Midsem Examination AI and Neuro-Fuzzy Theory AT74.05 October 14, 2021

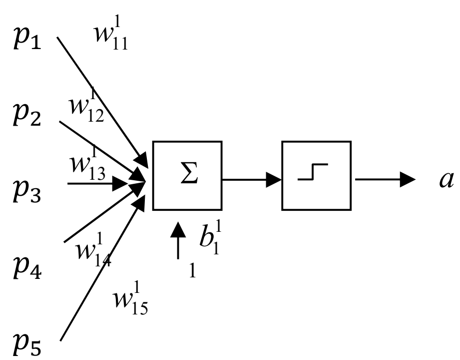
Time: 9:00-11:00 h. **Open Book**
Marks: 100

Attempt all questions.

Q.1 Design a neural network with appropriate parameters, which generates output of 1 when the input vector, $p = \begin{bmatrix}1\\2\\3\\4\\5\end{bmatrix}$, is applied, and the following points lie on the decision boundary $(1,1,1,1,1), (2,3,1,5,6), (9,7,4,2,1), (-4,-7,-3,-8,-2), (-5,-9,-3,-2,-1)$. Use the absolute value of the first element of the weight vector as 1. (25)

Solution

Single-Layer Perceptron with 5 inputs and one output is selected.



The weight vector must be orthogonal to any vectors lie on the decision boundary.

Consider a vector connecting $(1,1,1,1,1)$ and $(2,3,1,5,6)$,

$$[w^1_{11} \quad w^1_{12} \quad w^1_{13} \quad w^1_{14} \quad w^1_{15}]\begin{bmatrix}1\\2\\0\\4\\5\end{bmatrix} = [0]$$

Consider a vector connecting $(1,1,1,1,1)$ and $(9,7,4,2,1)$,

$$[w^1_{11} \quad w^1_{12} \quad w^1_{13} \quad w^1_{14} \quad w^1_{15}]\begin{bmatrix}8\\6\\3\\1\\0\end{bmatrix} = [0]$$

Consider a vector connecting $(1,1,1,1,1)$ and $(-4,-7,-3,-8,-2)$,

$$\begin{bmatrix} w_{11}^1 & w_{12}^1 & w_{13}^1 & w_{14}^1 & w_{15}^1 \end{bmatrix} \begin{bmatrix} 5 \\ 8 \\ 4 \\ 9 \\ 3 \end{bmatrix} = [0]$$

Consider a vector connecting $(1,1,1,1,1)$ and $(-5,-9,-3,-2,-1)$,

$$\begin{bmatrix} w_{11}^1 & w_{12}^1 & w_{13}^1 & w_{14}^1 & w_{15}^1 \end{bmatrix} \begin{bmatrix} 6 \\ 10 \\ 4 \\ 3 \\ 2 \end{bmatrix} = [0]$$

If w_{11}^1 is selected as 1, consider the above 4 equations in matrix form,

$$\begin{bmatrix} 2 & 0 & 4 & 5 \\ 6 & 3 & 1 & 0 \\ 8 & 4 & 9 & 3 \\ 10 & 4 & 3 & 2 \end{bmatrix} \begin{bmatrix} w_{12}^1 \\ w_{13}^1 \\ w_{14}^1 \\ w_{15}^1 \end{bmatrix} = \begin{bmatrix} -1 \\ -8 \\ -5 \\ -6 \end{bmatrix}$$

$$\begin{bmatrix} w_{12}^1 \\ w_{13}^1 \\ w_{14}^1 \\ w_{15}^1 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 4 & 5 \\ 6 & 3 & 1 & 0 \\ 8 & 4 & 9 & 3 \\ 10 & 4 & 3 & 2 \end{bmatrix}^{-1} \begin{bmatrix} -1 \\ -8 \\ -5 \\ -6 \end{bmatrix}$$

$$\begin{bmatrix} w_{12}^1 \\ w_{13}^1 \\ w_{14}^1 \\ w_{15}^1 \end{bmatrix} = \begin{bmatrix} 4.1042 \\ -11.5833 \\ 2.1250 \\ -3.5417 \end{bmatrix}$$

Thus, a possible weight vector is

$$\begin{bmatrix} w_{11}^1 & w_{12}^1 & w_{13}^1 & w_{14}^1 & w_{15}^1 \end{bmatrix} = \begin{bmatrix} 1 & 4.1042 & -11.5833 & 2.1250 & -3.5417 \end{bmatrix}$$

Consider point $(1,1,1,1,1)$ on the decision boundary. Determine the bias from

$$n_1^1 = \begin{bmatrix} 1 & 4.1042 & -11.5833 & 2.1250 & -3.5417 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} + [b_1^1] = [-7.8958 + b_1^1] = [0]$$

$$[b_1^1] = 7.8958$$

Consider the input vector $p = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{bmatrix}$ which should create the output of 1.

$$n_1^1 = \begin{bmatrix}1 & 4.1042 & -11.5833 & 2.1250 & -3.5417\end{bmatrix}\begin{bmatrix}1\\2\\3\\4\\5\end{bmatrix} + [7.8958] = [-26.8542]$$

However, this set of weight creates the output of 0 by hardlim function.

The actual weight vector should point in the opposite direction of the current one.

Thus, a weight vector is

$$\begin{bmatrix}\mathrm{w}_{11}^1 & \mathrm{w}_{12}^1 & \mathrm{w}_{13}^1 & \mathrm{w}_{14}^1 & \mathrm{w}_{15}^1\end{bmatrix} = \begin{bmatrix}-1 & -4.1042 & 11.5833 & -2.1250 & 3.5417\end{bmatrix}$$

Consider point (1,1,1,1,1) on the decision boundary. Determine the bias from

$$n_1^1 = \begin{bmatrix}-1 & -4.1042 & 11.5833 & -2.1250 & 3.5417\end{bmatrix}\begin{bmatrix}1\\1\\1\\1\\1\end{bmatrix} + [\mathrm{b}_1^1] = [7.8958 + \mathrm{b}_1^1] = [0]$$

$$[\mathrm{b}_1^1] = -7.8958$$

Consider the input vector $p = \begin{bmatrix}1\\2\\3\\4\\5\end{bmatrix}$ which should create the output of 1.

$$n_1^1 = \begin{bmatrix}-1 & -4.1042 & 11.5833 & -2.1250 & 3.5417\end{bmatrix}\begin{bmatrix}1\\2\\3\\4\\5\end{bmatrix} + [-7.8958] = [26.8542]$$

This set of weight creates the output of 1 by hardlim function correctly.

In conclusion, a weight vector is

$$\begin{bmatrix}\mathrm{w}_{11}^1 & \mathrm{w}_{12}^1 & \mathrm{w}_{13}^1 & \mathrm{w}_{14}^1 & \mathrm{w}_{15}^1\end{bmatrix} = \begin{bmatrix}-1 & -4.1042 & 11.5833 & -2.1250 & 3.5417\end{bmatrix}$$

The bias is

$$[\mathrm{b}_1^1] = -7.8958$$

Q.2 Design a network with appropriate parameters, that minimized the mean squared error when the data are presented equally of the relation between the inputs; x_1, x_2, x_3, and the output, y, of a system which is expressed by

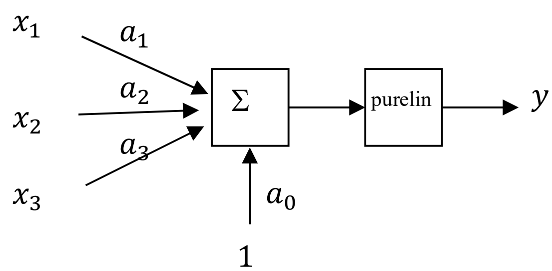
$$y = a_0 + a_1 x_1 + a_2\, x_2 + a_3\, x_3$$

When the data with noises are collected and expressed as shown in the below table. (25)

x_1	1	1	1	3	5
x_2	1	1	3	3	5
x_3	1	3	5	7	9
y	0.1	-7.8	-6.1	-9.7	-4.2

Solution

ADALINE Network is selected.



$$F(x) = E[t^2] - 2x^T E[tz] + x^T E[zz^T]x = c - 2x^T h + x^T R x \quad (1)$$

$$c = \frac{1}{5}((0.1)^2 + (-7.8)^2 + (-6.1)^2 + (-9.7)^2 + (-4.2)^2) = 41.96 \quad (2)$$

$$h = \frac{1}{5}\left(0.1\begin{bmatrix}1\\1\\1\\1\end{bmatrix} - 7.8\begin{bmatrix}1\\1\\3\\1\end{bmatrix} - 6.1\begin{bmatrix}1\\3\\5\\1\end{bmatrix} - 9.7\begin{bmatrix}3\\3\\7\\1\end{bmatrix} - 4.2\begin{bmatrix}5\\5\\9\\1\end{bmatrix}\right) = \begin{bmatrix}-12.78\\-15.22\\-31.90\\-5.54\end{bmatrix} \quad (3)$$

$$R = \frac{1}{5}\left(\begin{bmatrix}1\\1\\1\\1\end{bmatrix}\begin{bmatrix}1\\1\\1\\1\end{bmatrix}^T + \begin{bmatrix}1\\1\\3\\1\end{bmatrix}\begin{bmatrix}1\\1\\3\\1\end{bmatrix}^T + \begin{bmatrix}1\\3\\5\\1\end{bmatrix}\begin{bmatrix}1\\3\\5\\1\end{bmatrix}^T + \begin{bmatrix}3\\3\\7\\1\end{bmatrix}\begin{bmatrix}3\\3\\7\\1\end{bmatrix}^T + \begin{bmatrix}5\\5\\9\\1\end{bmatrix}\begin{bmatrix}5\\5\\9\\1\end{bmatrix}^T\right) = \begin{bmatrix}7.4 & 7.8 & 15.0 & 2.2\\7.8 & 9.0 & 17.0 & 2.6\\15.0 & 17.0 & 33.0 & 5.0\\2.2 & 2.6 & 5.0 & 1.0\end{bmatrix} \quad (4)$$

The parameters are determined.

$$\begin{bmatrix}a_1\\a_2\\a_3\\a_0\end{bmatrix} = \begin{bmatrix}7.4 & 7.8 & 15.0 & 2.2\\7.8 & 9.0 & 17.0 & 2.6\\15.0 & 17.0 & 33.0 & 5.0\\2.2 & 2.6 & 5.0 & 1.0\end{bmatrix}^{-1}\begin{bmatrix}-12.78\\-15.22\\-31.90\\-5.54\end{bmatrix} = \begin{bmatrix}2.03\\4.68\\-3.88\\-2.78\end{bmatrix} \quad (5)$$

Q.3 Design a network that can implement the following training set.

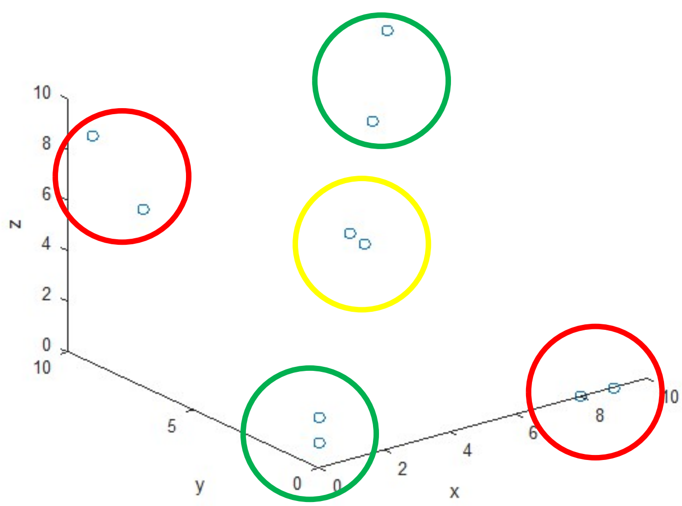
$$\left\{p_1=\begin{bmatrix}0\\7\\7\end{bmatrix},t_1=\begin{bmatrix}1\\0\\0\end{bmatrix}\right\},\ \left\{p_2=\begin{bmatrix}0\\9\\9\end{bmatrix},t_2=\begin{bmatrix}1\\0\\0\end{bmatrix}\right\},\left\{p_3=\begin{bmatrix}8\\0\\0\end{bmatrix},t_3=\begin{bmatrix}1\\0\\0\end{bmatrix}\right\},\left\{p_4=\begin{bmatrix}9\\0\\0\end{bmatrix},t_4=\begin{bmatrix}1\\0\\0\end{bmatrix}\right\},$$

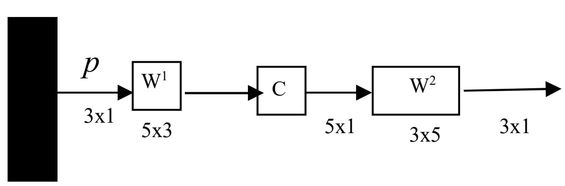
$$\left\{p_5=\begin{bmatrix}7\\7\\8\end{bmatrix},t_5=\begin{bmatrix}0\\1\\0\end{bmatrix}\right\},\left\{p_6=\begin{bmatrix}9\\9\\10\end{bmatrix},t_6=\begin{bmatrix}0\\1\\0\end{bmatrix}\right\},\left\{p_7=\begin{bmatrix}0\\0\\1\end{bmatrix},t_7=\begin{bmatrix}0\\1\\0\end{bmatrix}\right\},\left\{p_8=\begin{bmatrix}0\\0\\2\end{bmatrix},t_8=\begin{bmatrix}0\\1\\0\end{bmatrix}\right\},$$

$$\left\{p_9=\begin{bmatrix}4\\4\\6\end{bmatrix},t_9=\begin{bmatrix}0\\0\\1\end{bmatrix}\right\},\left\{p_{10}=\begin{bmatrix}6\\6\\4\end{bmatrix},t_{10}=\begin{bmatrix}0\\0\\1\end{bmatrix}\right\} \tag{25}$$

Solution

Based on the problem space as shown below. This is not a convex problem space, LVQ is used. Class 1, consisting of subclass no 1 with data no 1 and 2, and subclass no 2 with data no 3 and 4, is in red area. Class 2, consisting of subclass no 3 with data no 5 and 6, and subclass no 4 with data no 7 and 8, is in green area. Class 3, consisting of subclass no 5 with data no 9 and 10, is in yellow area.





For LVQ

From the training sets, there are 3 classes and 5 subclasses.

In the first layer of LVQ, the weights vector will be at the center of each subclass.

$$w_1^1 = 0.5\left(\begin{bmatrix}0\\7\\7\end{bmatrix} + \begin{bmatrix}0\\9\\9\end{bmatrix}\right) = \begin{bmatrix}0\\8\\8\end{bmatrix} \tag{1}$$

$$w_2^1 = 0.5\left(\begin{bmatrix}8\\0\\0\end{bmatrix} + \begin{bmatrix}9\\0\\0\end{bmatrix}\right) = \begin{bmatrix}8.5\\0\\0\end{bmatrix} \tag{2}$$

$$w_3^1 = 0.5\left(\begin{bmatrix}7\\7\\8\end{bmatrix} + \begin{bmatrix}9\\9\\10\end{bmatrix}\right) = \begin{bmatrix}8\\8\\9\end{bmatrix} \tag{3}$$

$$w_4^1 = 0.5\left(\begin{bmatrix}0\\0\\1\end{bmatrix} + \begin{bmatrix}0\\0\\2\end{bmatrix}\right) = \begin{bmatrix}0\\0\\1.5\end{bmatrix} \tag{4}$$

$$w_5^1 = 0.5\left(\begin{bmatrix}4\\4\\6\end{bmatrix} + \begin{bmatrix}6\\6\\4\end{bmatrix}\right) = \begin{bmatrix}5\\5\\5\end{bmatrix} \tag{5}$$

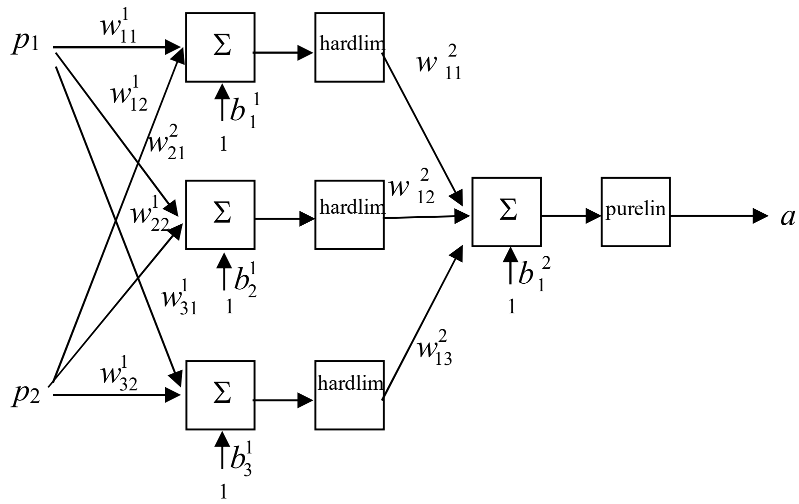
The weight matrix of the first layer is thus

$$W^1 = \begin{bmatrix}0 & 8 & 8\\8.5 & 0 & 0\\8 & 8 & 9\\0 & 0 & 1.5\\5 & 5 & 5\end{bmatrix} \tag{6}$$

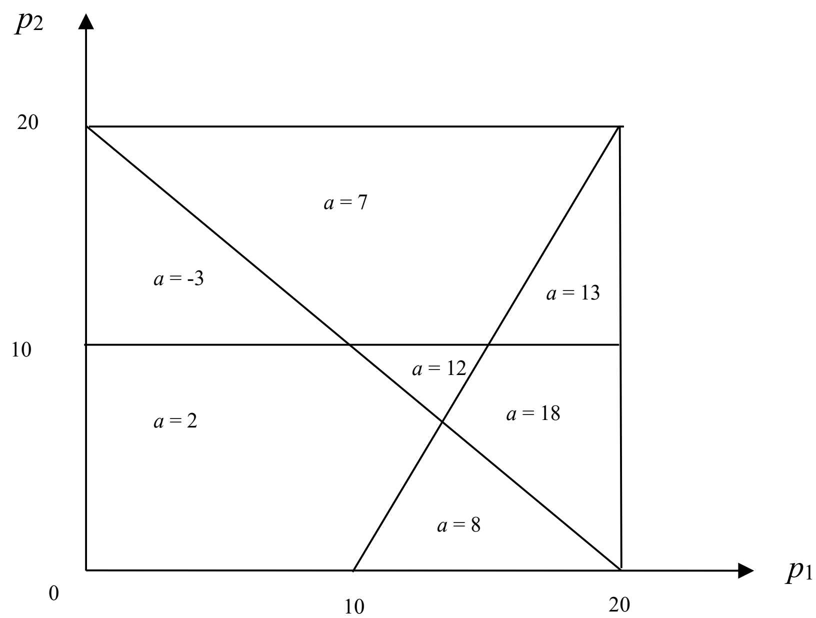
The weight matrix of the second layer combines and recognizes subclasses as class. Class 1 consists of subclasses 1 and 2. Class 2 consists of subclass 3 and 4. Class 3 consists of subclass 3. respectively.

$$W^2 = \begin{bmatrix}1 & 1 & 0 & 0 & 0\\0 & 0 & 1 & 1 & 0\\0 & 0 & 0 & 0 & 1\end{bmatrix} \tag{7}$$

Q.4 If a 2-3-1 MLP network as shown in the first figure is used to approximate the input-output relation as shown in the second figure, determine one set of the possible weights and biases of the network that generates the output correctly. (25)



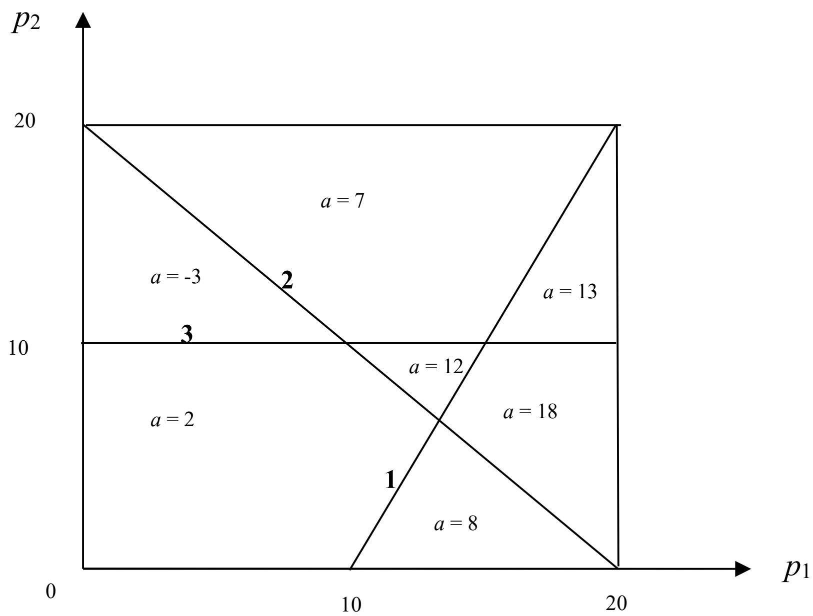
2-3-1 Network



Input-Output Relation

Solution

The function can be approximated as the superposition of three 2-D log-sigmoid function. The decision boundaries of the hardlim function are shown by straight lines labeling with 1, 2, and 3 respectively. The first decision boundary follows a straight line $2p_1 - p_2 - 20 = 0$ with the output span of 6. The second decision boundary follows a straight line $p_2 + p_1 - 20 = 0$ with the output span of 10. The third decision boundary follows a straight line $-p_2 + 10 = 0$ with the output span of 5.



Input-Output Relation

Weight and bias of the first decision boundary, which follows a line $2p_1 - p_2 - 20 = 0$, are determined from

$$n_1^1 = w_{11}^1 p_1 + w_{12}^1 p_2 + b_1^1 = 0 \tag{1}$$

Thus

$$w_{11}^1 = 2, w_{12}^1 = -1, b_1^1 = -20 \tag{2}$$

Weight and bias of the second decision boundary, which follows a line $p_2 + p_1 - 20 = 0$, are determined from

$$n_2^1 = w_{21}^1 p_1 + w_{22}^1 p_2 + b_2^1 = 0 \tag{3}$$

Thus

$$w_{21}^1 = 1, w_{22}^1 = 1, b_2^1 = -20 \tag{4}$$

Weight and bias of the third decision boundary, which follows a line $-p_2 + 10 = 0$, are determined from

$$n_3^1 = w_{31}^1 p_1 + w_{32}^1 p_2 + b_3^1 = 0 \quad (5)$$

Thus

$$w_{31}^1 = 0, w_{32}^1 = -1, b_3^1 = 10 \quad \underline{(6)}$$

Since output span of the first hardlim function is 6, thus

$$w_{11}^2 = 6 \quad \underline{(7)}$$

Since output span of the second hardlim function is 10, thus

$$w_{12}^2 = 10 \quad \underline{(8)}$$

Since output span of the third hardlim function is 5, thus

$$w_{13}^2 = 5 \quad \underline{(9)}$$

Since the plot starts from the output magnitude of 2, thus

$$b_1^2 = 2 \quad \underline{(10)}$$