = \begin{bmatrix} 1 & 1 \\ 3 & 3 \\ 4 & 2 \\ 5 & 4 \end{bmatrix} \quad (5)$$

The weight matrix of the second layer combines and recognizes subclasses as class. Class 1 consists of subclasses 1 and 4. Class 2 consists of subclass 2. Class 3 consists of subclass 3. respectively.

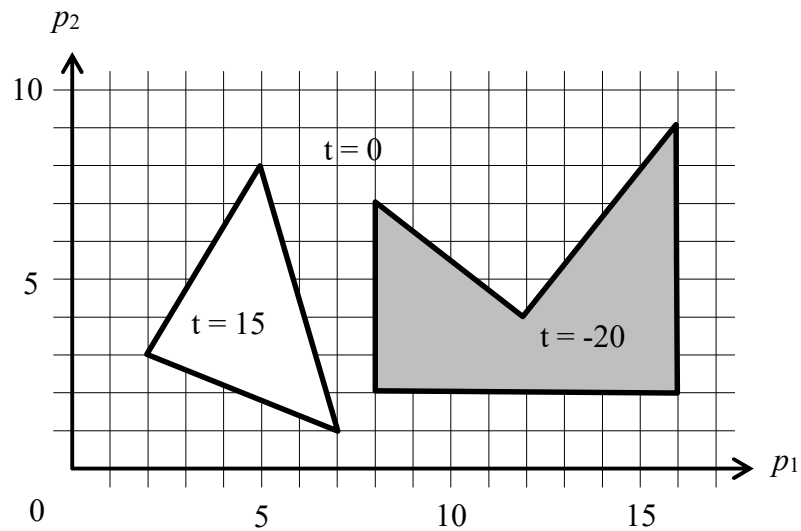
$$W^2 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \quad (6)$$

For Outstar

Parameters of Outstar network are designed as followings.

$$W^3 = [1 \quad 2 \quad 3] \quad (7)$$

Q.4 Consider the below figure, when the input is in the white area the target is 15, when the input is in the shaded area the target is -20, otherwise the target is 0.

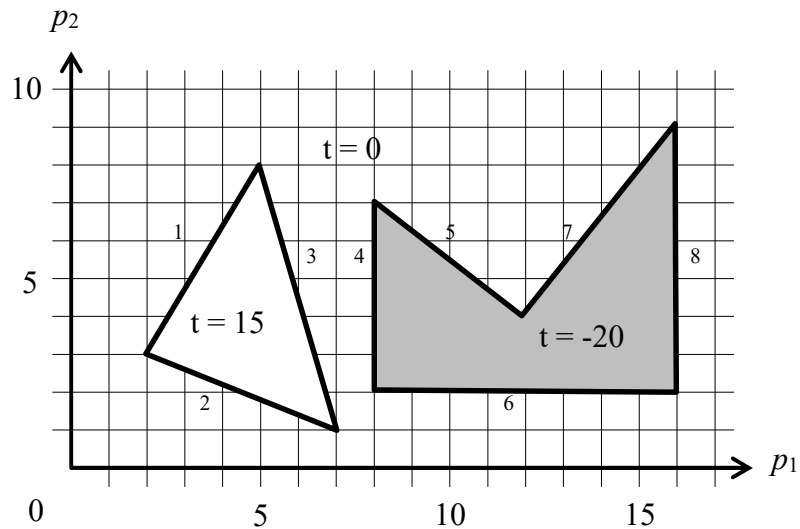


(a) Design the network which can perform this function. (5)

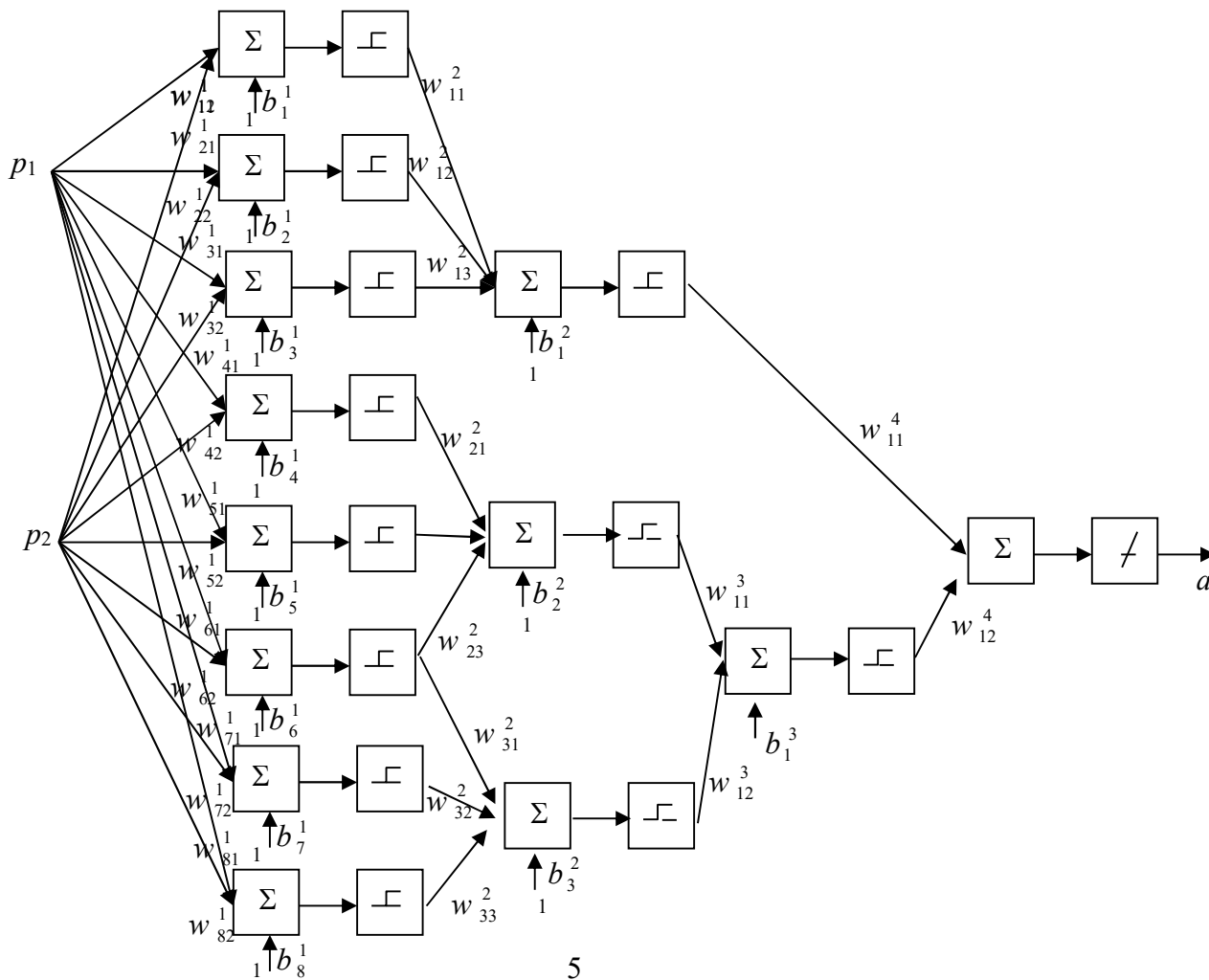
(b) Determine all the necessary parameters. (20)

Solution

(a) Each decision boundary is numbered as shown below.



Combination of Multi-Layer Perceptron and Outstar (replace satlins to purelin function) is selected.



(b)

For layer 1

Since \mathbf{w} should point into the area and it should be perpendicular to decision boundary. Thus, select

$$\mathbf{w}_1^1 = \begin{bmatrix} 5 \\ -3 \end{bmatrix} \quad (1)$$

$$\mathbf{w}_2^1 = \begin{bmatrix} 2 \\ 5 \end{bmatrix} \quad (2)$$

$$\mathbf{w}_3^1 = \begin{bmatrix} -7 \\ -2 \end{bmatrix} \quad (3)$$

$$\mathbf{w}_4^1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad (4)$$

$$\mathbf{w}_5^1 = \begin{bmatrix} -3 \\ -4 \end{bmatrix} \quad (5)$$

$$\mathbf{w}_6^1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad (6)$$

$$\mathbf{w}_7^1 = \begin{bmatrix} 5 \\ -4 \end{bmatrix} \quad (7)$$

$$\mathbf{w}_8^1 = \begin{bmatrix} -1 \\ 0 \end{bmatrix} \quad (8)$$

\mathbf{b} is determined from equating $\mathbf{n} = 0$; and solve for \mathbf{b}

$$n_1^1 = w_{11}^1 p_1 + w_{12}^1 p_2 + b_1^1 = 0 \quad (9)$$

at $p_1 = 2, p_2 = 3$;

$$5(2) - 3(3) + b_1^1 = 0; b_1^1 = -1 \quad (10)$$

$$n_2^1 = w_{21}^1 p_1 + w_{22}^1 p_2 + b_2^1 = 0 \quad (11)$$

at $p_1 = 2, p_2 = 3$;

$$2(2) + 5(3) + b_2^1 = 0; b_2^1 = -19 \quad (12)$$

$$n_3^1 = w_{31}^1 p_1 + w_{32}^1 p_2 + b_3^1 = 0 \quad (13)$$

at $p_1 = 7, p_2 = 1$;

$$-7(7) - 2(1) + b_3^1 = 0; b_3^1 = 51 \quad (14)$$

$$n_4^1 = w_{41}^1 p_1 + w_{42}^1 p_2 + b_4^1 = 0 \quad (15)$$

at $p_1 = 8, p_2 = 2$;

$$1(8) + 0(2) + b_4^1 = 0; b_4^1 = -8 \quad (16)$$

$$n_5^1 = w_{51}^1 p_1 + w_{52}^1 p_2 + b_5^1 = 0 \quad (17)$$

at $p_1 = 8, p_2 = 7$;

$$-3(8) - 4(7) + b_5^1 = 0; b_5^1 = 52 \quad (18)$$

$$n_6^1 = w_{61}^1 p_1 + w_{62}^1 p_2 + b_6^1 = 0 \quad (19)$$

at $p_1 = 8, p_2 = 2$;

$$0(8) + 1(2) + b_6^1 = 0; b_6^1 = -2 \quad (20)$$

$$n_7^1 = w_{71}^1 p_1 + w_{72}^1 p_2 + b_7^1 = 0 \quad (21)$$

at $p_1 = 16, p_2 = 9$;

$$5(16) - 4(9) + b_7^1 = 0; b_7^1 = -44 \quad (22)$$

$$n_8^1 = w_{81}^1 p_1 + w_{82}^1 p_2 + b_8^1 = 0 \quad (23)$$

at $p_1 = 16, p_2 = 2$;

$$-1(16) + 0(2) + b_8^1 = 0; b_8^1 = 16 \quad (24)$$

For layer 2

Since this is AND layer, if we select

$$w_{11}^2 = w_{12}^2 = w_{13}^2 = 1 \quad (25)$$

$$w_{21}^2 = w_{22}^2 = w_{23}^2 = 1 \quad (26)$$

$$w_{31}^2 = w_{32}^2 = w_{33}^2 = 1 \quad (27)$$

b must be selected such that only when all inputs are 1s, output is 1. Thus, select

$$b_1^2 = b_2^2 = b_3^2 = -2.5 \quad (28)$$

For layer 3

Since this is OR layer, if we select

$$w_{11}^3 = w_{12}^3 = 1 \quad (29)$$

b must be selected such that even one input is 1, output is 1. Thus, select

$$b_1^3 = -0.5 \quad (30)$$

For layer 4,

Outstar network is selected.

$$w_{11}^4 = 15 \quad (31)$$

$$w_{12}^4 = -20 \quad (32)$$