Midsem Examination AI and Neuro-Fuzzy Theory AT74.05 March 6, 2020

Time: 9:00-11:00 h. **Open Book**
Marks: 100

Attempt all questions.

Q.1 ADALINE network is applied to determine a quadratic relation between the input, $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$, and the output, $[y]$, of a system as expressed by

$$[y] = \frac{1}{2}[x_1 \quad x_2]\begin{bmatrix} a_1 & a_2 \\ a_2 & a_3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + [d_1 \quad d_2]\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + [c]$$

(a) Draw the network architecture. (5)

(b) Determine all the parameters in the quadratic relation when the data are collected and expressed as shown in the below table. (20)

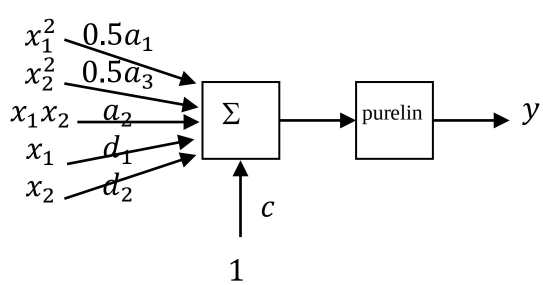
x_1	-2	-1	0	1	2	4	-4	-3	5
x_2	-4	-2	-1	-3	2	-4	5	8	0
y	16	2	3.5	10.5	38	34	39.5	104	56

Solution

(a)

Rewrite the quadratic relation

$$[y] = \frac{1}{2}[x_1 \quad x_2]\begin{bmatrix} a_1 & a_2 \\ a_2 & a_3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + [d_1 \quad d_2]\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + [c] = \frac{1}{2}a_1x_1^2 + \frac{1}{2}a_3x_2^2 + a_2x_1x_2 + d_1x_1 + d_2x_2 + c$$



(b)

$$F(x) = E[t^2] - 2x^TE[tz] + x^TE[zz^T]x = c - 2x^Th + x^TRx \quad (1)$$

$$c = \frac{1}{9}(16^2 + 2^2 + 3.5^2 + 10.5^2 + 38^2 + 34^2 + 39.5^2 + 104^2 + 56^2) = 2054.97 \quad (2)$$

$$h = \frac{1}{9}\left(16\begin{bmatrix}4\\16\\8\\-2\\-4\\1\end{bmatrix} + 2\begin{bmatrix}1\\4\\2\\-1\\-2\\1\end{bmatrix} + 3.5\begin{bmatrix}0\\1\\0\\0\\-1\\1\end{bmatrix} + 10.5\begin{bmatrix}1\\9\\-3\\1\\-3\\1\end{bmatrix} + 38\begin{bmatrix}4\\4\\4\\2\\2\\1\end{bmatrix} + 34\begin{bmatrix}16\\16\\-16\\4\\-4\\1\end{bmatrix} + 39.5\begin{bmatrix}16\\25\\-20\\-4\\5\\1\end{bmatrix} + 104\begin{bmatrix}9\\64\\-24\\-3\\8\\1\end{bmatrix} + 56\begin{bmatrix}25\\0\\0\\5\\0\\1\end{bmatrix}\right) = \begin{bmatrix}415.61\\966.83\\-397.50\\-0.17\\96.28\\33.72\end{bmatrix} \quad (3)$$

$$R = \frac{1}{9}\left(\begin{bmatrix}4\\16\\8\\-2\\-4\\1\end{bmatrix}\begin{bmatrix}4\\16\\8\\-2\\-4\\1\end{bmatrix}^T + \begin{bmatrix}1\\4\\2\\-1\\-2\\1\end{bmatrix}\begin{bmatrix}1\\4\\2\\-1\\-2\\1\end{bmatrix}^T + \begin{bmatrix}0\\1\\0\\0\\-1\\1\end{bmatrix}\begin{bmatrix}0\\1\\0\\0\\-1\\1\end{bmatrix}^T + \begin{bmatrix}1\\9\\-3\\1\\-3\\1\end{bmatrix}\begin{bmatrix}1\\9\\-3\\1\\-3\\1\end{bmatrix}^T + \begin{bmatrix}4\\4\\4\\2\\2\\1\end{bmatrix}\begin{bmatrix}4\\4\\4\\2\\2\\1\end{bmatrix}^T + \begin{bmatrix}16\\16\\-16\\4\\-4\\1\end{bmatrix}\begin{bmatrix}16\\16\\-16\\4\\-4\\1\end{bmatrix}^T + \begin{bmatrix}16\\25\\-20\\-4\\5\\1\end{bmatrix}\begin{bmatrix}16\\25\\-20\\-4\\5\\1\end{bmatrix}^T + \begin{bmatrix}9\\64\\-24\\-3\\8\\1\end{bmatrix}\begin{bmatrix}9\\64\\-24\\-3\\8\\1\end{bmatrix}^T + \begin{bmatrix}25\\0\\0\\5\\0\\1\end{bmatrix}\begin{bmatrix}25\\0\\0\\5\\0\\1\end{bmatrix}^T\right) = \begin{bmatrix}139.11 & 147.22 & -82.78 & 10.89 & 8.33 & 8.44\\147.22 & 594.11 & -240.78 & -27.44 & 53.44 & 15.44\\-82.78 & -240.78 & 147.22 & 8.33 & -27.44 & -5.44\\10.89 & -27.44 & 8.33 & 8.44 & -5.44 & 0.22\\8.33 & 53.44 & -27.44 & -5.44 & 15.44 & 0.11\\8.44 & 15.44 & -5.44 & 0.22 & 0.11 & 1.00\end{bmatrix} \quad (4)$$

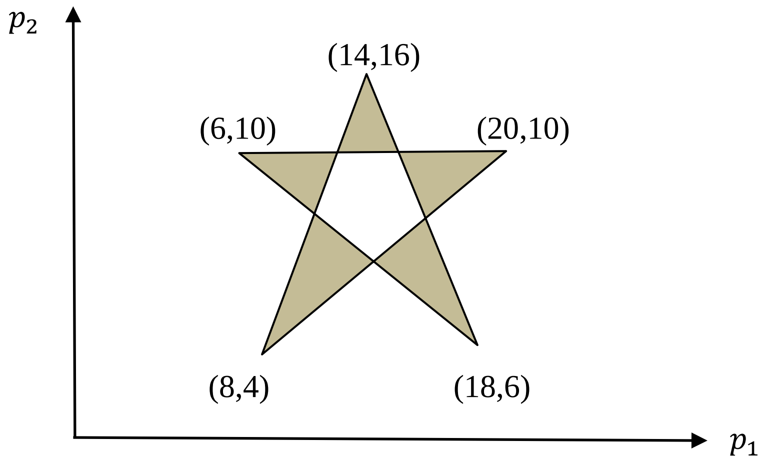
The minimum point is the stationary point of the quadratic function.

$$\begin{bmatrix}0.5a_1\\0.5a_3\\a_2\\d_1\\d_2\\c\end{bmatrix} = \begin{bmatrix}139.11 & 147.22 & -82.78 & 10.89 & 8.33 & 8.44\\147.22 & 594.11 & -240.78 & -27.44 & 53.44 & 15.44\\-82.78 & -240.78 & 147.22 & 8.33 & -27.44 & -5.44\\10.89 & -27.44 & 8.33 & 8.44 & -5.44 & 0.22\\8.33 & 53.44 & -27.44 & -5.44 & 15.44 & 0.11\\8.44 & 15.44 & -5.44 & 0.22 & 0.11 & 1.00\end{bmatrix}^{-1}\begin{bmatrix}415.61\\966.83\\-397.50\\-0.17\\96.28\\33.72\end{bmatrix} = \begin{bmatrix}1\\1.5\\1\\5\\4\\6\end{bmatrix} \quad (5)$$

$$\begin{bmatrix}a_1\\a_3\\a_2\\d_1\\d_2\\c\end{bmatrix} = \begin{bmatrix}2\\3\\1\\5\\4\\6\end{bmatrix} \quad (6)$$

$$[y] = \frac{1}{2}\begin{bmatrix}x_1 & x_2\end{bmatrix}\begin{bmatrix}2 & 1\\1 & 3\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix} + \begin{bmatrix}5 & 4\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix} + [6] \quad (7)$$

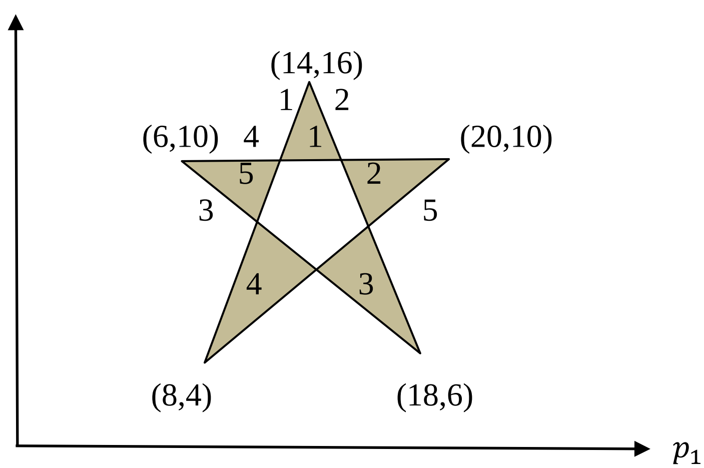
Q.2 MLP is applied to recognize the vectors in the shaded area from the other vectors when the apex coordinates of the star are given as shown in the below figure. If only 5 neurons are applied in the first layer. Design the MLP architecture, then determine all the weights and biases of the network. (25)



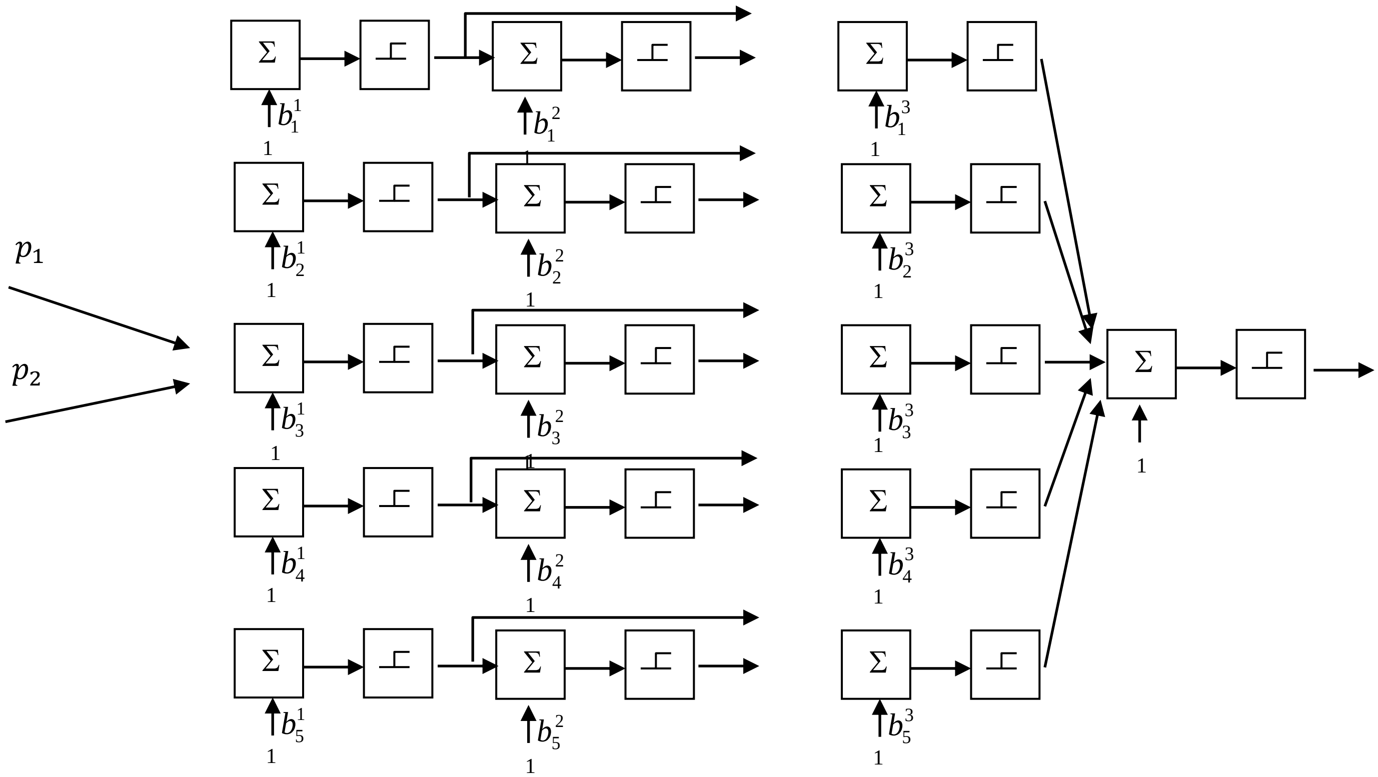
Solution

MLP is used to recognize the pattern. In the first layer, 5 neurons are used to create decision boundaries whose weight vectors point inward the star shape. In the second layer, 5 neurons are used to create decision boundaries whose weight vectors point outward the start shape. In the third layer, 5 neurons are used to AND the decision boundaries from both the first and the second layers. In the last layer, 1 neuron is used to OR the triangle polygon from the third layer.

Label decision boundaries 1 to 5 and polygon 1 to 5 as shown below.



DECISION BOUNDARY NOT AND OR



For layer 1

Weight vector is always perpendicular to decision boundary and points to group 1.

Thus, select

$$W_1^1 = [2 \quad -1] \text{ and } b_1^1 = -12$$

$$W_2^1 = [-5 \quad -2] \text{ and } b_2^1 = 102$$

$$W_3^1 = [1 \quad 3] \text{ and } b_3^1 = -36$$

$$W_4^1 = [0 \quad -1] \text{ and } b_4^1 = 10$$

$$W_5^1 = [-1 \quad 2] \text{ and } b_5^1 = 0$$

For layer 2

The neurons in the second layer perform NOT operation with one input and one output.

Select

$$W_1^2 = W_2^2 = W_3^2 = W_4^2 = W_5^2 = -1 \text{ and } b_1^2 = b_2^2 = b_3^2 = b_4^2 = b_5^2 = 0.5$$

For layer 3

The neurons in the third layer performs AND operation with three inputs and one output.

The first 5 weighs are from the first layer and the last 5 weights are from the second layer.

Select

$$W_1^3 = [1 \quad 1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 1 \quad 0] \text{ and } b_1^3 = -2.5$$

$$W_2^3 = [0 \quad 0 \quad 0 \quad 1 \quad 1 \quad 0 \quad 1 \quad 0 \quad 0 \quad 0] \text{ and } b_2^3 = -2.5$$

$$W_3^3 = [0 \quad 1 \quad 1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 1] \text{ and } b_3^3 = -2.5$$

$$W_4^3 = [1 \quad 0 \quad 0 \quad 0 \quad 1 \quad 0 \quad 0 \quad 1 \quad 0 \quad 0] \text{ and } b_4^3 = -2.5$$

$$W_5^3 = [0 \quad 0 \quad 1 \quad 1 \quad 0 \quad 1 \quad 0 \quad 0 \quad 0 \quad 0] \text{ and } b_5^3 = -2.5$$

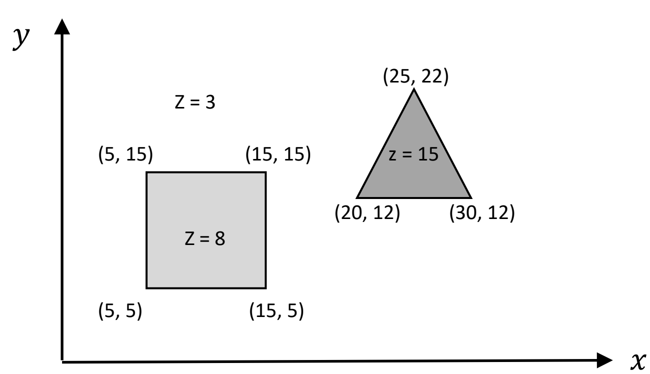
For layer 4

The neuron in the fourth layer performs OR operation with five inputs and one output.

Select

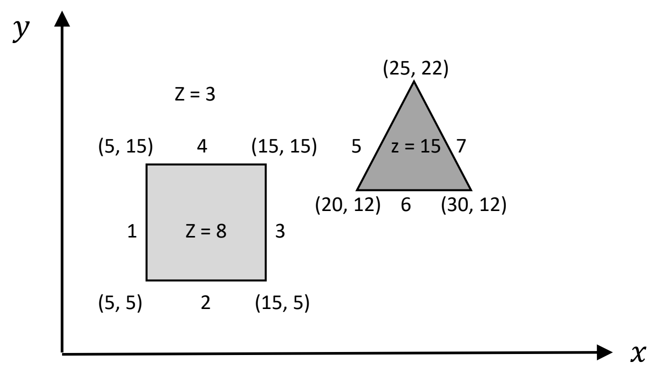
$$W_1^4 = [1 \quad 1 \quad 1 \quad 1 \quad 1] \text{ and } b_1^4 = -0.5$$

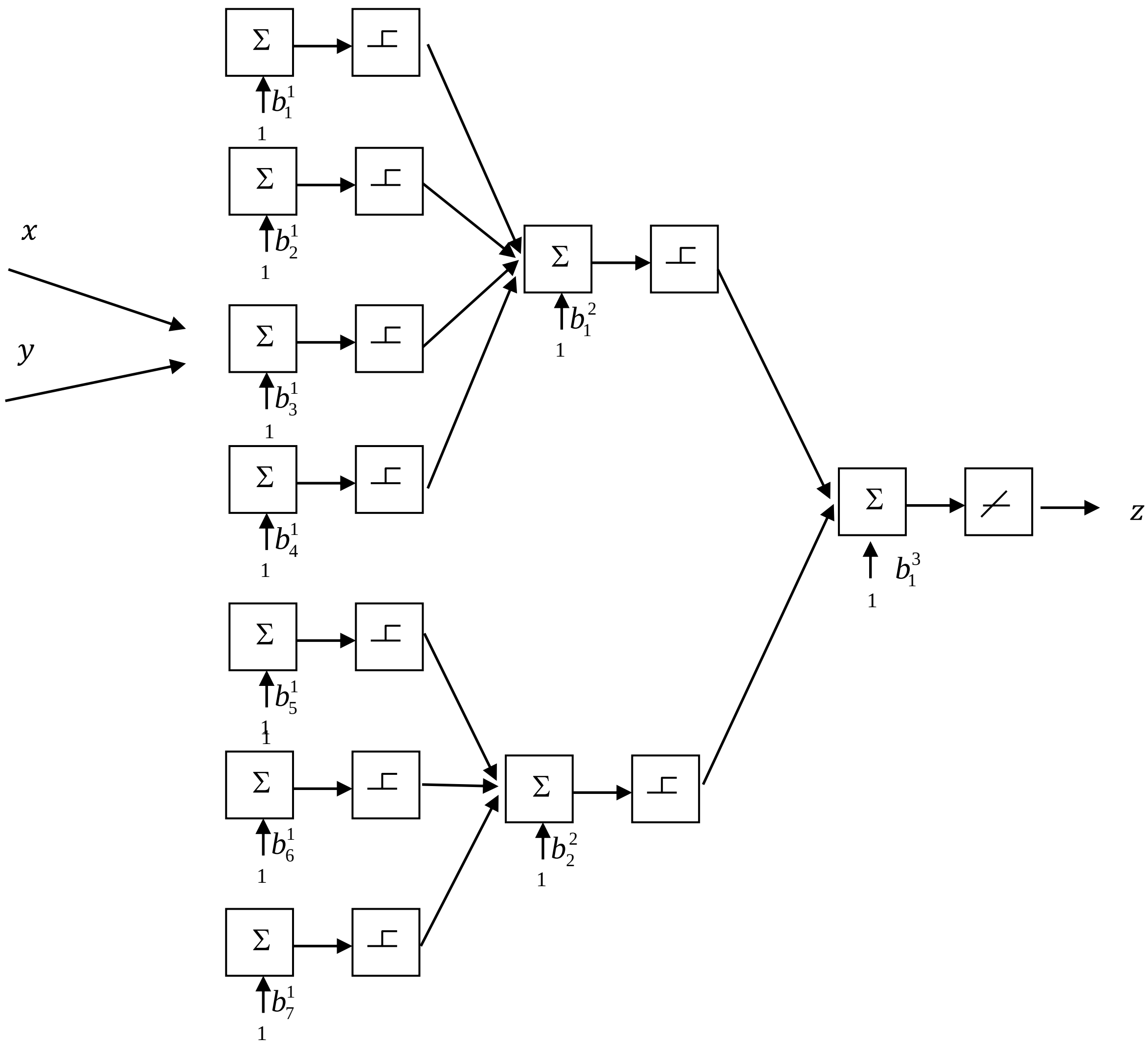
Q.3 Consider the relation of a system. When the input vector, $\begin{bmatrix} x \\ y \end{bmatrix}$, is inside the square area, the output, $[z]$, becomes 8, when the input vector is inside the rectangle area, the output becomes 15, otherwise the output becomes 3. The apex coordinates of the square and rectangle are given as shown in the below figure. Design the neural network which can create this relation. Determine all the parameters of the network. (25)



Solution

MLP can be applied. In the first layer the first 4 neurons are used to create decision boundaries for the square and the last 3 neurons are used to create decision boundaries for the triangle. In the second layer, the first neuron is used to AND the output of the neurons of the square and the second neuron is used to AND the output of the neurons of the triangle. The third layer is used to do linear superposition of the outputs from the second layer.





For layer 1

Weight vector is always perpendicular to decision boundary and points to group 1.

Thus, select

$$W_1^1 = [1 \quad 0] \text{ and } b_1^1 = -5$$

$$W_2^1 = [0 \quad 1] \text{ and } b_2^1 = -5$$

$$W_3^1 = [-1 \quad 0] \text{ and } b_3^1 = 15$$

$$W_4^1 = [0 \quad -1] \text{ and } b_4^1 = 15$$

$$W_5^1 = [2 \quad -1] \text{ and } b_5^1 = -28$$

$$W_6^1 = [0 \quad 1] \text{ and } b_2^1 = -12$$

$$W_7^1 = [-2 \quad -1] \text{ and } b_5^1 = 72$$

<u>For layer 2</u>

In the second layer, the first neuron is used to AND the output of the neurons of the square and the second neuron is used to AND the output of the neurons of the triangle.

Select

$$W_1^2 = [1 \quad 1 \quad 1 \quad 1 \quad 0 \quad 0 \quad 0] \text{ and } b_1^3 = -3.5$$

$$W_2^2 = [0 \quad 0 \quad 0 \quad 0 \quad 1 \quad 1 \quad 1] \text{ and } b_1^3 = -2.5$$

<u>For layer 3</u>

The third layer is used to do linear superposition of the outputs from the second layer. Because the output starts from 3, thus,

$$b_1^3 = 3$$

When the output from the square is 1, the output is 8. When the output from the rectangle is 1, the output is 15. Thus,

$$W_1^3 = [5 \quad 12]$$

Q.4 Linear associator network with Pseudoinverse rule is used to control a line-tracking mobile robot. An array of 3 LDR (Light Dependent Resistor), installed at the front part of the robot body, is used to detect the line. After processing the signal from LDR, the signal indicates 1 if the line is detected, and -1 if the line is not detected. The output from the network is the commands sending to 2 motors at the left and right wheels. The motor command varies from 100 (move forward at full speed) to 0 (stop). Assume that the following format of training set is applied.

$$\left\{ p = \begin{bmatrix} \text{Left LDR} \\ \text{Middle LDR} \\ \text{Right LDR} \end{bmatrix}, t = \begin{bmatrix} \text{Left Motor} \\ \text{Right Motor} \end{bmatrix} \right\}$$

(a) If the training set consists of

$$\left\{ p_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \ t_1 = \begin{bmatrix} 100 \\ 100 \end{bmatrix} \right\}, \left\{ p_2 = \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}, \ t_2 = \begin{bmatrix} 100 \\ 50 \end{bmatrix} \right\}, \left\{ p_3 = \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}, \ t_3 = \begin{bmatrix} 100 \\ 0 \end{bmatrix} \right\}$$

What is the weight from Pseudoinverse rule? Determine the output from (a) and (b) when the LDR pattern indicates

$$(1)\begin{bmatrix}-1\\-1\\-1\end{bmatrix},(2)\begin{bmatrix}1\\1\\-1\end{bmatrix},(3)\begin{bmatrix}1\\-1\\-1\end{bmatrix}. \tag{12.5}$$

(b) If the training set consists of

$$\left\{p_1=\begin{bmatrix}1\\1\\1\end{bmatrix},\ t_1=\begin{bmatrix}100\\100\end{bmatrix}\right\},\left\{p_2=\begin{bmatrix}-1\\1\\1\end{bmatrix},\ t_2=\begin{bmatrix}100\\50\end{bmatrix}\right\},\left\{p_3=\begin{bmatrix}1\\1\\-1\end{bmatrix},\ t_3=\begin{bmatrix}50\\100\end{bmatrix}\right\}$$

What is the weight from Pseudoinverse rule? Determine the output from (a) and (b) when the LDR pattern indicates

$$(1)\begin{bmatrix}-1\\-1\\-1\end{bmatrix},(2)\begin{bmatrix}-1\\-1\\1\end{bmatrix},(3)\begin{bmatrix}1\\-1\\-1\end{bmatrix}. \tag{12.5}$$

Solution

By Pseudoinverse rule,

$$W=TP^{+} \tag{1}$$

Where

$$P^{+}=(P^TP)^{-1}P^T \tag{2}$$

(a)

$$T=\begin{bmatrix}100 & 100 & 100\\100 & 50 & 0\end{bmatrix},\ P=\begin{bmatrix}1 & -1 & -1\\1 & 1 & -1\\1 & 1 & 1\end{bmatrix} \tag{3}$$

$$P^TP=\begin{bmatrix}1 & -1 & -1\\1 & 1 & -1\\1 & 1 & 1\end{bmatrix}^T\begin{bmatrix}1 & -1 & -1\\1 & 1 & -1\\1 & 1 & 1\end{bmatrix}=\begin{bmatrix}3 & 1 & -1\\1 & 3 & 1\\-1 & 1 & 3\end{bmatrix} \tag{4}$$

$$(P^TP)^{-1}=\begin{bmatrix}3 & 1 & -1\\1 & 3 & 1\\-1 & 1 & 3\end{bmatrix}^{-1}=\begin{bmatrix}0.5 & -0.25 & 0.25\\-0.25 & 0.5 & -0.25\\0.25 & -0.25 & 0.5\end{bmatrix} \tag{5}$$

$$(P^TP)^{-1}P^T=\begin{bmatrix}0.5 & -0.25 & 0.25\\-0.25 & 0.5 & -0.25\\0.25 & -0.25 & 0.5\end{bmatrix}\begin{bmatrix}1 & -1 & -1\\1 & 1 & -1\\1 & 1 & 1\end{bmatrix}^T=\begin{bmatrix}0.5 & 0 & 0.5\\-0.5 & 0.5 & 0\\0 & -0.5 & 0.5\end{bmatrix} \tag{6}$$

$$W=T(P^TP)^{-1}P^T=\begin{bmatrix}100 & 100 & 100\\100 & 50 & 0\end{bmatrix}\begin{bmatrix}0.5 & 0 & 0.5\\-0.5 & 0.5 & 0\\0 & -0.5 & 0.5\end{bmatrix}=\begin{bmatrix}0 & 0 & 100\\25 & 25 & 50\end{bmatrix} \tag{7}$$

(1) when the input is $\begin{bmatrix}-1\\-1\\-1\end{bmatrix}$,

$$a = \begin{bmatrix}0 & 0 & 100\\25 & 25 & 50\end{bmatrix}\begin{bmatrix}-1\\-1\\-1\end{bmatrix} = \begin{bmatrix}-100\\-100\end{bmatrix} \quad \underline{(8)}$$

(2) when the input is $\begin{bmatrix}1\\1\\-1\end{bmatrix}$,

$$a = \begin{bmatrix}0 & 0 & 100\\25 & 25 & 50\end{bmatrix}\begin{bmatrix}1\\1\\-1\end{bmatrix} = \begin{bmatrix}-100\\0\end{bmatrix} \quad \underline{(9)}$$

(3) when the input is $\begin{bmatrix}1\\-1\\-1\end{bmatrix}$,

$$a = \begin{bmatrix}0 & 0 & 100\\25 & 25 & 50\end{bmatrix}\begin{bmatrix}1\\-1\\-1\end{bmatrix} = \begin{bmatrix}-100\\-50\end{bmatrix} \quad \underline{(10)}$$

(b)

$$T = \begin{bmatrix}100 & 100 & 50\\100 & 50 & 100\end{bmatrix}, P = \begin{bmatrix}1 & -1 & 1\\1 & 1 & 1\\1 & 1 & -1\end{bmatrix} \quad (11)$$

$$P^T P = \begin{bmatrix}1 & -1 & 1\\1 & 1 & 1\\1 & 1 & -1\end{bmatrix}^T \begin{bmatrix}1 & -1 & 1\\1 & 1 & 1\\1 & 1 & -1\end{bmatrix} = \begin{bmatrix}3 & 1 & 1\\1 & 3 & -1\\1 & -1 & 3\end{bmatrix} \quad (12)$$

$$(P^T P)^{-1} = \begin{bmatrix}3 & 1 & 1\\1 & 3 & -1\\1 & -1 & 3\end{bmatrix}^{-1} = \begin{bmatrix}0.5 & -0.25 & -0.25\\-0.25 & 0.5 & 0.25\\-0.25 & 0.25 & 0.5\end{bmatrix} \quad (13)$$

$$(P^T P)^{-1} P^T = \begin{bmatrix}0.5 & -0.25 & -0.25\\-0.25 & 0.5 & 0.25\\-0.25 & 0.25 & 0.5\end{bmatrix}\begin{bmatrix}1 & -1 & 1\\1 & 1 & 1\\1 & 1 & -1\end{bmatrix}^T = \begin{bmatrix}0.5 & 0 & 0.5\\-0.5 & 0.5 & 0\\0 & 0.5 & -0.5\end{bmatrix} \quad (14)$$

$$W = T(P^T P)^{-1} P^T = \begin{bmatrix}100 & 100 & 50\\100 & 50 & 100\end{bmatrix}\begin{bmatrix}0.5 & 0 & 0.5\\-0.5 & 0.5 & 0\\0 & 0.5 & -0.5\end{bmatrix} = \begin{bmatrix}0 & 75 & 25\\25 & 75 & 0\end{bmatrix} \quad \underline{(15)}$$

(1) when the input is $\begin{bmatrix}-1\\-1\\-1\end{bmatrix}$,

$$a = \begin{bmatrix}0 & 75 & 25\\25 & 75 & 0\end{bmatrix}\begin{bmatrix}-1\\-1\\-1\end{bmatrix} = \begin{bmatrix}-100\\-100\end{bmatrix} \quad \underline{(16)}$$

(2) when the input is $\begin{bmatrix}-1\\-1\\1\end{bmatrix}$,

$$a = \begin{bmatrix} 0 & 75 & 25 \\ 25 & 75 & 0 \end{bmatrix} \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} -50 \\ -100 \end{bmatrix} \quad (17)$$

(3) when the input is $\begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix}$,

$$a = \begin{bmatrix} 0 & 75 & 25 \\ 25 & 75 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix} = \begin{bmatrix} -100 \\ -50 \end{bmatrix} \quad (18)$$