

Time: 9:00-11:00 h.
Marks: 100

Open Book

Attempt all questions.

Q.1 A neural network as shown in figure 1 is used to classify the shaded area (output of 1) from the white area (output of 0) of the problem as shown in figure 2. Determine all the parameters of the network.

(25)

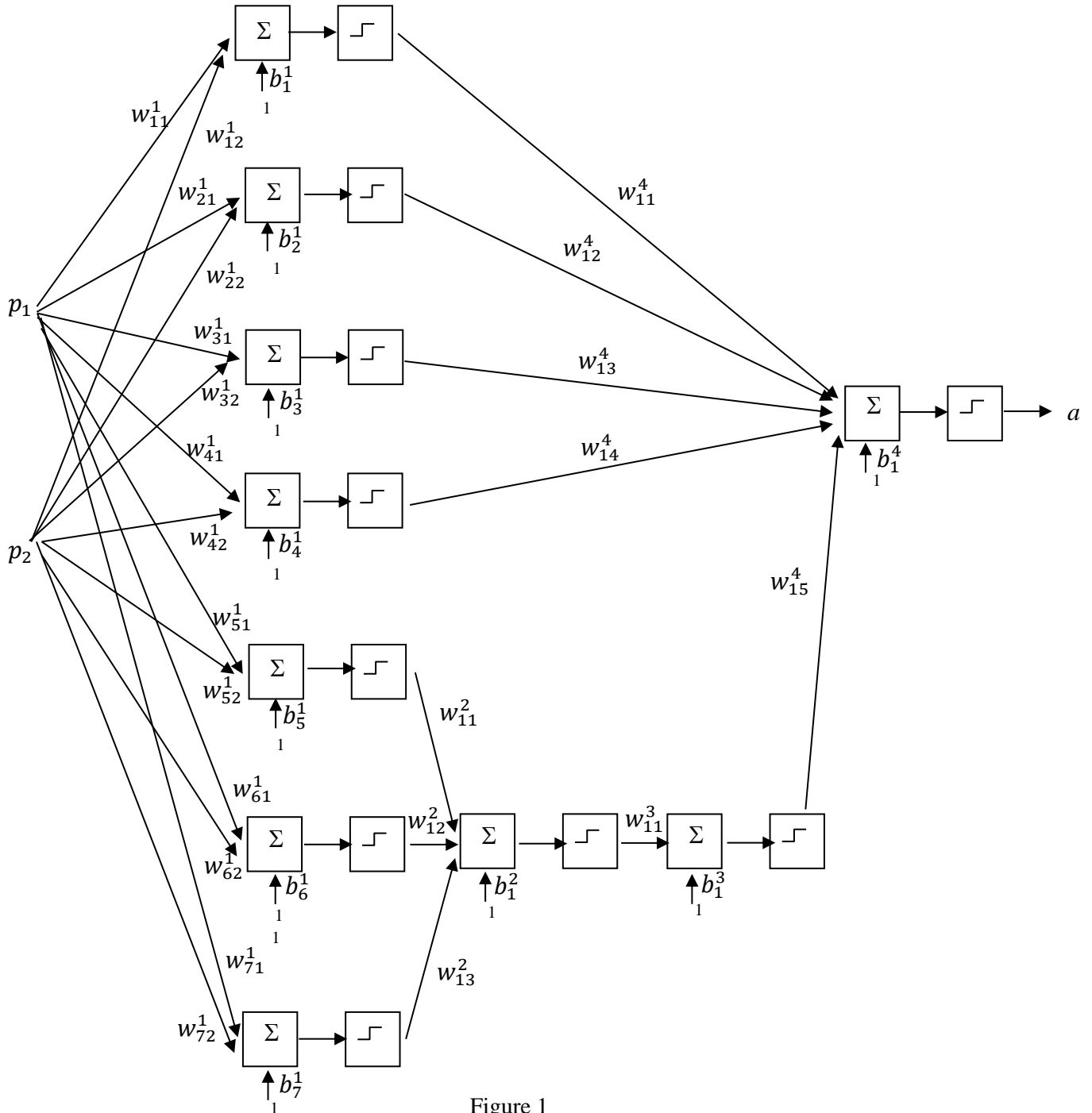


Figure 1

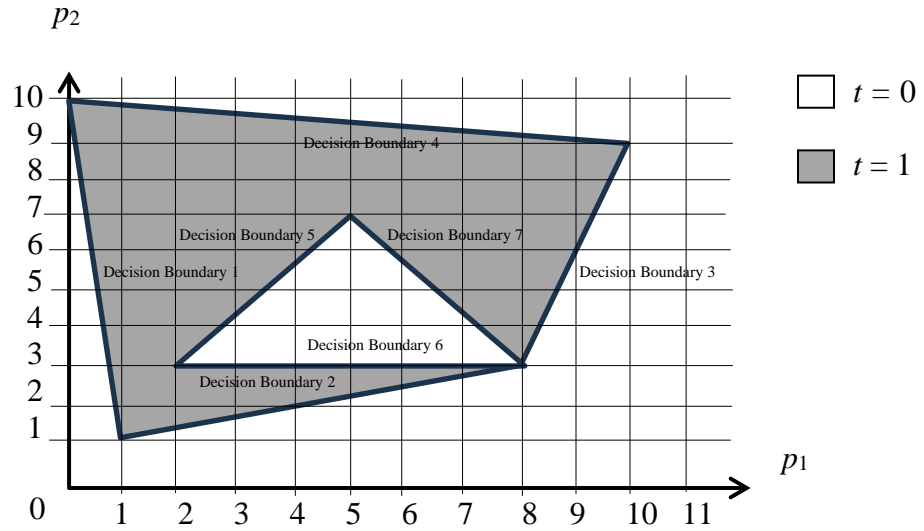


Figure 2

Solution

Layer 1: There are 7 decision boundaries in layer 1.

Decision boundary 1

$$[w_{11}^1 \ w_{12}^1] = [9 \ 1]$$

$$[b_1^1] = [-10]$$

Decision boundary 2

$$[w_{21}^1 \ w_{22}^1] = [-2 \ 7]$$

$$[b_2^1] = [-5]$$

Decision boundary 3

$$[w_{31}^1 \ w_{32}^1] = [-6 \ 2]$$

$$[b_3^1] = [42]$$

Decision boundary 4

$$[w_{41}^1 \ w_{42}^1] = [-1 \ -10]$$

$$[b_4^1] = [100]$$

Decision boundary 5

$$[w_{51}^1 \ w_{52}^1] = [4 \ -3]$$

$$[b_5^1] = [-3]$$

Decision boundary 6

$$[w_{61}^1 \quad w_{62}^1] = [0 \quad 1]$$

$$[b_6^1] = [-3]$$

Decision boundary 7

$$[w_{71}^1 \quad w_{72}^1] = [-4 \quad -3]$$

$$[b_7^1] = [45]$$

Layer 2: AND Layer

$$[w_{11}^2 \quad w_{12}^2 \quad w_{13}^2] = [1 \quad 1 \quad 1]$$

$$[b_1^2] = [-2.5]$$

Layer 3: NOT Layer

$$[w_{11}^3] = [-1]$$

$$[b_1^3] = [0.5]$$

Layer 4: AND Layer

$$[w_{11}^4 \quad w_{12}^4 \quad w_{13}^4 \quad w_{14}^4 \quad w_{15}^4] = [1 \quad 1 \quad 1 \quad 1 \quad 1]$$

$$[b_1^4] = [-4.5]$$

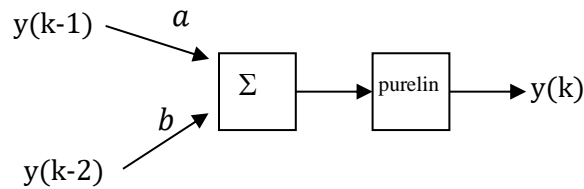
Q.2 ADALINE network is used to identify discrete model of an unforced second-order system which has the relation in the form of $y(k) = ay(k - 1) + by(k - 2)$; when $y(k)$ denotes the output at iteration k . The data are collected as shown in the table below.

k	0	1	2	3	4	5
y	10.00	5.00	4.50	3.25	2.53	1.91

- (a) Draw the network when the inputs are $y(k - 1)$ and $y(k - 2)$ and the output is $y(k)$. (5)
- (b) If all the data are presented equally and consider the output from iteration 2 onward, Determine the parameters a, b of the relation by LMS algorithm. (15)
- (c) Estimate the output $y(6)$. (5)

Solution

(a)



(b)

$$F(x) = E[t^2] - 2x^T E[tz] + x^T E[zz^T]x = c - 2x^T h + x^T R x \quad (1)$$

$$c = \frac{1}{4}(4.50^2 + 3.25^2 + 2.53^2 + 1.91^2) = 10.22 \quad (2)$$

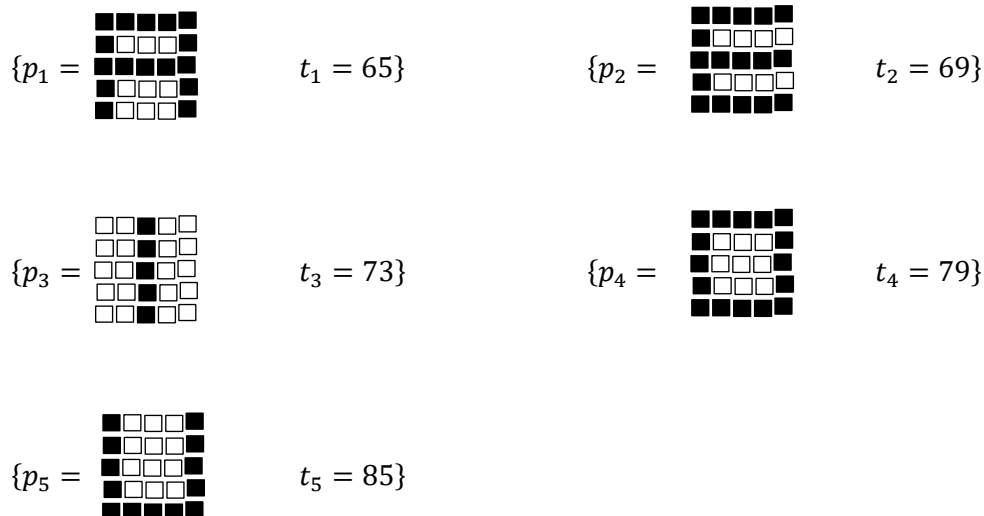
$$h = \frac{1}{4} \left(4.50 \begin{bmatrix} 5.00 \\ 10.00 \end{bmatrix} + 3.25 \begin{bmatrix} 4.50 \\ 5.00 \end{bmatrix} + 2.53 \begin{bmatrix} 3.25 \\ 4.50 \end{bmatrix} + 1.91 \begin{bmatrix} 2.53 \\ 3.25 \end{bmatrix} \right) = \begin{bmatrix} 12.54 \\ 19.71 \end{bmatrix} \quad (3)$$

$$R = \frac{1}{4} \left(\begin{bmatrix} 5.00 \\ 10.00 \end{bmatrix} \begin{bmatrix} 5.00 \\ 10.00 \end{bmatrix}^T + \begin{bmatrix} 4.50 \\ 5.00 \end{bmatrix} \begin{bmatrix} 4.50 \\ 5.00 \end{bmatrix}^T + \begin{bmatrix} 3.25 \\ 4.50 \end{bmatrix} \begin{bmatrix} 3.25 \\ 4.50 \end{bmatrix}^T + \begin{bmatrix} 2.53 \\ 3.25 \end{bmatrix} \begin{bmatrix} 2.53 \\ 3.25 \end{bmatrix}^T \right) = \begin{bmatrix} 15.55 & 23.84 \\ 23.84 & 38.95 \end{bmatrix} \quad (4)$$

The parameters are determined from the minimum point of the mean square error function.

$$\begin{bmatrix} a \\ b \end{bmatrix} = R^{-1}h = \begin{bmatrix} 15.55 & 23.84 \\ 23.84 & 38.95 \end{bmatrix}^{-1} \begin{bmatrix} 12.54 \\ 19.71 \end{bmatrix} = \begin{bmatrix} 0.5 \\ 0.2 \end{bmatrix} \quad (5)$$

Q.3 In an English OCR, a neural network is used to convert the images of English vowels A, E, I, O and U to ascii codes of 65, 69, 73, 79 and 85 respectively as shown in the below figure. Design the neural network with only one layer and its parameters that can perform the task correctly.



Encode the input vector by using 1 for black pixel and -1 for white segment and follow the order of row by row. (25)

Solution

Linear associator with Pseudoinverse rule can be applied.

$$\mathbf{W} = \mathbf{TP}^+ \quad (1)$$

Where

$$\mathbf{P}^+ = (\mathbf{P}^T \mathbf{P})^{-1} \mathbf{P}^T \quad (2)$$

and

$$\mathbf{P} = \begin{bmatrix} 1 & 1 & -1 & 1 & 1 \\ 1 & 1 & -1 & 1 & -1 \\ 1 & 1 & 1 & 1 & -1 \\ 1 & 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & 1 & 1 \\ 1 & 1 & -1 & 1 & 1 \\ -1 & -1 & -1 & -1 & -1 \\ -1 & -1 & 1 & -1 & -1 \\ -1 & -1 & -1 & -1 & -1 \\ 1 & -1 & -1 & 1 & 1 \\ 1 & 1 & -1 & 1 & 1 \\ 1 & 1 & -1 & -1 & -1 \\ 1 & 1 & 1 & -1 & -1 \\ 1 & 1 & -1 & -1 & -1 \\ 1 & 1 & -1 & 1 & 1 \\ 1 & 1 & -1 & 1 & 1 \\ -1 & -1 & -1 & -1 & -1 \\ -1 & -1 & 1 & -1 & -1 \\ -1 & -1 & -1 & -1 & -1 \\ 1 & -1 & -1 & 1 & 1 \\ 1 & 1 & -1 & 1 & 1 \\ -1 & 1 & -1 & 1 & 1 \\ -1 & 1 & 1 & 1 & 1 \\ -1 & 1 & -1 & 1 & 1 \\ 1 & 1 & -1 & 1 & 1 \end{bmatrix}, \mathbf{T} = [65 \quad 69 \quad 73 \quad 79 \quad 85] \quad (3)$$

$$\mathbf{W} = \begin{bmatrix} 2.3412 & -4.5323 & 6.0645 & -4.5323 & 2.3412 & 2.3412 & -12.9380 & -2.3412 & -12.9380 \\ -0.0819 & 2.3412 & -5.2535 & 5.3433 & -5.2535 & 2.3412 & 2.3412 & -12.9380 & -2.3412 \\ -12.9380 & -0.0819 & 2.3412 & -2.9202 & 7.6766 & -2.9202 & 2.3412 & & \end{bmatrix} \quad (4)$$

Q.4 If a 2-6-1 MLP network as shown in figure 1 is used to approximate the input-output relation as shown in figure 2, determine one set of the possible weights and biases of the network that generates the output correctly. (25)

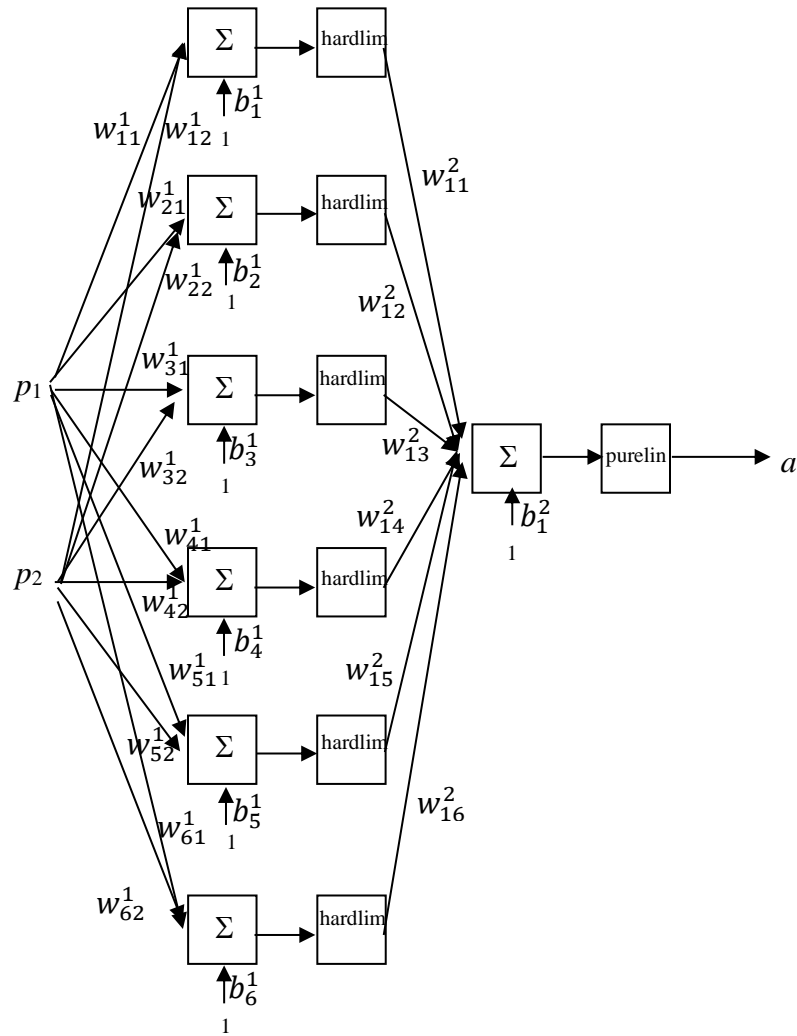


Figure 1: 2-6-1 Network

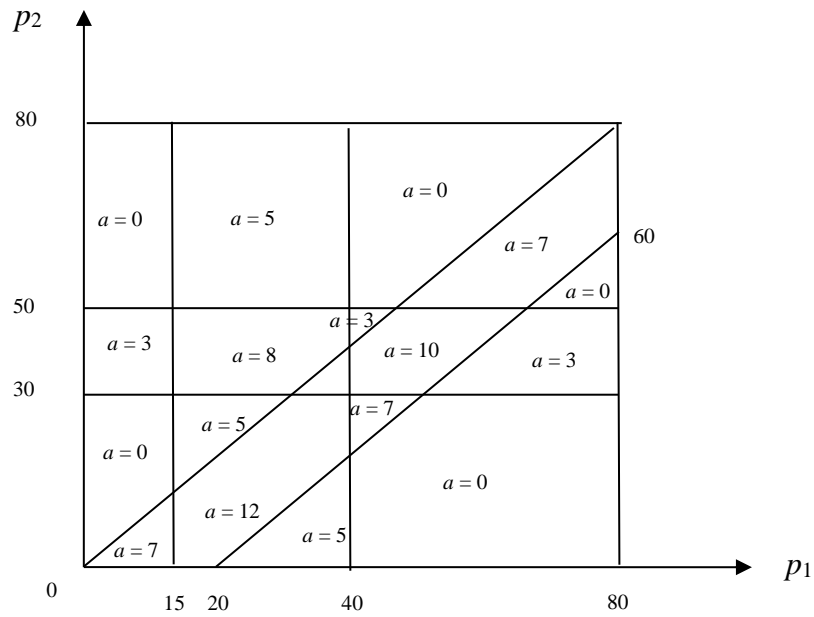
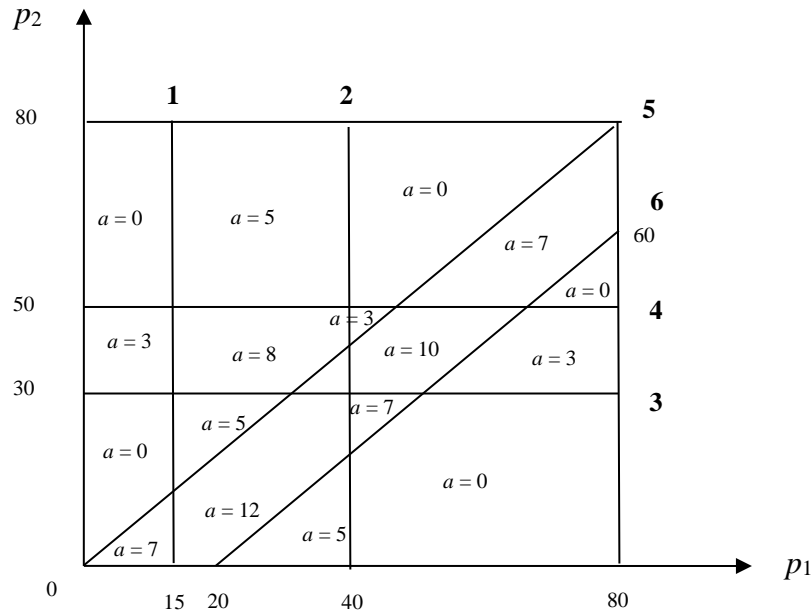


Figure 2: Input-Output Relation

Solution

The function can be approximated as the superposition of six 2-D hardlim function. The decision boundaries of the hardlim function are shown by straight lines labeling with 1, 2, 3, 4, 5, and 6 respectively. The first decision boundary follows a straight line $p_1 - 15 = 0$ with the output span of 5. The second decision boundary follows a straight line $p_1 - 40 = 0$ with the output span of -5. The third decision boundary follows a straight line $p_2 - 30 = 0$ with the output span of 3. The fourth decision boundary follows a straight line $p_2 - 50 = 0$ with the output span of -3. The fifth decision boundary follows a straight line $p_1 - p_2 = 0$ with the output span of 7. The sixth decision boundary follows a straight line $p_1 - p_2 - 20 = 0$ with the output span of -7.



Weight and bias of the first decision boundary, which follows a line $p_1 - 15 = 0$, are determined from

$$n_1^1 = w_{11}^1 p_1 + w_{12}^1 p_2 + b_1^1 = 0 \quad (1)$$

Thus

$$w_{11}^1 = 1, w_{12}^1 = 0, b_1^1 = -15 \quad (2)$$

Weight and bias of the second decision boundary, which follows a line $p_1 - 40 = 0$, are determined from

$$n_2^1 = w_{21}^1 p_1 + w_{22}^1 p_2 + b_2^1 = 0 \quad (3)$$

Thus

$$w_{21}^1 = 1, w_{22}^1 = 0, b_2^1 = -40 \quad (4)$$

Weight and bias of the third decision boundary, which follows a line $p_2 - 30 = 0$, are determined from

$$n_3^1 = w_{31}^1 p_1 + w_{32}^1 p_2 + b_3^1 = 0 \quad (5)$$

Thus

$$w_{31}^1 = 0, w_{32}^1 = 1, b_3^1 = -30 \quad (6)$$

Weight and bias of the fourth decision boundary, which follows a line $p_2 - 50 = 0$, are determined from

$$n_4^1 = w_{41}^1 p_1 + w_{42}^1 p_2 + b_4^1 = 0 \quad (7)$$

Thus

$$w_{41}^1 = 0, w_{42}^1 = 1, b_4^1 = -50 \quad (8)$$

Weight and bias of the fifth decision boundary, which follows a line $p_1 - p_2 = 0$, are determined from

$$n_5^1 = w_{51}^1 p_1 + w_{52}^1 p_2 + b_5^1 = 0 \quad (9)$$

Thus

$$w_{51}^1 = 1, w_{52}^1 = -1, b_5^1 = 0 \quad (10)$$

Weight and bias of the sixth decision boundary, which follows a line $p_1 - p_2 - 20 = 0$, are determined from

$$n_6^1 = w_{61}^1 p_1 + w_{62}^1 p_2 + b_6^1 = 0 \quad (11)$$

Thus

$$w_{61}^1 = 1, w_{62}^1 = -1, b_6^1 = -20 \quad (12)$$

Since output span of the first hardlim function is 5, thus

$$w_{11}^2 = 5 \quad (13)$$

Since output span of the second hardlim function is -5, thus

$$w_{12}^2 = -5 \quad (14)$$

Since output span of the third hardlim function is 3, thus

$$w_{13}^2 = 3 \quad (15)$$

Since output span of the fourth hardlim function is -3, thus

$$w_{14}^2 = -3 \quad (16)$$

Since output span of the fifth hardlim function is 7, thus

$$w_{15}^2 = 7 \quad (17)$$

Since output span of the sixth hardlim function is -7, thus

$$w_{16}^2 = -7 \quad (18)$$

Since the plot starts from the output magnitude of 0, thus

$$b_1^2 = 0 \quad (19)$$