

## Actuation

### 1. Introduction to Electro-Magnetic Principle

- Electromechanical energy conversion involves the interchange of energy between an electrical system and a mechanical system through the medium of a coupling magnetic field.
- When the conversion takes place from electrical to mechanical form the device is called a *motor*.
- When mechanical energy is converted to electrical energy the device is called a *generator*.
- When the electrical system is characterized by alternating current the devices are referred to as *ac motors* and *ac generators*, respectively.
- When the electrical system is characterized by direct current the electromechanical conversion devices are called *dc motors* and *dc generators*.
- Any magnet produces around itself a magnetic field. The amount of magnetism present in the region occupied by the magnet and its surroundings is described by the magnetic flux  $\phi$  (in webers).
- The density of the flux and its direction at a given point in space is termed the magnetic flux density  $B$  (in webers/square meter), a vector field.  $B$  is always called as magnetic field.

## 1.1 Conductor at Rest in a Magnetic Field

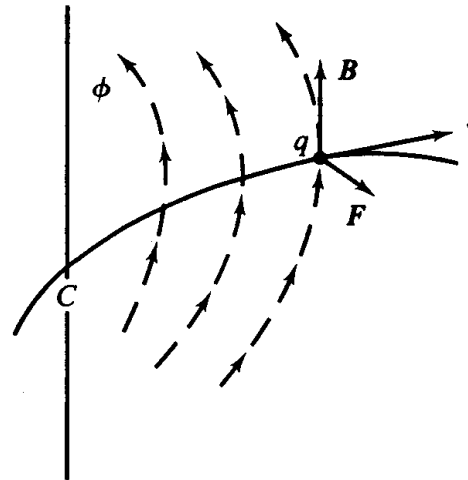


Figure 1.1-1 Force on a Charged Particle

- A charged particle with charge  $q$ , moving along a curve  $C$  with velocity  $v$  in a magnetic field  $B$ , is subject to an induced force.

$$\mathbf{F} = q\mathbf{v} \times \mathbf{B} \quad (1.1-1)$$

The current,

$$i = \frac{dq}{dt} = \frac{dq}{ds} \frac{ds}{dt} = \frac{dq}{ds} v \quad (1.1-2)$$

$$ids = v dq \quad (1.1-3)$$

The force differential,

$$d\mathbf{F} = dq\mathbf{v} \times \mathbf{B} = ids \times \mathbf{B} \quad (1.1-4)$$

The resultant force  $\mathbf{F}$  exerted by the field  $\mathbf{B}$  on the conductor of length  $L$ , carrying a constant current  $i$ ,

$$\mathbf{F} = \int_0^L ids \times \mathbf{B} = \int_0^L i(dz\hat{k} \times B\hat{i}) = iB\hat{j} \int_0^L dz = iBL\hat{j} \quad (1.1-5)$$

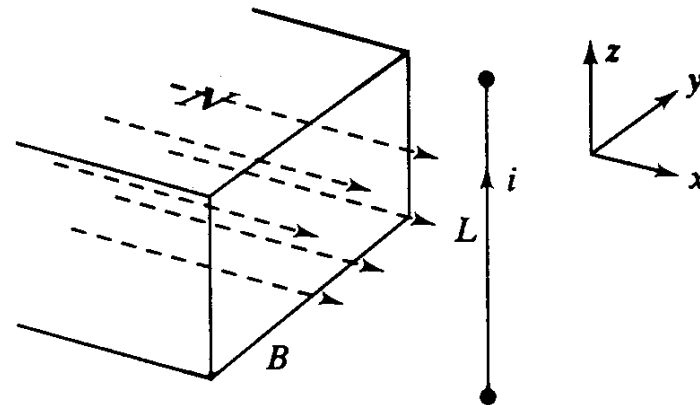


Figure 1.1-2 Force on a Conductor

- The basic law that is used to deduce the strength of the field at a given radial distance  $r$  from the conductor is the Biot-Savart law.

$$d\mathbf{B} = \frac{\mu}{4\pi} \frac{ids \times \mathbf{r}}{r^3} \quad (1.1-6)$$

where  $\mu = \mu_0 \mu_r$  in tesla·meters/ampere (T·m/A): the permeability of the medium in which the field exists

$\mu_0 = 4\pi \times 10^{-7}$  T·m/A: the permeability of a vacuum or the permeability constant

$\mu_r$ : the relative permeability.

- The permeability of a medium is its capacity for sustaining a magnetic field. For many ordinary materials, including air,  $\mu_r \approx 1$ ; for ferromagnetic materials  $\mu_r$  may be 1000 or more.

For a long (infinite) straight wire, the flux density at a perpendicular distance  $r$  from the wire,

$$B = \frac{\mu i}{2\pi r} \quad (1.1-7)$$

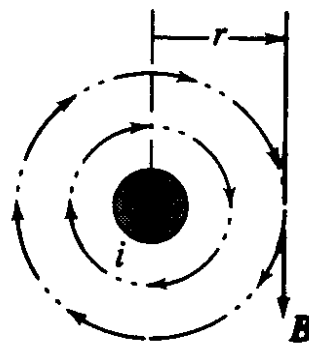


Figure 1.1-3 Field due to a Conductor

The flux density sustained inside the solenoid, the helical coil of wire wound around the cylinder of radius  $r$  and height  $h$ , with  $h \gg r$ ,

$$B = \frac{\phi}{A} = \frac{\mu Ni}{h} \quad (1.1-8)$$

where  $N$  : the number of turns in the coil.

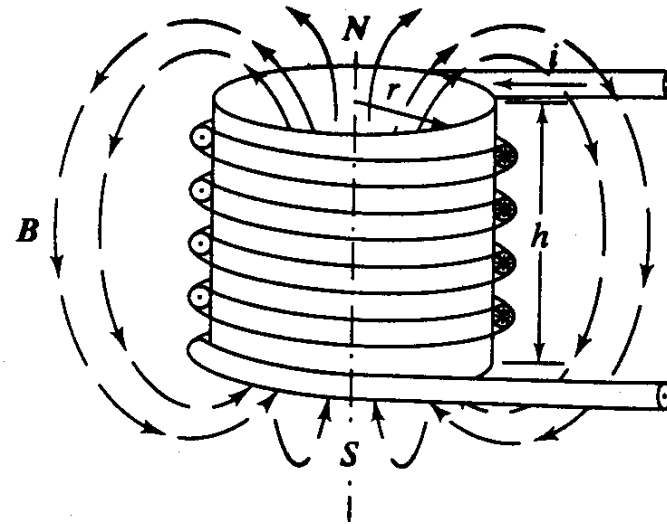


Figure 1.1-4 A Solenoid

1.2 Conductor Moving in a Magnetic Field

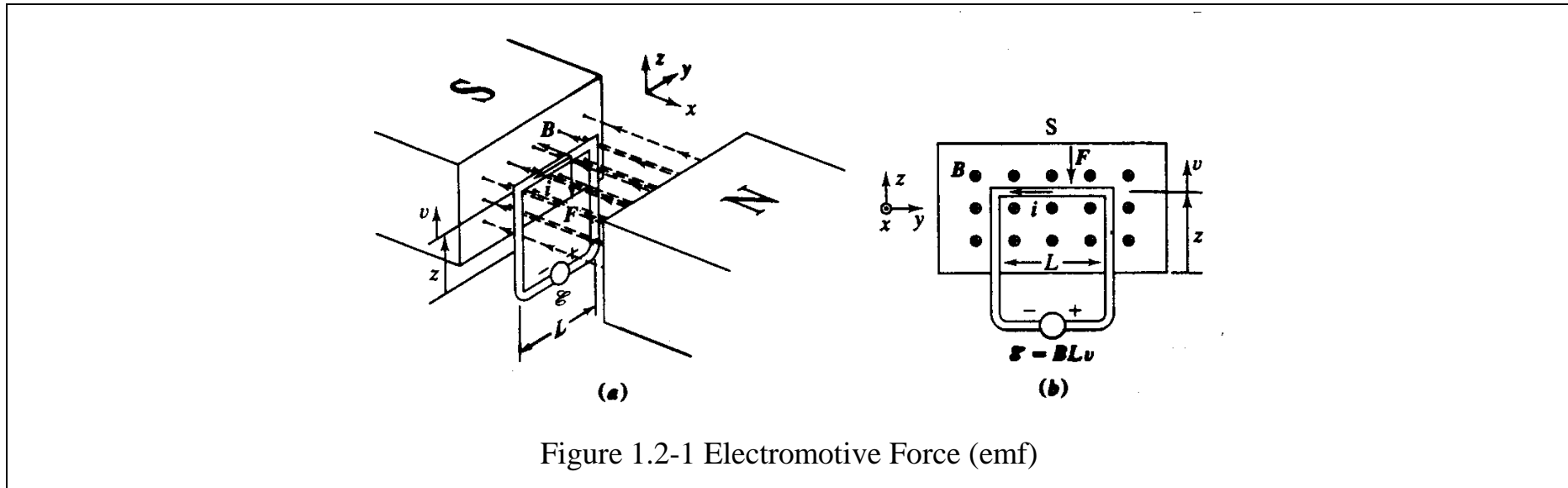


Figure 1.2-1 Electromotive Force (emf)

- The motion of a conducting loop through a magnetic field produces a current in the loop.
- According to Lenz's law: *An induced current in a closed conducting loop will appear in such a direction that it opposes the change that produced it.* No motion, no current.

When a conductor of length  $L$  (the horizontal segment of the loop) moving through a constant magnetic field  $B$  with a constant vertical velocity  $v$ , the opposing force,

$$F = -F\hat{k} = (iL\hat{j}) \times (-B\hat{i}) = iLB\hat{k} \tag{1.2-1}$$

$$i = -\frac{F}{LB} \quad (1.2-2)$$

In accordance with Faraday's law, the electrical potential,

$$E = \frac{d\phi}{dt} \quad (1.2-3)$$

The magnetic flux,

$$\phi_B = \oint \mathbf{B} \cdot d\mathbf{A} = BA \quad (1.2-4)$$

$$\phi_B = BLz \quad (1.2-5)$$

$$E = BLv \quad (1.2-6)$$

If  $R$  is the total resistance of the loop material,

$$E = iR = BLv \quad (1.2-7)$$

$$i = \frac{BLv}{R} \quad (1.2-8)$$

The corresponding force magnitude,

$$F = iLB = \frac{B^2 L^2 v}{R} \quad (1.2-9)$$



### 1.3 Classification of Electric Motors

Electric motors can be classified, by using the method of driving standard as:

1. Direct-Current Motors
  - 1.1 Series Motors
  - 1.2 Shunted Motors
  - 1.3 Separated Motors
  - 1.4 Mixed Motors
2. Alternating-Current Motors
  - 2.1 Induction Motors
  - 2.2 Synchronous Motors
3. Others
  - 3.1 Stepper Motors
  - 3.2 Brushless DC Motors

## 2. Direct-Current Motors

### 2.1 Construction of DC Motor

- Components in a dc motor
  - Stator, field winding
  - Rotor, armature winding
  - Commutator, mechanical rectifier used to switch the voltage provided to the armature for continuous rotation

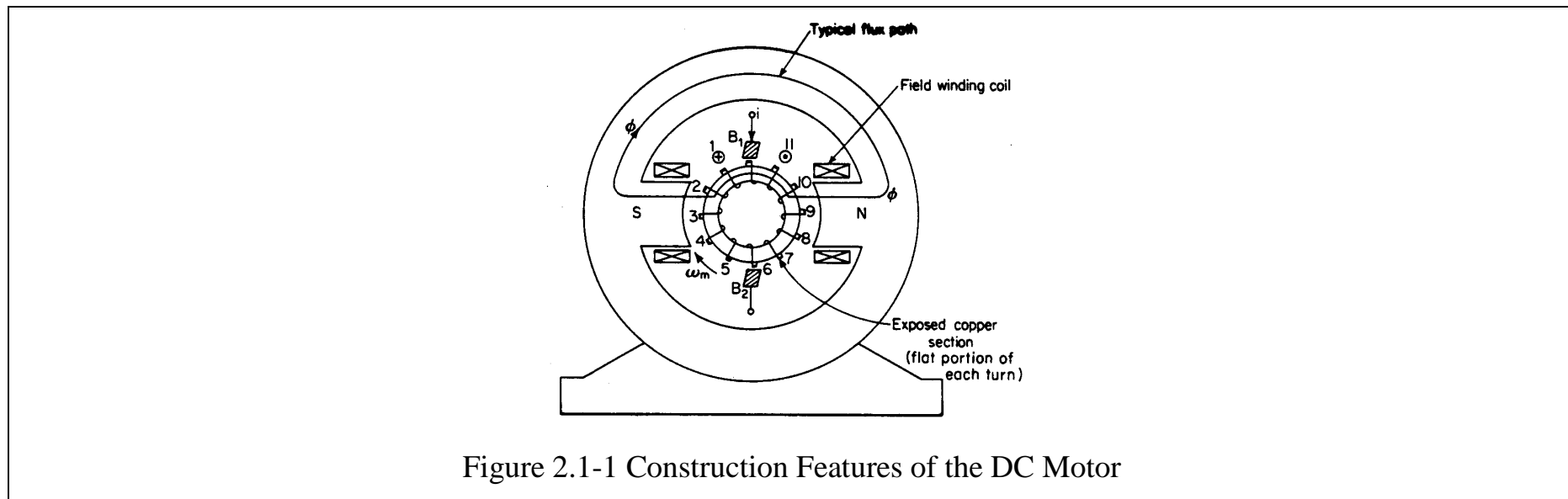
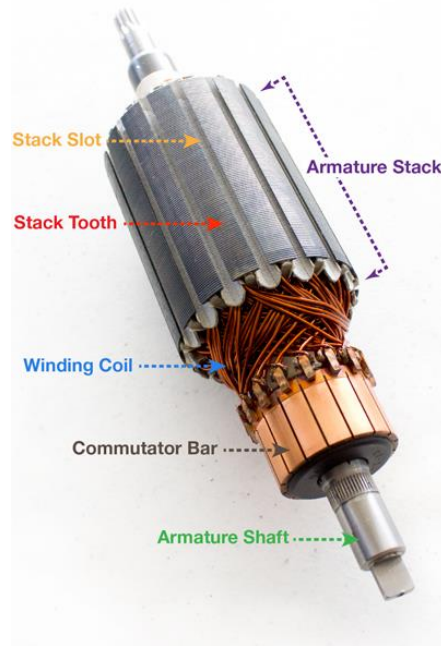


Figure 2.1-1 Construction Features of the DC Motor



(a)



(b)

Figure 2.1-2 DC Motor (a) Armature Winding (b) Stator Laminations

- The **stator** consists of a laminated ferromagnetic material equipped with a protruding structure around which coils are wrapped.
- The flow of direct current through the coils establishes a magnetic field distribution along the periphery of the air gap in much the same manner as occurs in the rotor of the synchronous motor.
- The **rotor** is composed of a laminated core, which is slotted to accommodate the armature winding.
- The **commutator** is composed of a series of copper segments insulated from one another and arranged in cylindrical fashion. Riding on the commutator are appropriately placed carbon brushes, which serve to conduct direct current to the armature winding.
- The parts of the armature winding, which lie directly below the brush width, are assumed to have the insulation removed, the copper is exposed. This allows current to be conducted to the armature winding through the brush as the rotor revolves.
- By placing the brushes on a line perpendicular to the field axis, all conductors contribute in producing a unidirectional torque.

## 2.2 DC Motor Model

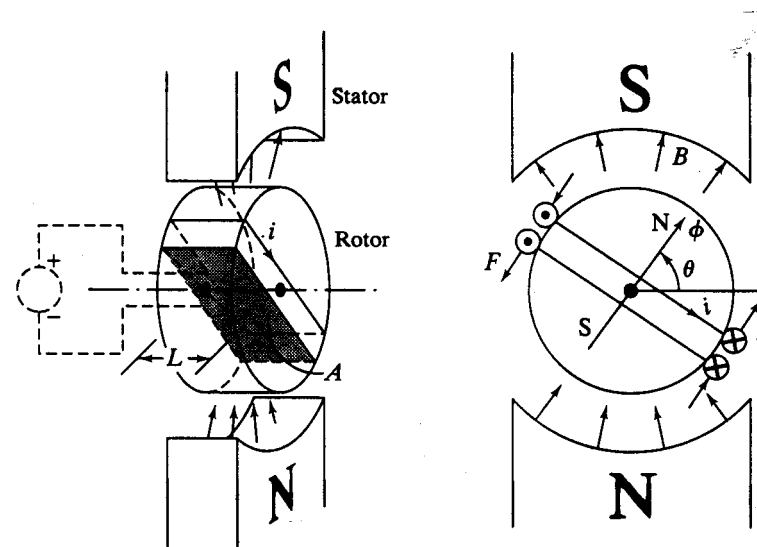


Figure 2.2-1 DC Motor Model

The resultant torque exerted on the rotor,

$$T = 2(2rF) = 4riLB = 2ABi = k_T i \quad (2.2-1)$$

where  $A$  : the area surrounded by the coil,  $k_T$  : the torque constant

The back emf or counter emf,

$$e_b = 4BLr\omega = 2AB\omega = k_b\omega \quad (2.2-2)$$

Euler's second law is used to determine the actual angular speed  $\omega$  of the rotating armature.

$$T = k_T i = J \frac{d\omega}{dt} \quad (2.2-3)$$

where  $J$  : the polar mass moment of inertia of the rotor about its axis of rotation when friction and damping are neglected.

Armature circuit equation is used to determine the armature current,

$$e_L(t) + e_R(t) + e_b(t) = e(t) \quad (2.2-4)$$

$$L \frac{di}{dt} + Ri + k_b\omega = e(t) \quad (2.2-5)$$

$$\frac{LJ}{k_T} \frac{d^2\omega}{dt^2} + \frac{RJ}{k_T} \frac{d\omega}{dt} + k_b\omega = e(t) \quad (2.2-6)$$

- For a DC motor, a continuously applied torque to provide a smooth rotation of the rotor can be accomplished is by switching the current direction as the coil passes through the horizontal position.
- The greater the number of slots, the smaller in the direction of the flux resultant. With a further increase in the number of slots, one eventually attains an armature field virtually fixed in space, with the stator and the rotor field mutually perpendicular.

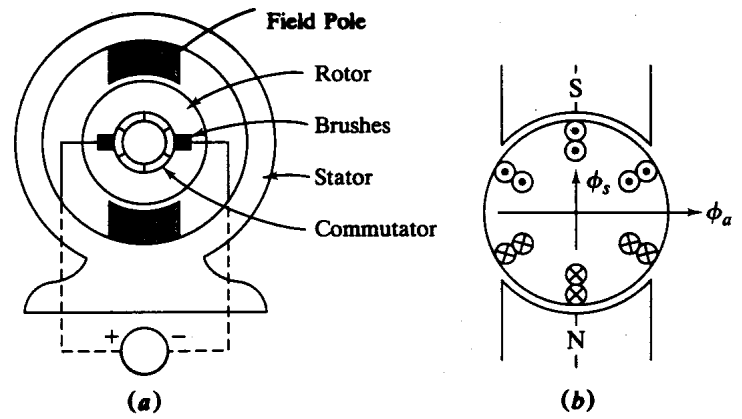


Figure 2.2-2 (a) DC Motor (b) Six Single-Turn Coils Schematic

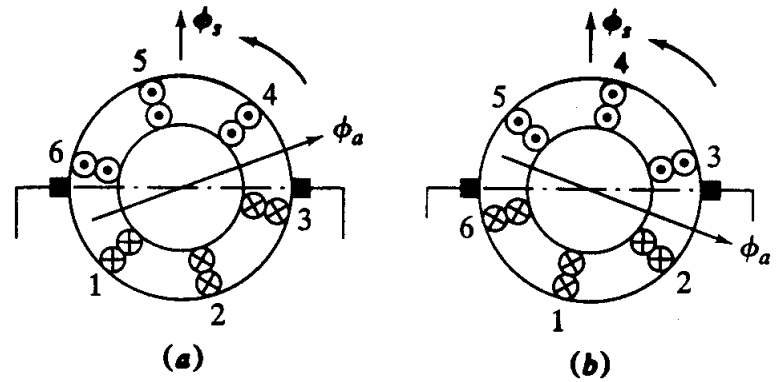


Figure 2.2-3 The Switch (a) Just Before (b) Just After

## 2.3 DC Motor Classification

- DC motors are classified in term of power as:
  - Motors with power ratings of 1 hp or more
  - Motors with fractional horsepower
- DC motors are classified in term of type of flux as:
  - Variable flux, provided by external sources
    - Field and armature separately excited
    - Field and armature in series
    - Field and armature in parallel
    - Field and armature connected in a mixed fashion
  - Constant flux, generally provided by permanent magnets
- An inductor is an electrical device, which may be used to set up a known magnetic field within some specified region.

The inductance has the unit of tesla·meters squared/ampere ( $T \cdot m^2/A$ ), henry,

$$L = \frac{N\phi}{i} \quad (2.3-1)$$

$i$  : the current in the device,  $N$  : the number of turns in the coil, and  $\phi$  : the flux.



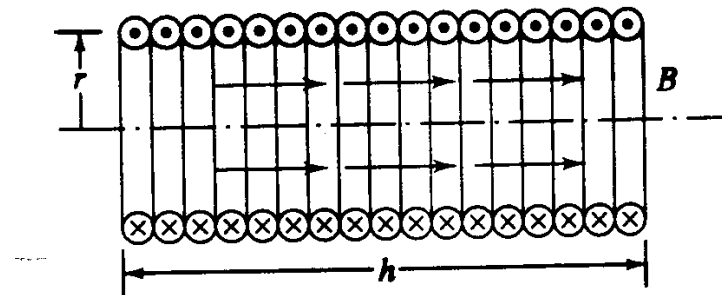


Figure 2.3-1 A Long Solenoid

The inductance of a long circular cylindrical solenoid of length  $h$ , when the interior of the solenoid is air and the solenoid has  $N$  turns,

$$L = \frac{NAB}{i} \quad (2.3-2)$$

where  $A = \pi r^2$  : the cross-sectional area of the solenoid.

The magnetic field  $B$  in the interior, in a central region,

$$B = \mu_0 i \frac{N}{h} \quad (2.3-3)$$

where  $\mu_0$  : the permeability constant and  $i$  : the current in the solenoid windings.

$$L = \mu_0 \pi r^2 \frac{N^2}{h} \quad (2.3-4)$$

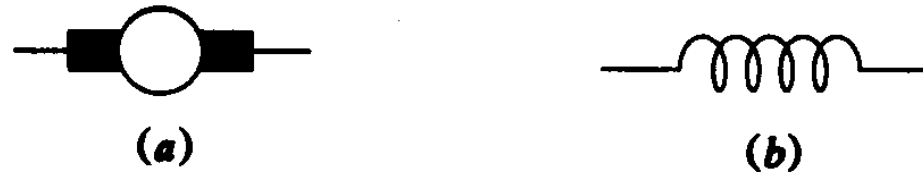


Figure 2.3-2 (a) Armature (b) Field

The field flux is proportional to the field current  $i_f(t)$ ,

$$\phi(t) = k_f i_f(t) \quad (2.3-4)$$

The corresponding motor torque is proportional to the field flux and the armature current  $i_a(t)$ ,

$$T_m(t) = k_m \phi(t) i_a(t) \quad (2.3-5)$$

The back emf, generated by the motion of the conductors through the magnetic field, is proportional to the flux and the angular velocity of the motor,  $\omega_m(t)$ ,

$$e_b(t) = k_b \phi(t) \omega_m(t) \quad (2.3-6)$$

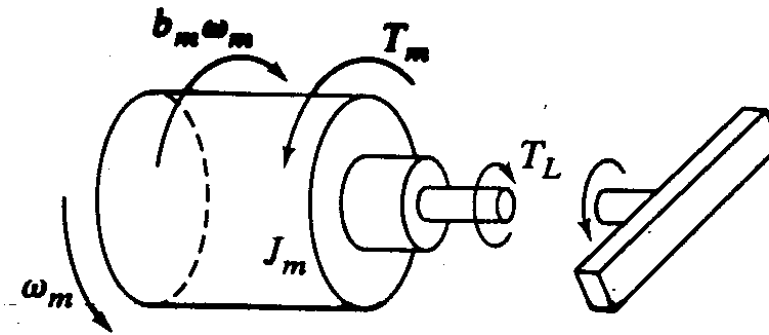


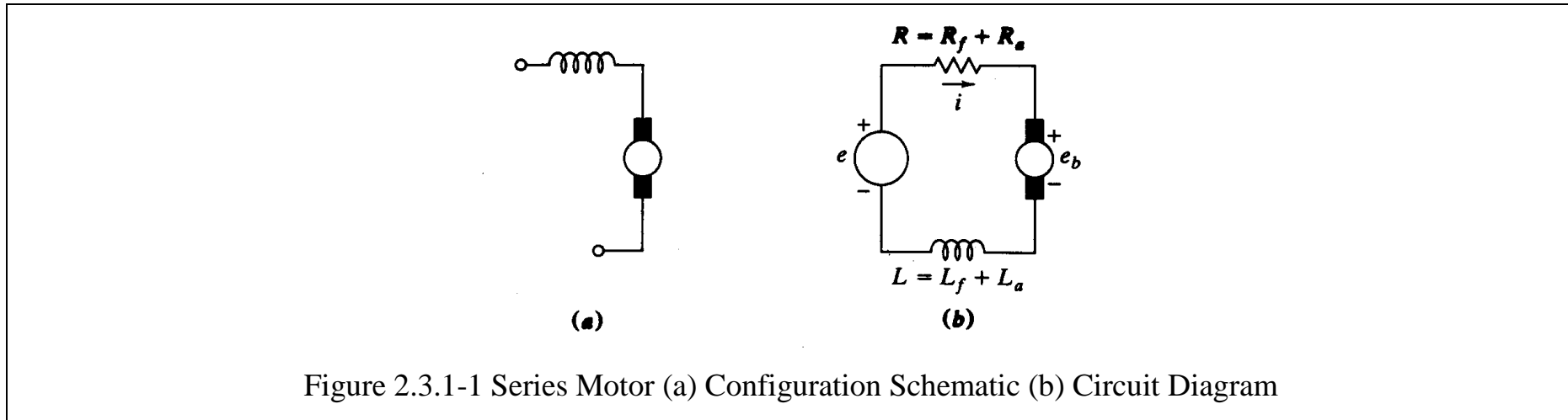
Figure 2.3-3 Rotor and Load

- The load on the motor consists of the polar mass moment of inertia of the rotor, some viscous resistance proportional to the angular velocity of the rotor, and a torque  $T_L$  due to the presence of some external load.

$$T_m - T_L - b_m \omega_m = J_m \frac{d\omega_m}{dt} \quad (2.3-7)$$

where  $J_m$  : the mass moment of inertia of the rotor,  $\omega_m$  : its angular velocity,  $T_m$  : the magnetic torque applied to the rotor, and  $b_m$  : the damping coefficient

### 2.3.1 Armature and Field in Series



- This kind of DC motor is usually employed when large start-up torques are required.
- The required current then decreases with an increase in angular velocity, with a corresponding decrease in magnetic flux. Since the field winding carries the full armature current, it generally consists of only a few turns of heavy gauge wire.
- This kind of motor would run away in a no-load condition; friction and losses, however, keep it in a safe operating range.
- The field current is the same as the armature current,  $i_a = i_f = i$ .

The air gap flux is proportional to the field current,

$$\phi(t) = k_f i(t) \quad (2.3.1-1)$$

The motor torque,

$$T_m(t) = k_m \phi(t) i_a(t) = k_s i^2(t) \quad (2.3.1-2)$$

where  $k_s$  : the torque constant for the series motor.

The back emf,

$$e_b(t) = k_c i(t) \omega_m(t) \quad (2.3.1-3)$$

Euler's second law for the rotor,

$$J_m \frac{d\omega_m}{dt} + b_m \omega_m = k_s i^2 - T_L \quad (2.3.1-4)$$

The circuit equation,

$$L \frac{di}{dt} + Ri = e - e_b = e - k_c i \omega_m \quad (2.3.1-5)$$

where  $R = R_a + R_f$  : the combined field and armature resistance and  $L = L_a + L_f$  : the combined field and armature inductance, and  $e_b$  : the back emf generated in the armature.

### 2.3.2 Armature and Field in Parallel

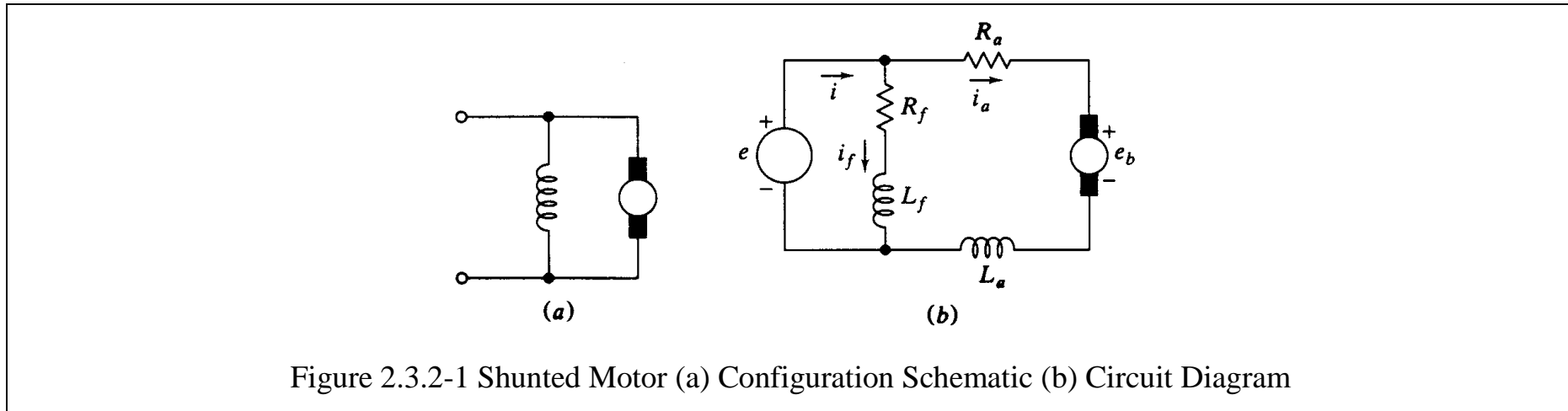


Figure 2.3.2-1 Shunted Motor (a) Configuration Schematic (b) Circuit Diagram

- This type of motor is termed a shunted DC motor.
- It is wide use for both fixed and variable speed applications.

The air gap flux,

$$\phi(t) = k_f i_f(t) \quad (2.3.2-1)$$

The corresponding torque,

$$T_m(t) = k_p i_f(t) i_a(t) \quad (2.3.2-2)$$

where  $k_p$  : the torque constant for the shunted (parallel) case.

The back emf,

$$e_b(t) = k_c i_f(t) \omega_m(t) \quad (2.3.2-3)$$

Euler's second law for the rotor,

$$J_m \frac{d\omega_m}{dt} + b_m \omega_m = k_p i_f i_a - T_L \quad (2.3.2-4)$$

The corresponding circuit equations,

$$L_f \frac{di_f}{dt} + R_f i_f = e \quad (2.3.2-5)$$

$$L_a \frac{di_a}{dt} + R_a i_a = e - k_c i_f \omega_m \quad (2.3.2-6)$$

2.3.3 Separate Armature and Field

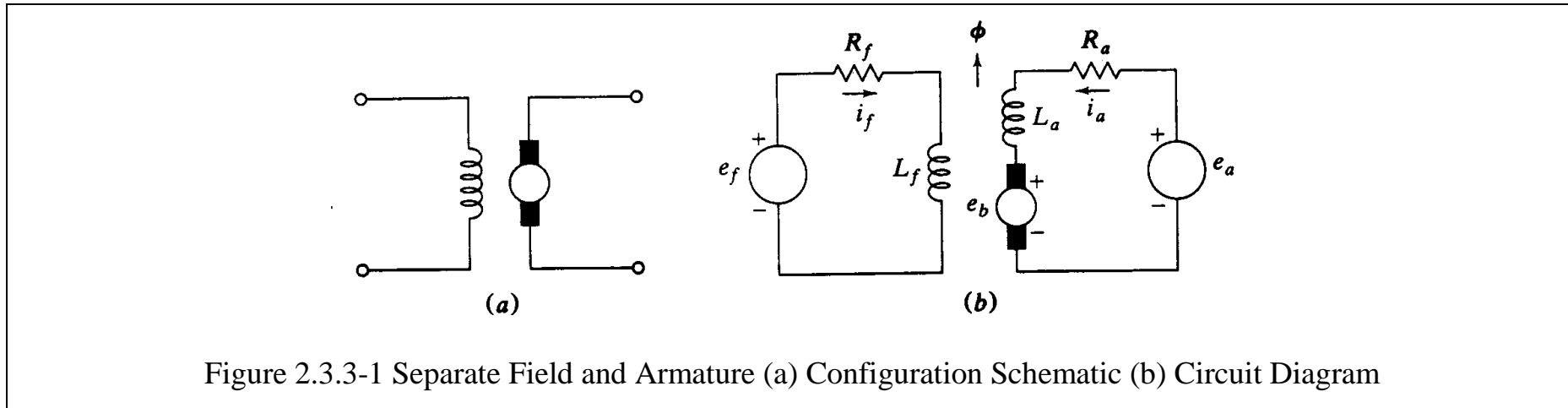


Figure 2.3.3-1 Separate Field and Armature (a) Configuration Schematic (b) Circuit Diagram

- Two separate potentials are used to power the armature and field. It is common to fix field potential and use the armature potential to control the motor.

The field circuit equation,

$$L_f \frac{di_f}{dt} + R_f i_f = e \tag{2.3.3-1}$$

- A constant field potential  $E_f$ , at steady state  $E_f = I_f R_f$ , yields a constant field current  $I_f$ .

The air gap flux,

$$\phi(t) = k_f I_f(t) \tag{2.3.3-2}$$



- The motor torque is proportional to the armature current with  $k_T$  denoting the torque constant.

$$T_m(t) = k_m k_f I_f i_a(t) = k_T i_a(t) \quad (2.3.3-3)$$

Euler's second law for the rotor,

$$J_m \frac{d\omega_m}{dt} + b_m \omega_m = k_T i_a - T_L \quad (2.3.3-4)$$

The circuit equation for the armature,

$$L_a \frac{di_a}{dt} + R_a i_a = e_a - k_b \omega_m \quad (2.3.3-5)$$

## 2.4 DC Motor Driver (Servo Amplifier)

- A servo amplifier is used to convert the low-power command signals from the computer/processor to levels that can be used to drive the motor.

### 2.4.1 Linear Servo Amplifier

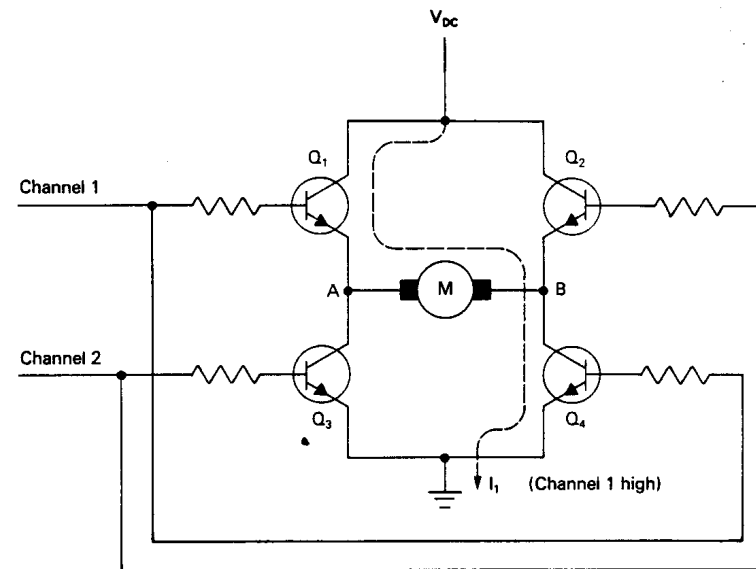


Figure 2.4.1-1 H-Type Servo Amplifier

- Two basic classes of linear servo amplifiers
  - H type
  - T type
- The H type, sometimes called a bridge amplifier, has the advantage of requiring a single or unipolar dc supply.
- It is not always easy to operate in a linear fashion.
- Because the motor must be floated with respect to the system ground, current and/or voltage feedback is not easy to achieve.
- By applying a positive control voltage to channel 1 (and grounding channel 2), the motor will turn in a direction, e.g., in the clockwise direction.
- When the control signals on channels 1 and 2 are reversed, the motor will now turn in the opposite or counterclockwise direction.
- The actual size of the armature voltage, and hence the motor speed, will depend on the amount of base current supplied by the control circuitry that precedes the power amplifier stage, e.g., a preamplifier.

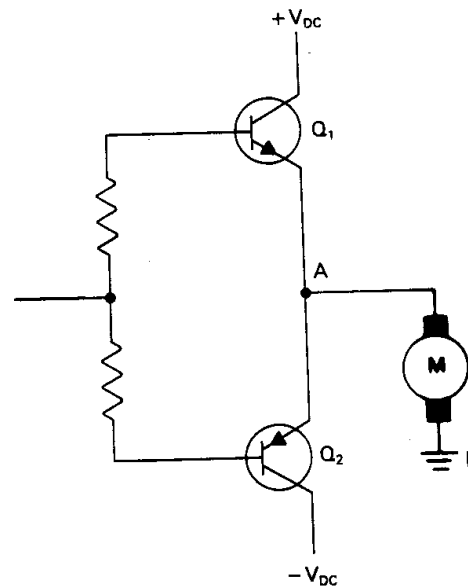


Figure 2.4.1-2 T-Type Servo Amplifier

- The T type requires a bipolar dc supply.
- It is easy to drive and since the motor does not have to float with respect to ground, current and/or voltage feedback is easy to implement.
- Since complementary power transistors are employed, Q1 and Q2, a single bipolar control signal can be used to turn on either Q1 or Q2, producing the desired bidirectional rotation.

- An undesirable characteristic of a T servo amplifier is the deadband or crossover distortion that exists around zero output voltage.
- This produces an armature drive voltage that is a nonlinear function of the servo amplifier input for small positive and negative inputs signal.
- Flyback protection is required since the inductance in the servomotor armature can produce an inductive kick when the power amplifier transistors are either suddenly all turned off or when the motor is plugged, i.e., the armature voltage is rapidly reversed to provide dynamic braking.
- Flyback diodes (or some other method of protecting the power transistors from breakdown) must be placed across the collector- emitter terminals of the output transistors.
- Failure to do this risks a collector-to-emitter short circuit, which can cause a runaway condition.

### 2.4.2 Pulse-Width-Modulated Amplifier

- When the output is only a fraction of the total supply voltage, the power transistors are in the active, linear, regions which means that the collector-to-emitter voltage drop  $V_{CE}$  of the transistors that are conducting is significant. The power dissipated in the collector (the product of collector current and the collector-to-emitter voltage) is large.
- The major advantage of pulse-width modulation (PWM) is that the power transistor is either off or in saturation.
- The power dissipated in the collector of PWM is less than in an equivalent linear amplifier since little or no collector current flows when the transistor is turned off or when current flows, the transistor is in saturation, which means that the drop across its collector is only 1 or 2 V. Thus the dissipation is still quite small.
- PWM devices can be of the H or T type, however, the output voltage of the T or H circuit will be almost equal to the full value of either the positive or negative dc supply voltage.
- The motor performs as a low-pass filter. With  $T_s$  defined as the period of the switching signal waveform, then if the radian switching frequency  $\omega_s = 2\pi/T_s \gg \omega_E$ , the electrical pole of the motor, the filtering action of the motor will cause the effective armature voltage to be the average value of the waveforms.

$$(V_{arm})_{ave} = \frac{1}{T_s} \int_0^{T_s} V_{arm}(t) dt \quad (2.4.2-1)$$

- Practical PWM switching rate is 1 to 15 kHz.

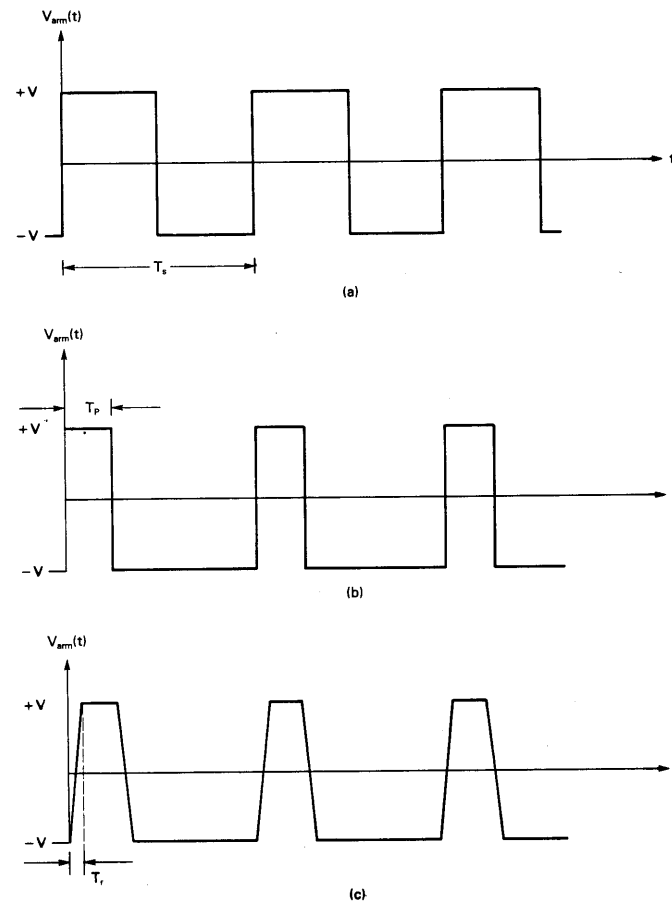


Figure 2.4.2-1 Typical PWM Waveforms (a) No Load PWM Output, Ideal Switch  $(V_{arm})_{ave} = 0$  (b) Loaded PWM Output, Ideal Switch  $(V_{arm})_{ave} = -V/2$ , CCW Turning (c) Nonideal Switch of (b) and Power Transistor in Active Region during  $T_r$

2.4.3 Effects of Feedback in Servo Amplifiers

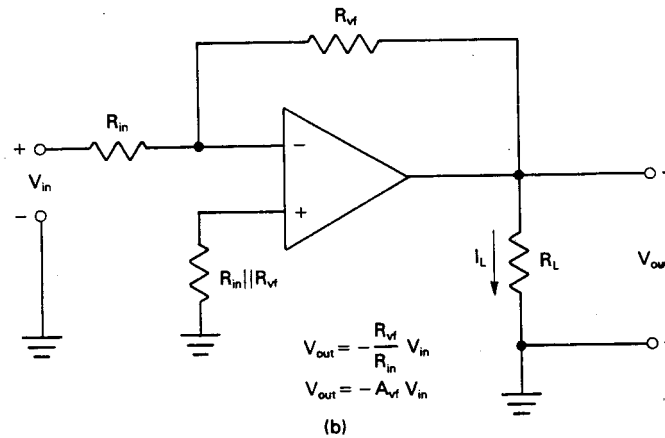
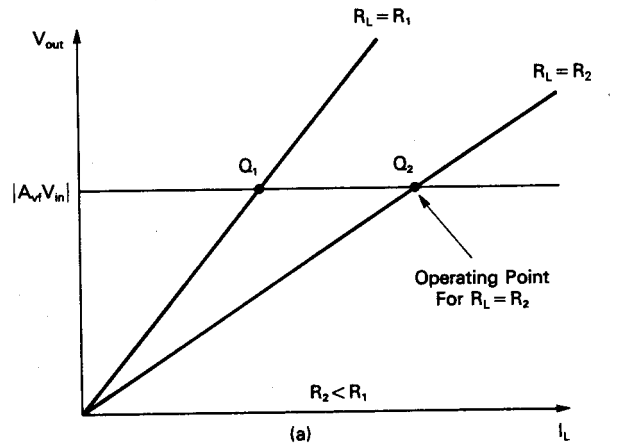


Figure 2.4.3-1 Voltage Feedback Amplifier (a)  $V-I$  Operating Characteristic (b) Circuit Diagram



- In the voltage feedback, an amplifier's output voltage is held constant regardless of changes in the load's impedance. This is sometimes referred to as a voltage-stabilized amplifier.
- The intersection of any single constant resistance line with a particular constant-voltage output curve defines an operating point.
- Regardless of the value of the load resistance, the voltage-stabilized amplifier will produce an output  $V_{out}$  which is proportional to both the input  $V_{in}$  and the gain factor  $V_{vf}$ .

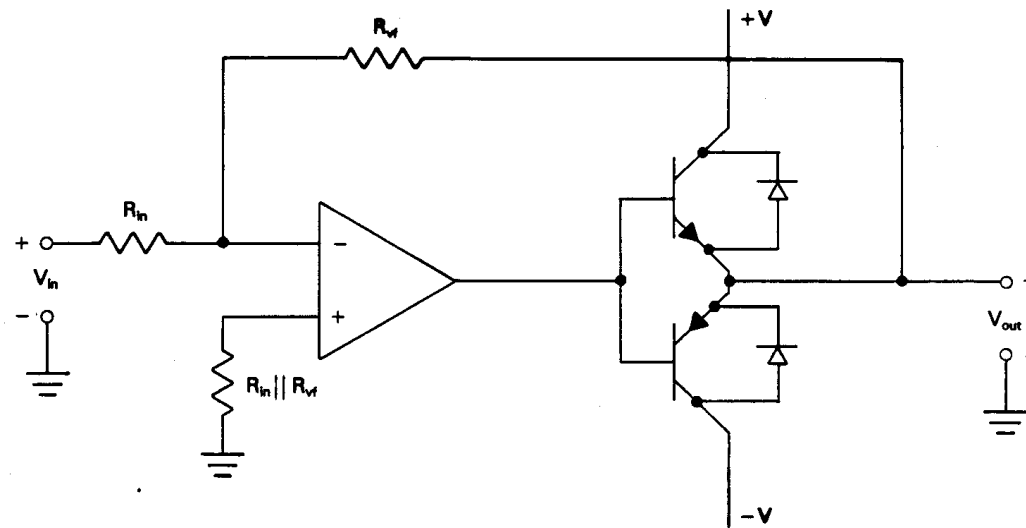


Figure 2.4.3-2 T-Type Power Stage Driven by an Op-Amp with Voltage Feedback

- The use of voltage feedback can reduce the deadband in a T amplifier configuration. The feedback will cause the appropriate drive signals to be applied to the transistors so that the output voltage will be a linear function of the input voltage and the gain.
- The advantage of using an amplifier with voltage feedback for motor control is that the voltage delivered to the motor terminals will be kept at the value commanded by the input for position or velocity control of the motor shaft.

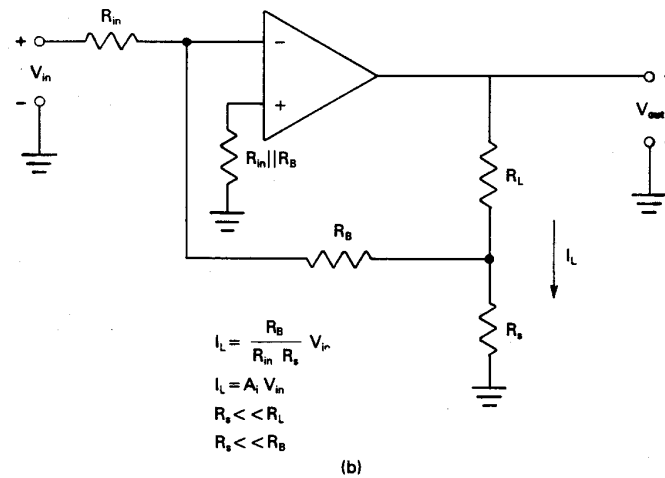
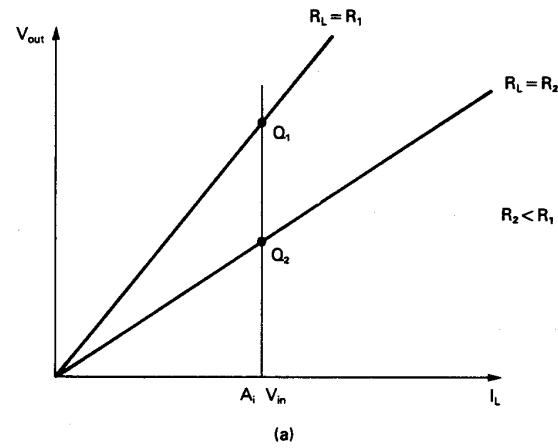


Figure 2.4.3-3 Current Feedback Amplifier (a) V-I Operating Characteristic (b) Circuit Diagram

- In an amplifier that has a stabilized current output, the output of the amplifier is a constant current defined by the product of the amplifier's gain  $A_i$  and the input voltage. As the load varies, the output voltage changes in order to keep the output current constant.
- This type of amplifier is used when it is desired to adjust the current across the motor terminals for a torque control application.
- One advantage of using such a device with dc servomotors is the fact that the current delivered will be the same regardless of changes in the motor's armature resistance. In addition, the voltage drops inherent in the wiring from the amplifier to the motor will not affect the power delivered to the motor.

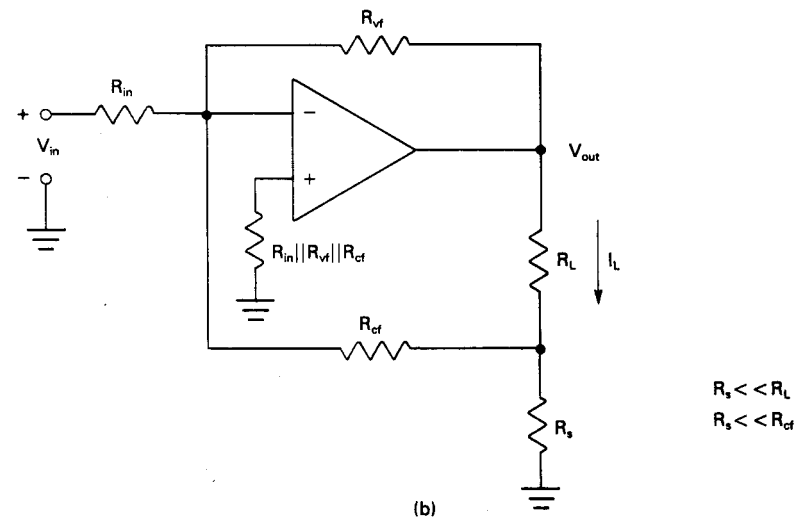
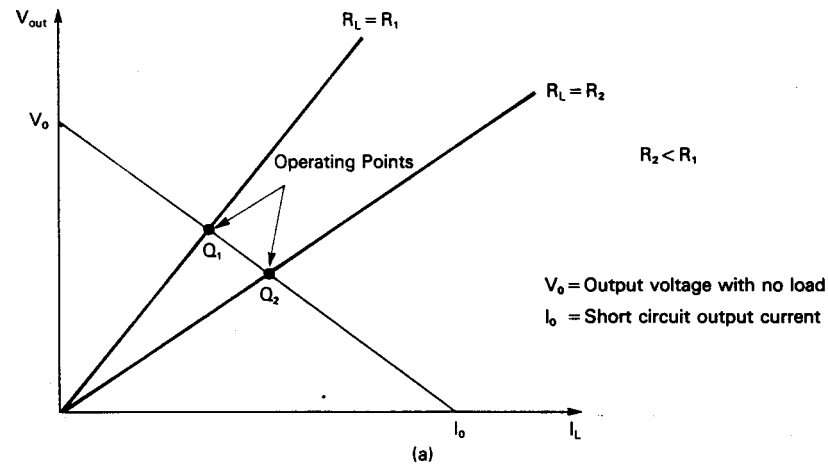


Figure 2.4.3-4 Combined Voltage and Current Feedback Amplifier (a) V-I Operating Characteristic (b) Circuit Diagram

- In a voltage-stabilized amplifier, a fixed series resistor has been added. The resistance value is defined as  $(V_o/I_o)$  ohms, where  $V_o$  is the amplifier's open-circuit voltage and  $I_o$  the short-circuit current.
- A major advantage of combining the two types of feedback is that the power dissipated in the armature or the torque produced under stall conditions may be controlled.
- If it is desired to limit the torque at stall, possibly to prevent damage to a transmission or coupler attached to the motor's shaft, it is necessary to limit the output current. The simultaneous use of voltage and current feedback with an amplifier can achieve this without the use of an external power resistor.

The input-output relationship for the amplifier,

$$V_{out} = -V_{in} \frac{R_{vf}}{R_{in}} - I_L R_s \frac{R_{vf}}{R_{cf}} \quad (2.4.3-1)$$

- The configuration is that of an inverting summing amplifier. Current limiting results because the two input voltages ( $V_{in}$  and  $I_L R_s$ ) have opposite signs.

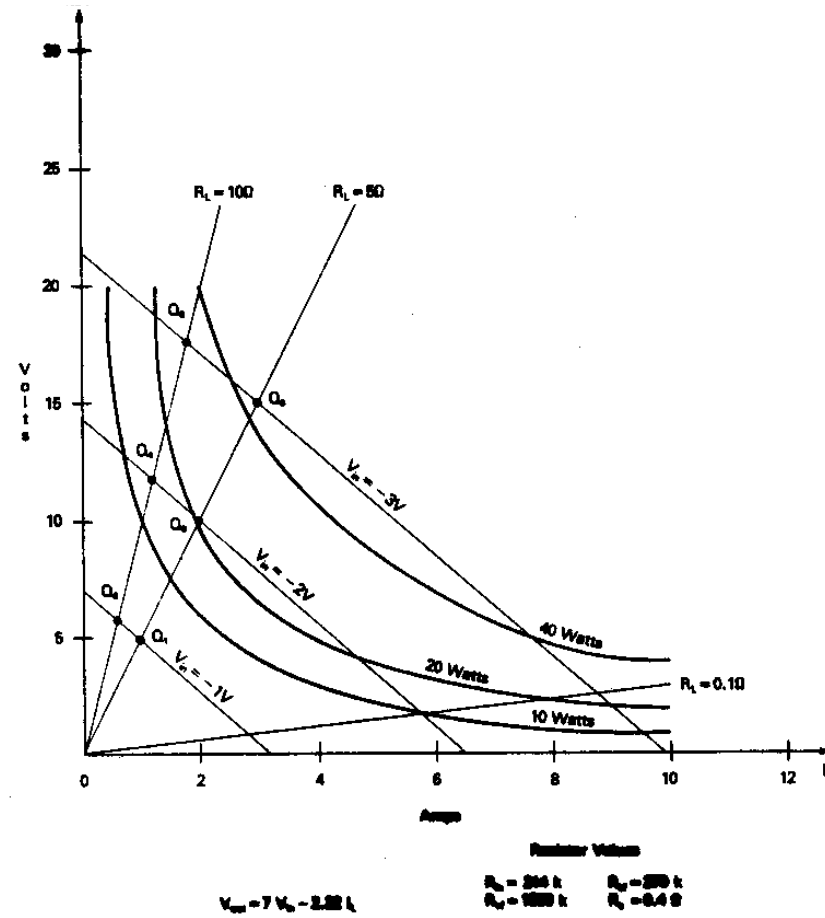


Figure 2.4.3-5 Load-Line Analysis for Amplifier with Combined Current and Voltage Feedback

2.4.3.1 Voltage Amplifier Driving a Servomotor

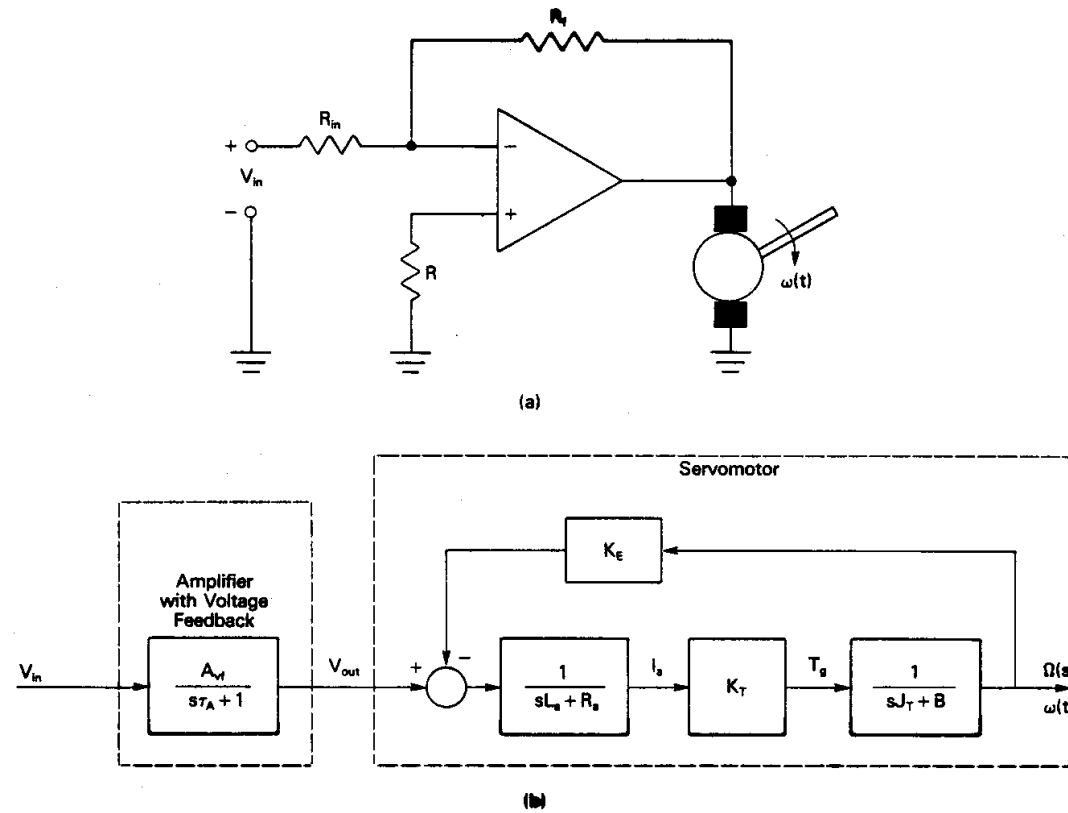


Figure 2.4.3.1-1 (a) Servomotor Driven by an Amplifier Utilizing Voltage Feedback (b) Block Diagram



The transfer function of the voltage amplifier,

$$\frac{V_{out}(s)}{V_{in}(s)} = \frac{A_{vf}}{1 + s\tau_A} \quad (2.4.3.1-1)$$

where  $\tau_A$  : the amplifier time constant, and  $A_{vf}$  : the magnitude of the gain.

$$|A_{vf}| = \frac{R_f}{R_{in}} \quad (2.4.3.1-2)$$

The overall transfer function,

$$\frac{\Omega(s)}{V_{in}(s)} = \frac{[K_T / (L_a J_T)]}{s^2 + [(R_a J_T + L_a B) / (L_a J_T)]s + [(K_T K_E + R_a B) / (L_a J_T (1 + s\tau_A))]} \cdot \frac{A_{vf}}{(1 + s\tau_A)} \quad (2.4.3.1-3)$$

- The use of voltage feedback has not affected the location of the motor poles. No loading exists between the two devices, which is true for a zero-output impedance amplifier.

2.4.3.2 Current Amplifier Driving a Servomotor

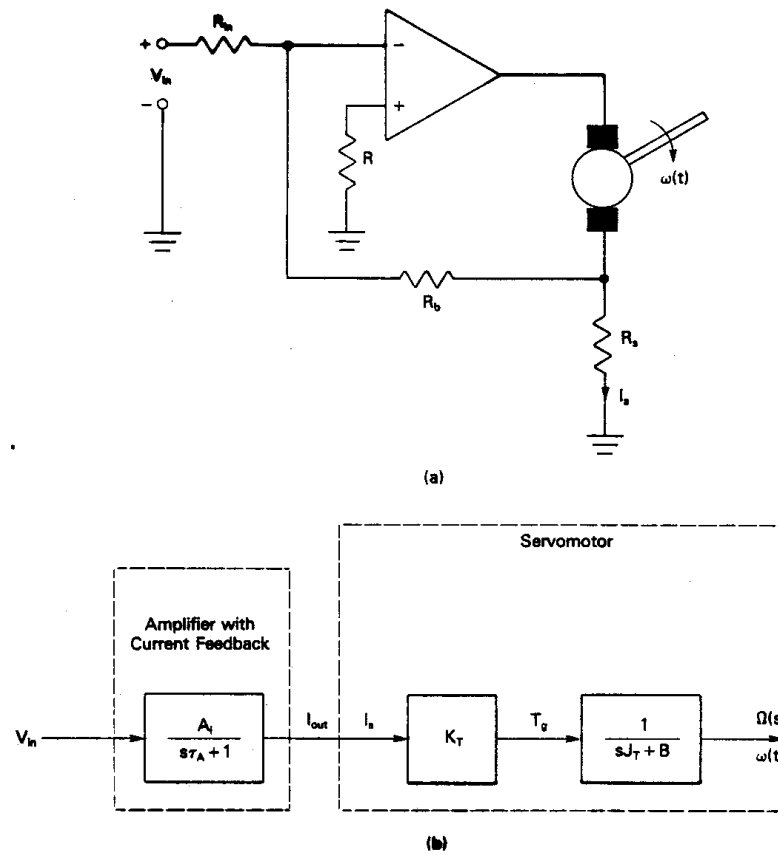


Figure 2.4.3.2-1 (a) Servomotor Driven by an Amplifier with Current Feedback (b) Block Diagram

The relationship between motor velocity and current,

$$K_T I_a(s) = (sJ_T + B)\Omega(s) \quad (2.4.3.2-1)$$

The transfer function relating armature current to shaft velocity,

$$\frac{\Omega(s)}{I_a(s)} = \frac{K_T}{sJ_T + B} \quad (2.4.3.2-2)$$

Transfer function of the current amplifier,

$$\frac{I_{out}(s)}{V_{in}(s)} = \frac{A_i}{1 + s\tau_A} \quad (2.4.3.2-3)$$

where  $\tau_A$  : the amplifier time constant and  $A_i$ : the gain of the amplifier,  $R_s \ll R_b$ ,

$$|A_i| = \frac{R_b}{R_{in}R_s} \quad (2.4.3.2-4)$$

The overall transfer function of a motor driven by a current amplifier,

$$\frac{\Omega(s)}{V_{in}(s)} = \frac{K_T A_i}{(sJ_T + B)(1 + s\tau_A)} \quad (2.4.3.2-5)$$

- The poles of the motor have been altered by the use of current feedback. In fact, the motor is seen to behave like a one-pole, rather than a two-pole device. Only the mechanical parameters of the system, the total inertia  $J_T$  and the viscous damping  $B$  have an effect on the behavior of the servo.

2.4.3.3 Current and Voltage Feedback Amplifier Driving a Servomotor

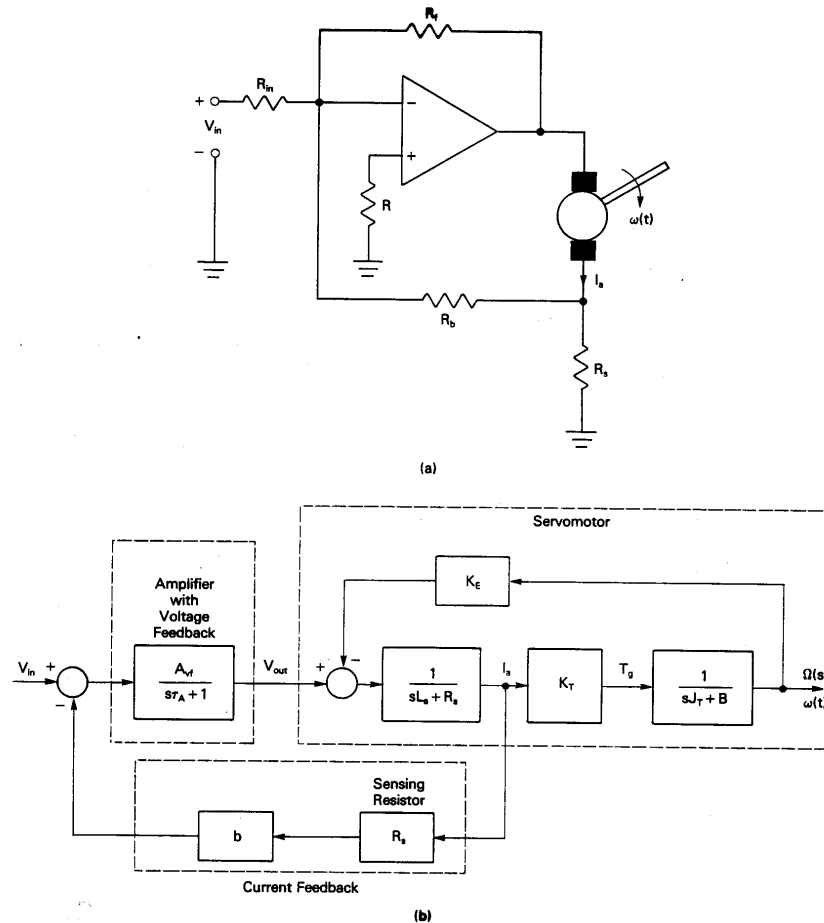


Figure 2.4.3.3-1 (a) Servomotor Driven by Amplifier Employing Both Voltage and Current Feedback (b) Block Diagram

The transfer function relating the shaft velocity to the input voltage,

$$\frac{\Omega(s)}{V_{in}(s)} = \frac{A_{vf} K_T}{(1 + s\tau_A)[(sL_a + R_a)(sJ_T + B) + K_E K_T] + AbR_s(sJ + B)} \quad (2.4.3.3-1)$$

where  $b = (R_{in}/R_b)$  and  $A_{vf} = (R_f/R_{in})$ .

- The presence of an additional term in the denominator can affect the pole locations.

## 2.4.4 AC Driver of DC Motor

### 2.4.4.1 Single-Phase, Half-Wave Thyristor Drive

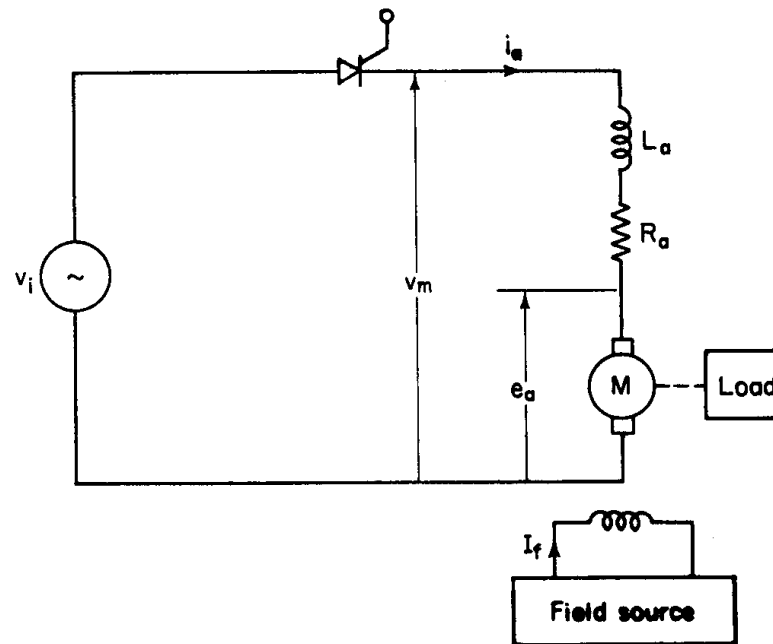


Figure 2.4.4.1-1 Armature Voltage Control of a DC Motor through Use of a Single-Phase, Half-Wave Controlled Rectifier

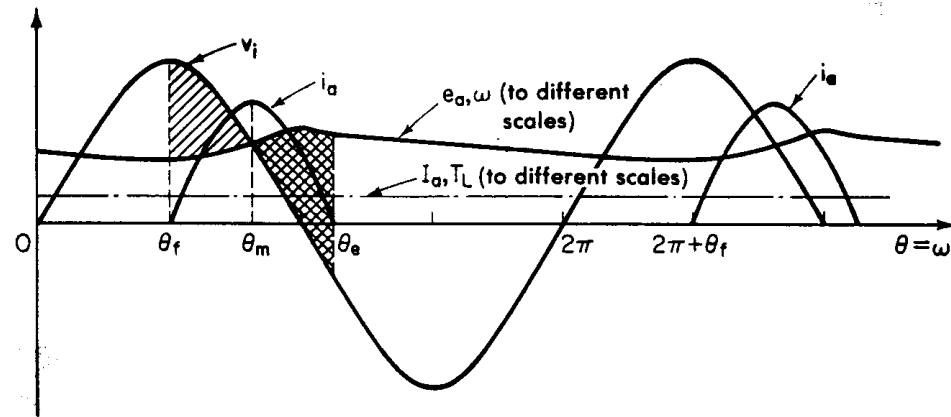


Figure 2.4.4.1-2 Wave Shapes of the Various Electrical Quantities, Motor Speed, and Load Torque

- The load torque to be supplied by the motor is an average armature current of  $I_a$  projected over a full cycle.
- The conduction period is the time during which the thyristor is conducting.

$$\theta_c = \theta_e - \theta_f \tag{2.4.4.1-1}$$

where  $\theta_f$ : the angle where the gate signal fires the thyristor and  $\theta_e$ : the extinction angle of the thyristor.

By Faraday's law,

$$\Delta i_a = \frac{1}{L_a} \int_{\theta_f}^{\theta_m} (v_i - e_a) dt \tag{2.4.4.1-2}$$

where  $L_a$ : the armature winding inductance.

2.4.4.2 Three-Phase, Half-Wave Thyristor Drive

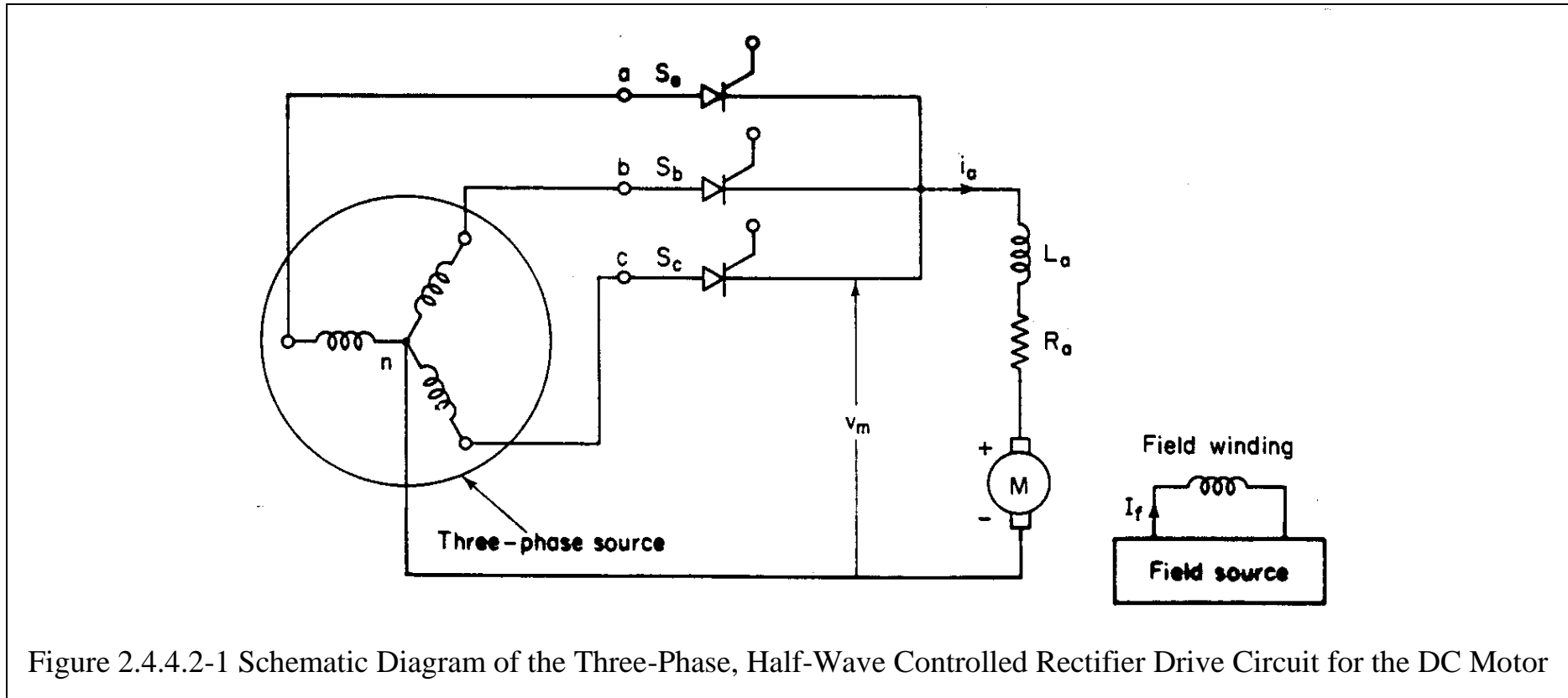


Figure 2.4.4.2-1 Schematic Diagram of the Three-Phase, Half-Wave Controlled Rectifier Drive Circuit for the DC Motor



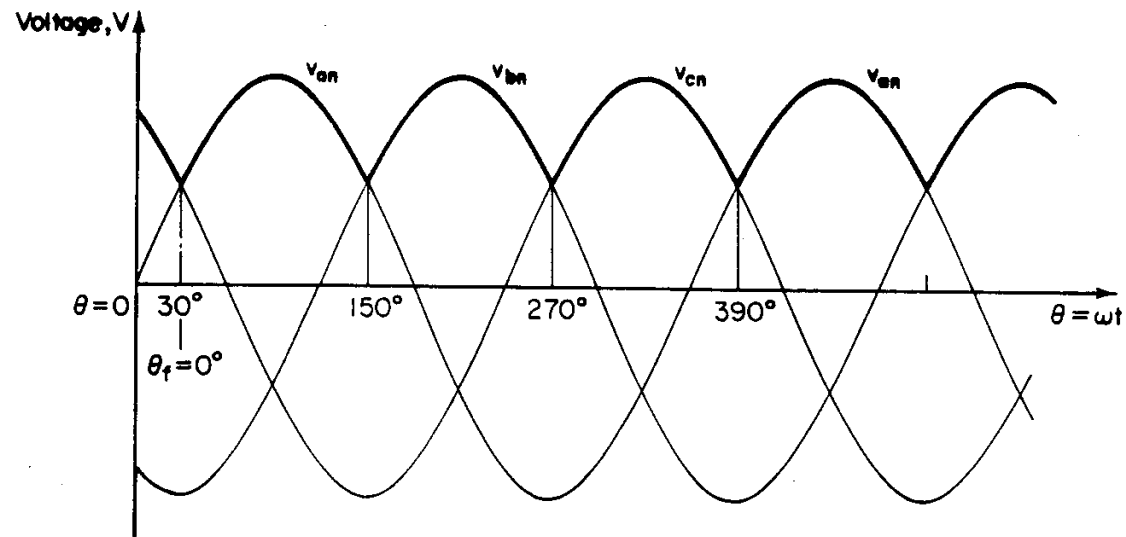


Figure 2.4.4.2-2 Wave Forms of the Balanced Three-Phase Source

The thyristor firing angle,  $\theta_f = 0^\circ$  corresponds to  $\theta = \omega t = 30^\circ$ .

The average voltage, when the thyristor is on for a period of  $120^\circ$ , corresponding to firing at angle  $\theta_f$ ,

$$V_m = V_{dc} = \frac{3}{2\pi} \int_{\theta_f + \pi/6}^{\theta_f + 5\pi/6} V_p \sin(\omega t) d(\omega t) \quad (2.4.4.2-1)$$

where  $V_p$  : the peak value of the line to neutral voltage and  $V_m$  : to the motor armature voltage.

$$V_{dc} = V_m = 0.827V_p \cos \theta_f \quad (2.4.4.2-2)$$

- A setting of the thyristor gate circuit to yield  $\theta_f = 0^\circ$  means that a maximum dc voltage appears across the armature terminals. A setting of  $\theta_f = 90^\circ$  reduces the average voltage to zero.

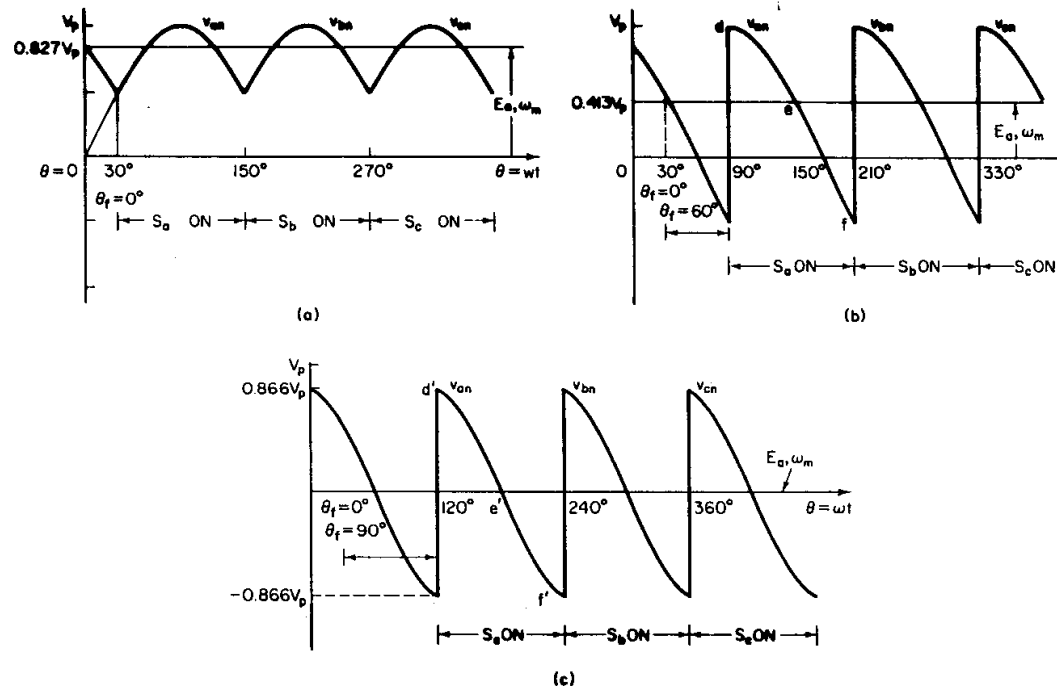


Figure 2.4.4.2-3 Wave Shapes of the Rectified Three-Phase Source for Three Values of the Thyristor Firing Angle, Average Values of the Armature Induced emf  $E_a$ , and Motor Speed  $\omega_m$  (a) Full-Firing of the Thyristor for  $\theta_f = 0^\circ$  (b)  $\theta_f = 60^\circ$  (c)  $\theta_f = 90^\circ$

### 3. Alternating-Current Motors

#### 3.1 Construction of AC Motor

##### 3.1.1 Induction Motor

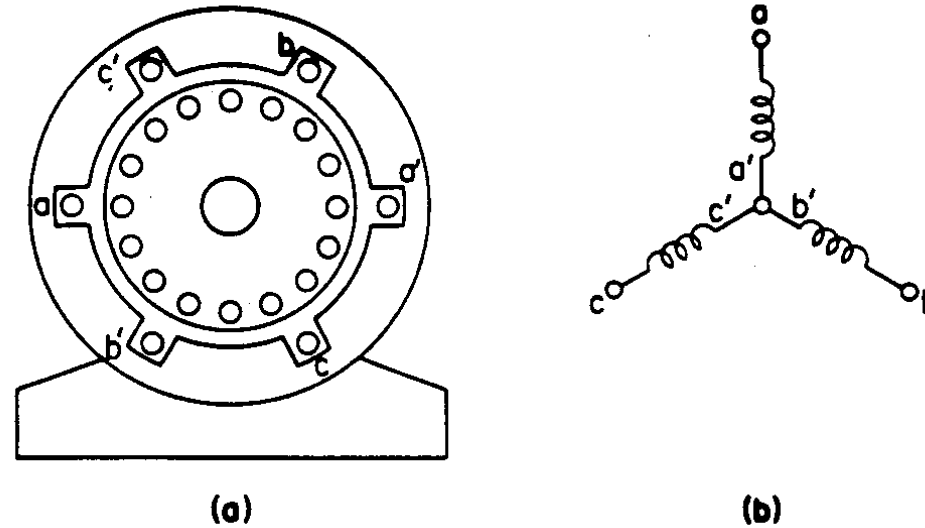


Figure 3.1.1-1 Three-Phase Induction Motor (a) Showing a Stator with Three-Phase Winding and Squirrel-Cage Rotor (b) Schematic Representation of a Three-Phase Y-Connected Stator Winding

- Induction motor is one of the most rugged and most widely used machines in industry.
- Its stator is composed of laminations of high-grade sheet steel.
- The inner surface is slotted to accommodate a three-phase winding.
- The three-phase winding is represented by three coils, the axes of which are 120 electrical degrees apart. One end of each phase is commonly connected, the three-phase stator winding is said to be Y-connected.

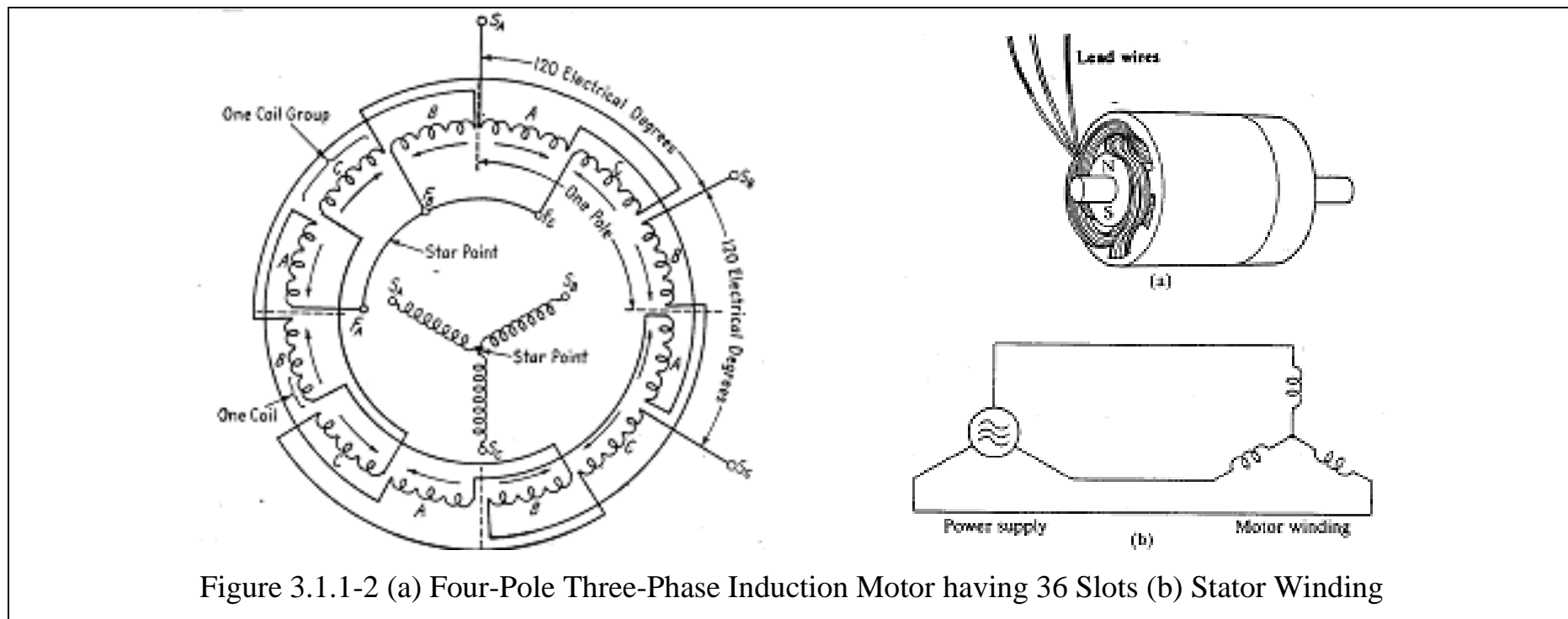


Figure 3.1.1-2 (a) Four-Pole Three-Phase Induction Motor having 36 Slots (b) Stator Winding

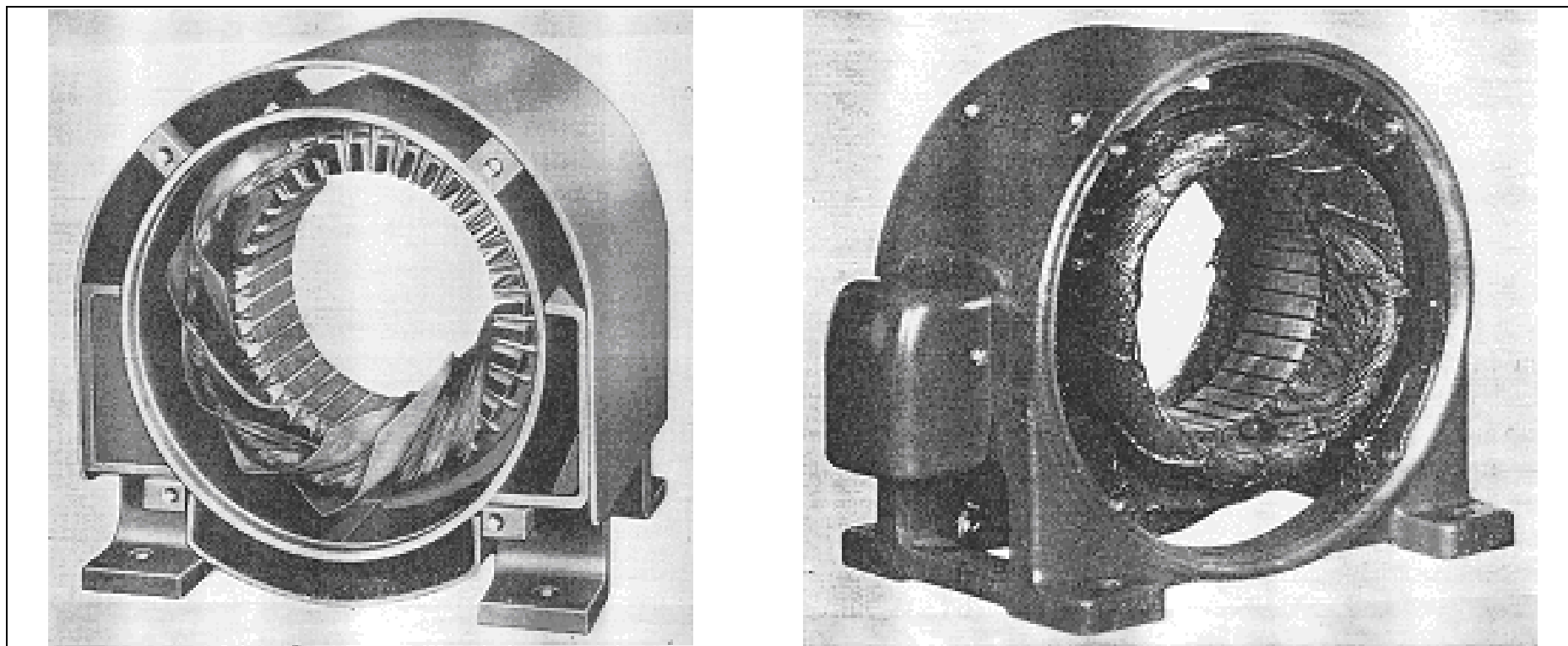


Figure 3.1.1-3 (a) Winding in the Stator Slot (b) Completely Wound Stator

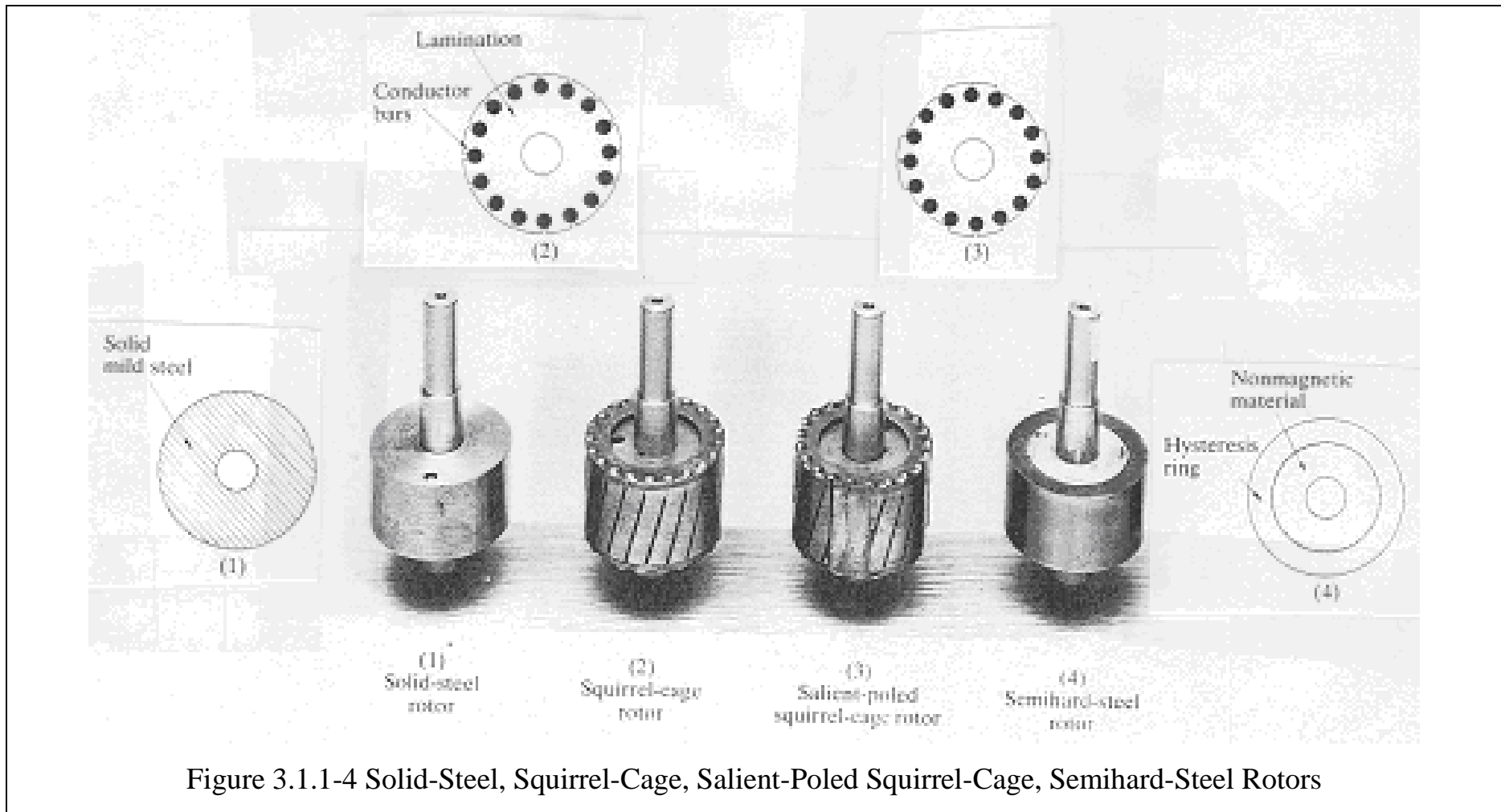


Figure 3.1.1-4 Solid-Steel, Squirrel-Cage, Salient-Poled Squirrel-Cage, Semihard-Steel Rotors

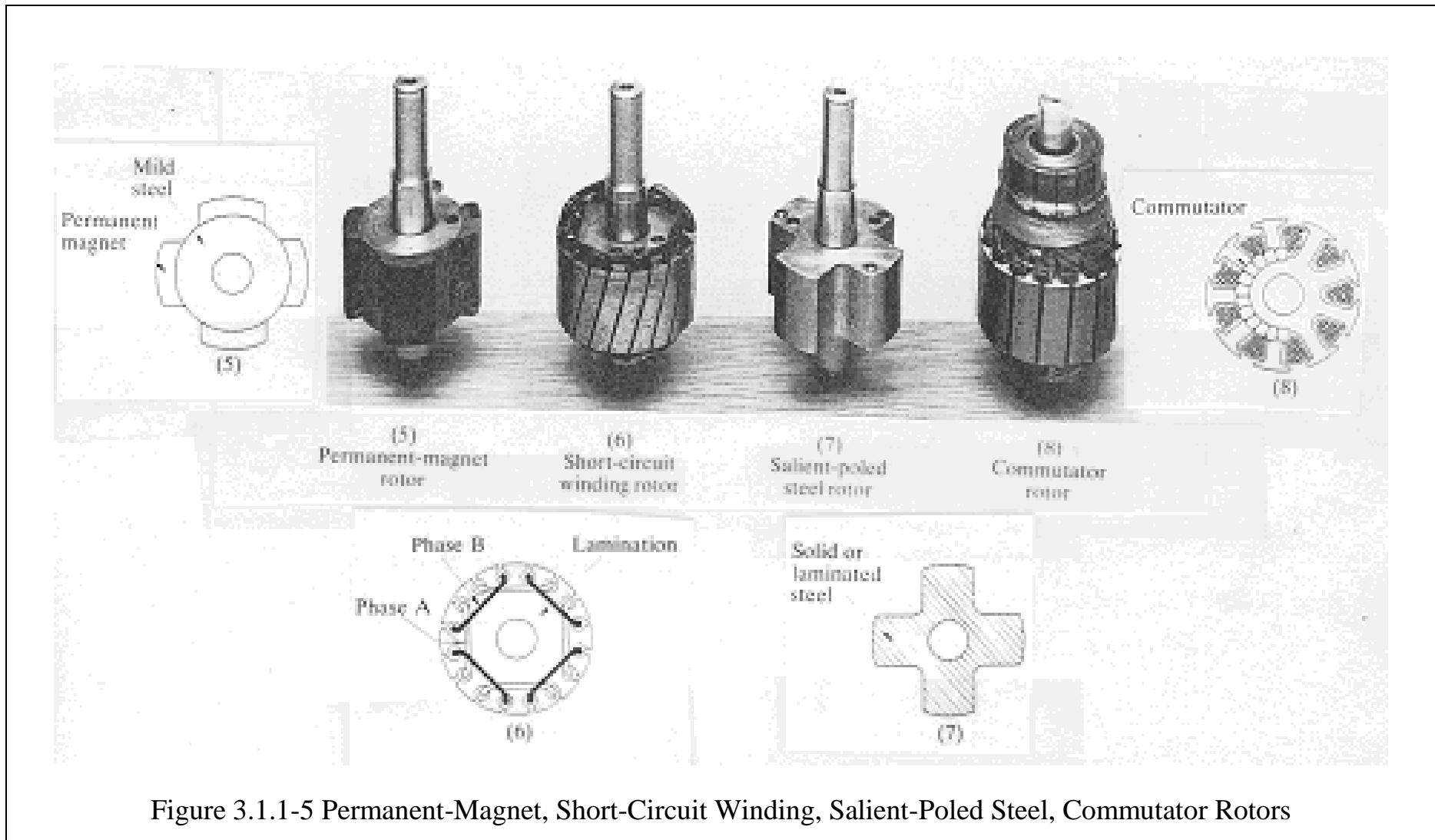
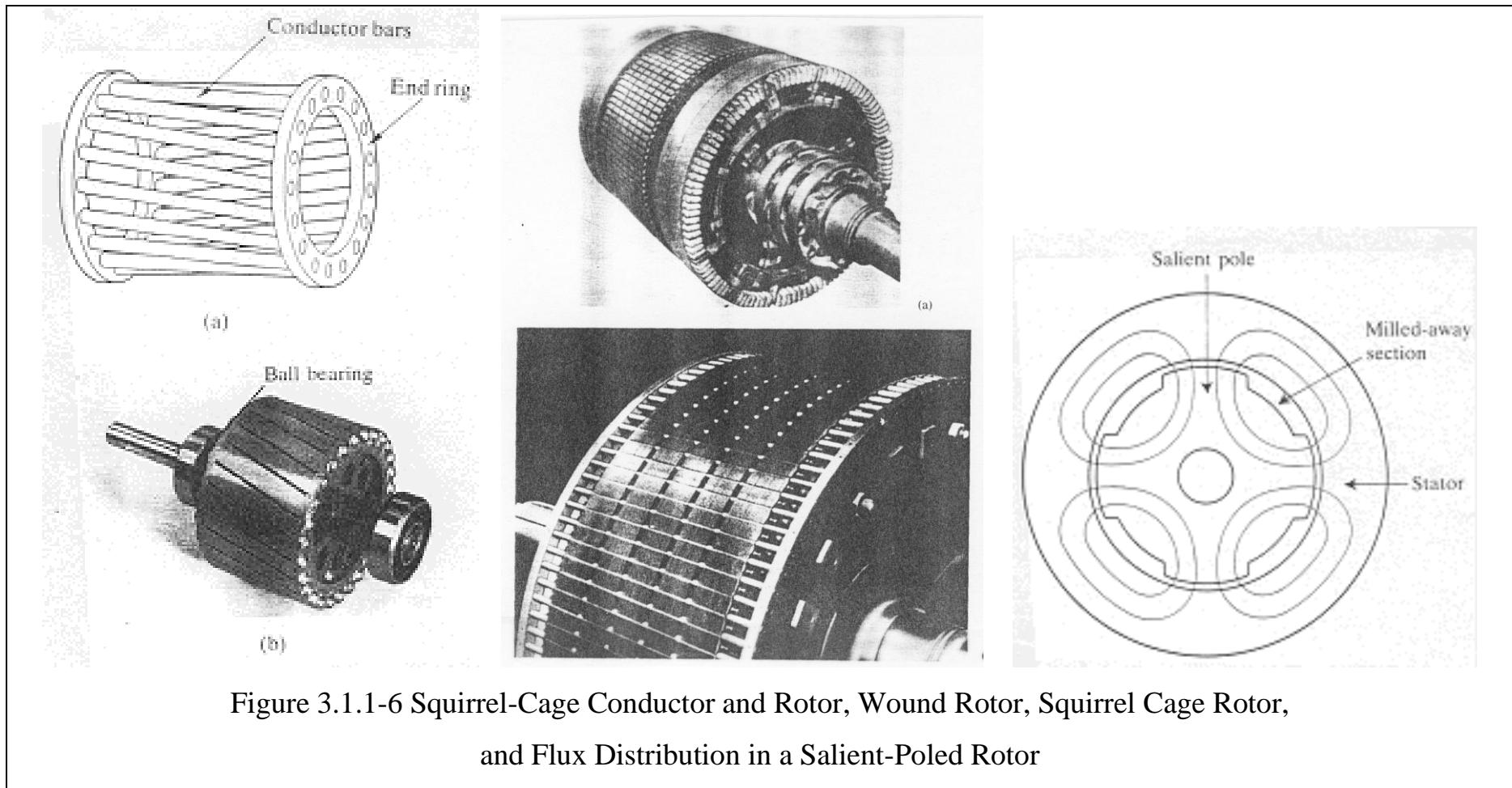


Figure 3.1.1-5 Permanent-Magnet, Short-Circuit Winding, Salient-Poled Steel, Commutator Rotors





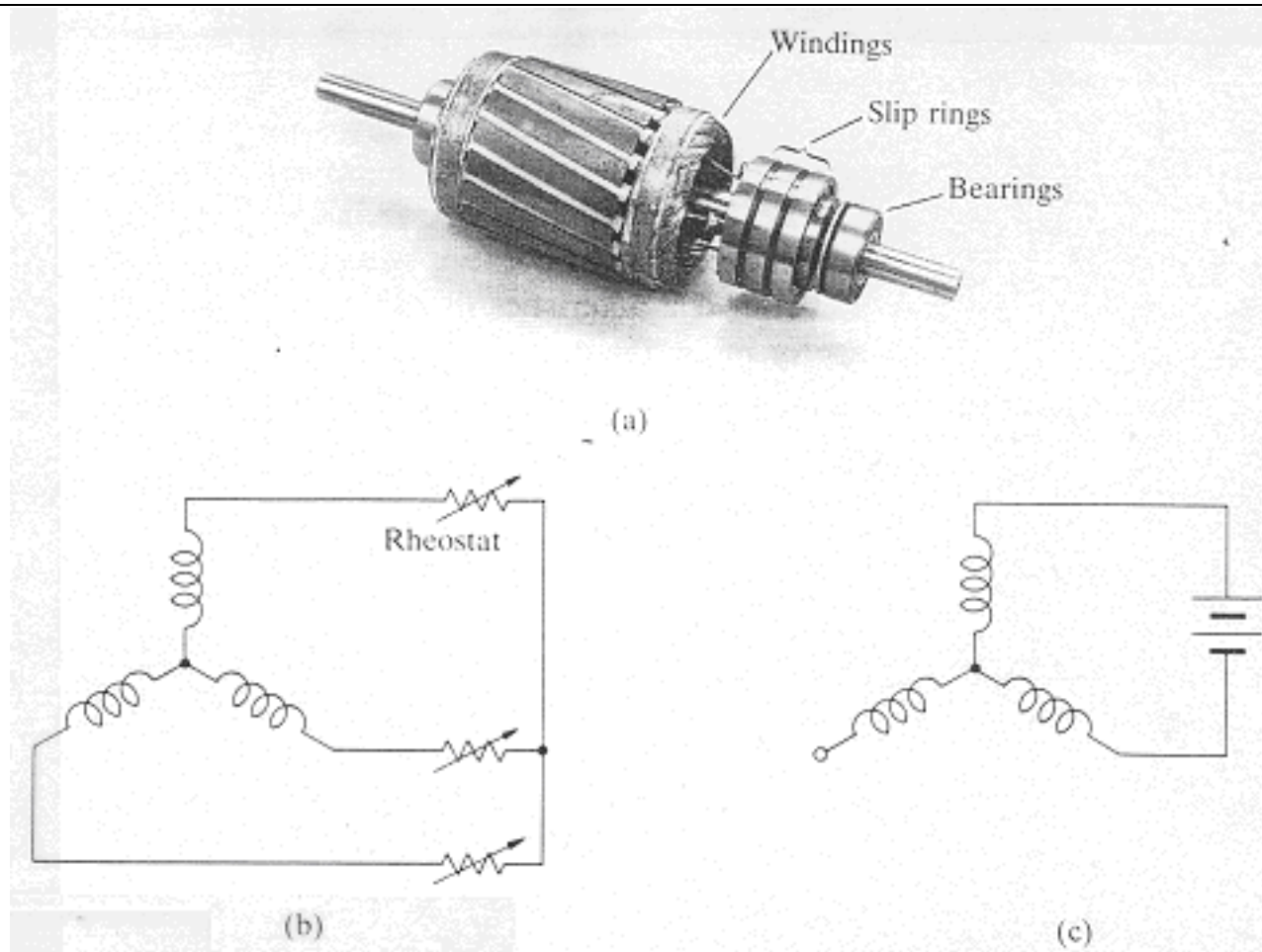


Figure 3.1.1-7 (a) Wound Robot (b) When Used as an Induction Motor (b) When Used as a Synchronous Motor

- There are various types of rotor of induction motor.
- The rotor may be made of laminations of slotted ferromagnetic material.
- Wound rotor always has three slip rings to allow an external three-phase resistor to be connected to the rotor winding for the purpose of providing speed control.
- The squirrel-cage winding consists merely of a number of copper bars imbedded in the rotor slots and connected at both ends by means of copper end rings. In some of the smaller sizes, aluminum is used.
- The squirrel-cage construction is not only simpler and economical than the wound-rotor type but more rugged as well.
- In normal operation, a three-phase voltage is applied to the stator winding at points *a-b-c*.
- Magnetizing currents flow in each phase, which together create a revolving magnetic field having two poles.
- The speed of the field is fixed by the frequency of the magnetizing currents and the number of poles for which the stator winding is designed.
- The revolving field produced by the stator winding cuts the rotor conductors, thereby inducing voltages. Since the rotor winding is short-circuited by the end rings, the induced voltages cause currents to flow which in turn react with the field to produce electromagnetic torque-and so motor action results.
- For the three-phase induction motor the field winding is located on the stator and the armature winding on the rotor.
- Induction motor is singly excited, electrical power is applied only to the stator winding. Current flows through the rotor winding by induction.

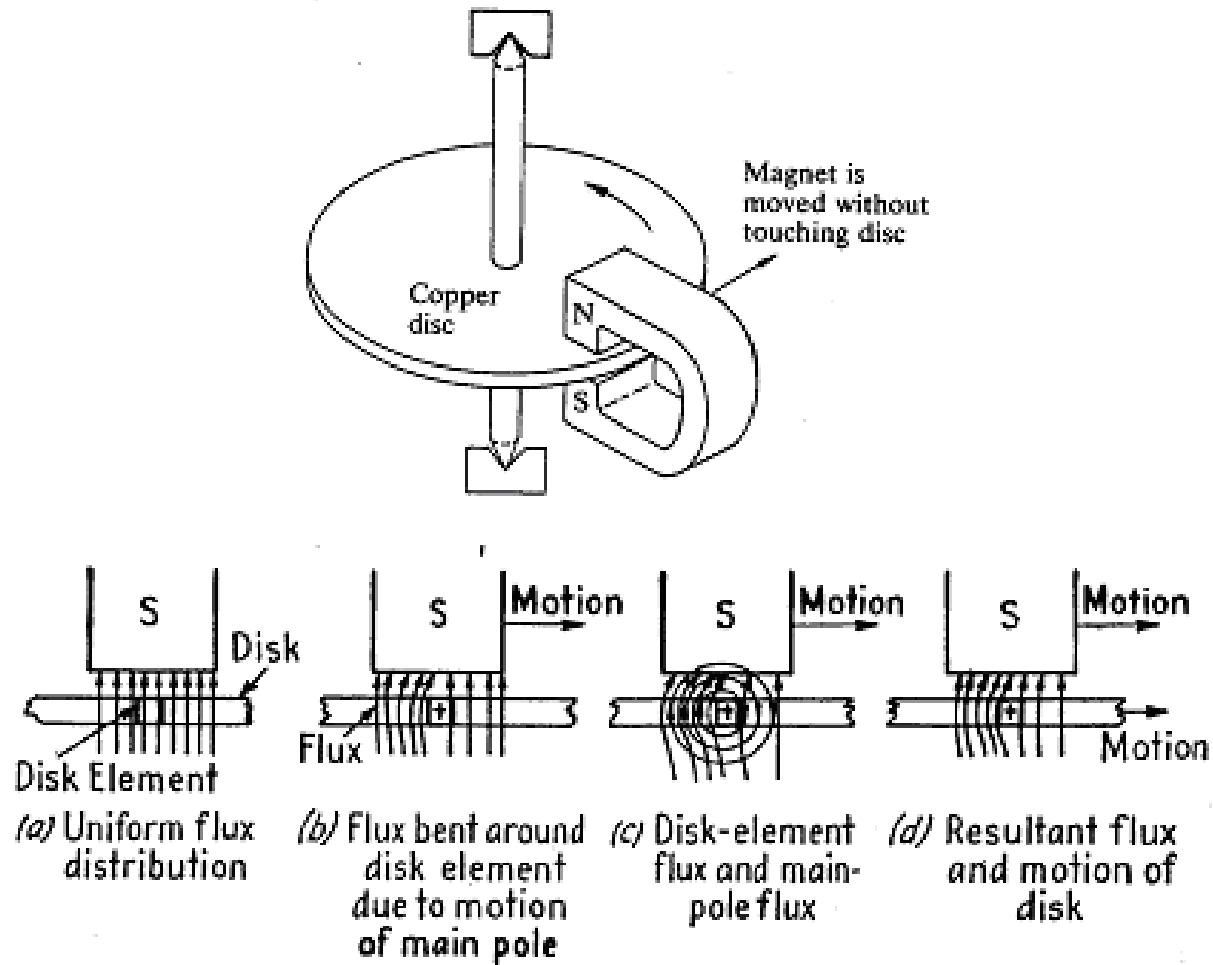


Figure 3.1.1-8 Arago's Rotation and Principle of Induction Motor

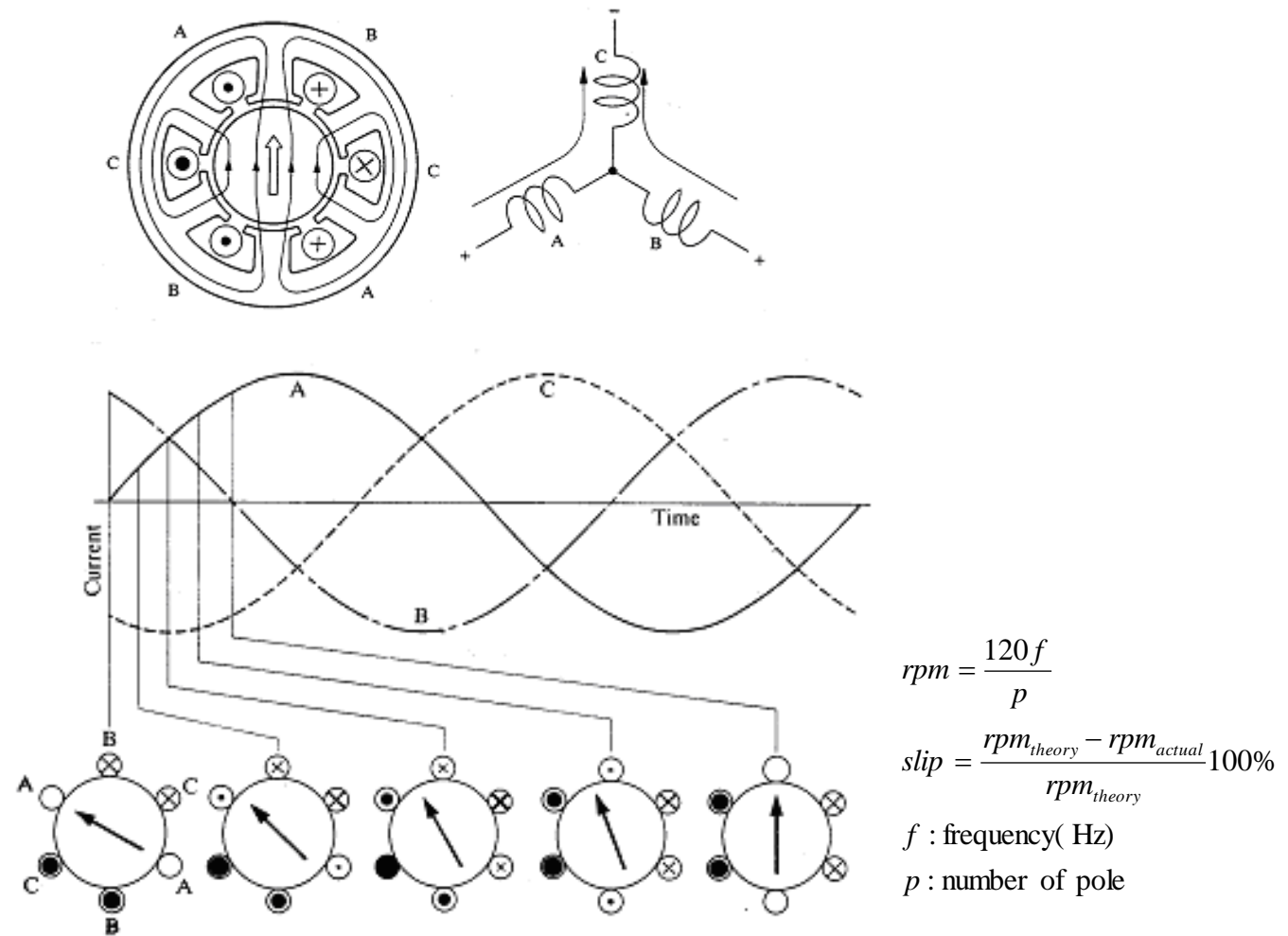


Figure 3.1.1-9 Current and Magnetic Flux Distribution and Revolving Magnetic Field

### 3.1.2 Synchronous Motor

- The stator consists of a stator frame, a slotted stator core, which provides a low-reluctance path for the magnetic flux, and a three-phase winding imbedded in the slots.
- The rotor either is cylindrical and equipped with a distributed winding or has salient poles with a coil wound on each leg.
- The cylindrical construction is used almost exclusively for turbogenerators, which operate at high speeds.
- The salient-pole construction is used exclusively for synchronous motors operating at speeds of 1800 rpm or less.
- The field winding is located on the rotor; the armature winding is located on the stator.
- Ac power is applied to the stator winding and dc power is applied to the rotor winding for the purpose of energizing the field poles.
- The synchronous motor is a doubly fed machine.
- The motor develops a nonzero torque at only one speed, synchronous speed.
- The synchronous speed refers to that rotor speed at which the rotor flux field and the armature ampere-conductor distribution (or the armature flux field) are stationary with respect to each other.
- The synchronous motor has no starting torque.

- To produce a continuous nonzero torque, it is first necessary to bring the dc excited rotor to synchronous speed by means of an auxiliary device. Most often, the auxiliary device is made of a squirrel-cage winding similar to that used in induction motors and imbedded in the pole faces. By means of this winding, the synchronous motor is brought up to almost synchronous speed. Then, if the field winding is energized at the right moment, a positive torque will be developed for a sufficiently long period to allow the armature poles to pull the rotor poles into synchronism.
- Because the magnetizing current for the synchronous machine originates from a separate source, the air-gap lengths are larger than those found in induction motors of comparable size and rating.
- The synchronous motors are more expensive and less rugged than induction motors in the smaller horsepower ratings because the rotor must be equipped with slip rings and brushes to allow the direct current to be conducted to the field winding.

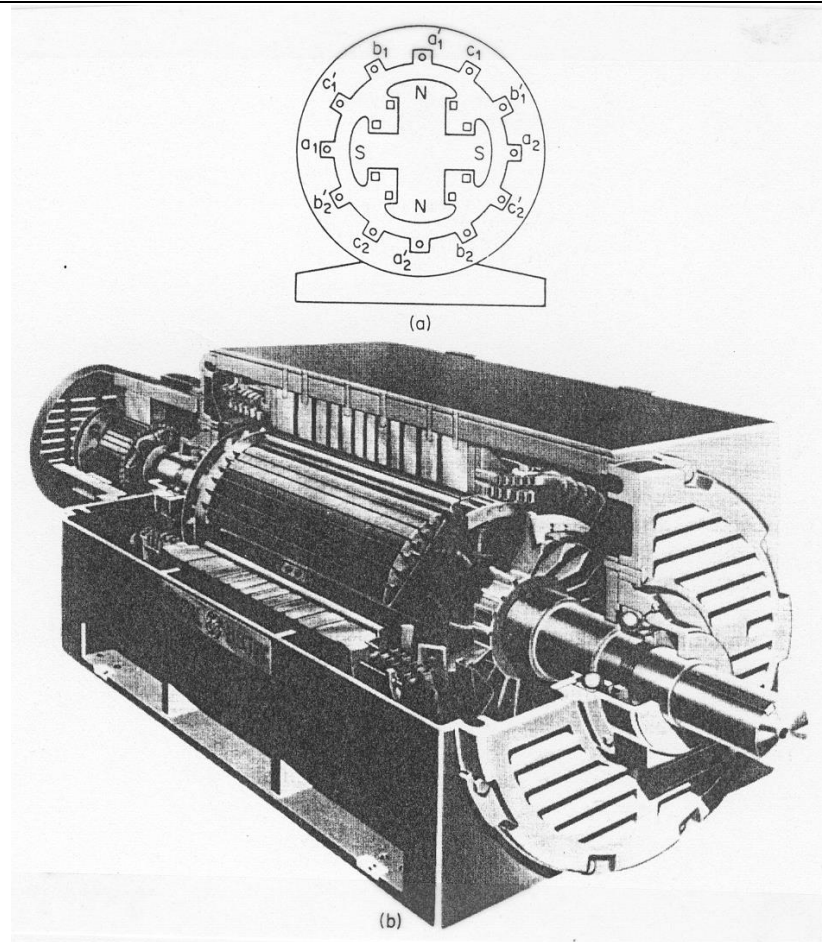


Figure 3.1.2-1 Salient-Pole Synchronous Motor

(a) Schematic Diagram (b) Showing Field Coils Wrapped around Salient Poles

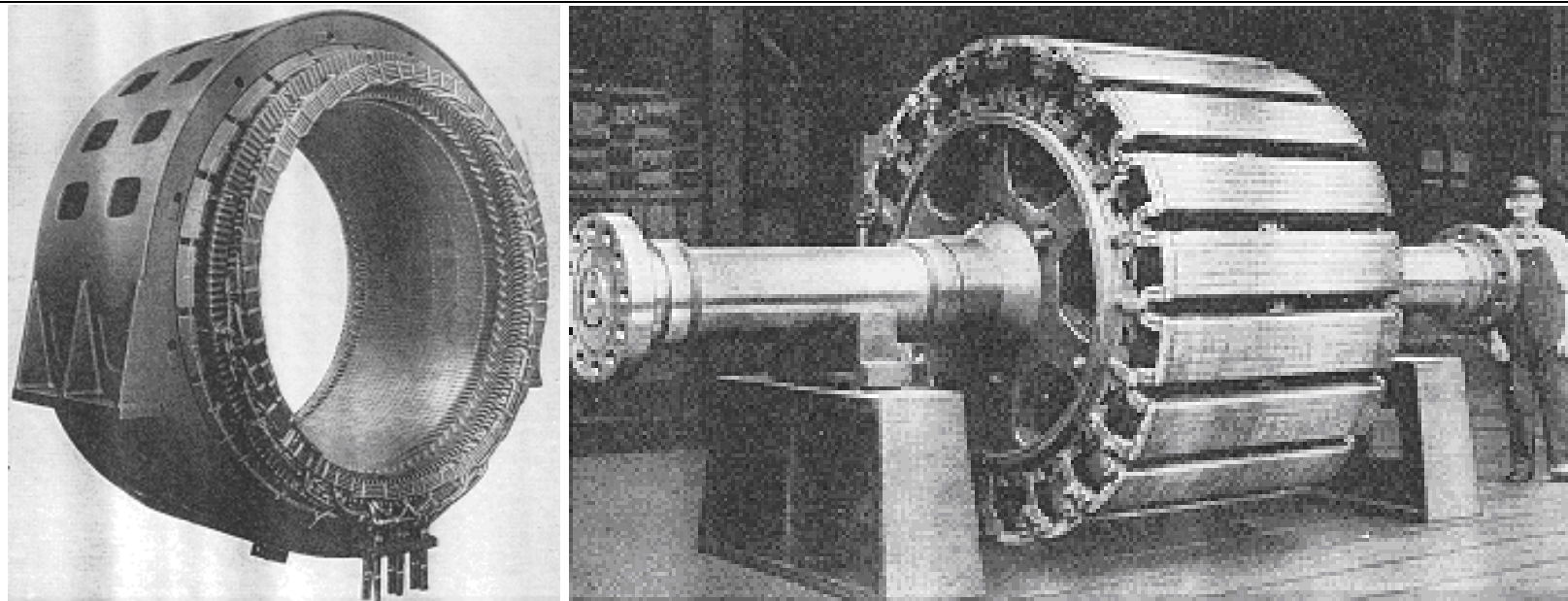


Figure 3.1.2-2 Completely Wound Stator and Completely Assembled 20-Pole Salient-Pole Rotor of an Induction Motor



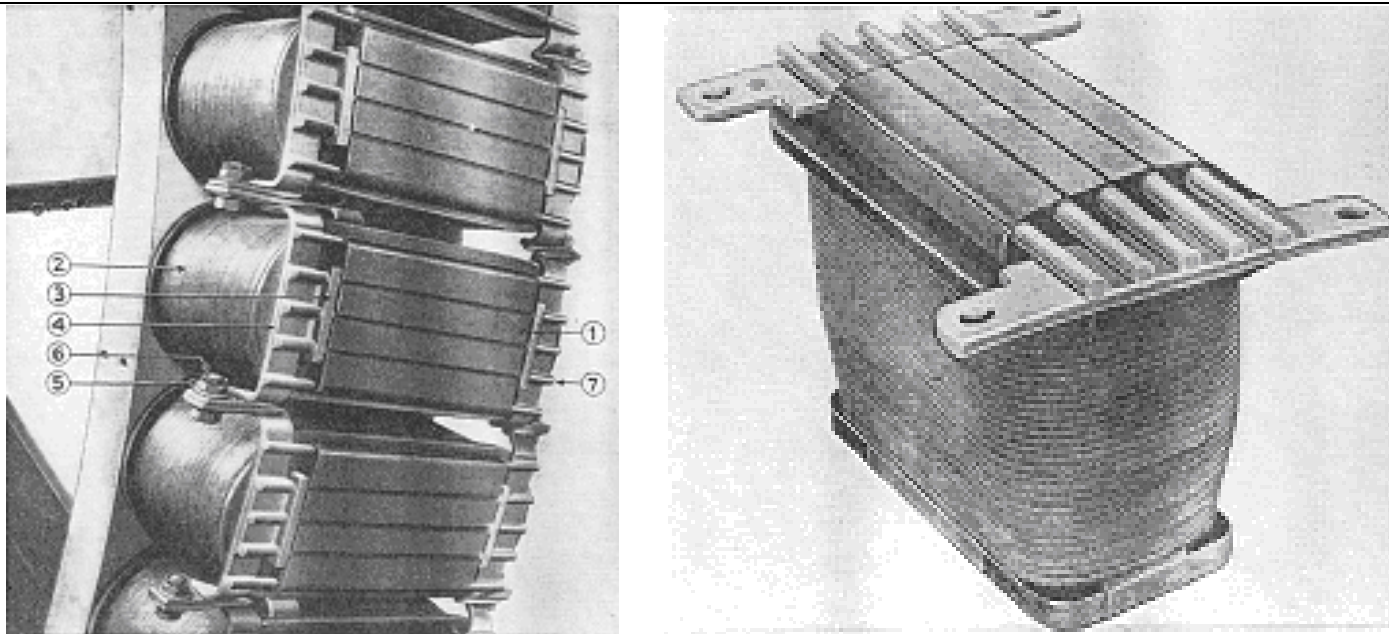


Figure 3.1.2-3 Squirrel-Cage Construction in the Rotor of a Synchronous Motor and the Squirrel Cage in the Pole Face

### 3.2 AC Motor Model

The instantaneous value of the emf induced in any one phase,

$$e = \omega N \phi \sin \omega t \quad (3.2-1)$$

where  $N$  : the total number of turns per phase of a three-phase winding,  $\phi$  : the total flux per pole,

$\omega$  : the relative cutting speed in electrical radians per second of the winding with respect to the flux-density wave.

The maximum value of this ac voltage,

$$E_{\max} = \omega N \phi \quad (3.2-2)$$

The corresponding rms value,

$$E = \frac{E_{\max}}{\sqrt{2}} = \sqrt{2} \pi f N \phi = 4.44 f N \phi \quad (3.2-3)$$

- The individual coils that make up the total  $N$  turns are intentionally designed to span not the full-pole pitch but rather only about 80% to 85% of a pole pitch. Such a coil is called a fractional-pitch coil which has the advantage of virtually eliminating the effects of all harmonics that may be present in the flux-density wave while only slightly reducing the fundamental component.
- The reduction in the fundamental component can be represented by a winding factor denoted by  $K_w$ . Usually  $K_w$  has values ranging from 0.85 to 0.95.

The final practical version of the induced rms voltage equation for an ac motor,

$$E = 4.44 f N K_w \phi \quad (3.2-4)$$

Electromagnetic torque produced by the interaction of the armature ampere-conductor distribution with the field distribution,

$$T = \frac{\pi}{8} p^2 \phi J_m \cos \psi \quad (3.2-5)$$

where  $p$  : number of poles,  $\phi$  : flux per pole in Wb,  $J_m$  : equivalent current sheet that represents an idealized ampere-conductor distribution expressed in A/rad,  $\psi$  : phase displacement angle between the start of the current sheet and the start of the flux density wave beneath a pole.

### 3.3 AC Motor Driver

#### 3.3.1 Speed Control of Induction Motor

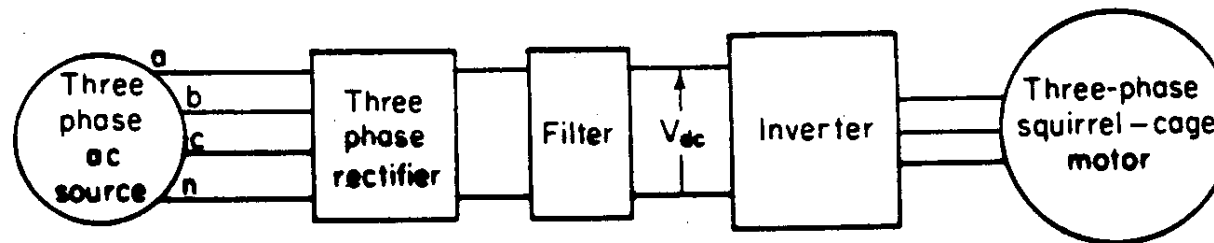


Figure 3.3.1-1 Principal Equipment Needed in the Speed Control of an Induction Motor

- The three-phase rectifier in the figure serves the purpose of converting the commonly available three-phase voltage to a dc source. Usually, the output of the rectifier contains higher harmonics of the fundamental frequency of the ac source and these are conveniently removed by an appropriate filter.
- The inverter generates a new three-phase voltage source which in general exhibits the properties of variable frequency, adjustable voltage, and even adjustable phase. This can be achieved by designing the speed control system so that it keeps the ratio of the inverter output voltage to the controlled frequency a constant.

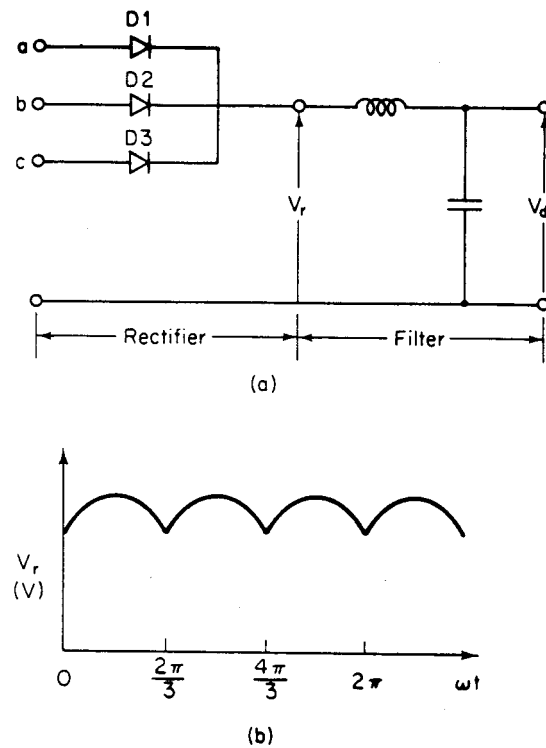
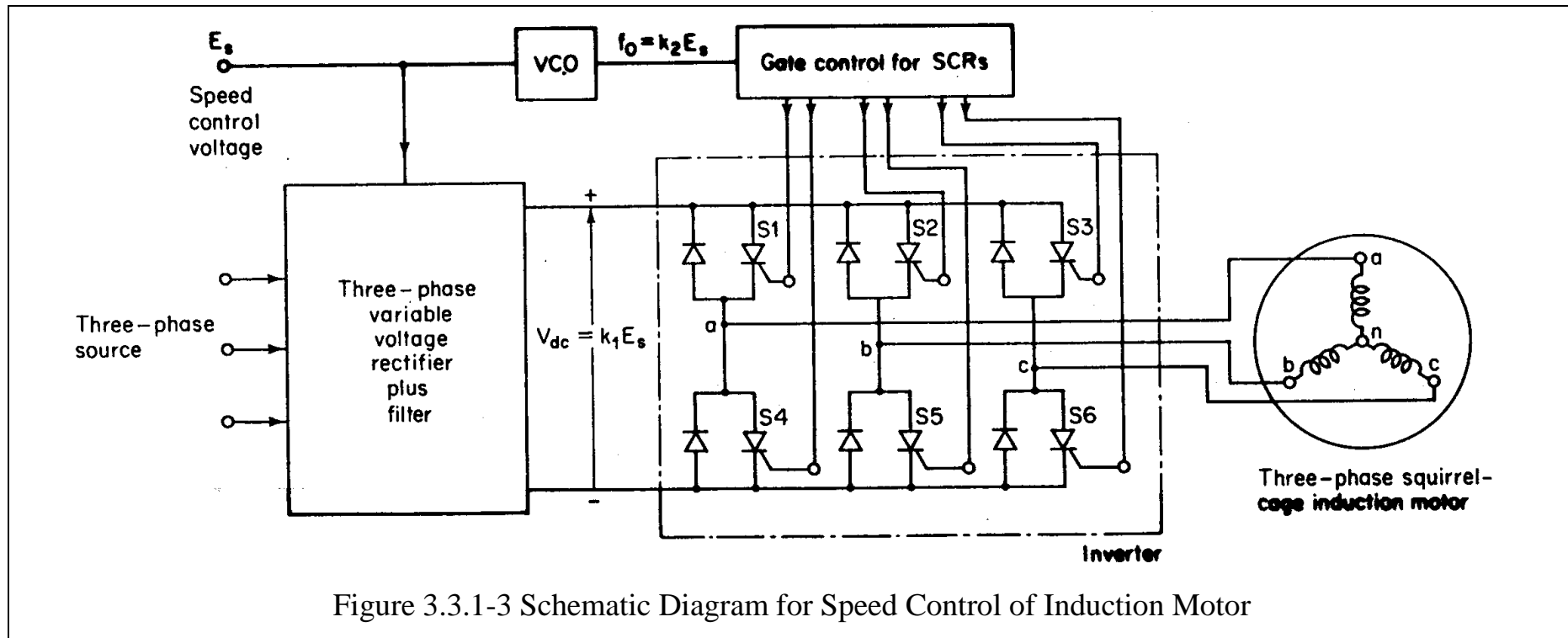


Figure 3.3.1-2 (a) Three-Phase Half-Wave Rectifier (b) Output Wave Shape of the Rectifier

- The output voltage can be made adjustable by replacing the diodes with silicon-controlled rectifiers (SCRs). By placing a suitable voltage pulse on the gate terminal of the SCR, the instant of firing of the diode section can be delayed, thereby reducing the value of the rectified output.



- The speed control voltage  $E_s$  serves two functions.
  - It controls the rectifying action of the three-phase rectifier in a fashion that allows the magnitude of the dc output to be directly proportional to  $E_s$ .
  - The speed control voltage also determines the output frequency of a voltage-controlled oscillator (VCO).

$$V_{dc} = k_1 E_s \quad (3.3.1-1)$$

$$f_o = k_2 E_s \quad (3.3.1-2)$$

where  $V_{dc}$  : the output voltage of the three-phase rectifier plus filter and  $f_o$  : the output frequency of the oscillator.

- The diodes that are placed in parallel with the SCRs in a reversed orientation are needed to provide a path for reactive currents when the corresponding SCR is gated off.
- The inverter takes the dc source and manipulates it to produce a three-phase ac voltage set.

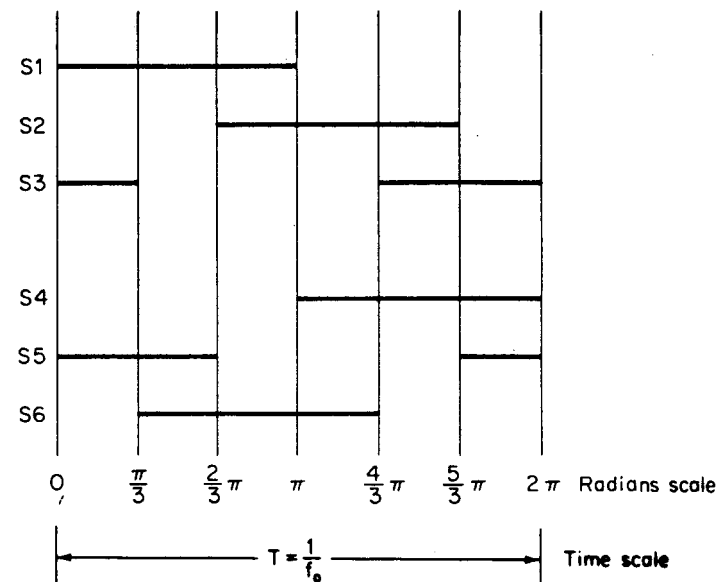


Figure 3.3.1-4 Switching Sequence Diagram for the SCRs

| Interval     | ON-gated SCR's | Voltage distribution on induction motor phases | Corresponding line voltages                             |
|--------------|----------------|--|---|
| (a) 0–60°    | S1, S3, S5     |  | $V_{ab} = V_{dc}$<br>$V_{bc} = -V_{dc}$<br>$V_{ca} = 0$ |
| (b) 60–120°  | S1, S5, S6     |  | $V_{ab} = V_{dc}$<br>$V_{bc} = 0$<br>$V_{ca} = -V_{dc}$ |
| (c) 120–180° | S1, S2, S6     |  | $V_{ab} = 0$<br>$V_{bc} = V_{dc}$<br>$V_{ca} = -V_{dc}$ |

Table 3.3.1-1 Distribution of the DC Input Voltage to the Inverter



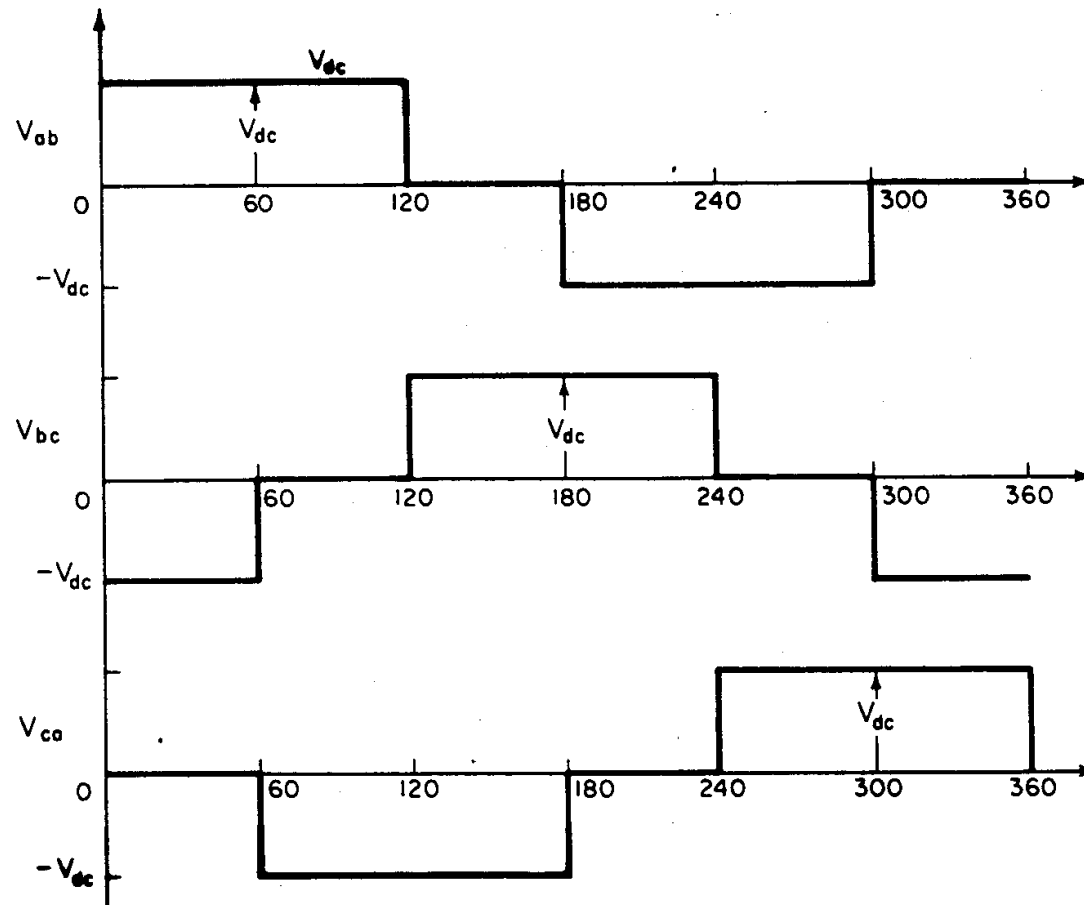


Figure 3.3.1-5 Variations of the Inverter Output Line-to-Line Voltages

From Fourier series analysis, the amplitude of fundamental component,

$$V_{1p} = \frac{2\sqrt{3}}{\pi} V_{dc} \quad (3.3.1-3)$$

The corresponding rms value of this voltage,

$$V_1 = \frac{V_{1p}}{\sqrt{2}} = \frac{\sqrt{6}}{\pi} V_{dc} = 0.78V_{dc} \quad (3.3.1-4)$$

The rms value of the voltage applied to the induction motor,

$$V_1 = 0.78V_{dc} = 0.78k_1E_s = k_0E_s \quad (3.3.1-5)$$

where  $k_0$  : a new constant =  $0.78k_1$ .

- The induced emf that occurs per phase in the induction motor for this type of speed control is normally  $1/\sqrt{3}$  of  $V_1$ .

$$E = \frac{V_1}{\sqrt{3}} = 4.44NK_w f_0 \phi = \frac{k_0}{\sqrt{3}} E_s \quad (3.3.1-6)$$

$$\phi = \frac{k_0 E_s}{\sqrt{3}(4.44)NK_w f_0} = \frac{k_0 E_s}{Kf_0} \quad (3.3.1-7)$$

where  $K = \sqrt{3}(4.44NK_w)$ .

- Speed control is achieved that ensures operation at constant flux.

$$\phi = \frac{k_0 E_s}{Kk_2 E_s} = \frac{k_0}{Kk_2} = \text{a constant} \quad (3.3.1-8)$$

### 3.3.2 Field Excitation by Rectified Source of Synchronous Motor

- One solution is to equip the synchronous motor with an auxiliary self-excited dc generator attached to the motor shaft.
- Once the synchronous motor reached nearly synchronous speed through the induction-motor action of its squirrel-cage winding, the dc generator field rheostat was adjusted to furnish the proper excitation for the synchronous motor, which in turn pulled the synchronous motor into synchronism with the line voltage.
- In applications of large synchronous motors, the exciter generator is replaced by a dc source obtained by electronic methods employing diodes and silicon-controlled rectifiers (SCRs).

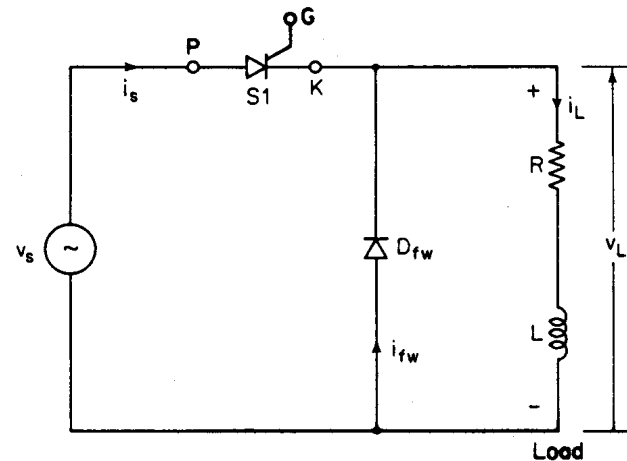


Figure 3.3.2-1 Single-Phase, Half-Wave Controlled Rectifier Circuit

- A single-phase, half-wave controlled rectifier can be used to adjust the field current in the field winding of a synchronous motor.
- The field winding is represented by the resistance  $R$  and the inductance  $L$  and is called the load.
- The diode  $D_{fw}$  serves to shunt the load.
- The silicon-controlled rectifier behaves as a controllable switch.
- When firing of the thyristor (SCR) is made to occur at  $\theta = 0^\circ$ , the voltage developed across the load is a maximum,  $V_m/\pi$ , where  $V_m$  denotes the peak value of the ac supply voltage.
- Action of the inductance causes the current to exist beyond the time when the thyristor ceases to conduct. As soon as the SCR cuts off at  $\omega t = \pi$ , the load current tries to change instantaneously. Then, by Lenz's law, an internally generated emf is induced in the inductor of a polarity that acts to sustain the original direction of the current. This action puts a positive potential across the shunt diode,  $D_{fw}$ , thus providing a path for the load current.

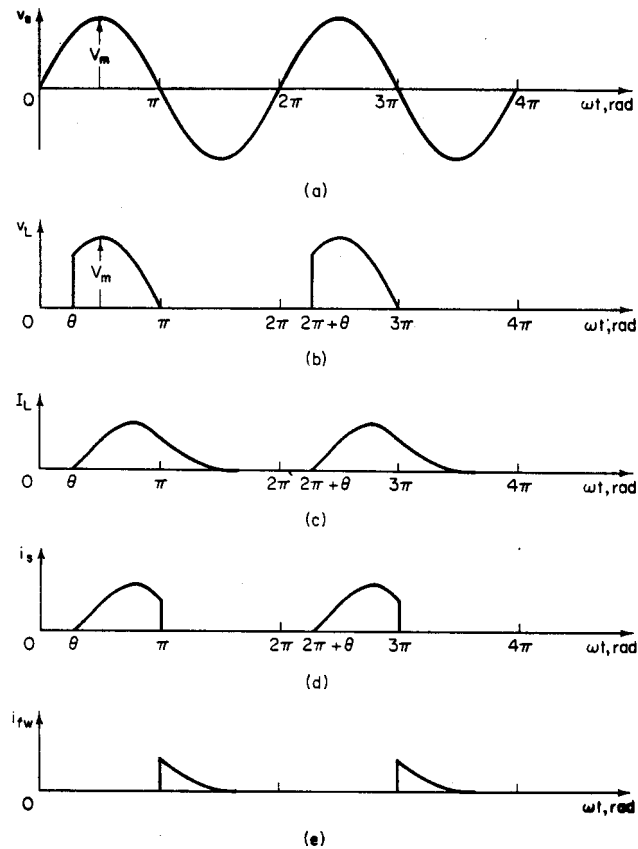


Figure 3.3.2-2 Time Variations of the Electrical Quantities (a) Source Voltage (b) Rectified Voltage across the  $R$ - $L$  Load (c) the  $R$ - $L$  Load Current Variation (d) Source Current (e) Current in the Free-Wheeling Shunt Diode

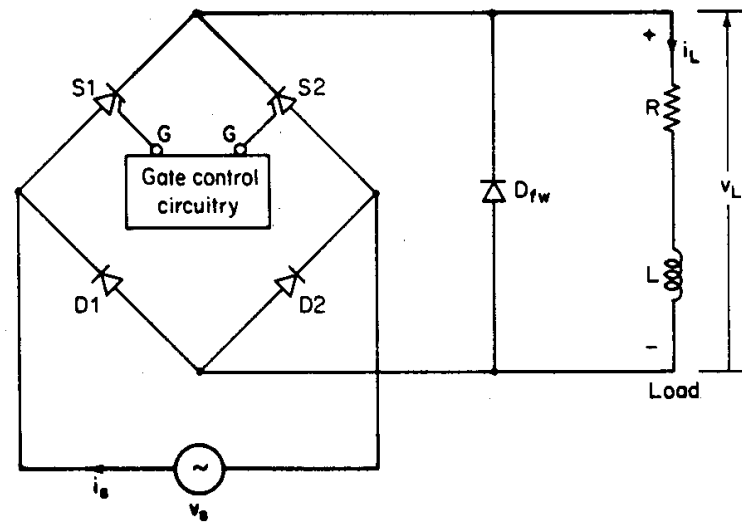


Figure 3.3.2-3 Single-Phase, Full-Wave Controlled Rectifier Circuit

- The average voltage to the load has a maximum value of  $2V_m/\pi$ .

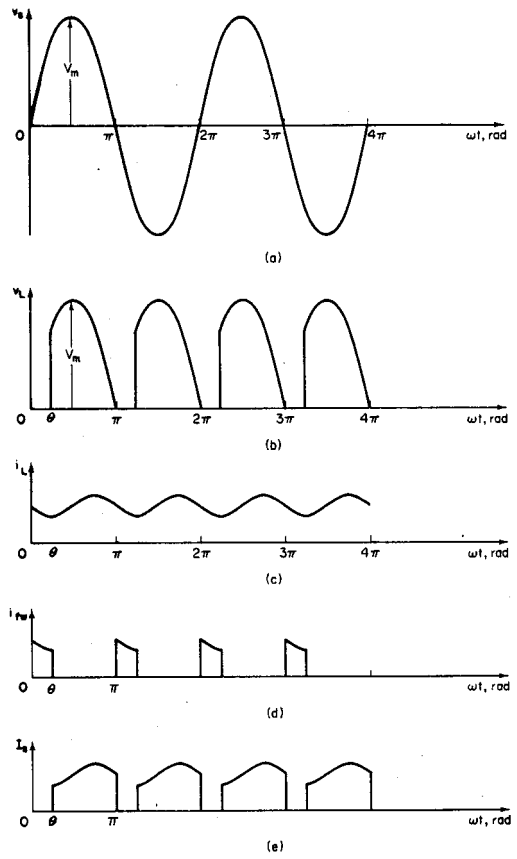


Figure 3.3.2-4 Time Variations of the Electrical Quantities (a) Source Voltage (b) Rectified Voltage across the  $R-L$  Load (c) the  $R-L$  Load Current Variation (d) Current in the Free-Wheeling Shunt Diode (e) Source Current

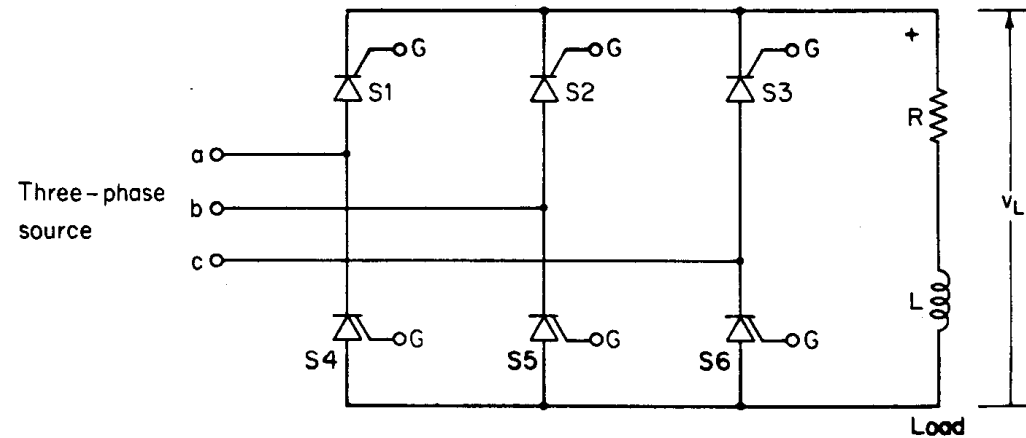


Figure 3.3.2-5 Three-Phase, Full-Wave Controlled Rectifier Circuit

- The availability of a rectifier circuit that uses a three-phase source is attractive for the reason that the three-phase source is already needed to drive the three-phase synchronous motor.
- A total of six thyristors (SCRs) is needed with this arrangement.
- It is possible to vary the average voltage across the field winding over a range that varies from zero to  $3V_m/\pi$ , where  $V_m$  denotes the maximum value of the ac voltage between lines.



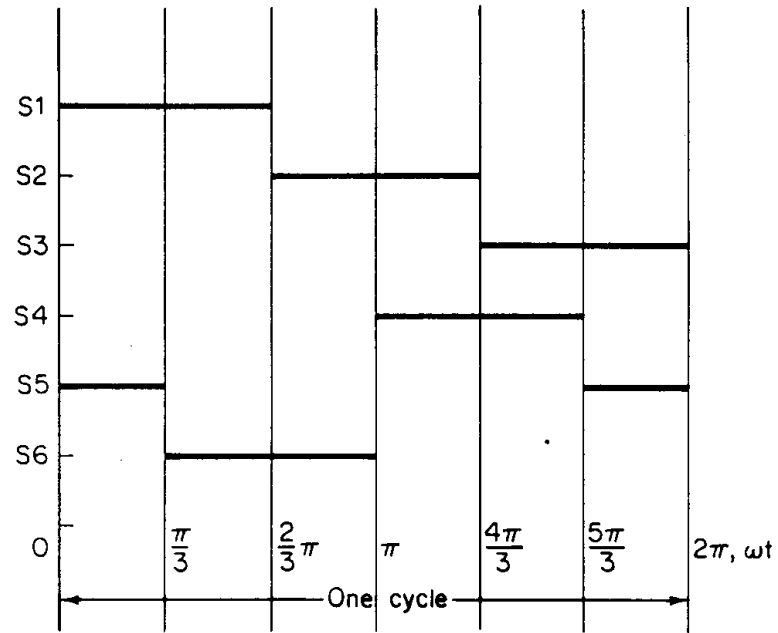


Figure 3.3.2-6 Switching Sequence Diagram for the Thyristors

## 4. Stepper Motors

### 4.1 Construction of Stepper Motor

- Magnetic reluctance is the analog of electrical resistance.
- Reluctance plays an essential part in stepper motor (also called stepping motor).

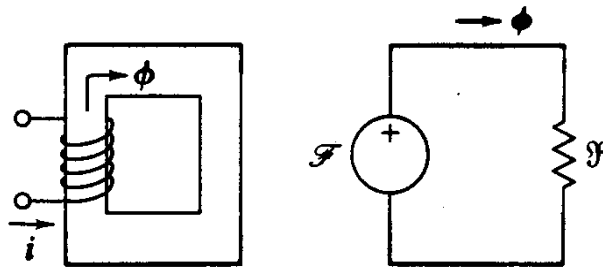


Figure 4.1-1 A Flux Loop

- The flux in the ferromagnetic core depends on the number of turns  $N$  of the coil around the core and on the current in the wire.

The magnetomotive force,  $E_{mmf}$ ,

$$E_{mmf} = Ni \quad (4.1-1)$$

The magnetic reluctance in the circuit,

$$E_{mnf} = R\phi \quad (4.1-2)$$

The magnetic reluctance of a core with cross section  $A(s)$  and permeability  $\mu(s)$ ,

$$R = \oint \frac{ds}{\mu(s)A(s)} = \sum \frac{l_i}{\mu_i A_i} \quad (4.1-3)$$

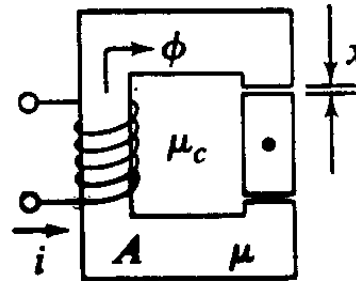


Figure 4.1-2 Magnetic Reluctance

The total reluctance,

$$R = \frac{L}{\mu_0 \mu_r A} + \frac{2x}{\mu_0 A} = \frac{1}{\mu_0 A} \left( \frac{L}{\mu_r} + 2x \right) \quad (4.1-4)$$

where  $\mu_r$ : relative permeability of ferromagnetic material,  $\mu_0$ : permeability of air

- The rotor rotates per step a fixed angle  $\theta_s$ , called the step angle.

The resolution or step number for a stepper motor,

$$S = \frac{360^\circ}{\theta_s} \quad (4.1-5)$$

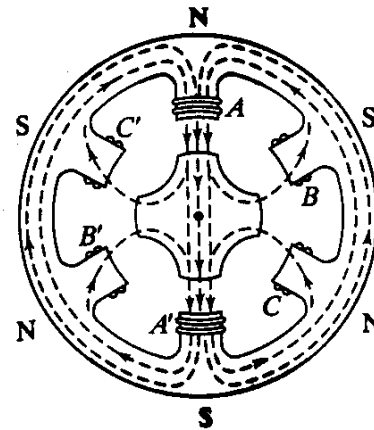


Figure 4.1-3 Basic Configuration of Three-Phase Stepper Motor

- There are several pole pairs, which are arranged at equal intervals around the stator. Each pole pair is called a phase.
- When one of the phases, say  $AA'$ , is excited, the rotor eventually positions itself to complete the flux path. There is a main flux path through the aligned rotor and stator teeth, with secondary flux paths occurring.
- The stepper motor rotates in such the way to minimize the reluctance, *variable reluctance stepper motor (VR motor)*.

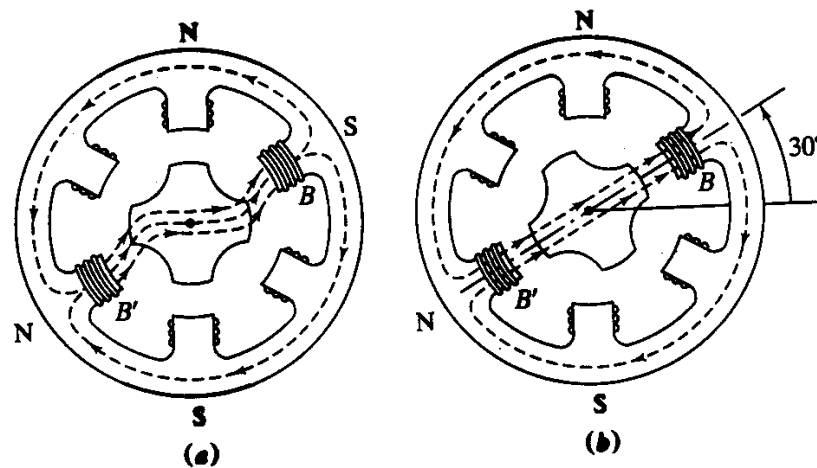


Figure 4.1-4 (a) Beginning of Step (b) Completed Loop

- To rotate the motor counterclockwise, phase  $AA'$  is turned off and phase  $BB'$  is excited. The Maxwell stress in the flux path, again acts to minimize the reluctance expressed in a decreasing of the air gap between rotor and stator tooth. This tension in the lines produces a counterclockwise torque until the rotor is again aligned with the stator poles, now corresponding to phase  $BB'$ . At the completion of this process, the rotor has executed one step.

The resolution of the motor may be expressed in terms of the number of phases  $p$  and the number of rotor teeth  $n_r$ ,

$$S = pn_r = \frac{n_s n_r}{n_s - n} \quad (4.1-6)$$

- The motor has a resolution of  $3 \times 4 = 12$  steps per revolution or a step angle of  $30^\circ$ .

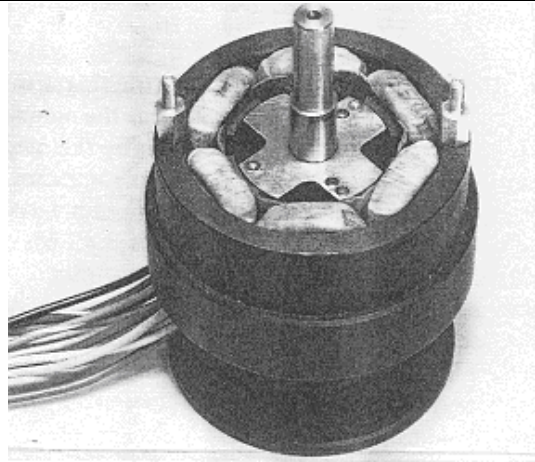


Figure 4.1-5 Three-Phase Variable Reluctance Stepper Motor

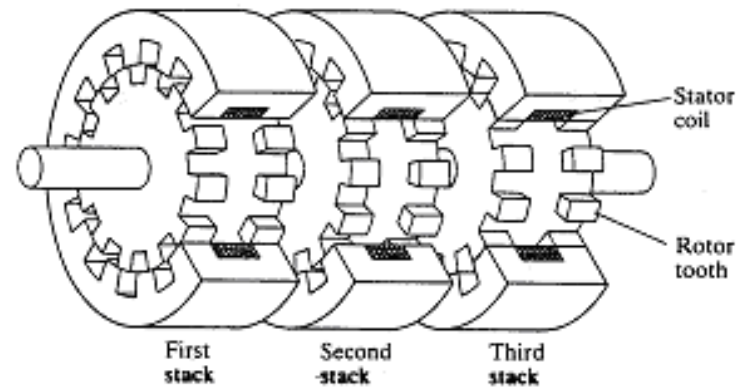
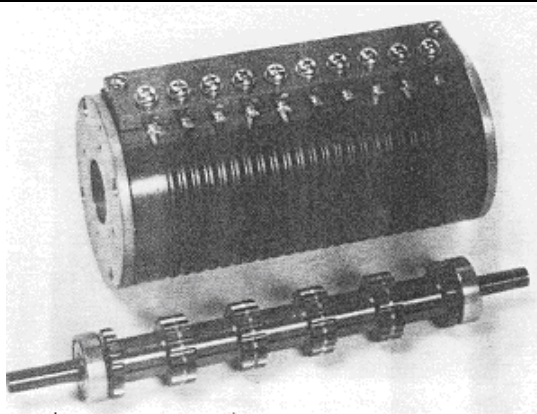
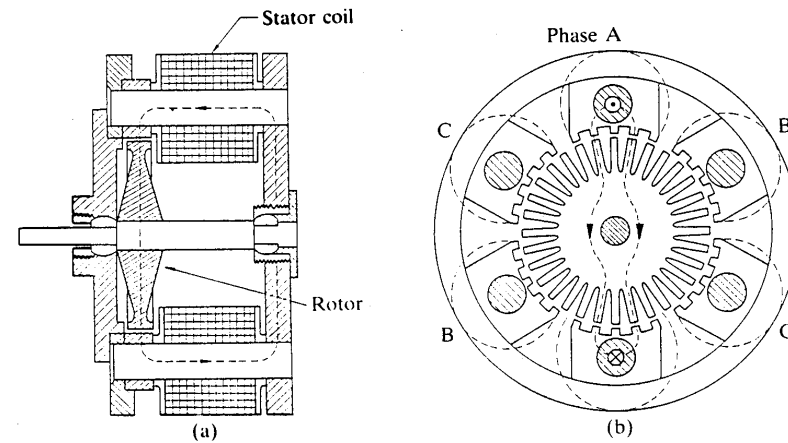
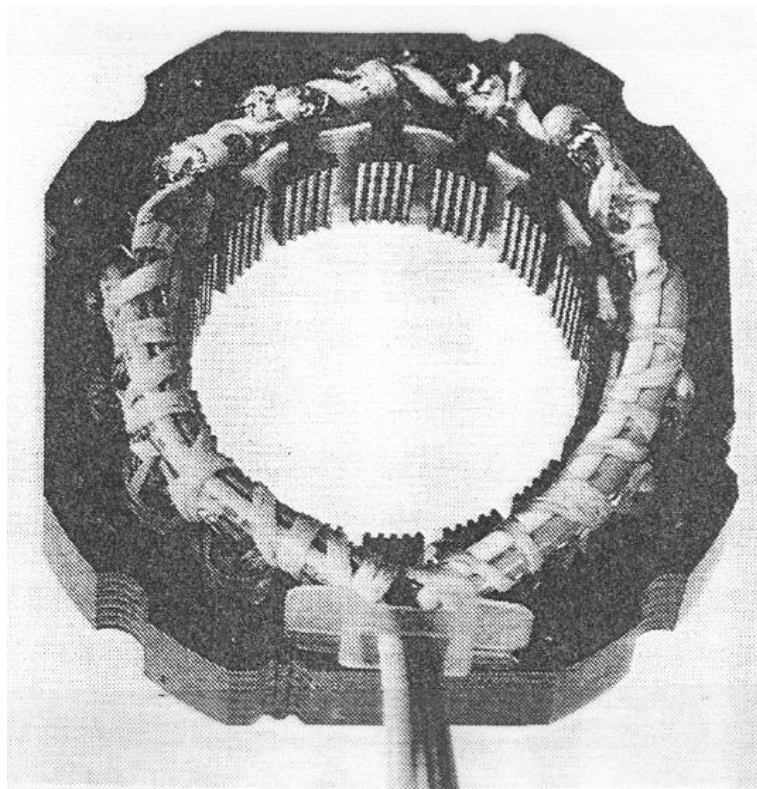
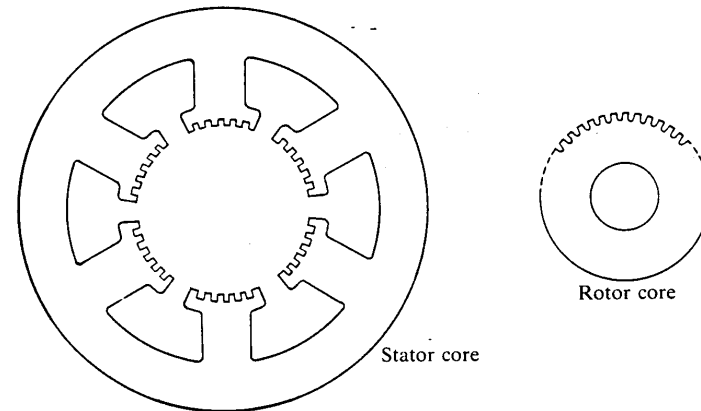


Figure 4.1-6 Multi-Stack Variable Stepper Motor



A three-phase stepping motor invented by C. L. Walker.



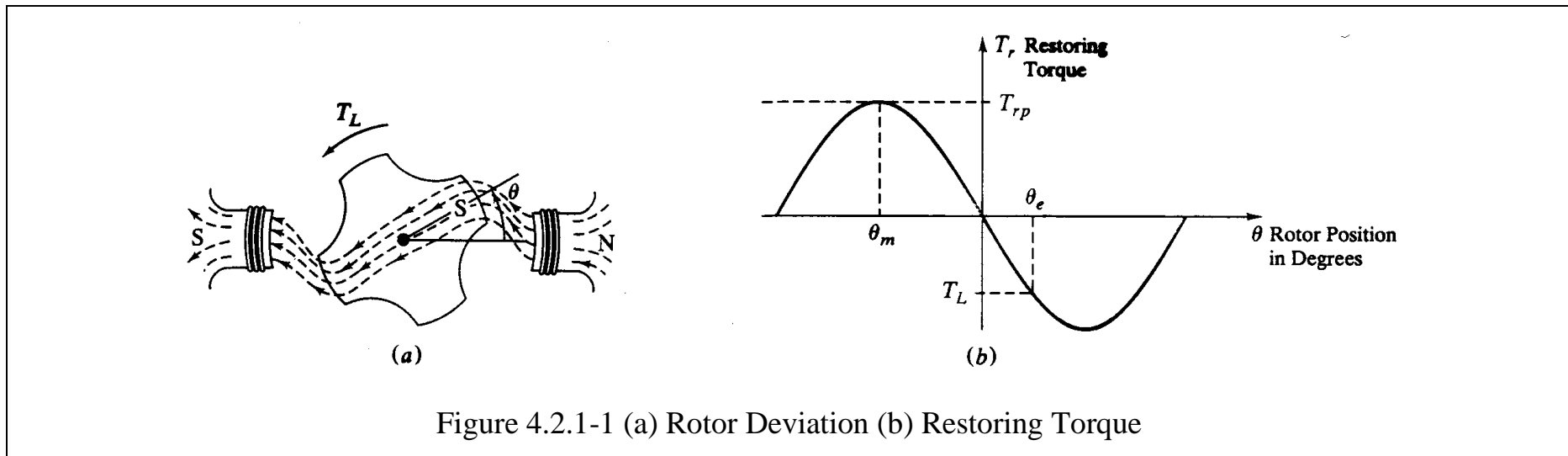
Cross-section of a modern VR stepping motor with a small step angle.

Figure 4.1-7 Three-Phase Stepper Motor with Small Step Angle

## 4.2 Stepper Motor Model

### 4.2.1 Static Holding Torque

- Generally, when the stepper motor stops, the load may remain as a static load on the motor. This always causes an angular deviation from the desired equilibrium position.



At static equilibrium,  $T_r = -T_L$ , the static positioning error with  $n_r$  being the number of rotor teeth,

$$\theta_e = \frac{1}{n_r} \arcsin \left( \frac{T_L}{T_{rp}} \right) \tag{4.2.1-1}$$

- Whenever  $T_L > T_{rp}$ , the rotor will rotate to the next position (or more) and a positioning error will result.



### 4.2.2 Single-Step Dynamic Response

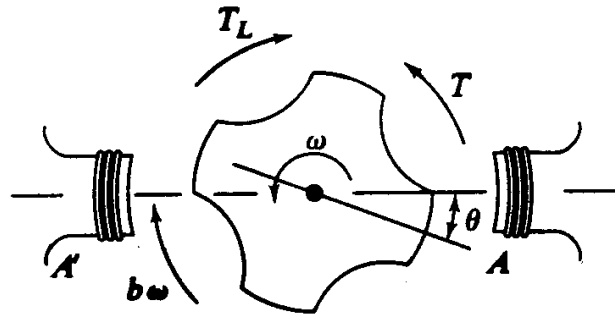


Figure 4.2.2-1 Dynamic Response of the Rotor

If the current in phase AA' is  $i_A$ , the torque exerted on the rotor by phase AA' is

$$T = \frac{1}{2} i_A^2 \frac{dL_A}{d\theta} \quad (4.2.2-1)$$

where  $L_A$  : the self-inductance of the coils comprising phase AA'.

- This inductance is a maximum when the rotor teeth are aligned with the stator teeth. It is a minimum when the AA'-axis is midway between the rotor teeth.

$$L_A = L_0 \cos n_r \theta \quad (4.2.2-2)$$

where  $n_r$  : the number of teeth in the rotor.

The exerted torque becomes

$$T = -\frac{1}{2}i_A^2 L n_r \sin n_r \theta \quad (4.2.2-3)$$

- The torque and  $\theta$  always have opposite signs.

From Euler's second law,

$$-T_L + T - b\omega = J_m \frac{d\omega}{dt} \quad (4.2.2-4)$$

where  $T_L$ : a load torque to the motor,  $J_m$ : the polar mass moment of inertia, and  $b$ : the viscous damping coefficient.

$$J_m \frac{d^2\theta}{dt^2} + b \frac{d\theta}{dt} + \frac{1}{2}i_A^2 L n_r \sin n_r \theta = -T_L \quad (4.2.2-5)$$

The circuit equation with a constant voltage  $v_0$  across the windings for phase  $AA'$ ,

$$v_0 - i_A R - \frac{d}{dt}(L_A i_A) = 0 \quad (4.2.2-6)$$

$\beta(t)$  is defined as the small deviation of the actual angle  $\theta_0(t)$  from the desired constant step angle  $\theta_i$ ,

$$\beta(t) = \theta_0(t) - \theta_i \quad (4.2.2-7)$$

The current  $i_A(t)$ ,

$$i_A(t) = i_0 + y(t) \quad (4.2.2-8)$$

where  $i_0$  : the constant stationary current required to maintain the angle  $\theta_i$ ,

$y(t)$ : the small deviation in current corresponding to the deviation  $\beta(t)$ .

To linearize the equation,  $\theta(t) = \beta(t)$ ,  $T_L = 0$ , and neglecting all products of small terms,

$$J_m \frac{d^2 \beta}{dt^2} + b \frac{d\beta}{dt} + \frac{1}{2} i_0^2 L n_r^2 \beta = 0 \quad (4.2.2-9)$$

$$(L_0 + L) \frac{dy}{dt} + Ry = 0 \quad (4.2.2-10)$$

$$J_m \frac{d^2 \theta_0}{dt^2} + b \frac{d\theta_0}{dt} + \frac{1}{2} i_0^2 L n_r^2 \theta_0 = \frac{1}{2} i_0^2 L n_r^2 \theta_i \quad (4.2.2-11)$$

$$\frac{d^2 \theta_0}{dt^2} + 2\zeta \omega_n \frac{d\theta_0}{dt} + \omega_n^2 \theta_0 = \omega_n^2 \theta_i \quad (4.2.2-12)$$

The natural frequency,

$$\omega_n = i_0 n_r \sqrt{\frac{L}{2J_m}} \quad (4.2.2-13)$$

The damping factor,

$$\zeta = \frac{b}{i_0 n_r \sqrt{2LJ_m}} \quad (4.2.2-14)$$

The corresponding system transfer function with output  $\theta_0$  and input  $\theta_i$ ,

$$G = \frac{\theta_0(s)}{\theta_i(s)} = \frac{\omega_n^2}{s^2 + 2\zeta \omega_n s + \omega_n^2} \quad (4.2.2-15)$$

The response with the use of an input  $\theta_i(t) = u(t)$ , the unit step function, along with the specification of zero initial conditions and  $0 < \zeta < 1$ , underdamping,

$$\theta_0(t) = 1 - \frac{1}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \sin\left(\omega_d t + \arctan \frac{\sqrt{1-\zeta^2}}{\zeta}\right) \quad (4.2.2-16)$$

where  $\omega_d = \omega_n \sqrt{1-\zeta^2}$ : the damped frequency.

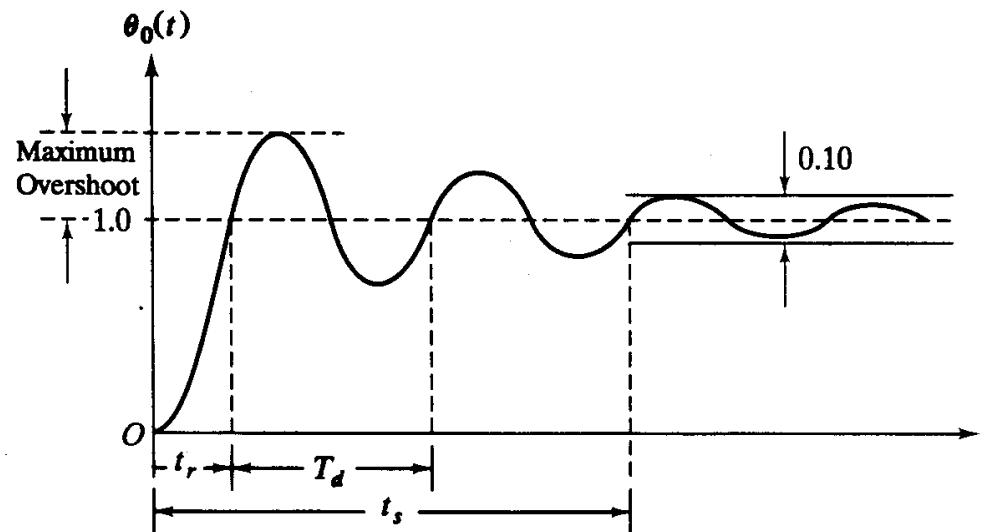


Figure 4.2.2-2 Unit-Step Response of Rotor of Stepper Motor

- Because of the response oscillations, for the successive stepping of the motor if the pulsing which urges the rotor from position to position is poorly chosen, it may amplify the oscillations until a step is skipped.

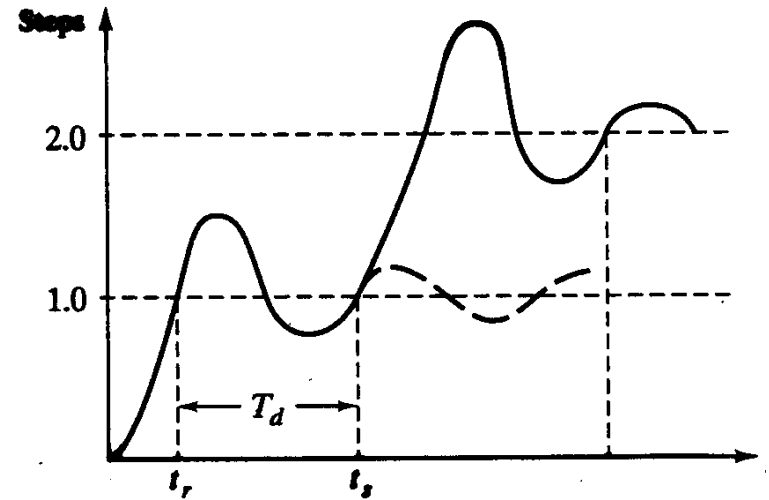


Figure 4.2.2-3 Sequential Stepping

### 4.3 Stepper Motor Driver (Open Loop Driver)

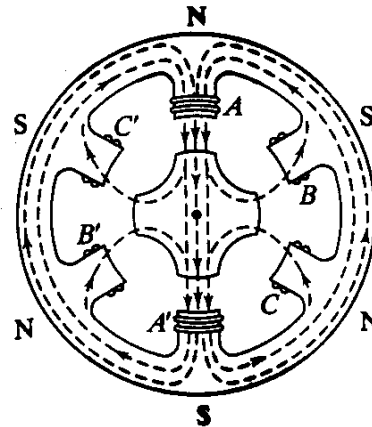
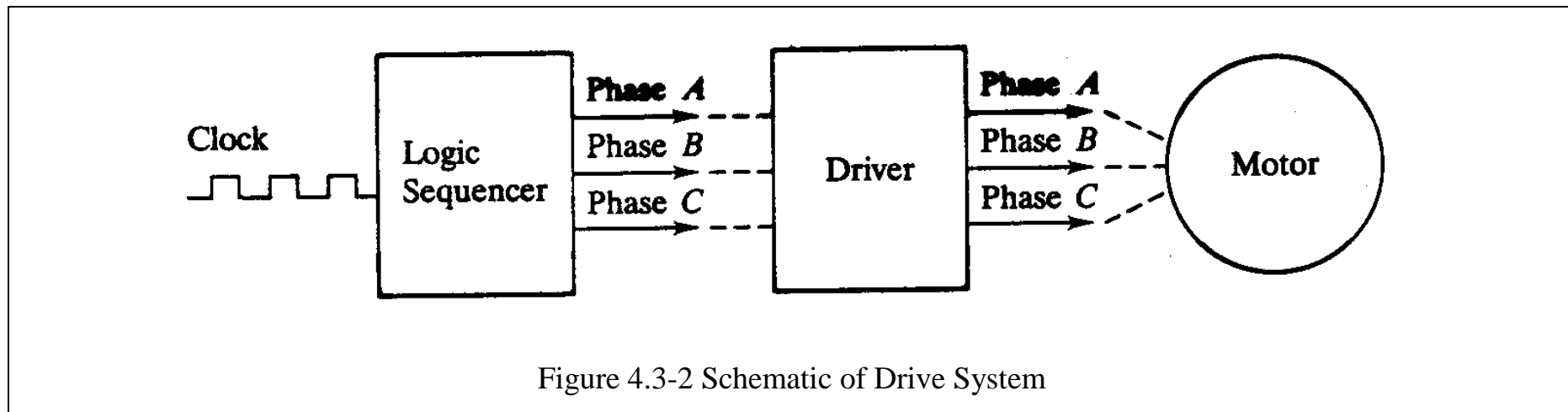


Figure 4.3-1 Three-Phase Stator and Four-Tooth Rotor Stepper Motor

- In a simple dc motor, the motor can be driven by providing only appropriate voltage and current. Which is not the case for stepper motor.
- In one-phase-on sequencing, the phase with which the rotor was aligned is turned off and the next phase is turned on to attract the next opposing set of rotor teeth in sequence.



- The logic sequencer is used to produce the switching sequence. It assures that the phases are powered in proper sequence.
- The driver provides the current needed.
- A clockwise sequencing of the phases results in a counterclockwise rotor rotation. For a clockwise rotation, the sequence is simply reversed.

|         | R | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---------|---|---|---|---|---|---|---|---|---|
| Phase A |   |   |   |   |   |   |   |   |   |
| Phase B |   |   |   |   |   |   |   |   |   |
| Phase C |   |   |   |   |   |   |   |   |   |

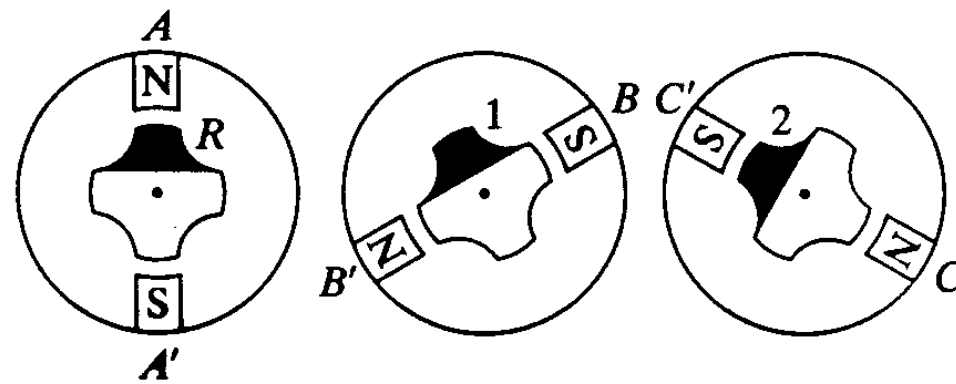
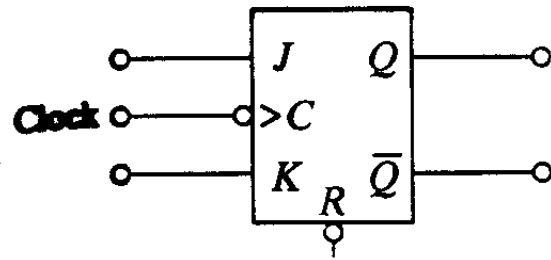


Figure 4.3-3 Phase Sequencing for Counterclockwise Rotor Rotation



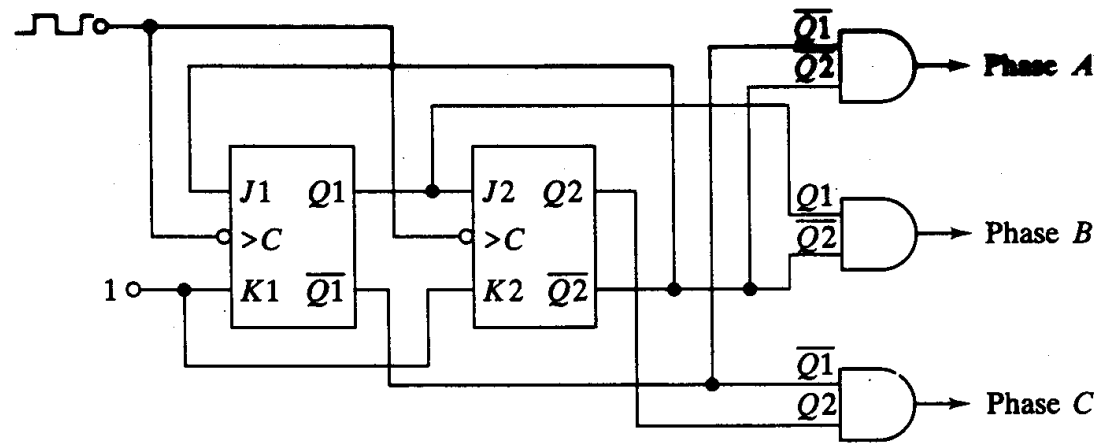


(a)

| $J$ | $K$ | $Q(t + 1)$   | Operation  |
|-----|-----|--------------|------------|
| 0   | 0   | $Q(t)$       | No Change  |
| 0   | 1   | 0            | Reset      |
| 1   | 0   | 1            | Set        |
| 1   | 1   | $\bar{Q}(t)$ | Complement |

(b)

Figure 4.3-4 (a) Negative Transition JK Flip-Flop (b) Characteristic Table



(a)

|                 | $t$ | $t+1$ | $t+2$ | $t+3$ | $t+4$ | $t+5$ |
|-----------------|-----|-------|-------|-------|-------|-------|
| $R$             |     | 1     | 2     | 3     | 4     | 5     |
| $Q1$            | 0   | 1     | 0     | 0     | 1     | 0     |
| $\overline{Q1}$ | 1   | 0     | 1     | 1     | 0     | 1     |
| $Q2$            | 0   | 0     | 1     | 0     | 0     | 1     |
| $\overline{Q2}$ | 1   | 1     | 0     | 1     | 1     | 0     |

(b)

Figure 4.3-5 (a) Logic Sequencer, (b) Truth Table

- The outputs from the AND gates activate the respective phase drivers. The simplest driver is a direct connection of the logic circuit to the base of the transistor switch. The resulting current may not be enough to saturate the transistor and the driver would then include some current amplifiers.
- Once the switch is saturated, the full constant phase voltage  $V_p$  is applied across the phase circuit, since the voltage drop across the transistor is negligible. Usually,  $V_p$  will be high enough to supply the rated phase current.
- Typical phase circuit for one-phase-on operation includes the self-inductance of the winding  $L_w$  and the winding resistance  $R_w$ . It also includes an external (external to the winding) resistance  $R_s$  and a diode.

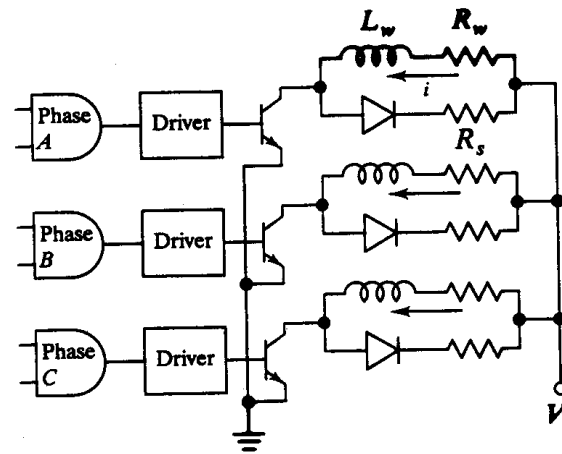


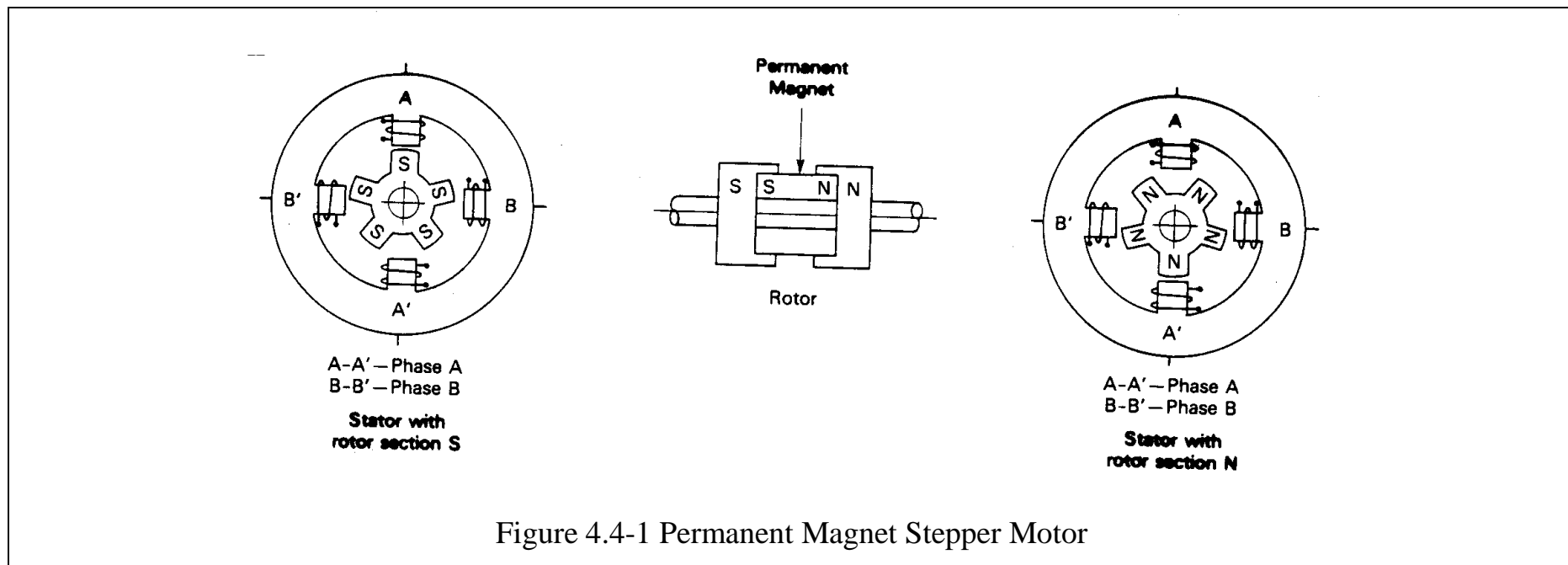
Figure 4.3-6 Phase Circuits and Drivers

## 4.4 Stepper Motor Classification

- Stepper motors are not suited to operations with heavy loads. They are suitable for open loop accurate position control at low torque.
- In **one-phase-on** operation of the VR motor depending on the pulsing intervals, this operation may produce overshoot and oscillation about the final equilibrium position.
- Overshoot and oscillation may be reduced with **two-phase-on** operation, involving the simultaneous excitation of two phases at a time.
- **Full-step, half-step and micro-step** operations depend upon the proportion of the voltages exciting two phases at a time.
- Classification of stepper motors
  1. Variable Reluctance Motors
  2. Permanent Magnet Motors
    - Permanent magnet motor has permanent magnets in the rotor. It is usually manufactured without rotor teeth and for step sizes larger than  $30^\circ$ , since it is difficult to provide very many permanent magnetic poles in the rotor.
    - The advantages of permanent magnet motors include a higher torque for a given size and coming to rest in a fixed position because of the presence of torque, even when the motor is off.
    - The permanent magnet motors tend to be costly and the flux density is limited by the magnet.

### 3. Hybrid Motors

- In both permanent magnet motors and hybrid motors, the torque is proportional to the current and a holding torque exists when the motor is off.
- Hybrid motors may include permanent magnets either in the stator or in the rotor.
- Hybrid motors are used when high resolution is required and a step angle of  $1.8^\circ$  is common. Their torque-to-weight ratio is high.

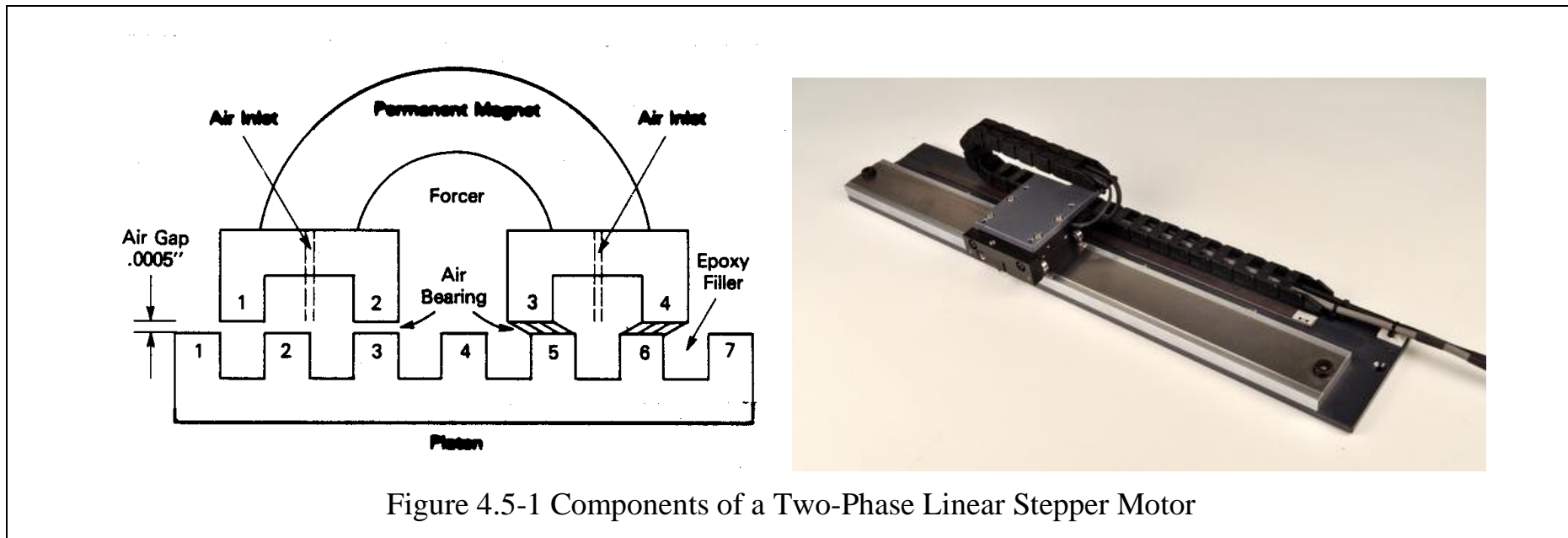


| Characteristic         | PM motor                                    | VR motor                                       |
|------------------------|---|--|
| 1. Motor               | Magnetized                                  | Not magnetized                                 |
| 2. Rotor position      | Depends on stator excitation polarity       | Independent of stator excitation polarity      |
| 3. Rotor inertia       | High due to magnet                          | Low (no magnet)                                |
| 4. Mechanical response | Not as good (due to high inertia)           | Good (low-inertia device)                      |
| 5. Inductance          | Low due to rotor offset                     | Generally high for same torque rating          |
| 6. Electrical response | Faster current rise (due to low inductance) | Slower current rise (due to higher inductance) |

Figure 4.4-2 Characteristics Comparison between PM and VR Motors

## 4.5 Linear Stepper Motors

- In linear stepper motor, a movable armature, which is referred to as a forcer, is suspended over the fixed stator, also called a platen.
- An air bearing is formed between the bottom of the forcer and the top of the platen.



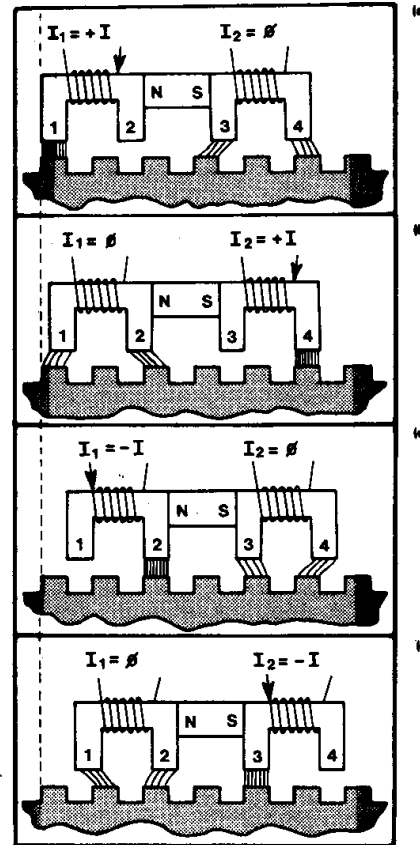


Figure 4.5-2 Motion in a Two-Phase Linear Stepper Motor

- Each new phase excitation produces a quarter pitch movement of the forcer relative to the platen. Thus a total motion of  $\frac{3}{4}$  of a full pitch is indicated



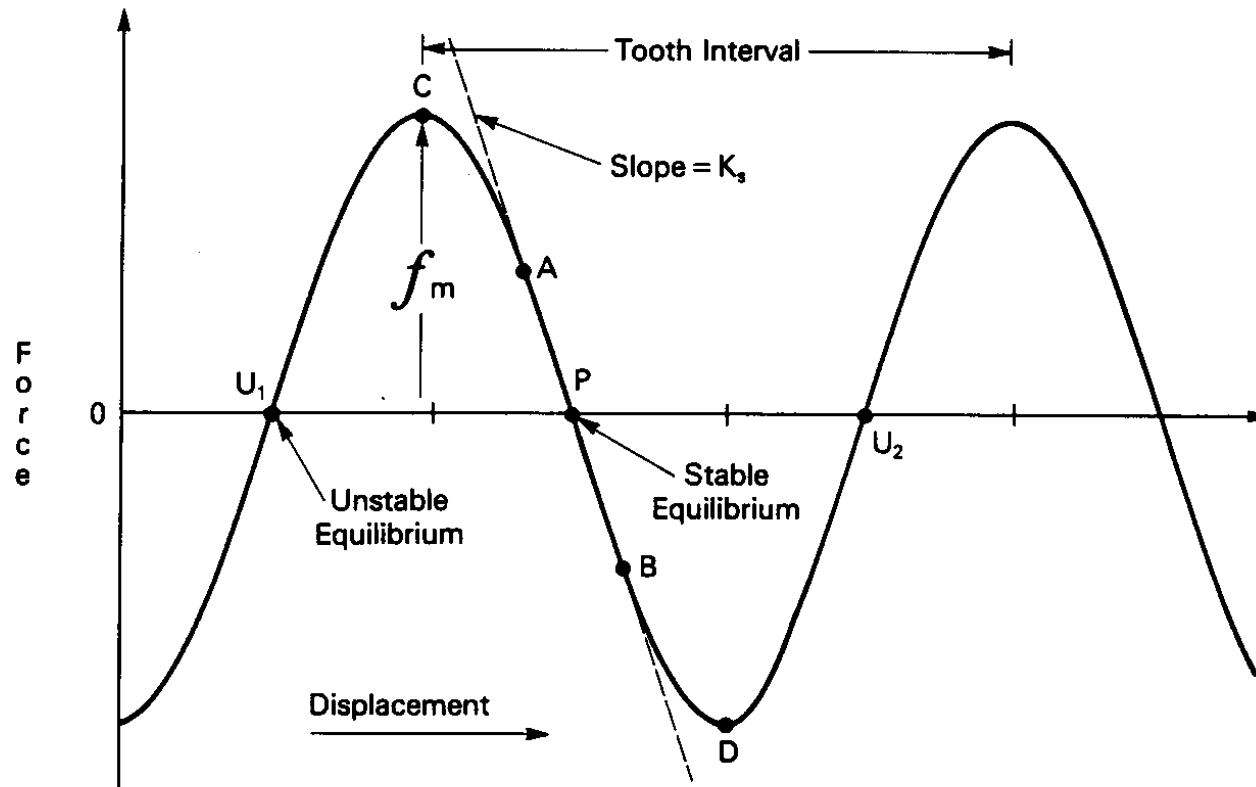


Figure 4.5-3 Motor Force versus Displacement for a Linear Stepper Motor

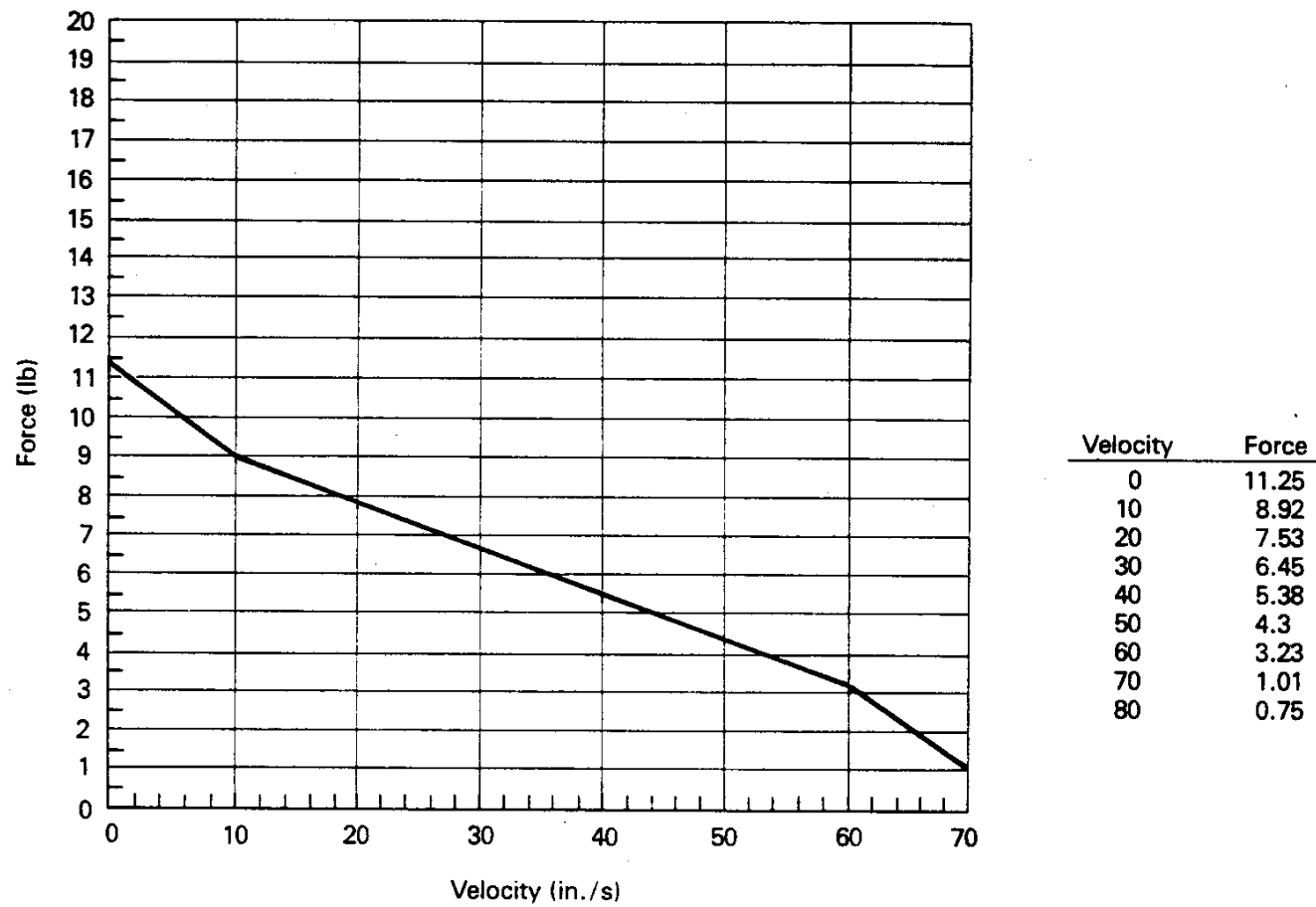
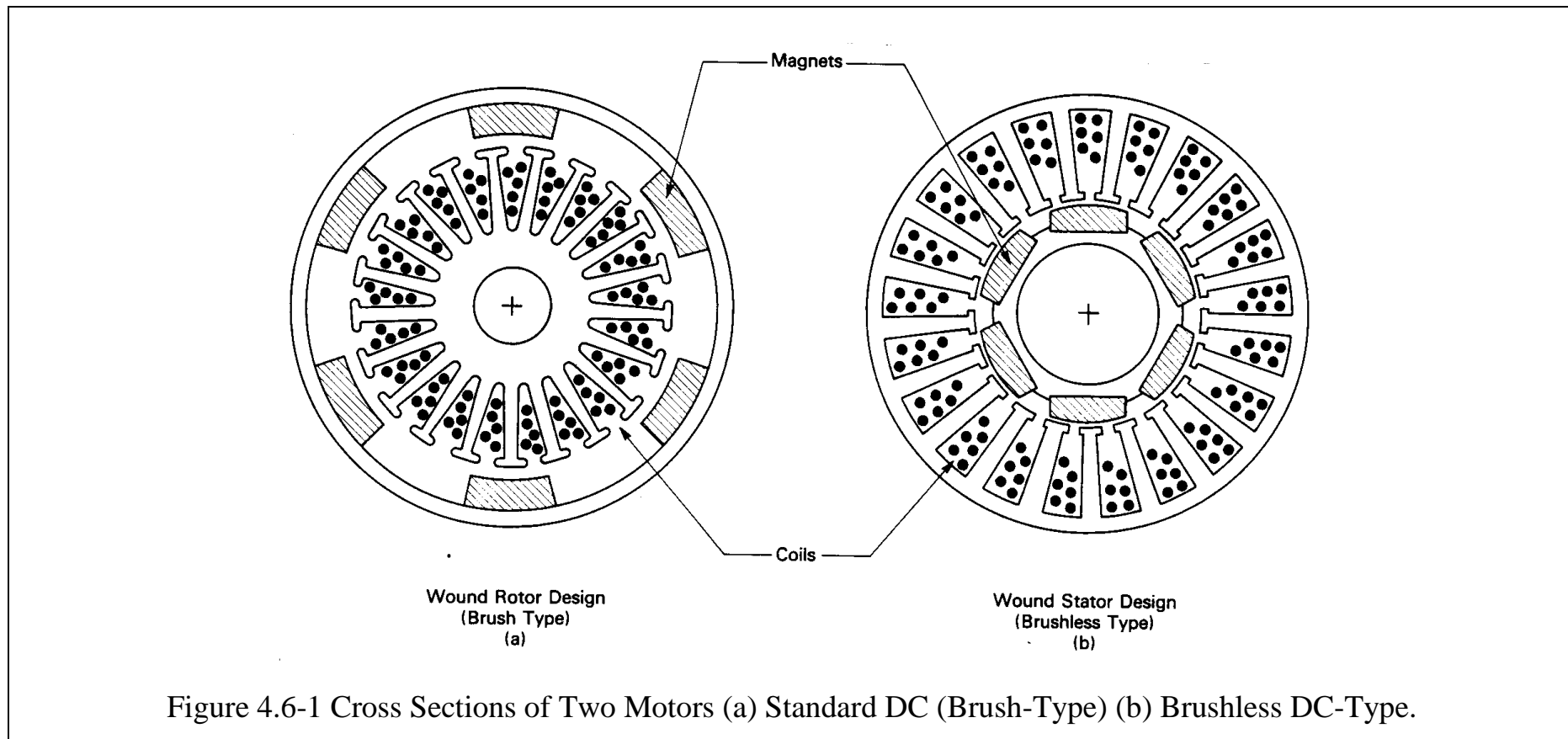


Figure 4.5-4 Force versus Speed Curve for a Linear Stepper Motor

## 4.6 Brushless DC Motors

- To get rid of the problems at brushes of dc motors, brushless dc motors are introduced.



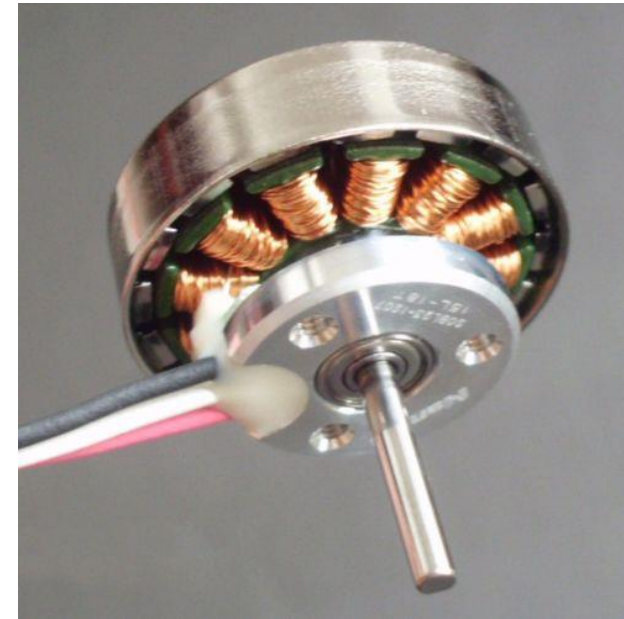
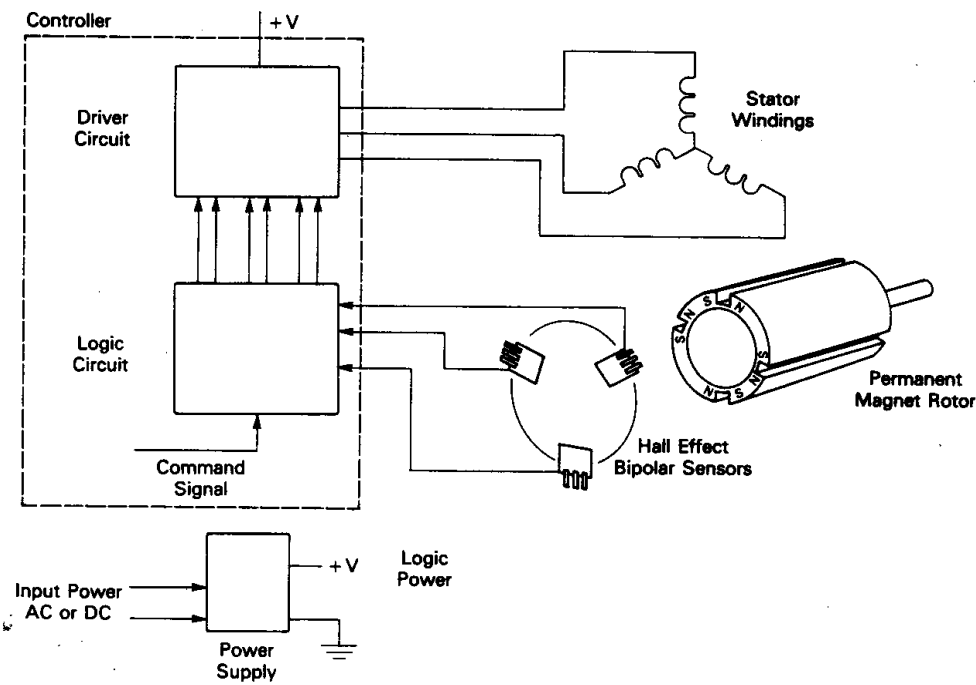


Figure 4.6-2 Sixteen-Pole, Three-Phase Winding Brushless DC Motor with Electronic Commutation Scheme Utilizing Three Hall Effect Sensors

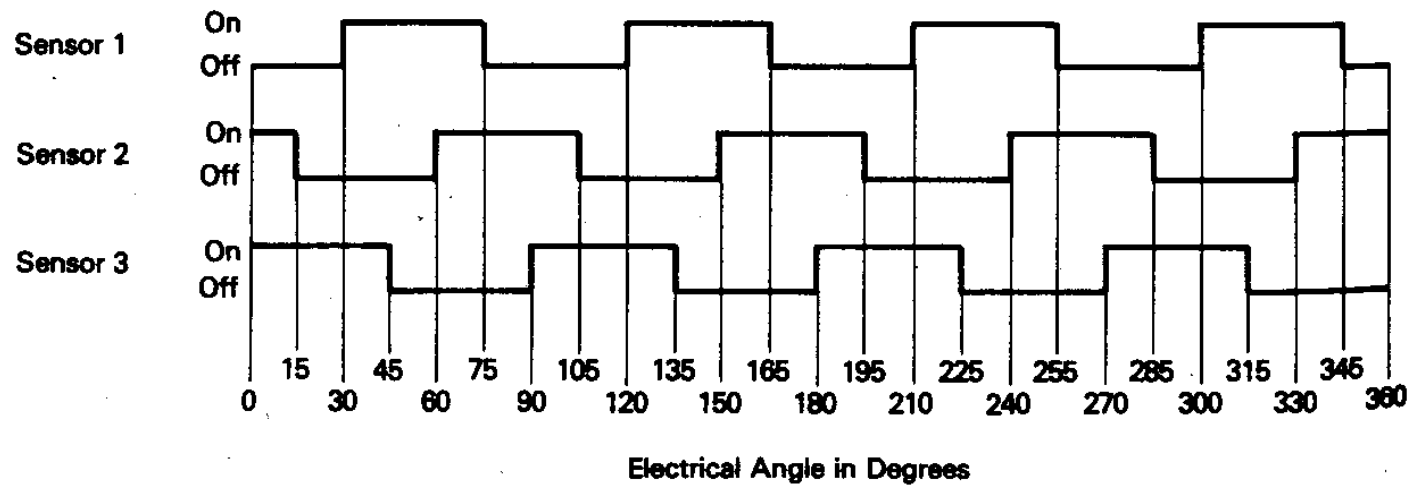


Figure 4.6-3 Outputs of Three Hall Effect Sensors Used in the 8-Pole, 3-Phase Brushless DC Motor

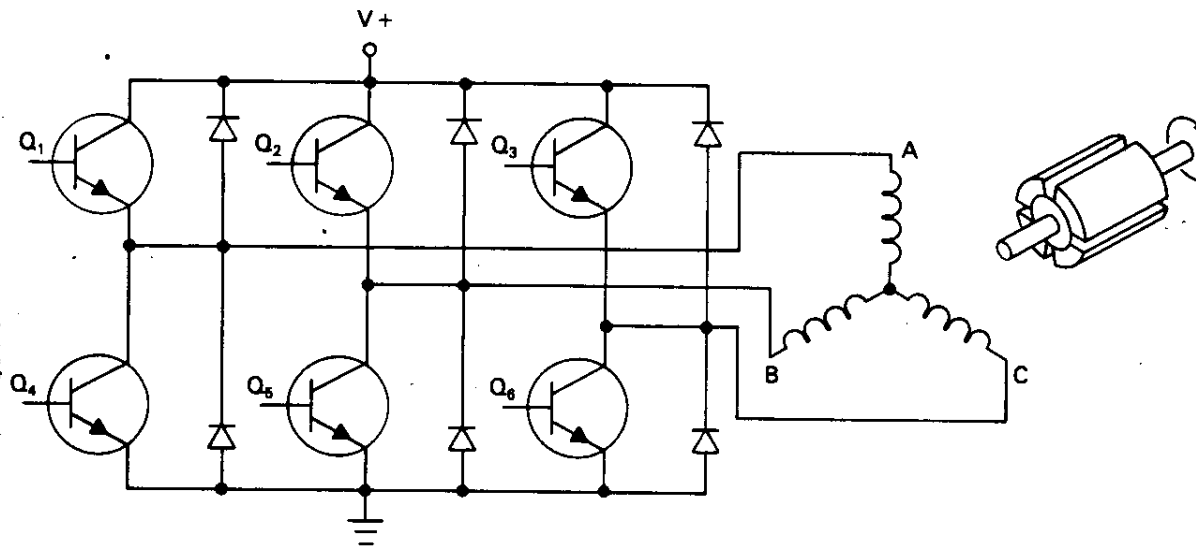


Figure 4.6-4 Three-Phase Driver Circuit for the Brushless DC Motor

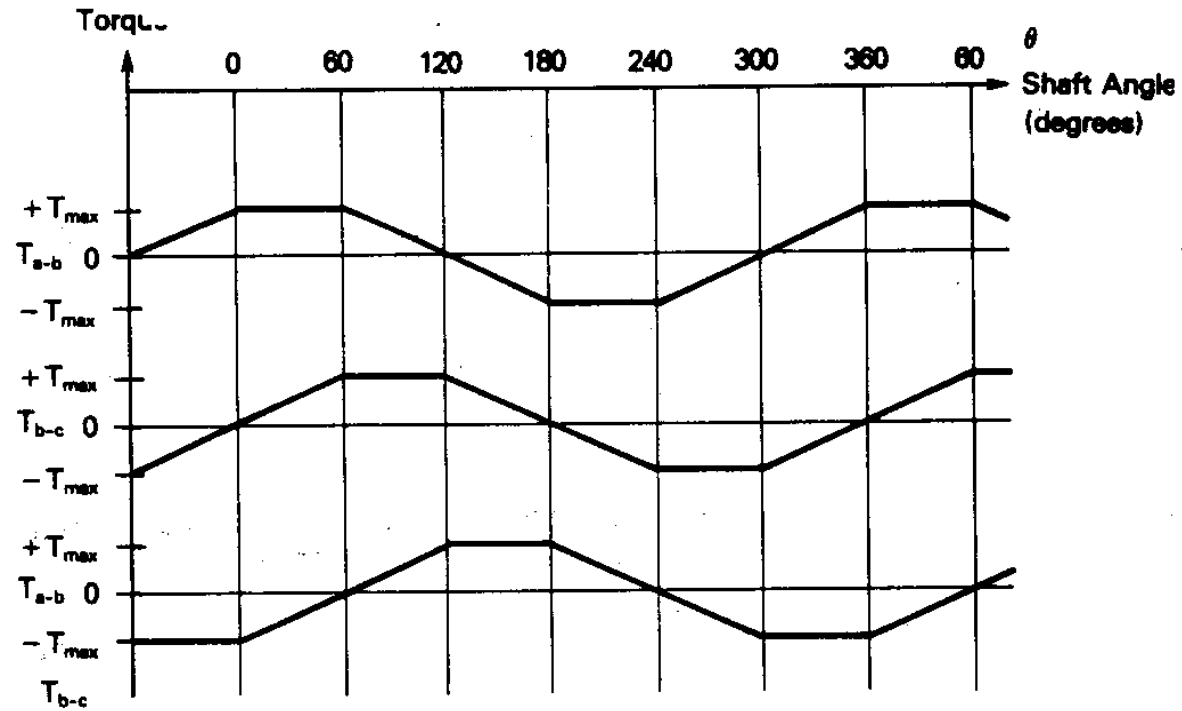


Figure 4.6-5 Torque versus Motor Shaft Angle for a Brushless DC Motor  
Assuming a Constant Current in Each of the Three Phases

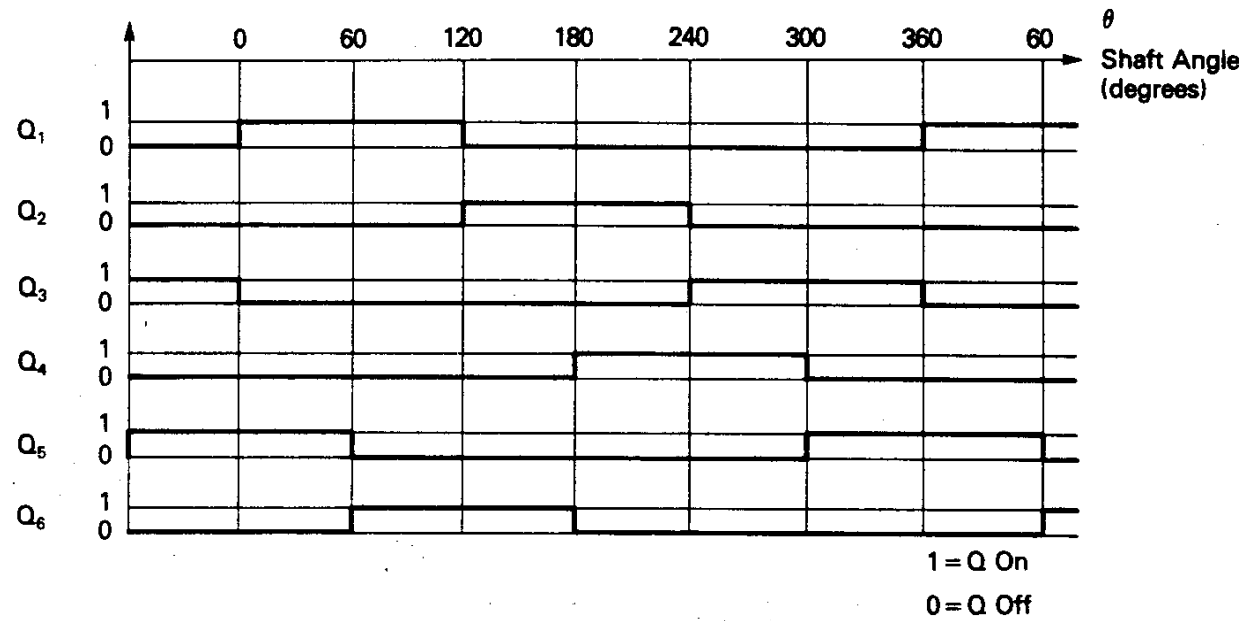


Figure 4.6-6 Transistor base Drive Signals for the Three-Phase Driver Circuit



## 4.7 Hydraulic Actuators

- Applied for moving sizable loads (normally more than 5 to 7hp) in a rapid and precisely controlled manner.
- For proper operation of a hydraulic system, the bulk modulus of the oil is extremely important attribute to be selected. A high bulk modulus implies a stiff, quickly responding system with a corresponding quick pressure buildup, while a low bulk modulus may result in a system that is too loose because of the high compressibility of the oil.

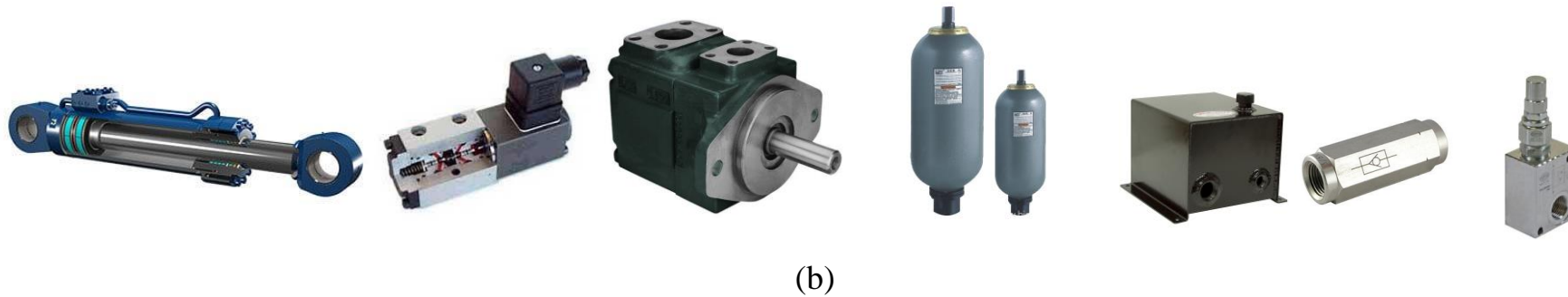
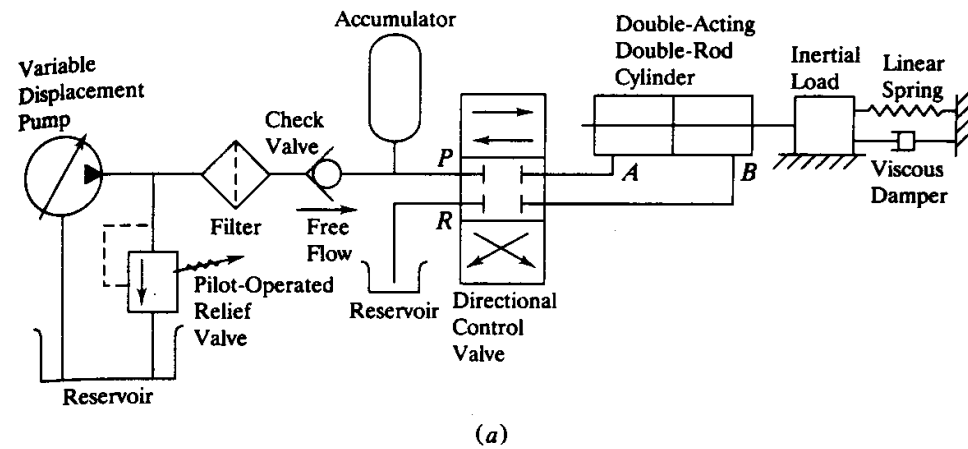


Figure 4.7-1 (a) Complete Hydraulic Circuit

(b) Hydraulic Components; Actuator, Valve, Pump, Accumulator, Reservoir, Check Valve, Relief Valve





















|   |   |   |   |
|---|---|---|---|
|    | Check valve; flow is free to the right, blocked to the left |    | Filter  |
|    | Simple relief valve   |    | Accumulator                                     |
|    | Pilot-operated relief valve                                 |    | Gas-pressurized accumulator                     |
|    | Pressure-compensated flow control valve                     |    | Variable displacement pump                      |
|    | Needle valve with variable restrictor                       |    | Pressure-compensated variable displacement pump |
|    | Directional control valve closed port-closed center         |    | Gear motor                                      |
|    | Directional control valve closed center-open port           |    | Variable displacement motor                     |
|    | Directional control valve open center-open port             |    | Single-action cylinder                          |
|   | Directional control valve open center-closed port           |   | Double-action single-rod cylinder               |
|  | Hydraulic reservoir   |  | Double-action double-rod cylinder               |

Table 4.7-1 Symbols for hydraulic system components

- Four necessary components of hydraulic system are reservoir, pump, valve, and actuator.
- Reservoir: Reservoir holds the fluid used in hydraulic operation and should contain ample fluid for all system operations. It also serves to cool and clean the returning hydraulic fluid and prevent turbulence.
- Variable displacement pump: Pump creates constant flow and/or pressure. Pump volume is the amount of fluid discharged or moved by the pump in one minute.
- Filter: Filter removes particles from the fluid.
- Accumulator: Accumulator serves two functions. For a given flow supplied to a hydraulic line, it can provide a suddenly required increase in fluid; or conversely, they can serve as shock absorber, accepting fluid which is forced back into the system due to an increase in load, for example.
- Pilot-operated relief valve: It responds to pressure. As long as the operating pressure is normal, the fluid flows toward the system load. If the pressure becomes too great, the relief valve opens and reroutes some of the fluid back to the reservoir until pressure returns to normal.
- Check valve: Check valve allows fluid to flow in only one direction.

- Actuator: There are three main types of linear actuators: single-action pistons, double-action piston, and differential piston. For a piston, we distinguish between the rod end and the blind end. For a single-action piston, fluid is pumped into the blind end only; the piston is returned by a spring or by the load. For the double-action piston, fluid is pumped into both ends of the cylinder. Since the piston rod is on the rod side, the piston areas exposed to the fluid are different, and different amounts of pressure are required to produce a given force. This asymmetry can be overcome by using piston rods on both sides of the cylinder. In the case of the differential piston, a fixed pressure is maintained in the rod end, while the blind end functions as before.
- Direction control valve: Depending on the desired motion of the piston, fluid must be directed either to the rod end or the blind end. This directing of the fluid flow is accomplished by directional control valve. Generally, this requires two outlets, or control ports, to supply the piston ends with fluid, and two valve inlets, one for the hydraulic fluid supply and the other to return fluid to the reservoir.

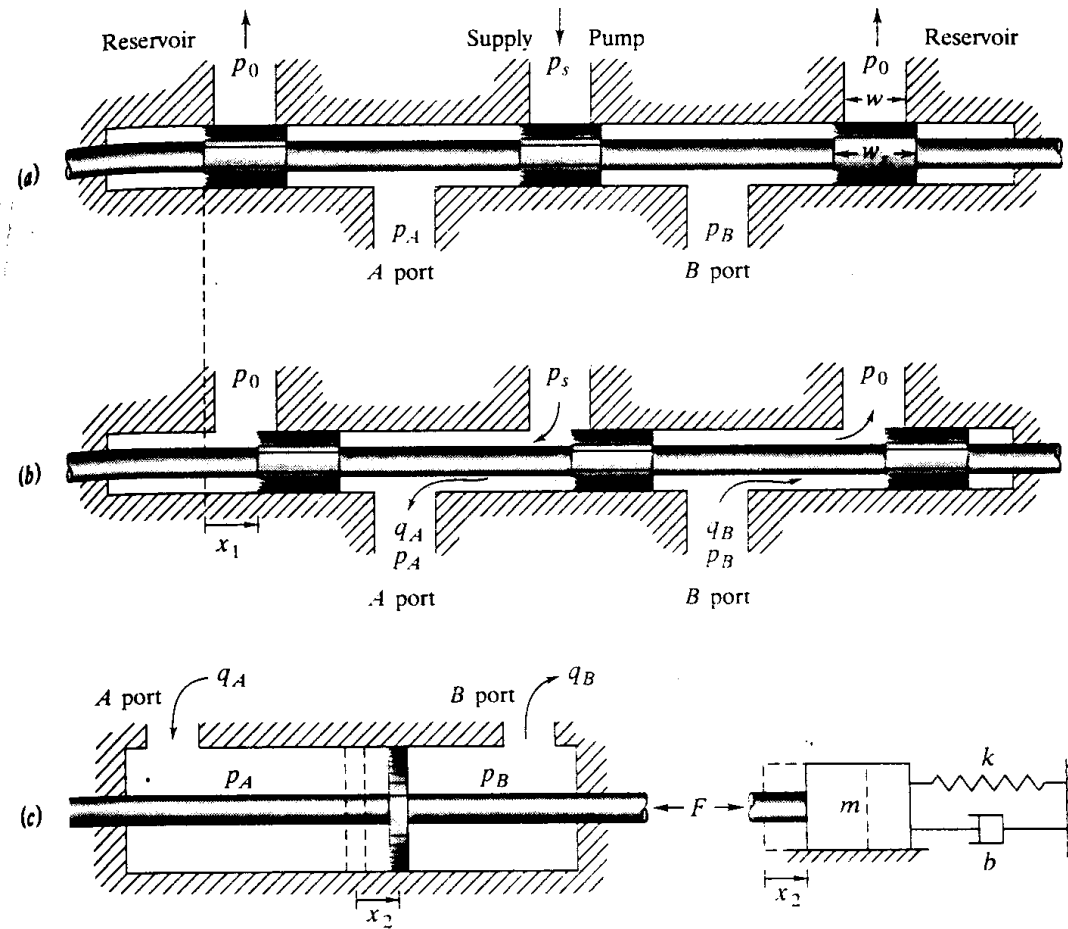


Figure 4.7-2 Free-Body Diagrams for the Valve-Actuator-Load Combination

4.8 Pneumatic Actuators

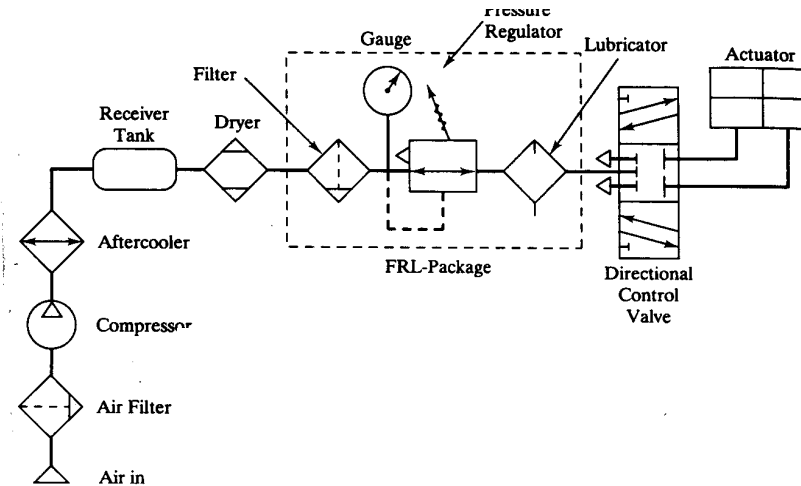


Figure 4.8-1 (a) Complete Pneumatic Circuit

(b) Pneumatic Components; Actuator, Valve, Compressor with Tank, FRL, Dryer

- Pneumatically powered robot components generally move at full speed from one mechanical stop to the next, with no intermediate motion control.
- Air is a relatively clean operating medium and may be released to the surroundings with little negative effect.
- The necessary elements are air intake, compressor, and actuator.
- Compressor: Its function is to compress air. The most common type of compressor is the reciprocating piston compressor, consisting of a piston and chamber. The piston is driven by a rod and crankshaft. The air chamber at the opposite end of the cylinder contains an intake valve and an outlet valve. When the piston motion increases the size of the air chamber, it creates a low-pressure region, and air is admitted through the intake valve. During the opposite motion, the volume of the air chamber is decreased, the air is compressed, and the compressed air is exhausted through the outlet valve when the air chamber volume is at its smallest.
- Aftercooler: It cools the air after it has been compressed to remove the water vapor and to maintain downstream pressure.
- Storage tank: It serves the same purpose accumulator does in hydraulic system, to supply a constant flow of high-pressure air to the system components.
- Desiccant dryer: It is used to remove moisture by chemical operation.
- Filter: Filter removes oil droplets, water droplets, and particles from the air stream. Normally pressure of the air drops a little here.



- Pressure regulator: Because different actuators in the system usually require different operating pressures, pressure regulators are used throughout the system. The simplest kind is a poppet valve, where the poppet is moved up or down by a spring-loaded diaphragm. When the back pressure from an actuator reaches a set level, it acts on the diaphragm, closing the poppet and thus stopping the air flow. When there is a pressure drop due to a movement of the load, for example, then the spring force overcomes the resultant pressure, displacing the diaphragm and opening the poppet, and air flow resumes.
- Lubricator: The dryness of the air can cause problems in pneumatic actuators. Lubricators inject atomized oil into the air stream to lubricate the motion of the actuator.
- Directional control valve: This valve controls the direction of air flow – in essence, the direction and, to some extent, the speed of the actuator motion.
- Actuator: Linear and rotary actuators are classified. Pistons may be single or double action, and pumps may also serve as motors.

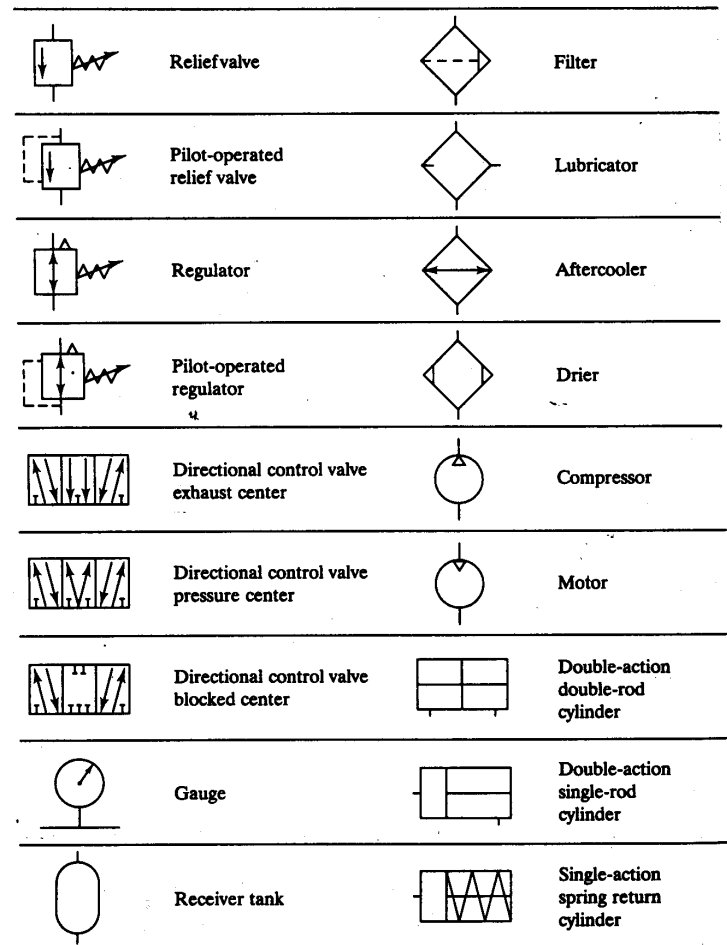


Figure 4.8-2 Symbols for Pneumatic System Components