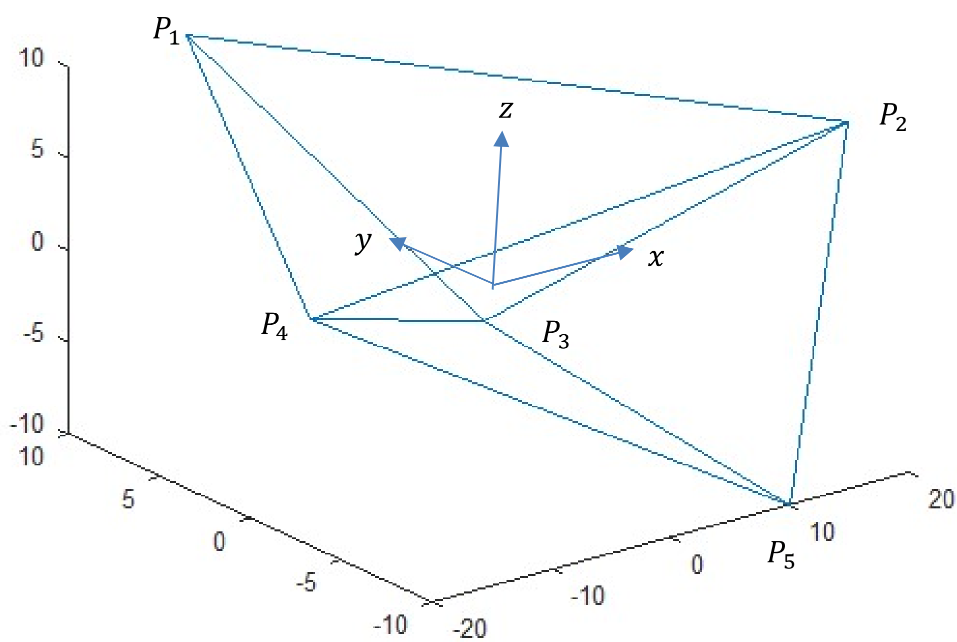
Midsem Examination **AI and Neuro-Fuzzy Theory AT74.05** **October 6, 2022**

Time: 9:00-11:00 h. **Open Book**
Marks: 100

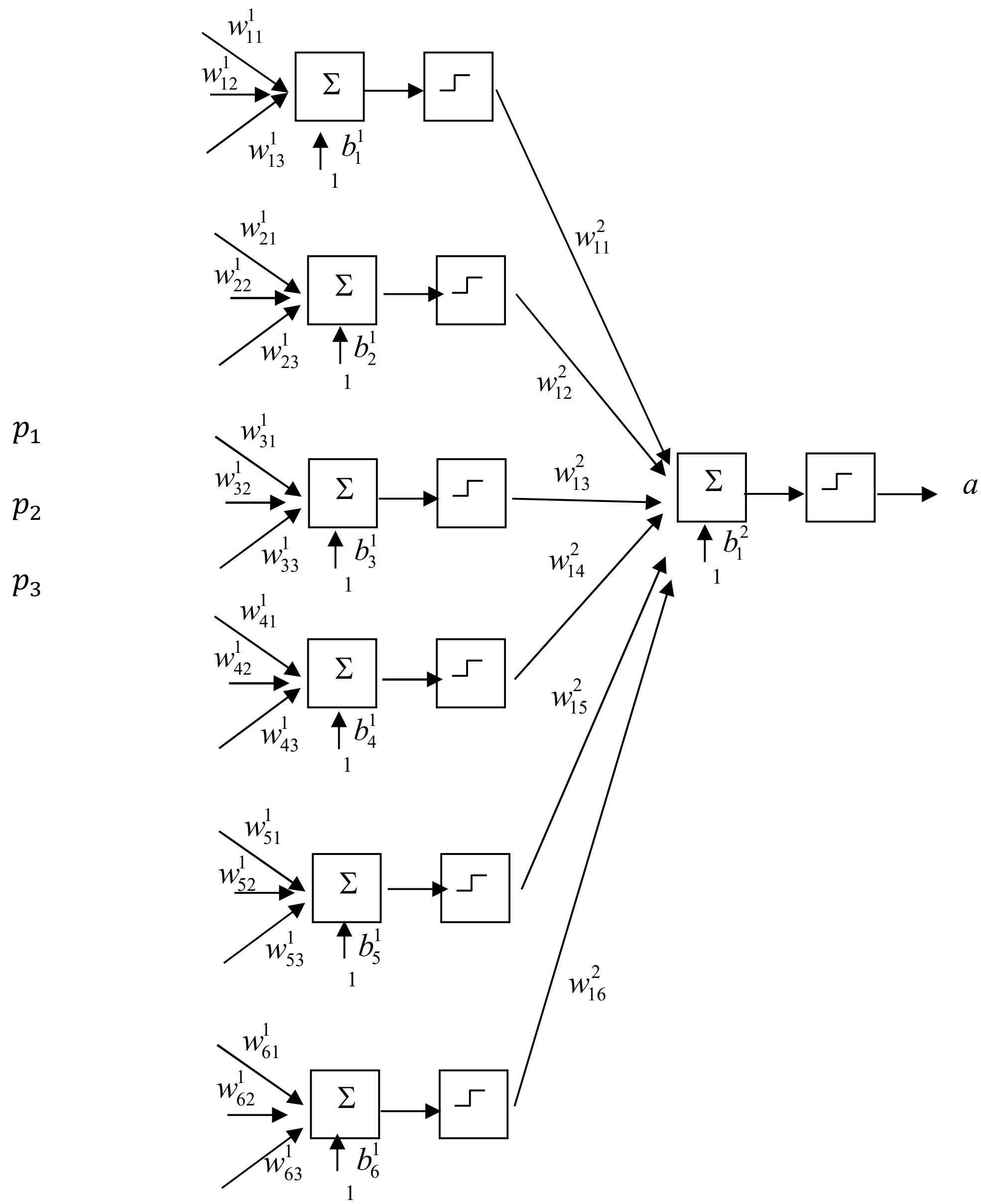
Attempt all questions.

Q.1 Design a neural network with all the parameters, which can generate the output of 1 when the input vector, p, is inside the polygon and the output of 0 when the input vector is outside the polygon. The coordinates of the apexes are given as follows: $P_1 = (-10,10,10), P_2 = (15,-10,10), P_3 = (15,10,-10), P_4 = (-15,0,0), P_5 = (10,-10,-10)$. Use only integer for each element in all the weight vectors. (25)



Solution

Since the polygon is convex, Two-Layer Perceptron (3-6-1) is selected.



Layer 1: There are 6 decision boundaries in layer 1.

Decision boundary 1 contains P_1, P_2, and P_3

The weight vector must be orthogonal to any vectors lie on the decision boundary.

Consider a vector connecting P_1 and P_2,

$$[w_{11}^1 \quad w_{12}^1 \quad w_{13}^1]\begin{bmatrix}25\\-20\\0\end{bmatrix} = [0]$$

Consider a vector connecting P_1 and P_3,

$$[w_{11}^1 \quad w_{12}^1 \quad w_{13}^1]\begin{bmatrix}25\\0\\-20\end{bmatrix} = [0]$$

If w_{11}^1 is firstly selected as -1, consider the above 2 equations in matrix form,

$$\begin{bmatrix}-20 & 0\\0 & -20\end{bmatrix}\begin{bmatrix}w_{12}^1\\w_{13}^1\end{bmatrix} = \begin{bmatrix}25\\25\end{bmatrix}$$

$$\begin{bmatrix}w_{12}^1\\w_{13}^1\end{bmatrix} = \begin{bmatrix}-20 & 0\\0 & -20\end{bmatrix}^{-1}\begin{bmatrix}25\\25\end{bmatrix}$$

$$\begin{bmatrix}w_{12}^1\\w_{13}^1\end{bmatrix} = \begin{bmatrix}-\frac{5}{4}\\-\frac{5}{4}\end{bmatrix}$$

Thus, a possible weight vector is

$$[w_{11}^1 \quad w_{12}^1 \quad w_{13}^1] = \begin{bmatrix}-1 & -\frac{5}{4} & -\frac{5}{4}\end{bmatrix}$$

Convert each element of the weight vector to integer

$$[w_{11}^1 \quad w_{12}^1 \quad w_{13}^1] = [-4 \quad -5 \quad -5]$$

Consider P_1 on the decision boundary. Determine the bias from

$$n_1^1 = [-4 \quad -5 \quad -5]\begin{bmatrix}-10\\10\\10\end{bmatrix} + [b_1^1] = [-60 + b_1^1] = [0]$$

$$[b_1^1] = [60]$$

Decision boundary 2 contains P_1, P_3, and P_4

The weight vector must be orthogonal to any vectors lie on the decision boundary.

Consider a vector connecting P_1 and P_3,

$$[w_{21}^1 \quad w_{22}^1 \quad w_{23}^1]\begin{bmatrix}25\\0\\-20\end{bmatrix} = [0]$$

Consider a vector connecting P_1 and P_4,

$$\begin{bmatrix} w_{21}^1 & w_{22}^1 & w_{23}^1 \end{bmatrix} \begin{bmatrix} 5 \\ 10 \\ 10 \end{bmatrix} = [0]$$

If w_{22}^1 is firstly selected as -1, consider the above 2 equations in matrix form,

$$\begin{bmatrix} 25 & -20 \\ 5 & 10 \end{bmatrix} \begin{bmatrix} w_{21}^1 \\ w_{23}^1 \end{bmatrix} = \begin{bmatrix} 0 \\ 10 \end{bmatrix}$$

$$\begin{bmatrix} w_{21}^1 \\ w_{23}^1 \end{bmatrix} = \begin{bmatrix} 25 & -20 \\ 5 & 10 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 10 \end{bmatrix}$$

$$\begin{bmatrix} w_{21}^1 \\ w_{23}^1 \end{bmatrix} = \begin{bmatrix} \frac{4}{7} \\ \frac{5}{7} \end{bmatrix}$$

Thus, a possible weight vector is $\begin{bmatrix} w_{21}^1 & w_{22}^1 & w_{23}^1 \end{bmatrix} = \begin{bmatrix} \frac{4}{7} & -1 & \frac{5}{7} \end{bmatrix}$

Convert each element of the weight vector to integer

$$\begin{bmatrix} w_{21}^1 & w_{22}^1 & w_{23}^1 \end{bmatrix} = \begin{bmatrix} 4 & -7 & 5 \end{bmatrix}$$

Consider P_1 on the decision boundary. Determine the bias from

$$n_2^1 = \begin{bmatrix} 4 & -7 & 5 \end{bmatrix} \begin{bmatrix} -10 \\ 10 \\ 10 \end{bmatrix} + [b_2^1] = [-60 + b_2^1] = [0]$$

$$[b_2^1] = [60]$$

Decision boundary 3 contains P_1, P_4, and P_2

The weight vector must be orthogonal to any vectors lie on the decision boundary.

Consider a vector connecting P_1 and P_4,

$$\begin{bmatrix} w_{31}^1 & w_{32}^1 & w_{33}^1 \end{bmatrix} \begin{bmatrix} 5 \\ 10 \\ 10 \end{bmatrix} = [0]$$

Consider a vector connecting P_1 and P_2,

$$\begin{bmatrix} w_{31}^1 & w_{32}^1 & w_{33}^1 \end{bmatrix} \begin{bmatrix} 25 \\ -20 \\ 0 \end{bmatrix} = [0]$$

If w_{32}^1 is firstly selected as 1, consider the above 2 equations in matrix form,

$$\begin{bmatrix} 5 & 10 \\ 25 & 0 \end{bmatrix} \begin{bmatrix} w_{31}^1 \\ w_{33}^1 \end{bmatrix} = \begin{bmatrix} -10 \\ 20 \end{bmatrix}$$

$$\begin{bmatrix} w_{31}^1 \\ w_{33}^1 \end{bmatrix} = \begin{bmatrix} 5 & 10 \\ 25 & 0 \end{bmatrix}^{-1} \begin{bmatrix} -10 \\ 20 \end{bmatrix}$$

$$\begin{bmatrix} w_{31}^1 \\ w_{33}^1 \end{bmatrix} = \begin{bmatrix} \frac{4}{5} \\ -\frac{7}{5} \end{bmatrix}$$

Thus, a possible weight vector is $\begin{bmatrix} w_{31}^1 & w_{32}^1 & w_{33}^1 \end{bmatrix} = \begin{bmatrix} \frac{4}{5} & 1 & -\frac{7}{5} \end{bmatrix}$

Convert each element of the weight vector to integer

$$\begin{bmatrix} w_{21}^1 & w_{22}^1 & w_{23}^1 \end{bmatrix} = \begin{bmatrix} 4 & 5 & -7 \end{bmatrix}$$

Consider P_1 on the decision boundary. Determine the bias from

$$n_3^1 = \begin{bmatrix} 4 & 5 & -7 \end{bmatrix} \begin{bmatrix} -10 \\ 10 \\ 10 \end{bmatrix} + [b_3^1] = [-60 + b_3^1] = [0]$$

$$[b_3^1] = [60]$$

Decision boundary 4 contains P_5, P_2, and P_3

The weight vector must be orthogonal to any vectors lie on the decision boundary.

Consider a vector connecting P_5 and P_2,

$$\begin{bmatrix} w_{41}^1 & w_{42}^1 & w_{43}^1 \end{bmatrix} \begin{bmatrix} 5 \\ 0 \\ 20 \end{bmatrix} = [0]$$

Consider a vector connecting P_5 and P_3,

$$\begin{bmatrix} w_{41}^1 & w_{42}^1 & w_{43}^1 \end{bmatrix} \begin{bmatrix} 5 \\ 20 \\ 0 \end{bmatrix} = [0]$$

If w_{41}^1 is firstly selected as -1, consider the above 2 equations in matrix form,

$$\begin{bmatrix} 0 & 20 \\ 20 & 0 \end{bmatrix} \begin{bmatrix} w_{42}^1 \\ w_{43}^1 \end{bmatrix} = \begin{bmatrix} 5 \\ 5 \end{bmatrix}$$

$$\begin{bmatrix} w_{42}^1 \\ w_{43}^1 \end{bmatrix} = \begin{bmatrix} 0 & 20 \\ 20 & 0 \end{bmatrix}^{-1} \begin{bmatrix} 5 \\ 5 \end{bmatrix}$$

$$\begin{bmatrix} w_{42}^1 \\ w_{43}^1 \end{bmatrix} = \begin{bmatrix} \frac{1}{4} \\ \frac{1}{4} \end{bmatrix}$$

Thus, a possible weight vector is $\begin{bmatrix} w_{41}^1 & w_{42}^1 & w_{43}^1 \end{bmatrix} = \begin{bmatrix} -1 & \frac{1}{4} & \frac{1}{4} \end{bmatrix}$

Convert each element of the weight vector to integer

$$[w^1_{41} \quad w^1_{42} \quad w^1_{43}] = [-4 \quad 1 \quad 1]$$

Consider P_5 on the decision boundary. Determine the bias from

$$n^1_4 = [-4 \quad 1 \quad 1]\begin{bmatrix} 10 \\ -10 \\ -10 \end{bmatrix} + [b^1_4] = [-60 + b^1_4] = [0]$$

$$[b^1_4] = [60]$$

Decision boundary 5 contains P_5, P_3, and P_4

The weight vector must be orthogonal to any vectors lie on the decision boundary.

Consider a vector connecting P_5 and P_3,

$$[w^1_{51} \quad w^1_{52} \quad w^1_{53}]\begin{bmatrix} 5 \\ 20 \\ 0 \end{bmatrix} = [0]$$

Consider a vector connecting P_5 and P_4,

$$[w^1_{51} \quad w^1_{52} \quad w^1_{53}]\begin{bmatrix} 25 \\ -10 \\ -10 \end{bmatrix} = [0]$$

If w^1_{53} is firstly selected as 1, consider the above 2 equations in matrix form,

$$\begin{bmatrix} 5 & 20 \\ 25 & -10 \end{bmatrix}\begin{bmatrix} w^1_{51} \\ w^1_{\mathbf{52}} \end{bmatrix} = \begin{bmatrix} 0 \\ 10 \end{bmatrix}$$

$$\begin{bmatrix} w^1_{51} \\ w^1_{\mathbf{52}} \end{bmatrix} = \begin{bmatrix} 5 & 20 \\ 25 & -10 \end{bmatrix}^{-1}\begin{bmatrix} 0 \\ 10 \end{bmatrix}$$

$$\begin{bmatrix} w^1_{51} \\ w^1_{\mathbf{52}} \end{bmatrix} = \begin{bmatrix} \frac{4}{11} \\ -\frac{1}{11} \end{bmatrix}$$

Thus, a possible weight vector is $[w^1_{51} \quad w^1_{52} \quad w^1_{53}] = \begin{bmatrix} \frac{4}{11} & -\frac{1}{11} & 1 \end{bmatrix}$

Convert each element of the weight vector to integer

$$[w^1_{51} \quad w^1_{52} \quad w^1_{53}] = [4 \quad -1 \quad 11]$$

Consider P_5 on the decision boundary. Determine the bias from

$$n^1_5 = [4 \quad -1 \quad 11]\begin{bmatrix} 10 \\ -10 \\ -10 \end{bmatrix} + [b^1_5] = [-60 + b^1_5] = [0]$$

$$[b_5^1] = [60]$$

Decision boundary 6 contains P_5, P_4, and P_2

The weight vector must be orthogonal to any vectors lie on the decision boundary.

Consider a vector connecting P_5 and P_4,

$$[w_{61}^1 \quad w_{62}^1 \quad w_{63}^1]\begin{bmatrix}25\\-10\\-10\end{bmatrix} = [0]$$

Consider a vector connecting P_5 and P_2,

$$[w_{61}^1 \quad w_{62}^1 \quad w_{63}^1]\begin{bmatrix}5\\0\\20\end{bmatrix} = [0]$$

If w_{62}^1 is firstly selected as 1, consider the above 2 equations in matrix form,

$$\begin{bmatrix}25 & -10\\5 & 20\end{bmatrix}\begin{bmatrix}w_{61}^1\\w_{63}^1\end{bmatrix} = \begin{bmatrix}10\\0\end{bmatrix}$$

$$\begin{bmatrix}w_{61}^1\\w_{63}^1\end{bmatrix} = \begin{bmatrix}25 & -10\\5 & 20\end{bmatrix}^{-1}\begin{bmatrix}10\\0\end{bmatrix}$$

$$\begin{bmatrix}w_{61}^1\\w_{63}^1\end{bmatrix} = \begin{bmatrix}\frac{4}{11}\\-\frac{1}{11}\end{bmatrix}$$

Thus, a possible weight vector is $[w_{61}^1 \quad w_{62}^1 \quad w_{63}^1] = \begin{bmatrix}\frac{4}{11} & 1 & -\frac{1}{11}\end{bmatrix}$

Convert each element of the weight vector to integer

$$[w_{61}^1 \quad w_{62}^1 \quad w_{63}^1] = [4 \quad 11 \quad -1]$$

Consider P_5 on the decision boundary. Determine the bias from

$$n_6^1 = [4 \quad 11 \quad -1]\begin{bmatrix}10\\-10\\-10\end{bmatrix} + [b_6^1] = [-60 + b_6^1] = [0]$$

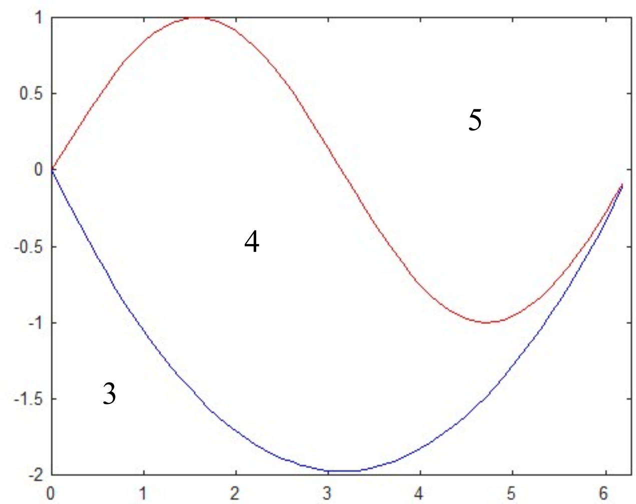
$$[b_6^1] = [60]$$

Layer 2: AND Layer

$$[w_{11}^2 \quad w_{12}^2 \quad w_{13}^2 \quad w_{14}^2 \quad w_{15}^2 \quad w_{16}^2] = [1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1]$$

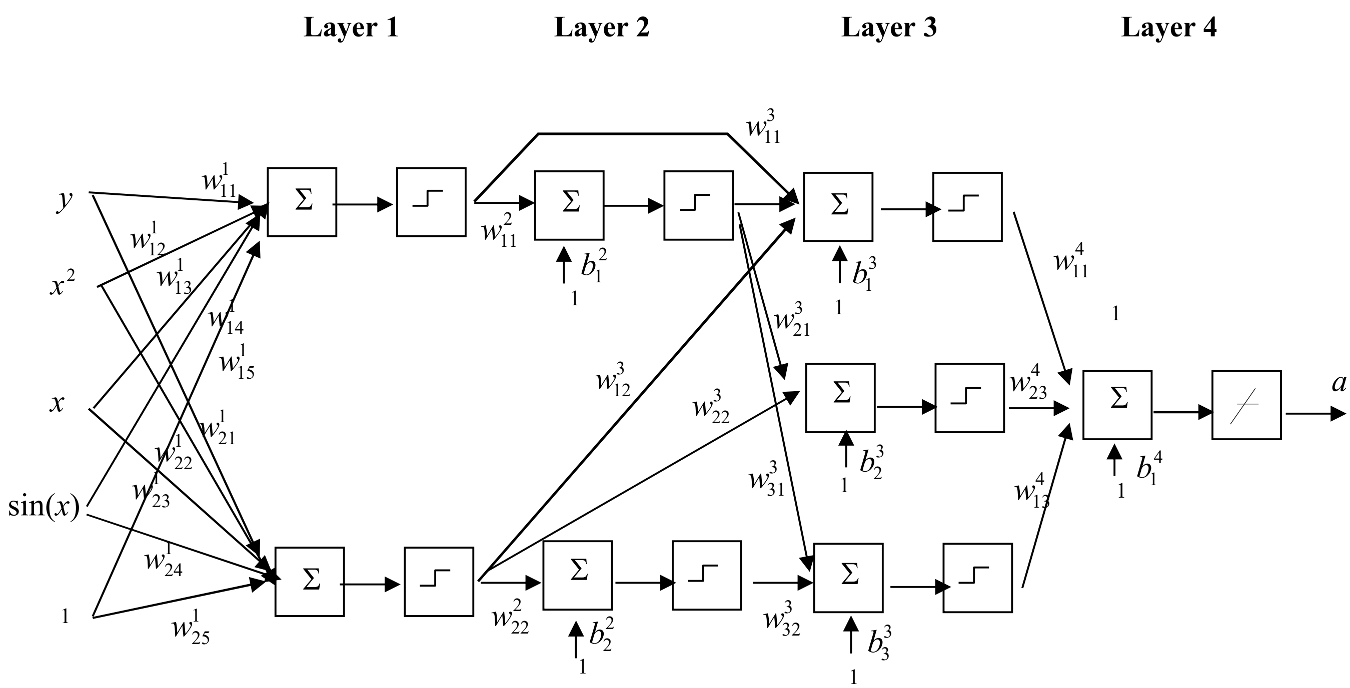
$$[b_1^2] = [-5.5]$$

Q.2 Design a neural network with all the parameters, which can generate the output of 5 when the coordinate of (x, y) is in the area above the graph , $y = sin(x)$, generate the output of 4 when is in the area inside the intersection of the graph, $y = 0.2x^2 - 0.4\pi x$, and the graph, $y = sin(x)$, and generate the output of 3 when is in the area beneath the graph $y = 0.2x^2 - 0.4\pi x$ as shown in the figure below. The neural network input vector is $\begin{bmatrix} y \\ x^2 \\ x \\ sin(x) \\ 1 \end{bmatrix}$. (25)



Solution

4 layers neural network can be applied. The first layer has 2 neurons is used to create 2 decision boundaries whose each output is 1 when the coordinate is above each function. The second layer has 2 neurons is used to create NOT of the decision boundaries. The third layer has 3 neurons is used to AND the selected decision boundaries from layer 1 and layer 2. The fourth layer has 1 neuron is used to superpose the outputs from layer 3 with different weights.



Layer 1

The first neuron in the first layer is used to create a sinusoidal decision boundary whose output is 1 when the coordinate is above.

$$\mathbf{w}_1^1 = \begin{bmatrix} 1 & 0 & 0 & -1 & 0 \end{bmatrix} \tag{1}$$

The second neuron in the first layer is used to create a quadratic decision boundary whose output is 1 when the coordinate is above.

$$\mathbf{w}_1^1 = \begin{bmatrix} 1 & -0.2 & 0.4\pi & 0 & 0 \end{bmatrix} \tag{2}$$

Layer 2

This is NOT layer, if we select

$$w_{11}^2 = w_{22}^2 = -1 \tag{3}$$

$$b_1^2 = b_2^2 = 0.5 \tag{4}$$

Layer 3

This is AND layer.

Neuron 1 is used to AND the output from both decision boundaries, if we select

$$w_{11}^3 = w_{12}^3 = 1 \quad \underline{(5)}$$

$$b_1^3 = -1.5 \quad \underline{(6)}$$

Neuron 2 is used to AND the output from NOT of the decision boundary 1 and the decision boundary 2, if we select

$$w_{21}^3 = w_{22}^3 = 1 \quad \underline{(7)}$$

$$b_2^3 = -1.5 \quad \underline{(8)}$$

Neuron 3 is used to AND the output from NOT of both decision boundaries, if we select

$$w_{31}^3 = w_{32}^3 = 1 \quad \underline{(9)}$$

$$b_3^3 = -1.5 \quad \underline{(10)}$$

Layer 4

This is a linear layer used to superpose outputs from layer 3 with different weights.

$$\mathbf{w}_1^4 = \begin{bmatrix} 5 & 4 & 3 \end{bmatrix} \quad \underline{(11)}$$

$$b_1^4 = 0 \quad \underline{(12)}$$

Q.3 ADALINE network is used to determine four parameters; *a*, *b,* *c* and *d*, of the function, $f = ax + by + cz + d$, when the relations of *x, y, z* and *f* are as follows.

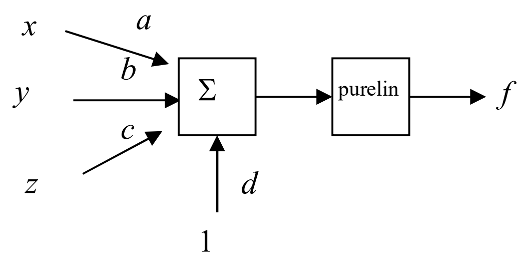
X	-6	-4	-2	1	2	4	6	8
Y	-12	-9	-6	-3	-1	3	6	9
Z	-2	-1	0	1	2	3	4	5
F	10.1	10.8	12.3	17.6	18.4	14.7	16.2	16.9

(a) Draw the ADALINE network, what are the input and output of the network. (5)

(b) If all the data are presented equally, determine the parameters by LMS algorithm. (20)

Solution

(a)



(b)

$$F(x) = E[t^2] - 2x^T E[tz] + x^T E[zz^T]x = c - 2x^T h + x^T Rx \quad (1)$$

$$c = \frac{1}{8}\left(10.1^2 + 10.8^2 + 12.3^2 + 17.6^2 + 18.4^2 + 14.7^2 + 16.2^2 + 16.9^2\right) = 222.8 \quad (2)$$

$$h = \frac{1}{8}\left(10.1\begin{bmatrix}-6\\-12\\-2\\1\end{bmatrix} + 10.8\begin{bmatrix}-4\\-9\\-1\\1\end{bmatrix} + 12.3\begin{bmatrix}-2\\-6\\0\\1\end{bmatrix} + 17.6\begin{bmatrix}1\\-3\\1\\1\end{bmatrix} + 18.4\begin{bmatrix}2\\-1\\2\\1\end{bmatrix} + 14.7\begin{bmatrix}4\\3\\3\\1\end{bmatrix} + 16.2\begin{bmatrix}6\\6\\4\\1\end{bmatrix} + 16.9\begin{bmatrix}8\\9\\5\\1\end{bmatrix}\right)$$

$$= \begin{bmatrix}27.15\\-8.75\\27.1\\14.625\end{bmatrix} \quad (3)$$

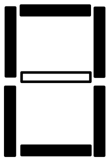
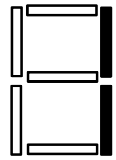
$$R = \frac{1}{8}\left(\begin{bmatrix}-6\\-12\\-2\\1\end{bmatrix}\begin{bmatrix}-6\\-12\\-2\\1\end{bmatrix}^T + \begin{bmatrix}-4\\-9\\-1\\1\end{bmatrix}\begin{bmatrix}-4\\-9\\-1\\1\end{bmatrix}^T + \begin{bmatrix}-2\\-6\\0\\1\end{bmatrix}\begin{bmatrix}-2\\-6\\0\\1\end{bmatrix}^T + \begin{bmatrix}1\\-3\\1\\1\end{bmatrix}\begin{bmatrix}1\\-3\\1\\1\end{bmatrix}^T + \begin{bmatrix}2\\-1\\2\\1\end{bmatrix}\begin{bmatrix}2\\-1\\2\\1\end{bmatrix}^T + \begin{bmatrix}4\\3\\3\\1\end{bmatrix}\begin{bmatrix}4\\3\\3\\1\end{bmatrix}^T + \begin{bmatrix}6\\6\\4\\1\end{bmatrix}\begin{bmatrix}6\\6\\4\\1\end{bmatrix}^T + \begin{bmatrix}8\\9\\5\\1\end{bmatrix}\begin{bmatrix}8\\9\\5\\1\end{bmatrix}^T\right)$$

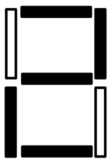
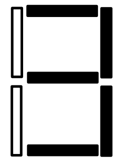
$$= \begin{bmatrix}22.125 & 29.375 & 12.125 & 1.125\\29.375 & 49.625 & 13.25 & -1.625\\12.125 & 13.25 & 7.5 & 1.5\\1.125 & -1.625 & 1.5 & 1\end{bmatrix} \quad (4)$$

The parameters are determined from the minimum point of the mean square error function

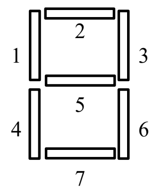
$$\begin{bmatrix}a\\b\\c\\d\end{bmatrix} = R^{-1}h = \begin{bmatrix}22.125 & 29.375 & 12.125 & 1.125\\29.375 & 49.625 & 13.25 & -1.625\\12.125 & 13.25 & 7.5 & 1.5\\1.125 & -1.625 & 1.5 & 1\end{bmatrix}^{-1}\begin{bmatrix}27.15\\-8.75\\27.1\\14.625\end{bmatrix} = \begin{bmatrix}4.6\\-4.4\\5.0\\-5.2\end{bmatrix} \quad (5)$$

Q.4 Design a neural network with only one layer and all its parameters which can convert 4 numbers shown by 7-segment LED to binary numbers correctly when the training sets consist of

$\{p_1 =$  $t_1 = [0 \quad 0 \quad 0]^T\}$ $\{p_2 =$  $t_2 = [0 \quad 0 \quad 1]^T\}$

$\{p_3 =$  $t_3 = [0 \quad 1 \quad 0]^T\}$ $\{p_4 =$  $t_4 = [0 \quad 1 \quad 1]^T\}$

Encode the input vector by using 1 for black segment and -1 for white segment, when the order is as shown in the figure below



(25)

Solution

Since there are 7 elements in the input and there are 4 patterns, Linear associator with Pseudoinverse rule can be applied.

$$\mathbf{W} = \mathbf{T}\mathbf{P}^{+} \tag{1}$$

Where

$$\mathbf{P}^{+} = (\mathbf{P}^{T}\,\mathbf{P})^{-1}\mathbf{P}^{T} \tag{2}$$

and

$$\mathbf{P} = \begin{bmatrix} 1 & -1 & -1 & -1 \\ 1 & -1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ -1 & -1 & 1 & 1 \\ 1 & 1 & -1 & 1 \\ 1 & -1 & 1 & 1 \end{bmatrix}, \ \mathbf{T} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix} \tag{3}$$

$$\mathbf{P}^{+} = \left(\begin{bmatrix} 1 & -1 & -1 & -1 \\ 1 & -1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ -1 & -1 & 1 & 1 \\ 1 & 1 & -1 & 1 \\ 1 & -1 & 1 & 1 \end{bmatrix}^{T} \begin{bmatrix} 1 & -1 & -1 & -1 \\ 1 & -1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ -1 & -1 & 1 & 1 \\ 1 & 1 & -1 & 1 \\ 1 & -1 & 1 & 1 \end{bmatrix} \right)^{-1} \begin{bmatrix} 1 & -1 & -1 & -1 \\ 1 & -1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ -1 & -1 & 1 & 1 \\ 1 & 1 & -1 & 1 \\ 1 & -1 & 1 & 1 \end{bmatrix}^{T}$$

$$= \begin{bmatrix} 0.150 & 0.100 & 0.150 & 0.150 & -0.200 & 0.150 & 0.100 \\ -0.225 & -0.150 & 0.275 & 0.025 & -0.200 & 0.025 & -0.150 \\ -0.250 & 0.000 & 0.250 & 0.250 & 0.000 & -0.250 & 0.000 \\ -0.025 & 0.150 & -0.025 & -0.275 & 0.200 & 0.225 & 0.150 \end{bmatrix} \quad (4)$$

$$\mathbf{W} = \mathbf{TP}^{+} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -0.275 & 0.150 & 0.225 & -0.025 & 0.200 & -0.025 & 0.150 \\ -0.250 & 0.000 & 0.250 & -0.250 & 0.000 & 0.250 & 0.000 \end{bmatrix} \quad \underline{(5)}$$