Sensing

1. Introduction to Measuring Instrument

1.1 Application of Measuring Instrument

Applications of measuring instruments:

1. Monitoring of processes and operations; thermometer, barometer, water and electric meters.
2. Control of processes and operations; thermostat in air conditioner and automotive coolant system, encoder in motor.
3. Experimental engineering analysis

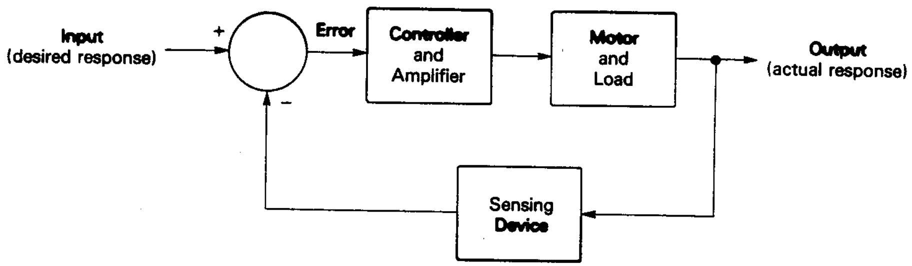


Figure 1.1-1 Feedback-Control System

Features of theoretical methods:

1. General use rather than for restricted application.
2. Simplifying assumptions requirement.
3. Complicated mathematical problems.
4. Only pencil, paper, computing machines, requirement.
5. No time delay.

Features of experimental methods:

1. Specific use.
2. No simplifying assumption requirement, but experimental and environmental setup requirement.
3. Experimental equipment requirement.
4. Actual system or a scale model requirement.
5. Considerable time requirement for design, construction, and debugging of apparatus.

1.2 Functional Elements of an Instrument

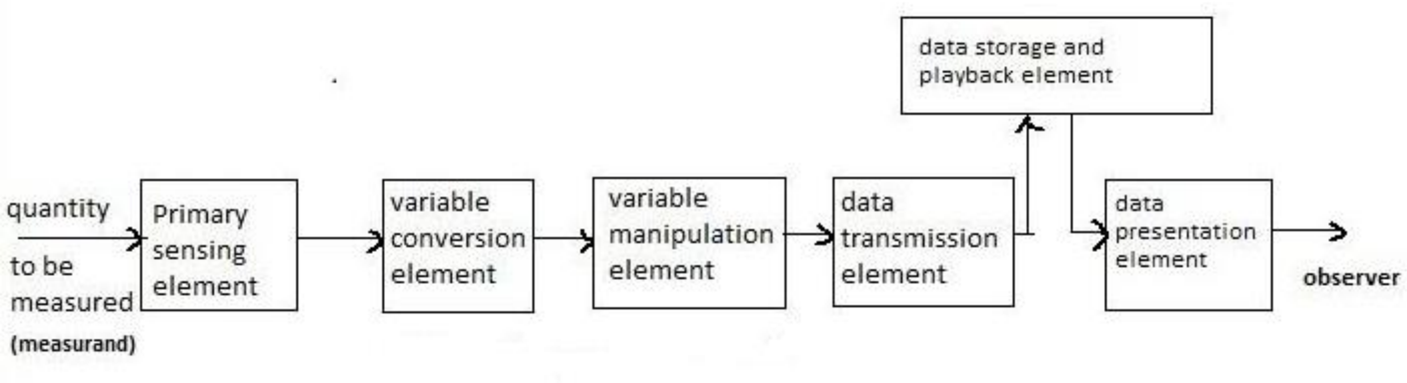


Figure 1.2-1 Functional Elements of an Instrument or a Measurement System

- Primary sensing element: extract the measured quantity from the measured medium
- Variable conversion element: convert the measured quantity to other form for easier manipulation
- Variable manipulation element: manipulate the measured data by amplifying, filtering, shifting, summing, etc
- Data transmission element: transmit the data to remote location
- Data storage/playback element: store the data for future use
- Data presentation element: present the data to the observer

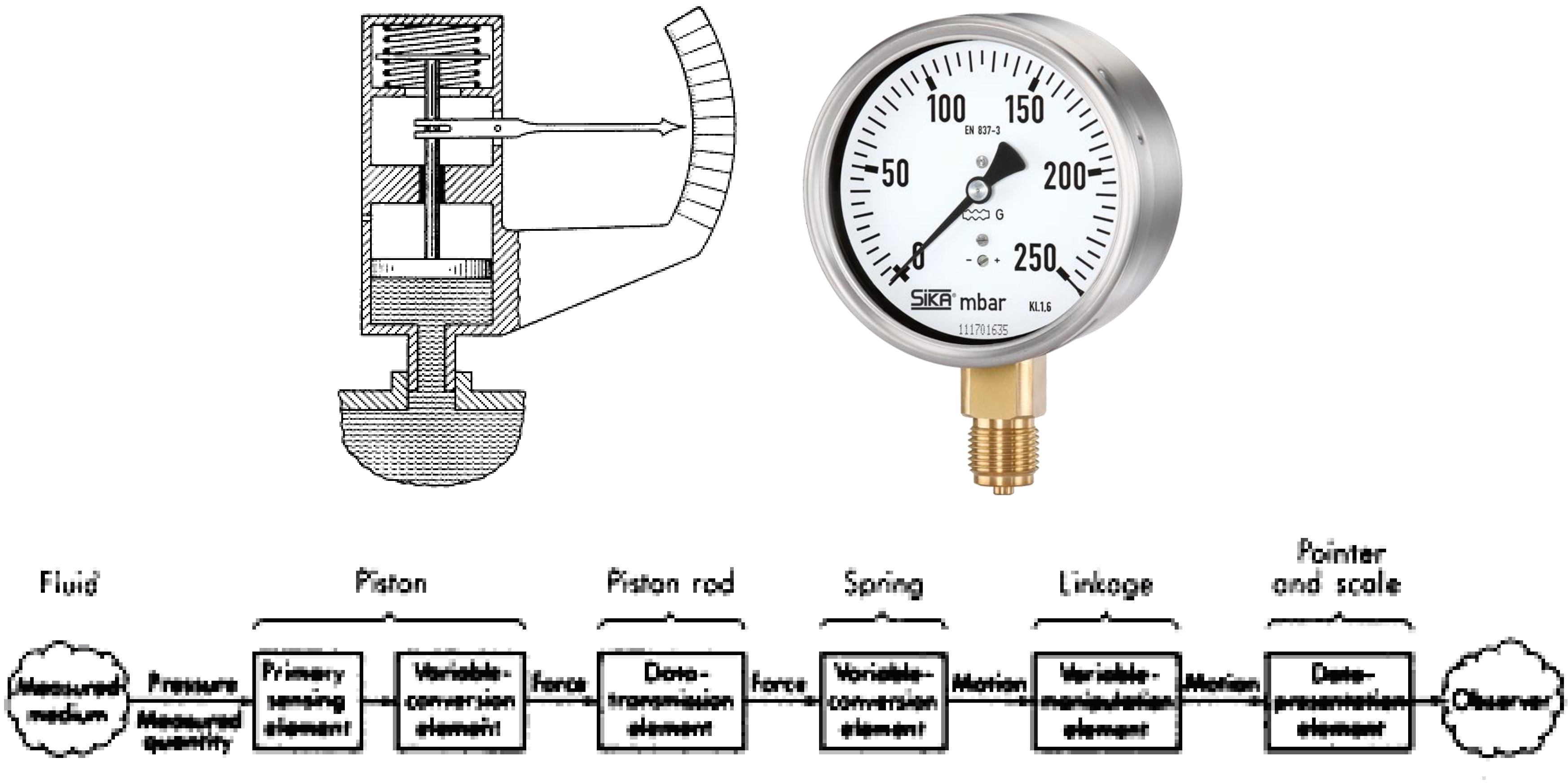


Figure 1.2-2 Pressure Gauge

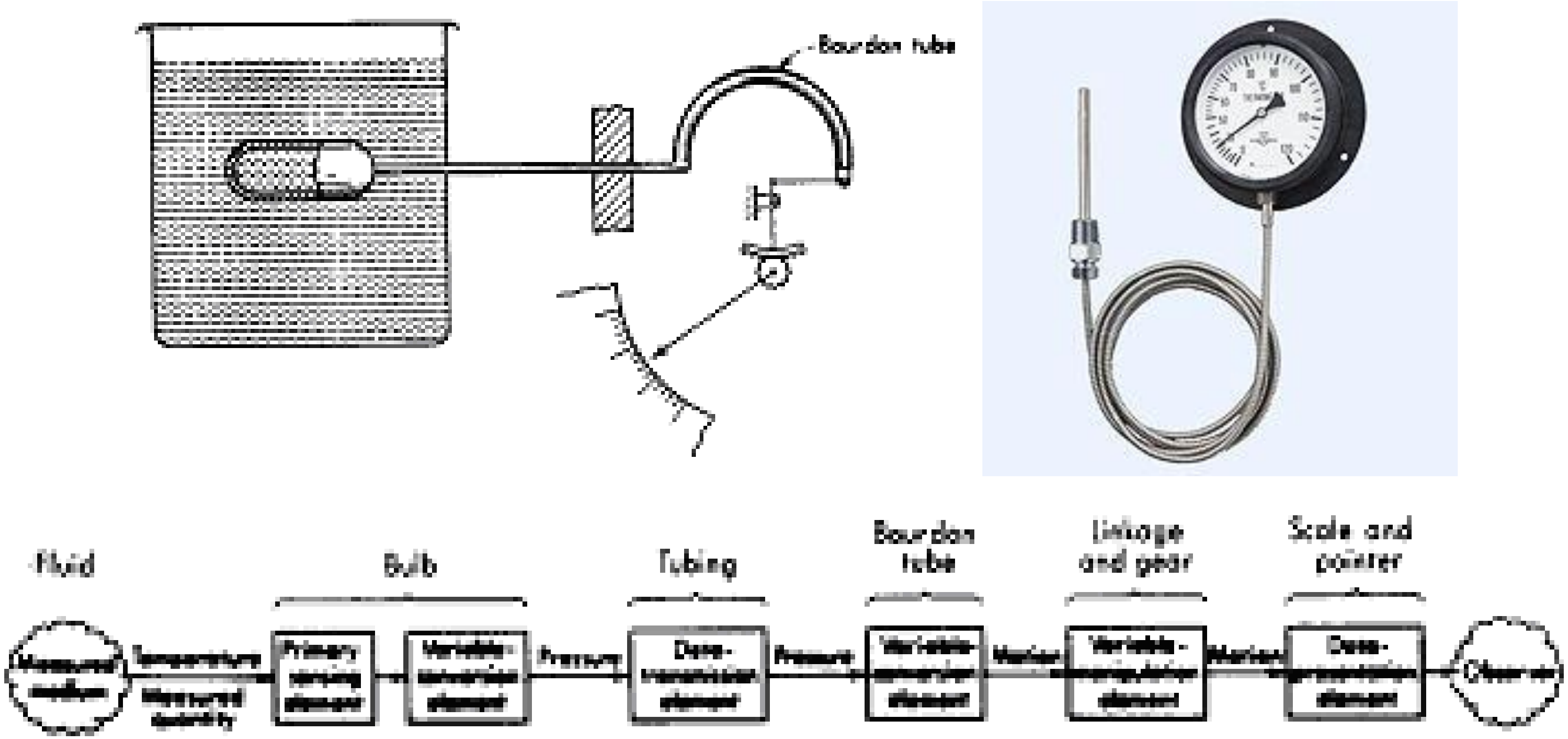


Figure 1.2-3 Pressure Thermometer

1.3 Instrument Classification

Active/Passive Instruments

Passive instrument: The instrument output is entirely produced by the quantity being measured.

Active instrument: The quantity being measured simply modulates the magnitude of some external power source.

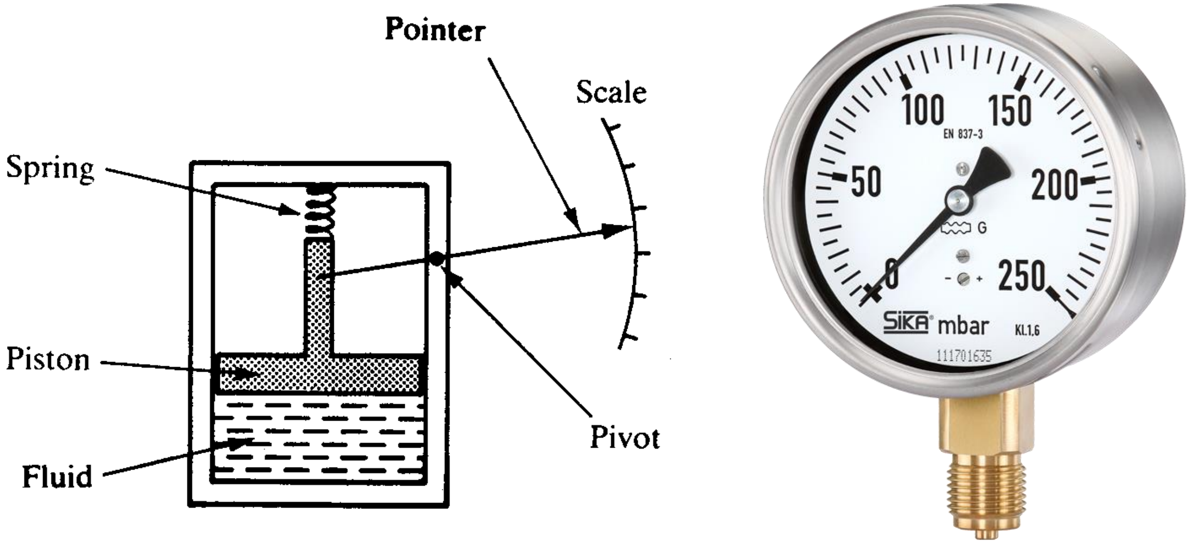


Figure 1.3-1 Pressure Gauge: Passive Instrument

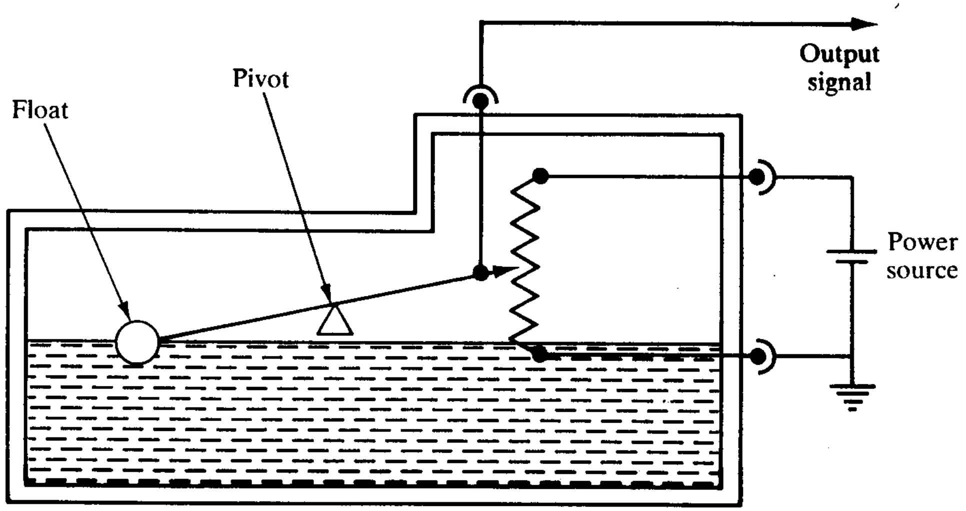


Figure 1.3-2 Petrol-Tank Level Indicator: Active Instrument

Active Instrument

- High resolution
- Expensive

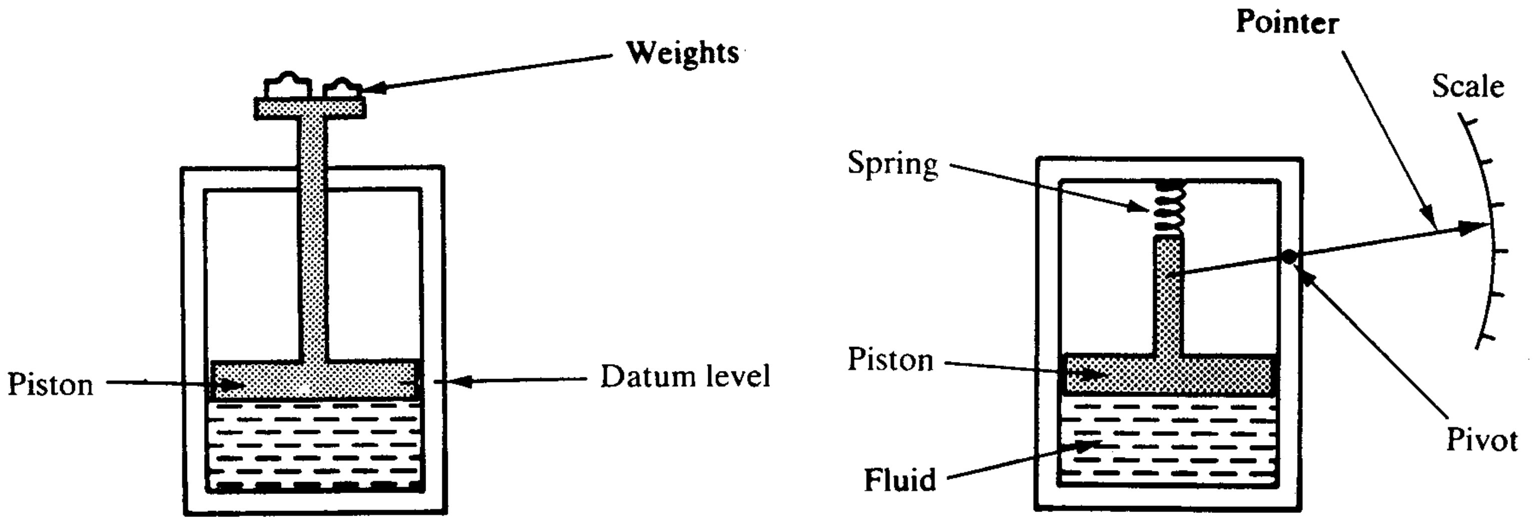
Passive Instrument

- Low resolution
- Cheap

Null/Deflection Type Instruments

Null type instrument: The measurement is obtained by balancing the instrument with the known quantity to reach the datum level, null point.

Deflection type instrument: The measurement is obtained from the deflection or movement of a pointer.



(a) (b)

Figure 1.3-3 (a) Dead-Weight Pressure Gauge: Null Type, (b) Pressure Gauge: Deflection Type

Analog/Digital Instruments

Analog instrument: The output varies continuously as the quantity being measured changes. The output can have an infinite number of values within the range that the instrument is designed to measure.

Digital instrument: The output varies in discrete steps and so can only have a finite number of values.

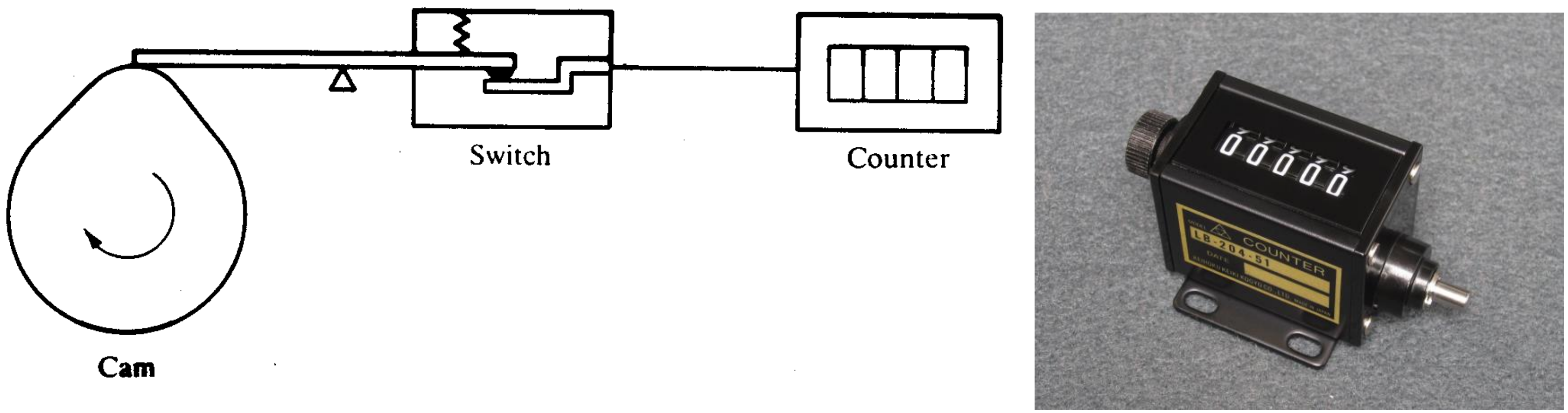


Figure 1.3-4 Rev-Counter: Digital Instrument

1.4 Static Characteristics of Instruments

- Static characteristics of a measuring instrument are concerned with the steady-state reading that the instruments settle down.

Accuracy

- Accuracy is the extent to which a reading might be wrong, and is often quoted as a percentage of the full-scale reading of an instrument.
- Example: A pressure gauge of range 0-10 bar has a quoted inaccuracy of ±1.0% f.s. (±1% of full-scale reading), then the maximum error to be expected in any reading is 0.1 bar.

Tolerance

- Tolerance is a term which is closely related to accuracy and defines the maximum error which is to be expected in some value.
- Tolerance is not a static characteristic of measuring instrument, however the accuracy of some instruments is sometimes quoted as a tolerance figure.
- Tolerance describes the maximum deviation of a manufactured component from some specified value.
- Example: Tolerance of a crankshaft diameter is ±0.1 mm. Tolerance of a resistor is 5% (silver stripe), 10% (golden stripe).

Range or Span

- The range or span of an instrument defines the minimum and maximum values of a quantity that the instrument is designed to measure.

Bias

- Bias describes a constant error which exists over the full range of measurement of an instrument.
- This error is normally removable by calibration.
- Example: Bathroom scale has a thumbwheel for calibration so that the output reading is zero with no load.

Linearity

- Non-linearity is defined as the maximum deviation of any of the output readings from the straight line representing the relation between measured quantity and output reading obtained from curve fitting.
- Non-linearity is usually expressed as a percentage of full-scale reading.

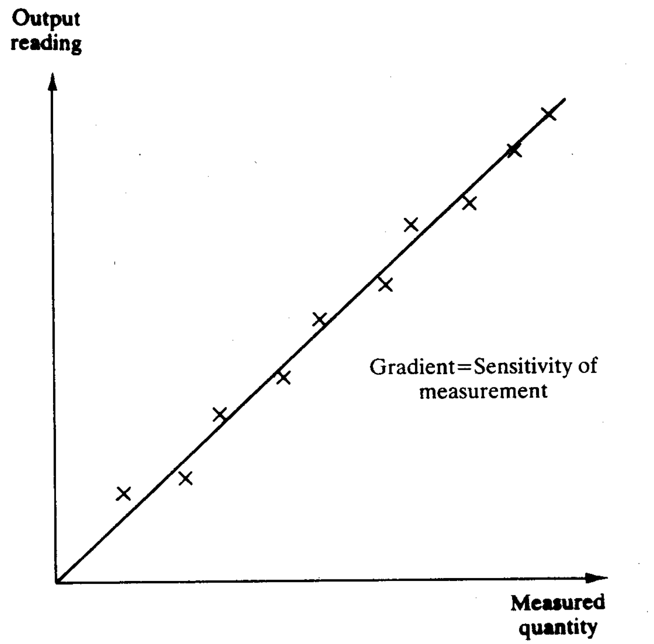


Figure 1.4-1 Instrument Output Characteristic

Sensitivity of Measurement

- The sensitivity of measurement is a measure of the change in instrument output which occurs when the quantity being measured changes by a given amount.

$$\text{Sensitivity} = \frac{\text{Scale Deflection}}{\text{Value of Measurand causing Deflection}}$$

- Sensitivity can be measured from slope of the straight line representing the relation between measured quantity and output reading.
- Example: A pressure of 2 bar produces deflection of 10 degrees in a pressure transducer, the sensitivity is 5 degrees/bar.

Sensitivity to Disturbance

Environmental (temperature, pressure, etc) changes affect instruments in two main ways, zero drift and sensitivity drift.

Zero Drift

- Zero drift describes the effect where the zero reading of an instrument is modified by a change in ambient conditions.
- The effect of zero drift is to impose a bias in the instrument output readings: this is normally removable by recalibration in the usual way.
- Example: Zero drift of a voltmeter affected by ambient temperature changes is measured in volts/°C.

Sensitivity Drift (Scale Factor Drift)

- Sensitivity drift defines the amount by which an instrument's sensitivity of measurement varies as ambient conditions change.
- An alternate name for this phenomenon is scale factor drift.
- Example: Sensitivity drift of a pressure transducer is measured in units of the form (degrees/bar)/°C.

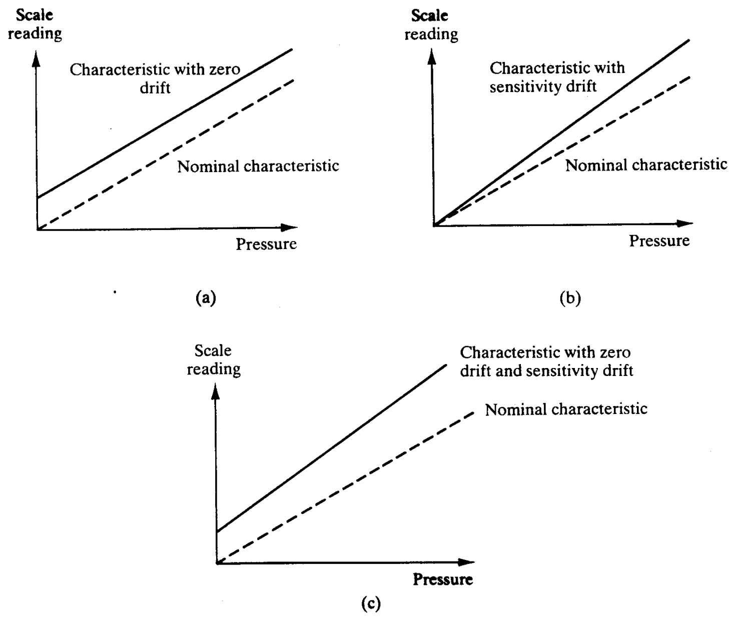


Figure 1.4-2 (a) Zero Drift (b) Sensitivity Drift (c) Zero Drift Plus Sensitivity Drift

Hysteresis

- Hysteresis is the non-coincidence between loading and unloading curves of an instrument.

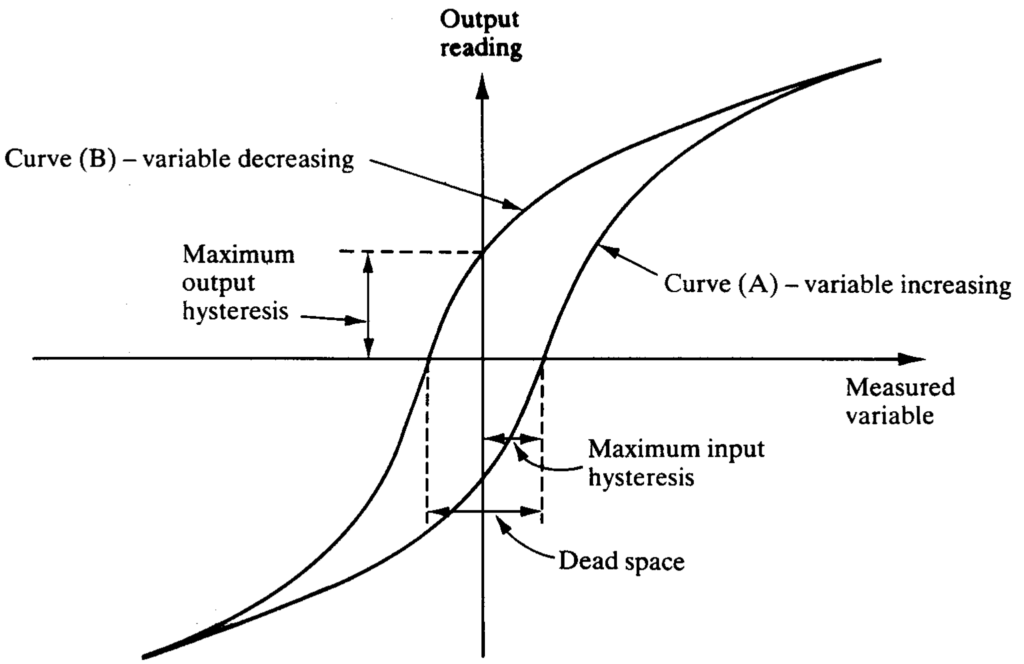


Figure 1.4-3 Instrument Characteristic with Hysteresis

Dead Space

- Dead space is defined as the range of different input values over which there is no change in output value.
- Example: Backlash in gears is a typical cause of dead space.

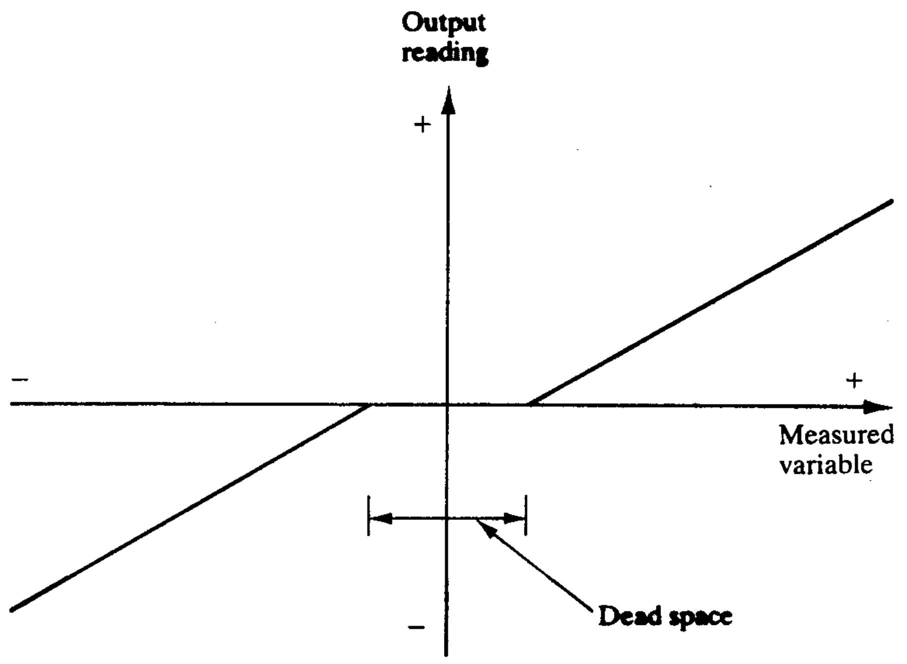


Figure 1.4-4 Instrument Characteristic with Dead Space

Threshold

- Threshold is the minimum level of input before the change in the instrument output reading.

Resolution

- Resolution is a lower limit on the magnitude of the change in the input measured quantity which produces an observable change in the instrument output.
- Like threshold, resolution is sometimes specified as an absolute value and sometimes as a percentage of f.s. deflection.

1.5 Dynamic Characteristics of Instruments

- The dynamic characteristics of a measuring instrument describe its behavior between the time a measured quantity changes value and the time when the instrument output attains a steady value in response.

In any linear, time-invariant measuring system, the general relation between input and output for time $t > 0$:

$$a_n \frac{d^n q_o}{dt^n} + a_{n-1} \frac{d^{n-1} q_o}{dt^{n-1}} + \cdots + a_1 \frac{dq_o}{dt} + a_0 q_o = b_m \frac{d^m q_i}{dt^m} + b_{m-1} \frac{d^{m-1} q_i}{dt^{m-1}} + \cdots + b_1 \frac{dq_i}{dt} + b_0 q_i \qquad (1.5\text{-}1)$$

q_i: the measured quantity

q_o: the output reading

$a_0, \ldots, a_n, b_0, \ldots, b_m$: constants

When only step changes in the measured quantity is considered,

$$a_n \frac{d^n q_o}{dt^n} + a_{n-1} \frac{d^{n-1} q_o}{dt^{n-1}} + \cdots + a_1 \frac{dq_o}{dt} + a_0 q_o = b_0 q_i \qquad (1.5\text{-}2)$$

1.5.1 Zero-Order Instrument

If all the coefficients a_1, …, a_n other than a_0 are assumed zero,

$$a_0 q_o = b_0 q_i \text{ or } q_0 = \frac{b_0 q_i}{a_0} = K q_i \quad (1.5.1\text{-}1)$$

K: a constant known as the instrument sensitivity.

Example: A potentiometer, which measures motion, is a zero-order instrument, where the output voltage changes instantaneously as the slider is displaced along the potentiometer track.

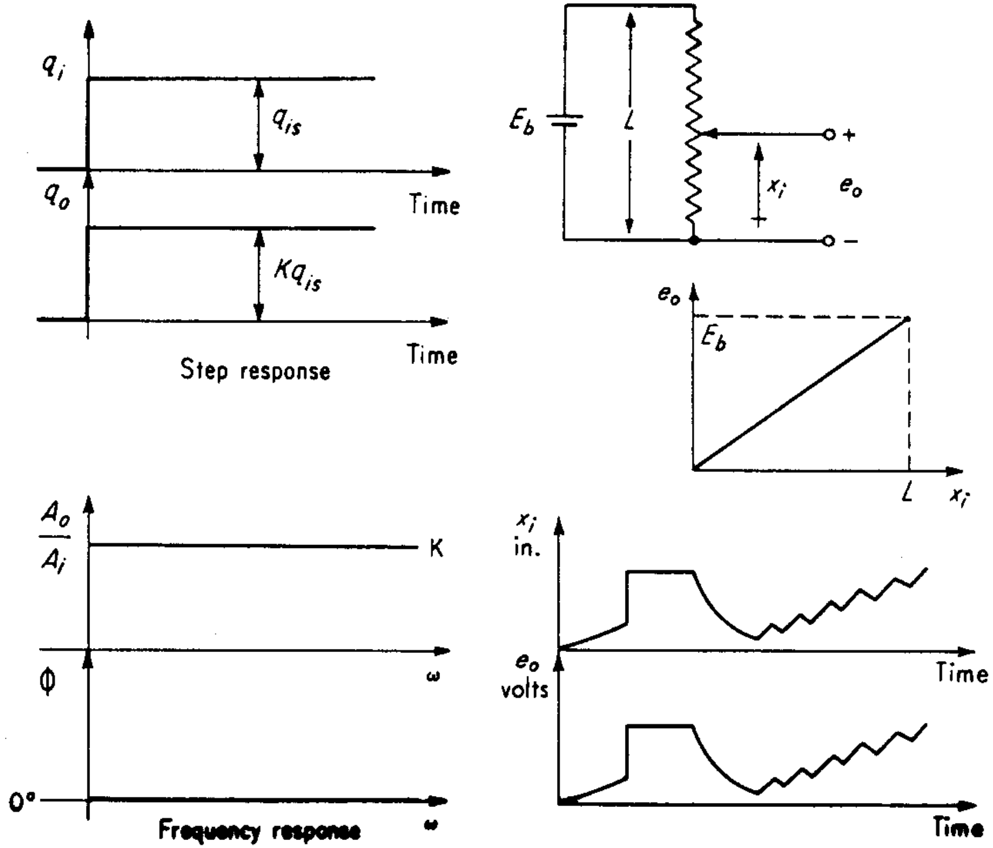


Figure 1.5.1-1 Zero-Order Instrument Characteristics

1.5.2 First-Order Instrument

If all the coefficients a_2, …, a_n except for a_0 and a_1 are assumed zero,

$$a_1 \frac{dq_o}{dt} + a_0 q_o = b_0 q_i \tag{1.5.2-1}$$

By taking Laplace transformation,

$$a_1 Q_o s + a_0 Q_o = b_0 Q_i \tag{1.5.2-2}$$

$$Q_o = \frac{(b_0 / a_0)}{(1 + (a_1 / a_0) s)} Q_i \tag{1.5.2-3}$$

$$Q_o = \frac{K}{(1 + \tau s)} Q_i \tag{1.5.2-4}$$

$K = b_0/a_0$: the static sensitivity

$\tau = a_1/a_0$: the time constant of the system

- The time constant, τ, of the step response is the time taken for the output quantity to reach 63% of its final value.
- The thermocouple is a first-order instrument. If a thermocouple at room temperature is plunged into boiling water, the output does not rise instantaneously to a level indicating 100°C but instead exponentially approaches a reading indicating 100°C.

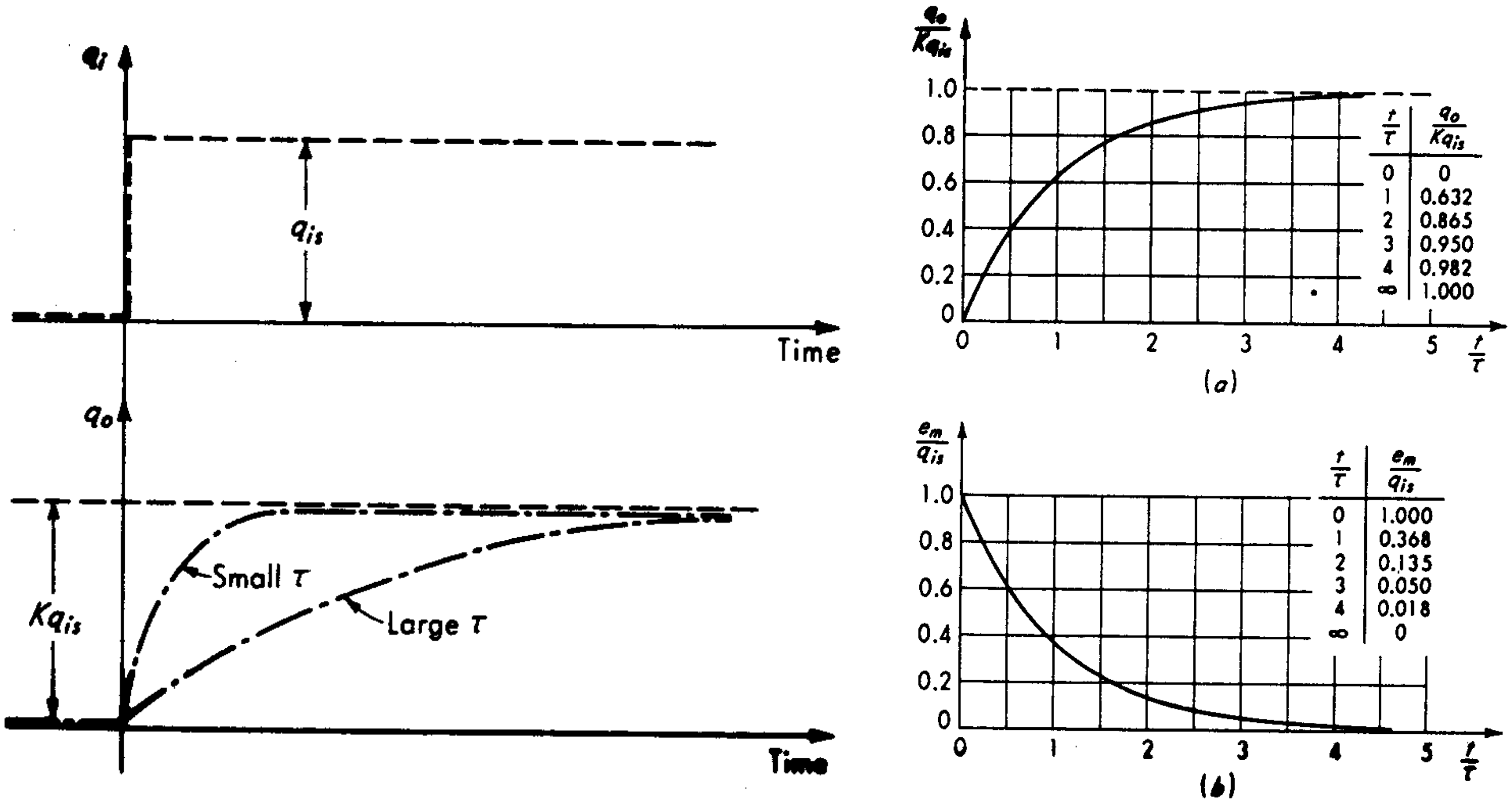


Figure 1.5.2-1 Step-Response of First-Order Instrument

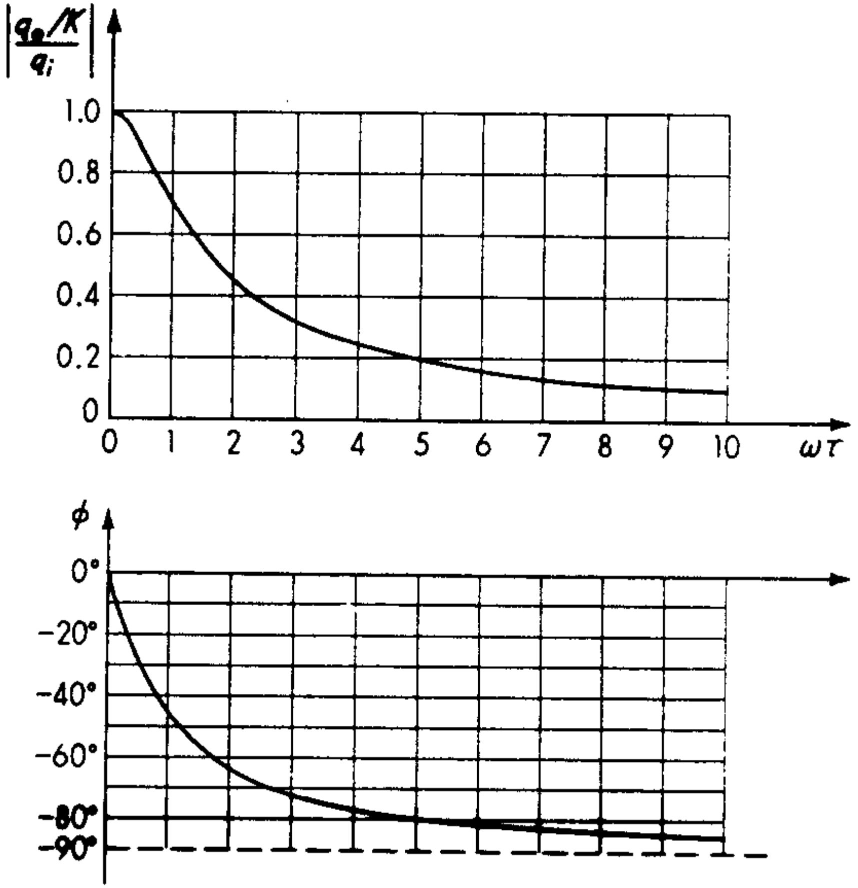


Figure 1.5.2-2 Frequency Response of First-Order Instrument

1.5.3 Second-Order Instrument

If all the coefficients a_3, …, a_n except for a_0, a_1, and a_2 are assumed zero,

$$a_2 \frac{d^2 q_o}{dt^2} + a_1 \frac{dq_o}{dt} + a_0 q_o = b_0 q_i \qquad (1.5.3\text{-}1)$$

By taking Laplace transformation,

$$a_2 Q_o s^2 + a_1 Q_o s + a_0 Q_o = b_0 Q_i \qquad (1.5.3\text{-}2)$$

$$Q_o = \frac{(b_0 / a_0)}{(1 + (a_1 / a_0)s + (a_2 / a_0)s^2)} Q_i \qquad (1.5.3\text{-}3)$$

$$Q_o = \frac{K}{(1 + (2\zeta / \omega)s + (1/\omega^2)s^2)} Q_i \qquad (1.5.3\text{-}4)$$

K (static sensitivity), ω (undamped natural frequency), and ζ (damping ratio):

$$K = b_0 / a_0 \qquad (1.5.3\text{-}5)$$

$$\omega = \sqrt{a_0 / a_2} \qquad (1.5.3\text{-}6)$$

$$\zeta = a_1 /(2\sqrt{a_0 a_2}) \qquad (1.5.3\text{-}7)$$

$$\tau = 2a_2 / a_1 \qquad (1.5.3\text{-}8)$$

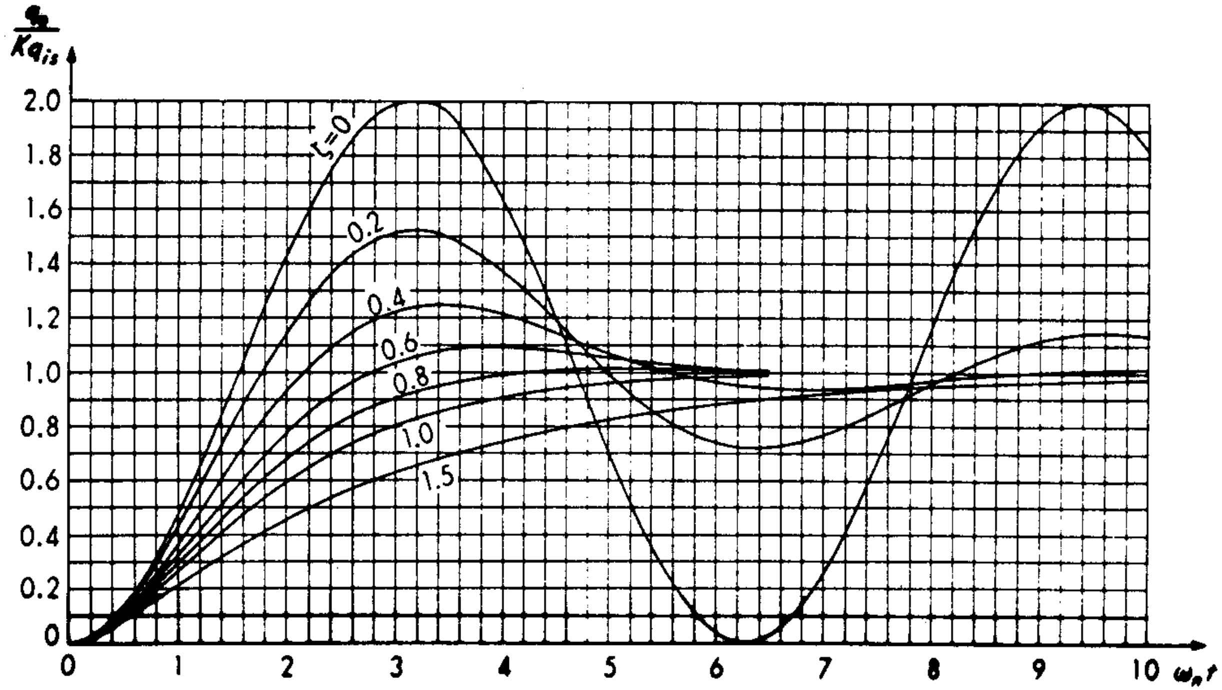


Figure 1.5.3-1 Non-Dimensional Step-Response of Second-Order Instrument

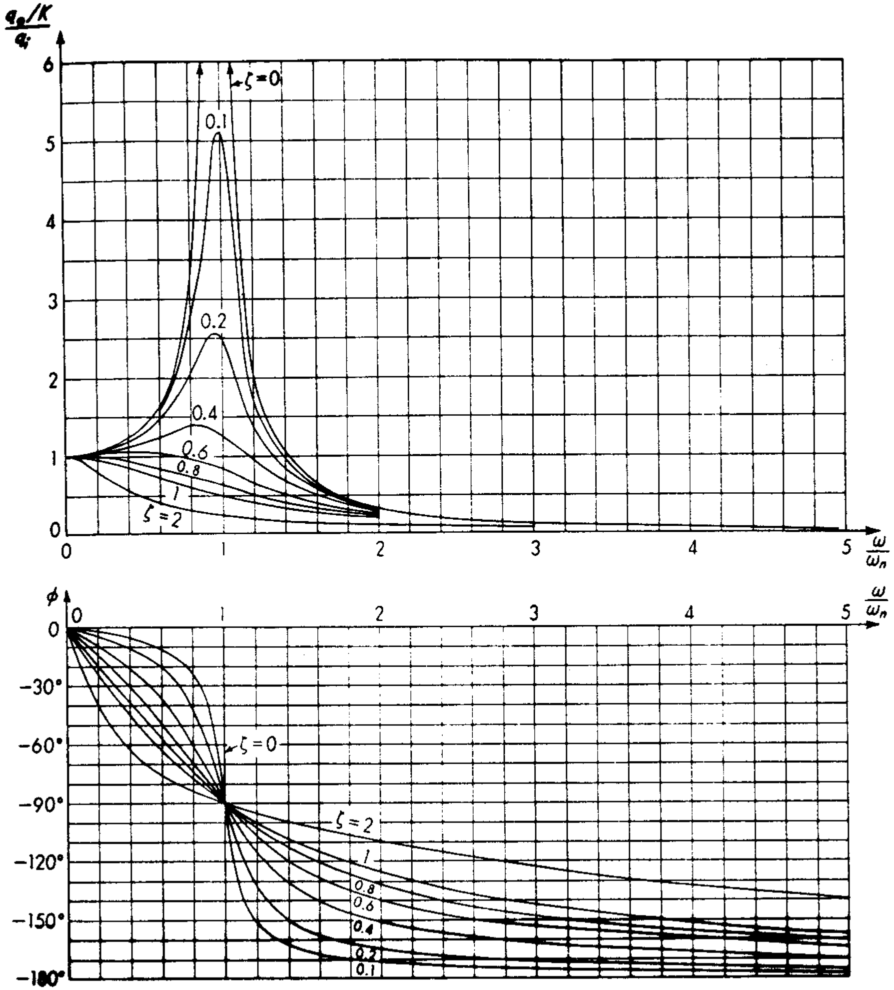


Figure 1.5.3-2 Frequency Response of Second-Order Instrument

1.6 Measurement System Errors

1.6.1 Random and Systematic Errors

Random errors

- Random errors usually arise when measurements are taken by human observation of an analog meter reading, especially where this involves interpolation between scale points.
- Electrical noise can also be a source of random errors.
- To some extent, random errors can be overcome by taking the same measurement a number of times and extracting a value by statistical means.

Systematic errors

- Two major sources of systematic errors are system disturbance during measurement and the effect of modifying inputs.
- Other sources of systematic error include bent meter needles, the use of uncalibrated instruments, cabling practices and thermal e.m.f.'s, error which is inherent in the manufacture of an instrument.
- These are quantified in the accuracy figure quoted in the published instrument specifications.

1.6.2 System Disturbance due to Measurement

- The process of measurement disturbs the system.

Thermometer: Some amount of heat is transferred to thermometer.

Pressure Gauge: Some amount of air flows into the gauge.

Measurements in Electric Circuits

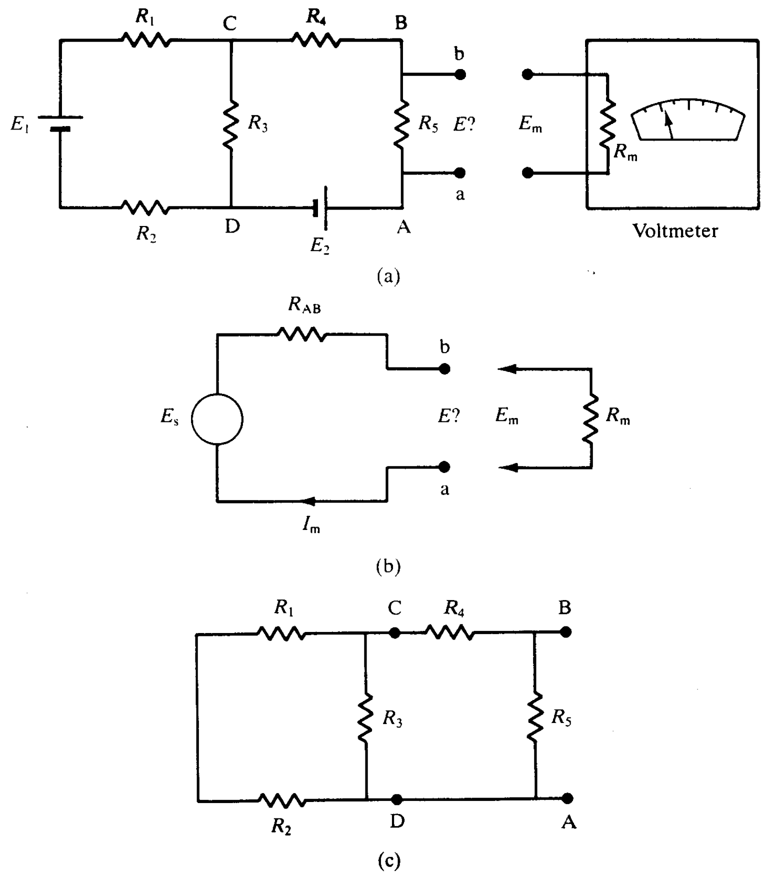


Figure 1.6.2-1 (a) A Circuit in Which the Voltage Across R_5 is to be Measured, (b) Equivalent Circuit by Thevenin's Theorem, (c) The Circuit Used to Find the Equivalent Single Resistance R_{AB}

$$R_{CD} = \frac{(R_1 + R_2)R_3}{R_1 + R_2 + R_3} \tag{1.6.2-1}$$

$$R_{AB} = \frac{(R_4 + R_{CD})R_5}{R_4 + R_{CD} + R_5} \tag{1.6.2-2}$$

$$R_{AB} = \frac{[(R_1 + R_2)R_3 /(R_1 + R_2 + R_3) + R_4]R_5}{(R_1 + R_2)R_3 /(R_1 + R_2 + R_3) + R_4 + R_5} \tag{1.6.2-3}$$

$$I = \frac{E_0}{R_{AB} + R_m} \tag{1.6.2-4}$$

$$E_m = \frac{E_0}{R_{AB} + R_m} R_m \tag{1.6.2-5}$$

$$\frac{E_m}{E_0} = \frac{R_m}{R_{AB} + R_m} \tag{1.6.2-6}$$

- As R_m gets larger, the ratio E_m/E_0 gets closer to unity, showing that the design strategy should be to make R_m as high as possible to minimize disturbance of the measured system.
- The simplest way of increasing the input impedance (the resistance) of the meter is either to increase the number of turns in the coil or to construct the same number of coil turns with a higher-resistance material.
- Higher input impedance decreases the current flowing in the coil, however, giving less magnetic torque and thus decreasing the measurement sensitivity of the instrument.
- The sensitivity can be improved by changing the spring constant of the restraining springs of the instrument, such that less torque is required to turn the pointer by a given amount.

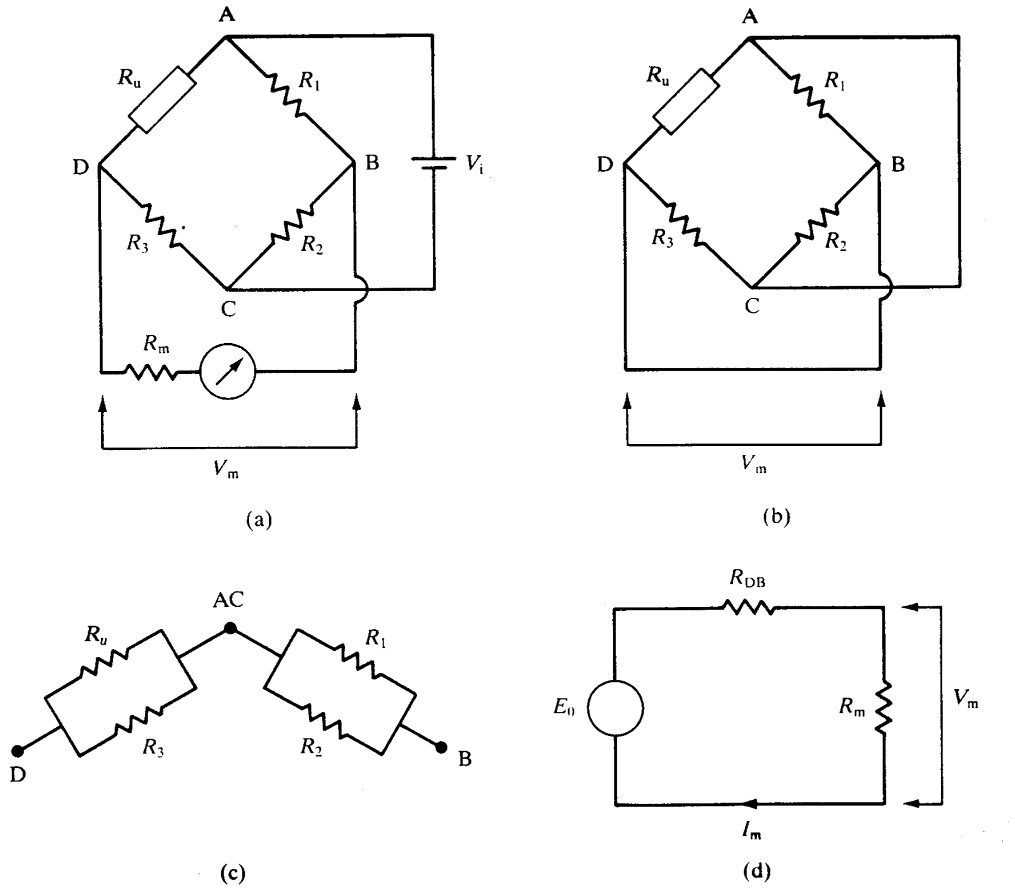


Figure 1.6.2-2 (a)A bridge Circuit (b) Equivalent Circuit by Thevenin's Theorem

(c) Alternate Representation (d) Equivalent Circuit via Thevenin's Theorem

$$R_{DB} = \frac{R_1 R_2}{R_1 + R_2} + \frac{R_u R_3}{R_u + R_3} \qquad (1.6.2\text{-}7)$$

$$E_0 = V_i \left(\frac{R_u}{R_u + R_3} - \frac{R_1}{R_1 + R_2} \right) \qquad (1.6.2\text{-}8)$$

$$I_m = \frac{E_0}{R_{DB} + R_m} \qquad (1.6.2\text{-}9)$$

$$V_m = I_m R_m = \frac{E_0 R_m}{R_{DB} + R_m} \qquad (1.6.2\text{-}10)$$

$$V_m = \frac{V_i [R_u /(R_u + R_3) - R_1 /(R_1 + R_2)] R_m}{R_1 R_2 /(R_1 + R_2) + R_u R_3 /(R_u + R_3) + R_m} \qquad (1.6.2\text{-}11)$$

$$V_m = \frac{V_i R_m (R_u R_2 - R_1 R_3)}{R_1 R_2 (R_u + R_3) + R_u R_3 (R_1 + R_2) + R_m (R_1 + R_2)(R_u + R_3)} \qquad (1.6.2\text{-}12)$$

1.6.3 Modifying Inputs in Measurement Systems

Careful Instrument Design

- Reducing the sensitivity of an instrument to modifying inputs to as low a level as possible.
- Example: In the design of strain gauges, the element should be constructed from a material whose resistance has a very low temperature coefficient.

Method of Opposing Inputs

- Introducing an equal and opposite modifying input which cancels it out.
- Example: Millivoltmeter consists of a coil suspended in a fixed magnetic field produced by a permanent magnet.
    - When an unknown voltage is applied to the coil, the magnetic field due to the current interacts with the fixed field and causes the coil (and a pointer attached to the coil) to turn.
    - If the coil resistance is sensitive to temperature, then any modifying input to the system in the form of a temperature change will alter the value of the coil current for a given applied voltage and so alter the pointer output reading.
    - Compensation for this is made by introducing a compensating resistance R_{comp} into the circuit, where R_{comp} has a temperature coefficient which is equal in magnitude but opposite in sign to that of the coil.

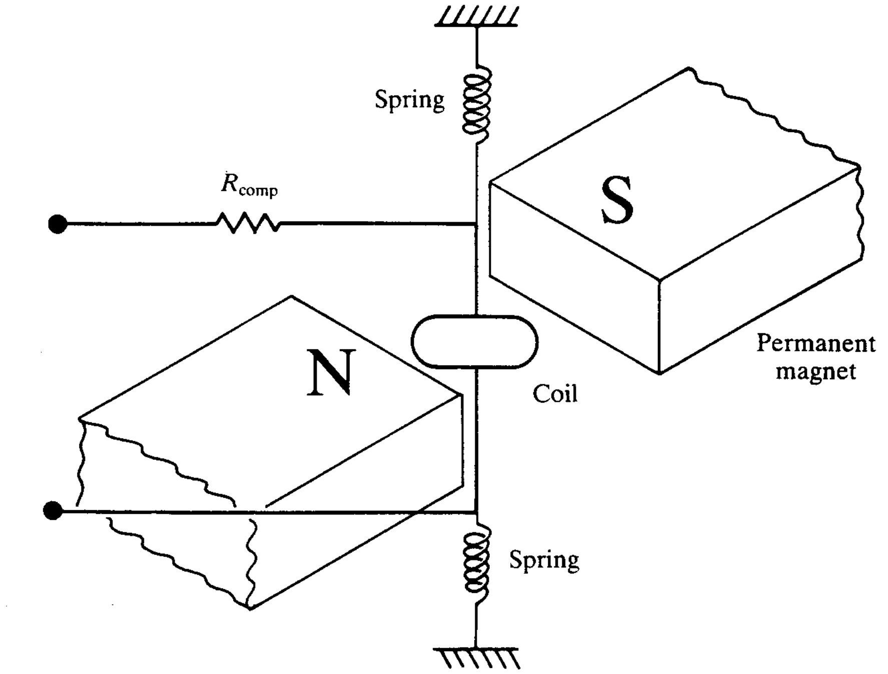


Figure 1.6.3-1 Millivoltmeter

High-Gain Feedback

- In this system, the unknown voltage E_i is applied to a motor of torque constant K_m, and the torque induced turns a pointer against the restraining action of a spring with spring constant K_s. The effect of modifying inputs on the motor and spring constants are represented by variables D_m and D_s.

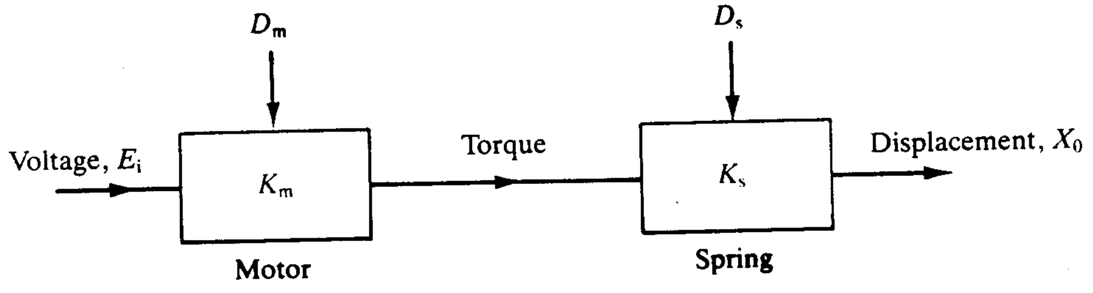


Figure 1.6.3-2 Block Diagram of Voltage-Measuring Instrument

$$X_0 = K_m K_s E_i \tag{1.6.3-1}$$

- D_m and D_s have large effect to output displacement.

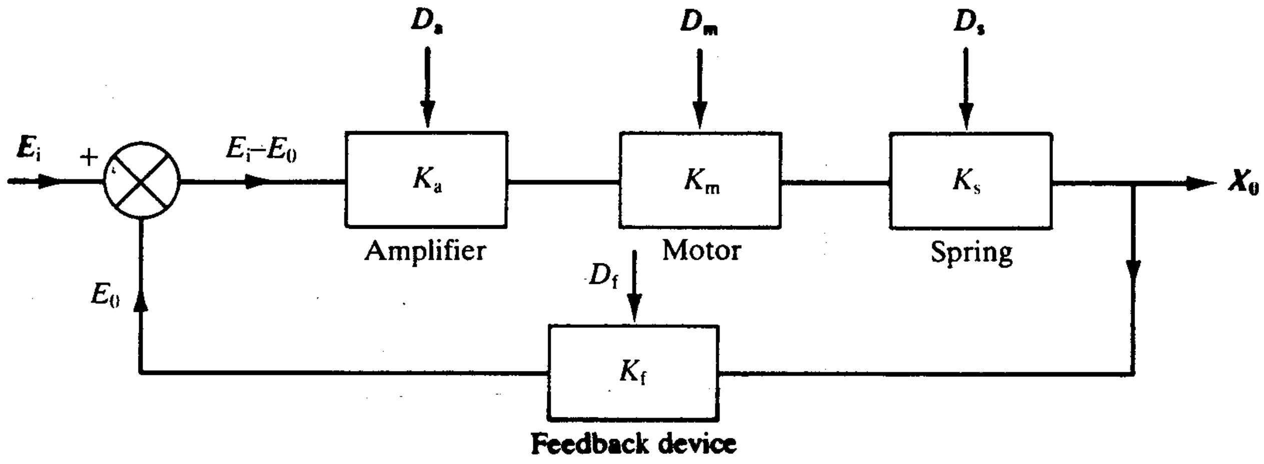


Figure 1.6.3-3 Conversion of System of Figure 1.6.3-2 into a High-Gain, Closed Loop One

$$E_0 = K_f X_0 \tag{1.6.3-2}$$

$$X_0 = (E_i - E_0)K_a K_m K_s = (E_i - K_f X_0)K_a K_m K_s \tag{1.6.3-3}$$

$$X_0 = \frac{K_a K_m K_s}{1 + K_f K_a K_m K_s} E_i \tag{1.6.3-4}$$

If K_a is very large (it is a high-gain amplifier),

$$X_0 = \frac{1}{K_f} E_i \tag{1.6.3-5}$$

- D_m, D_s, and D_a have small effect to output displacement.
- It is usually an easy matter to design a feedback device which is insensitive to modifying inputs.
- There is a possibility that high-gain feedback will cause instability in the system. Any application of this method must therefore include careful stability analysis of the system.

Signal Filtering

- One frequent problem in measurement systems is corruption of the output reading by periodic noise, often at a frequency of 50 Hz caused by pick-up through the close proximity of the measurement system to apparatus or current-carrying cables operating on a mains supply.
- Periodic noise corruption at higher frequencies is also often introduced by mechanical oscillation or vibration within some component of a measurement system.
- The amplitude of all such noise components can be substantially attenuated by the inclusion of filtering of an appropriate form in the system.
- Band-stop filters can be especially useful where corruption is of one particular known frequency.
- Low-pass filters are employed to attenuate all noise in the range of 50 Hz and higher frequencies.
- Measurement systems with a low-level output, such as a bridge circuit measuring a strain-gauge resistance, are particularly prone to noise.

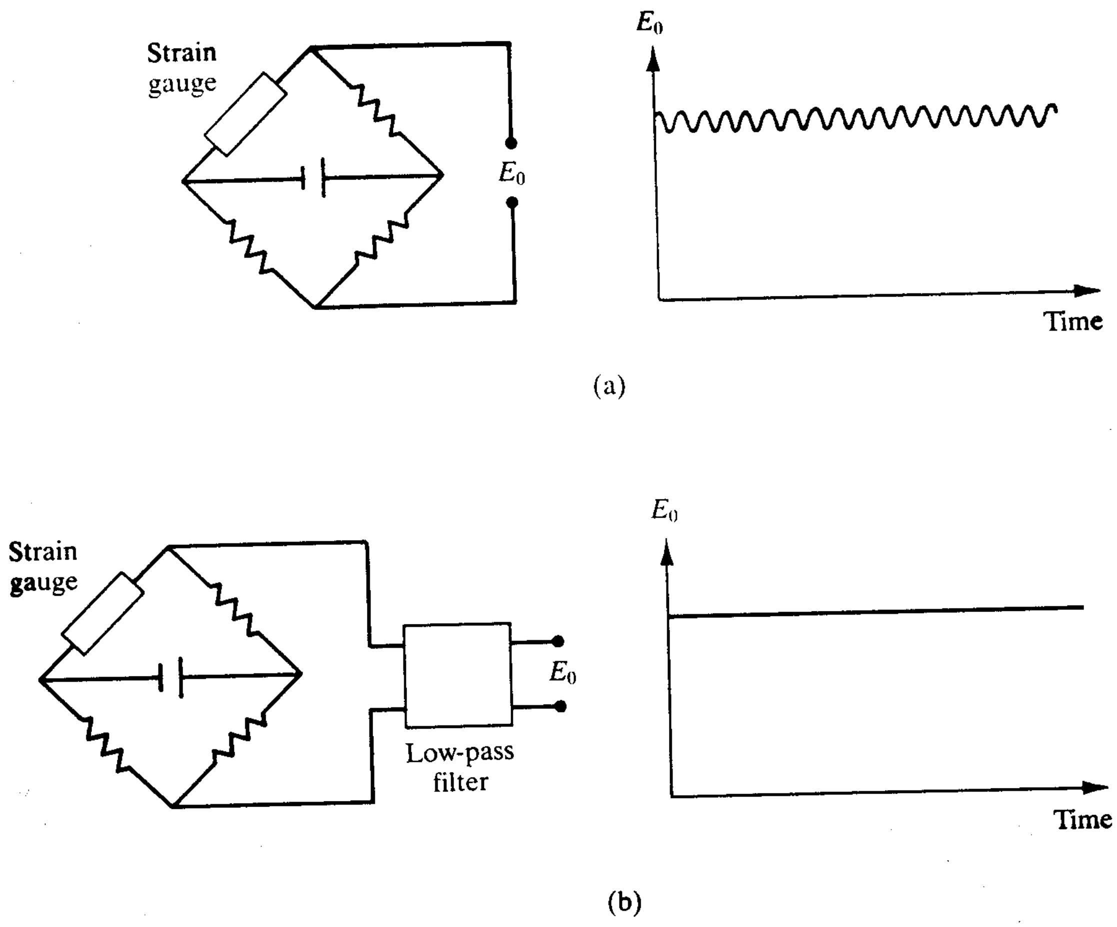


Figure 1.6.3-4 (a) Noise-Corrupted Output of Bridge Circuit Measuring Resistance of Strain Gauge

(b) Effect of Adding Low-Pass Filter

1.6.4 Other Sources of Error

Connecting Leads

- Resistance of connecting lead is affected by ambient condition; temperature, magnetic field.
- Minimize resistance of connecting lead.
- Shield connecting lead from temperature change, magnetic field, etc.

Thermal e.m.f's

- Based on thermocouple principle, whenever metals of two different types are connected together, a thermal e.m.f. is generated according to the temperature of the joint.
- The thermal e.m.f.'s are only of a few millivolts in magnitude and so this effect is only significant when typical voltage output signals of a measurement system are of a similar low magnitude.
- The temperature at both joints should be the same because the thermal e.m.f.'s will be equal and opposite and so cancel out.

1.6.5 Estimation of Total Measurement System Errors

Error in a Product

When the outputs *y* and *z* of two measurement system components are multiplied together,

$$P = yz \quad (1.6.5\text{-}1)$$

When the possible error in *y* is ±*ay* and in *z* is ±*bz*,

$$P_{max} = (y + ay)(z + bz) = yz + ayz + byz + aybz \quad (1.6.5\text{-}2)$$

$$P_{min} = (y - ay)(z - bz) = yz - ayz - byz + aybz \quad (1.6.5\text{-}3)$$

- For typical measurement system components with output errors of up to 1-2% in magnitude, $a \ll 1$ and $b \ll 1$ and terms in *aybz* are negligible compared with the other terms.

$$P_{max} = yz(1 + a + b) \quad (1.6.5\text{-}4)$$

$$P_{min} = yz(1 - a - b) \quad (1.6.5\text{-}5)$$

- The error in the product *P* is ±(*a* + *b*).

Error in a Quotient

When the output measurement *y* of one system component with possible error ±*ay* is divided by the output measurement *z* of another system component with possible error ±*bz*,

$$Q_{max} = \frac{y+ay}{z-bz} = \frac{(y+ay)(z+bz)}{(z-bz)(z+bz)} = \frac{yz+ayz+byz+abyz}{z^2-b^2z^2} \qquad (1.6.5\text{-}6)$$

$$Q_{min} = \frac{y-ay}{z+bz} = \frac{(y-ay)(z-bz)}{(z+bz)(z-bz)} = \frac{yz-ayz-byz+abyz}{z^2-b^2z^2} \qquad (1.6.5\text{-}7)$$

- For *a* « 1 and *b* « 1, terms in *ab* and b^2 are negligible compared with the other terms.

$$Q_{max} = \frac{yz(1+a+b)}{z^2} \qquad (1.6.5\text{-}8)$$

$$Q_{min} = \frac{yz(1-a-b)}{z^2} \qquad (1.6.5\text{-}9)$$

$$Q = \frac{y}{z} \pm \frac{y}{z}(a+b) \qquad (1.6.5\text{-}10)$$

- The error in the quotient is ±(*a* + *b*).

Error in a Sum

When the two outputs *y* and *z* of separate measurement system components are to be added together,

$$S = y + z \tag{1.6.5-11}$$

When the maximum errors in *y* and *z* are ±*ay* and ±*bz* respectively,

$$S_{max} = y + ay + z + bz \tag{1.6.5-12}$$

$$S_{min} = y - ay + z - bz \tag{1.6.5-13}$$

$$S = y + z \pm (ay + bz) \tag{1.6.5-14}$$

Maximum error, *e*,

$$e = \left[(ay)^2 + (bz)^2\right]^{1/2} \tag{1.6.5-15}$$

$$S = (y + z) \pm e \tag{1.6.5-16}$$

$$S = (y + z)(1 \pm f) \tag{1.6.5-17}$$

where $f = e/(y + z)$.

Error in a Difference

If the two outputs y and z of separate measurement systems are to be subtracted from one another, and the possible errors are $\pm ay$ and $\pm bz$,

$$S = (y - z) \pm e \qquad (1.6.5\text{-}18)$$

$$S = (y - z)(1 \pm f) \qquad (1.6.5\text{-}19)$$

where $f = e/(y - z)$.

2. Signal Manipulation

2.1 The Potentiometer Circuit

$$E_o = \frac{R_1}{R_1 + R_2} E_i = \frac{1}{1+r} E_i \quad (2.1\text{-}1)$$

E_i : the input voltage, r : the resistance ratio R_2/R_1

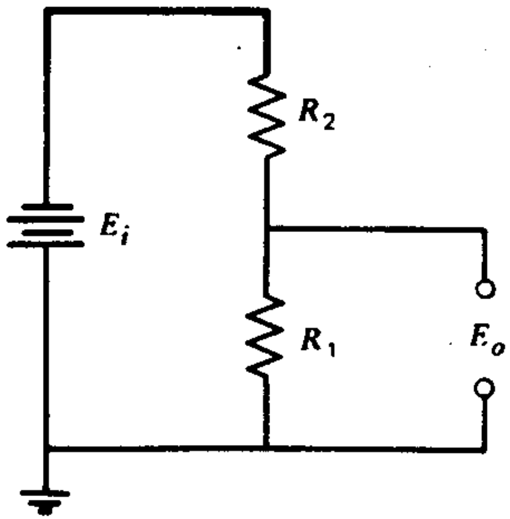


Figure 2.1-1 Constant-Voltage Potentiometer Circuit

If the resistors R_1 and R_2 are varied by ΔR_1 and ΔR_2, the change ΔE_o in the output voltage

$$E_o + \Delta E_o = \frac{R_1 + \Delta R_1}{R_1 + \Delta R_1 + R_2 + \Delta R_2} E_i \tag{2.1-2}$$

$$\Delta E_o = \left[\frac{R_1 + \Delta R_1}{R_1 + \Delta R_1 + R_2 + \Delta R_2} - \frac{R_1}{R_1 + R_2} \right] E_i \tag{2.1-3}$$

$$\Delta E_o = \frac{\frac{r}{(1+r)^2}\left(\frac{\Delta R_1}{R_1} - \frac{\Delta R_2}{R_2}\right)}{1 + \frac{1}{1+r}\left(\frac{\Delta R_1}{R_1} + r\frac{\Delta R_2}{R_2}\right)} E_i \tag{2.1-4}$$

The nonlinear term η,

$$\eta = 1 - \frac{1}{1 + \frac{1}{1+r}\left(\frac{\Delta R_1}{R_1} + r\frac{\Delta R_2}{R_2}\right)} \tag{2.1-5}$$

$$\Delta E_o = \frac{r}{(1+r)^2}\left(\frac{\Delta R_1}{R_1} - \frac{\Delta R_2}{R_2}\right)(1-\eta)E_i \tag{2.1-6}$$

With r = 9 and ΔR_2 = 0,

$$\eta = 1 - \frac{1}{1 + \left(0.1\frac{\Delta R_1}{R_1}\right)} = \left(0.1\frac{\Delta R_1}{R_1}\right) - \left(0.1\frac{\Delta R_1}{R_1}\right)^2 + \left(0.1\frac{\Delta R_1}{R_1}\right)^3 + \cdots \tag{2.1-7}$$

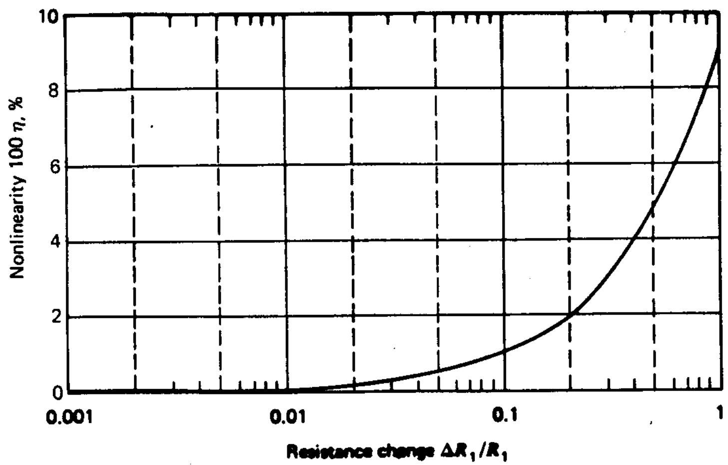


Figure 2.1-2 Nonlinear Term η as a Function of Resistance Change $\Delta R_1/R_1$ for a Constant-Voltage Potentiometer Circuit with r = 9 and $\Delta R_2 = 0$

The sensitivity of the potentiometer circuit for a case where $\Delta R_2 = 0$,

$$S_{cv} = \frac{\Delta E_o}{\frac{\Delta R_1}{R_1}} = \frac{r}{(1+r)^2} E_i \qquad (2.1\text{-}8)$$

- All transducers have limited power dissipation, P_T, capabilities that restrict the input voltage.

$$P_T = \frac{E_T^2}{R_T} \qquad (2.1\text{-}9)$$

E_T : the voltage across the transducer, R_T : the transducer resistance

$$E_T = \frac{1}{1+r} E_i \quad (2.1\text{-}10)$$

$$E_{i\max} = (1+r)\sqrt{P_T R_T} \quad (2.1\text{-}11)$$

$$S_{cv} = \frac{r}{1+r}\sqrt{P_T R_T} \quad \text{with } \Delta R_2 = 0 \quad (2.1\text{-}12)$$

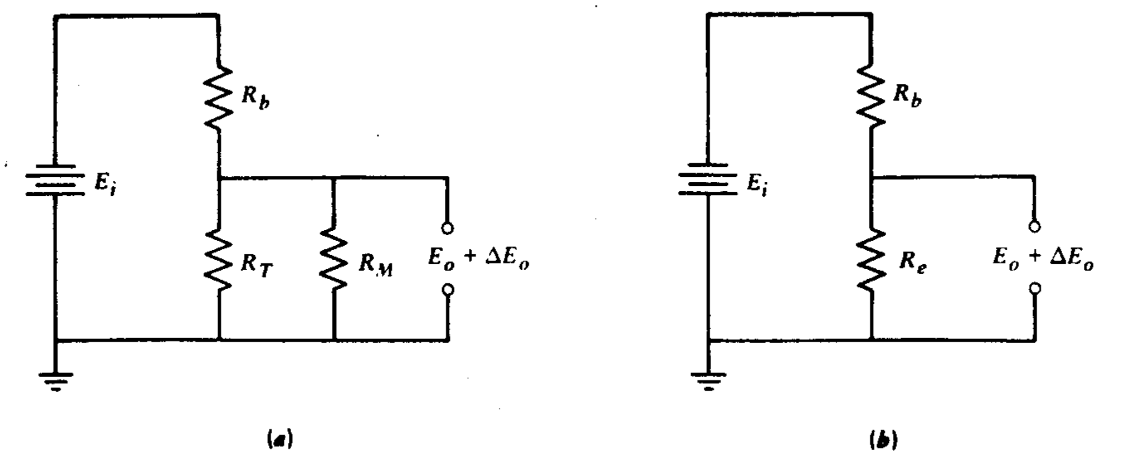


Figure 2.1-3 Constant-Voltage Potentiometer Circuit with a Recording Instrument

(a) Resistive Load Associated with the Recording Instrument (b) The Equivalent Circuit

$$R_e = \frac{R_T R_M}{R_T + R_M} \tag{2.1-13}$$

With $\Delta R_b = 0$ and $\eta \approx 0$,

$$\Delta E_o\Big]_{R_M} = \frac{R_b R_e}{(R_b + R_e)^2} \frac{\Delta R_e}{R_e} E_i \tag{2.1-14}$$

$$\Delta E_o\Big]_{R_M} = \frac{R_b R_T}{\left[R_T + R_b\left(\frac{R_T}{R_M}\right) + R_b\right]^2} \frac{\Delta R_T}{R_T} E_i \tag{2.1-15}$$

For the open-circuit voltage ($R_M = \infty$),

$$\Delta E_o\Big]_{R_M \to \infty} = \frac{R_b R_T}{\left[R_T + R_b\right]^2} \frac{\Delta R_T}{R_T} E_i \tag{2.1-16}$$

$$\Delta E_o\Big]_{R_M} = \Delta E_o\Big]_{R_M \to \infty} (1 - L) \tag{2.1-17}$$

L: the loss in output due to the presence of R_M.

$$L = 2r \frac{R_T}{R_M} \frac{\left[1 + r\left(1 + \frac{1}{2}\frac{R_T}{R_M}\right)\right]}{\left[1 + r\left(1 + \frac{R_T}{R_M}\right)\right]^2} \tag{2.1-18}$$

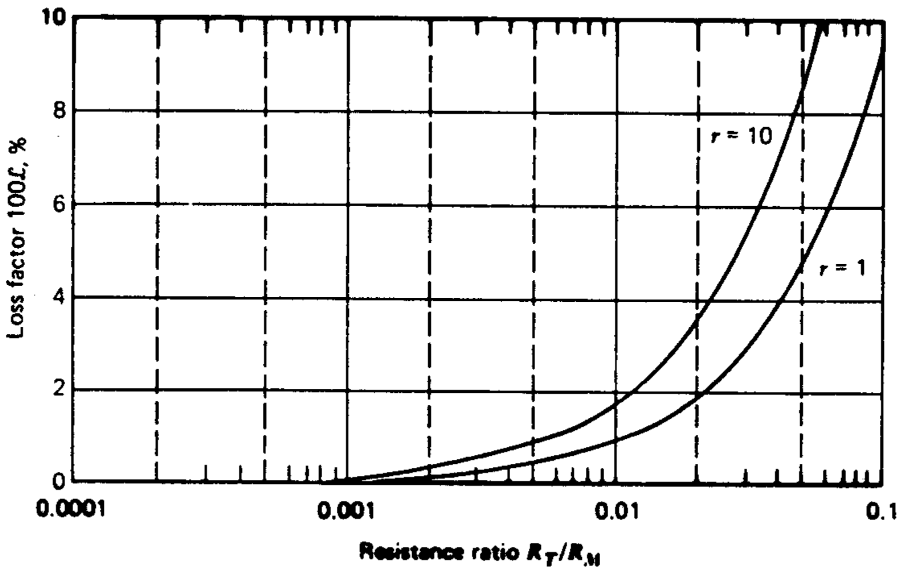


Figure 2.1-4 Loss Factor L as a Function of R_T/R_M for a Potentiometer Circuit Loaded with a Voltage Measuring Instrument

2.2 Bridge Circuit

2.2.1 Null-Type D.C. Wheatstone Bridge

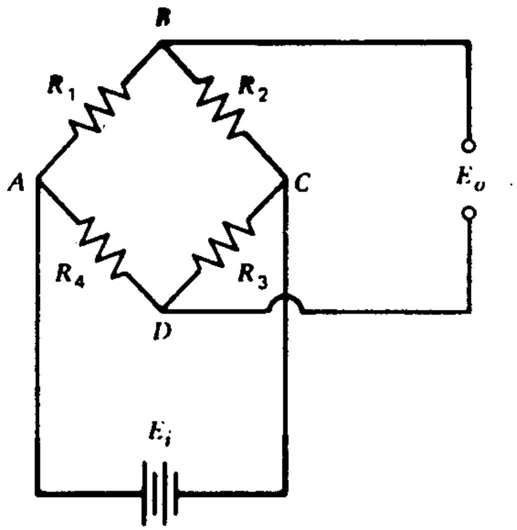


Figure 2.2.1-1 Constant-Voltage Wheatstone Bridge Circuit

$$E_{AB} = \frac{R_1}{R_1 + R_2} E_i \qquad (2.2.1\text{-}1)$$

$$E_{AD} = \frac{R_4}{R_3 + R_4} E_i \qquad (2.2.1\text{-}2)$$

$$E_o = E_{AB} - E_{AD} = E_A - E_B - (E_A - E_D) = E_{DB} \quad (2.2.1\text{-}3)$$

$$E_o = \frac{R_1 R_3 - R_2 R_4}{(R_1 + R_2)(R_3 + R_4)} E_i \quad (2.2.1\text{-}4)$$

$E_o = 0$, the bridge is balanced, if

$$R_1 R_3 = R_2 R_4 \text{ or } \frac{R_1}{R_2} = \frac{R_4}{R_3} \text{ or } R_1 = \frac{R_2 R_4}{R_3} \quad (2.2.1\text{-}5)$$

- For null-type Wheatstone bridge, R_1 is unknown resistance, R_2 is value adjustable resistance, R_3 and R_4 are fixed.
- When the circuit is balanced, the value of R_1 can be obtained.

2.2.2 Deflection-Type D.C. Wheatstone Bridge

- It is much easier to measure small values of ΔE_o from a zero voltage base (E_o = 0), thus the bridge circuit must be balanced initially, $R_1R_3 = R_2R_4$.

$$\Delta E_o = \frac{(R_1+\Delta R_1)(R_3+\Delta R_3)-(R_2+\Delta R_2)(R_4+\Delta R_4)}{(R_1+\Delta R_1+R_2+\Delta R_2)(R_3+\Delta R_3+R_4+\Delta R_4)}E_i \tag{2.2.2-1}$$

By expanding and neglecting higher-order terms,

$$\Delta E_o = \frac{R_1R_2}{(R_1+R_2)^2}\left(\frac{\Delta R_1}{R_1}-\frac{\Delta R_2}{R_2}+\frac{\Delta R_3}{R_3}-\frac{\Delta R_4}{R_4}\right)E_i \tag{2.2.2-2}$$

$$\Delta E_o = \frac{r}{(1+r)^2}\left(\frac{\Delta R_1}{R_1}-\frac{\Delta R_2}{R_2}+\frac{\Delta R_3}{R_3}-\frac{\Delta R_4}{R_4}\right)E_i \tag{2.2.2-3}$$

where $r = R_2/R_1$.

If the higher-order terms are retained,

$$\Delta E_o = \frac{r}{(1+r)^2}\left(\frac{\Delta R_1}{R_1}-\frac{\Delta R_2}{R_2}+\frac{\Delta R_3}{R_3}-\frac{\Delta R_4}{R_4}\right)(1-\eta)E_i \tag{2.2.2-4}$$

$$\eta = \frac{1}{1+\dfrac{r+1}{\dfrac{\Delta R_1}{R_1}+\dfrac{\Delta R_4}{R_4}+r\left(\dfrac{\Delta R_2}{R_2}+\dfrac{\Delta R_3}{R_3}\right)}} \tag{2.2.2-5}$$

For the case of $R_1 = R_2 = R_3 = R_4$,

$$\eta = \frac{\sum_{i=1}^{4} \frac{\Delta R_i}{R_i}}{\sum_{i=1}^{4} \frac{\Delta R_i}{R_i} + 2} \tag{2.2.2-6}$$

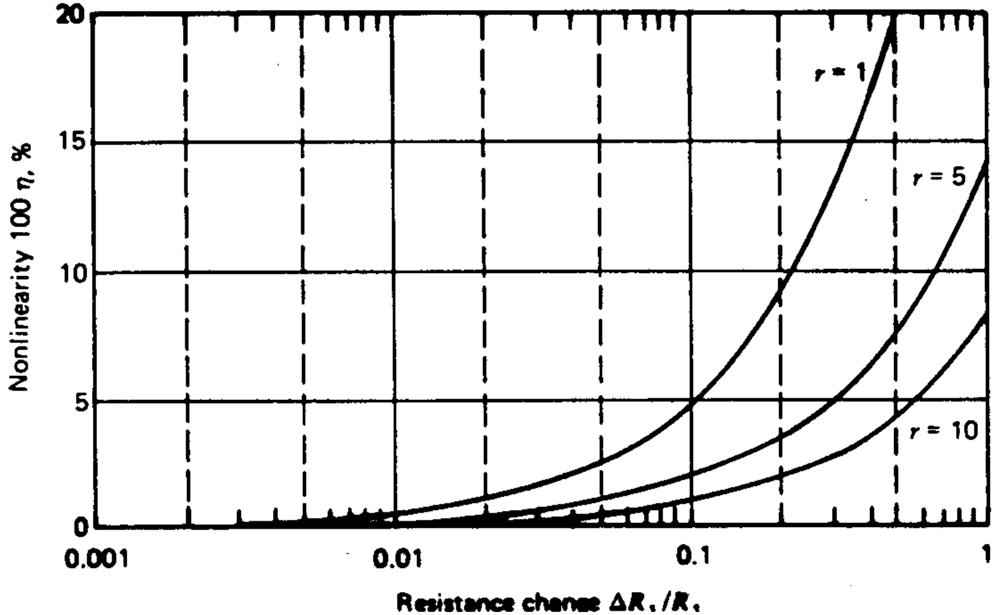


Figure 2.2.2-1 Nonlinear Term η as a Function of Resistance Change $\Delta R_1/R_1$ for a Constant-Voltage Wheatstone Bridge Circuit with One Active Gauge.

The sensitivity of a Wheatstone bridge with a constant-voltage power supply and a single active arm,

$$S_{cv} = \frac{\Delta E_o}{\frac{\Delta R_1}{R_1}} = \frac{r}{(1+r)^2} E_i \tag{2.2.2-7}$$

- The power P_T that can be dissipated by the transducer limits the bridge voltage E_i.

$$E_i = I_T(R_1 + R_2) = I_T R_T (1+r) = (1+r)\sqrt{P_T R_T} \tag{2.2.2-8}$$

$$S_{cv} = \frac{r}{1+r}\sqrt{P_T R_T} \tag{2.2.2-9}$$

- The output impedance R_B of the bridge can be determined by using Thevenin's theorem.

$$R_B = \frac{R_1 R_2}{R_1 + R_2} + \frac{R_3 R_4}{R_3 + R_4} \tag{2.2.2-10}$$

2.2.3 Apex Balancing

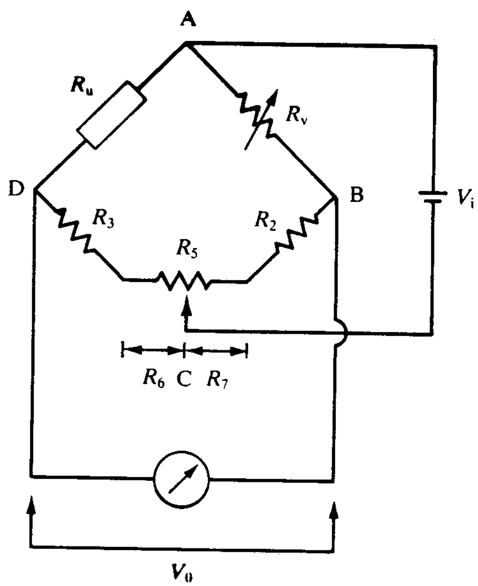


Figure 2.2.3-1 Apex Balancing

- One form of apex balancing consists of placing an additional variable resistor R_5 at the junction C between the resistances R_2 and R_3, and applying the excitation voltage V_i to the wiper of this variable resistance.
- In calibration, R_u and R_v are replaced by two equal resistances, and R_5 is varied until the output voltage V_o is zero.

$$R_3 + R_6 = R_2 + R_7 \tag{2.2.3-1}$$

2.2.4 Null-Type A.C. Bridge

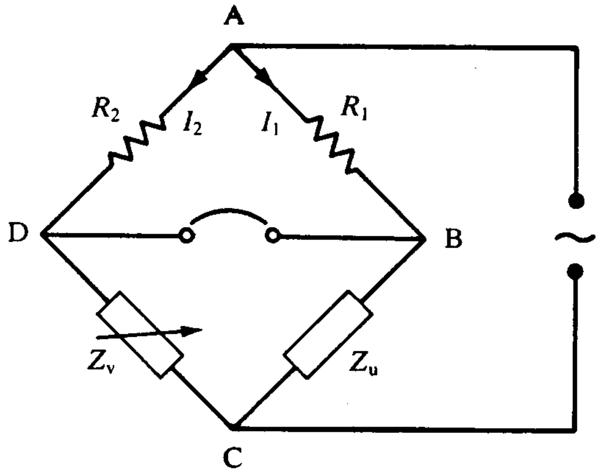


Figure 2.2.4-1 Null-Type Impedance Bridge

At the null point,

$$I_1 R_1 = I_2 R_2 \tag{2.2.4-1}$$

$$I_1 Z_u = I_2 Z_v \tag{2.2.4-2}$$

$$Z_u = \frac{Z_v R_1}{R_2} \tag{2.2.4-3}$$

If Z_u is capacitive,

$$Z_u = R_u + \frac{1}{j\omega C_u} \quad (2.2.4\text{-}4)$$

then Z_v must consist of a variable-resistance box and a variable-capacitance box in series

If Z_u is inductive, then

$$Z_u = R_u + j\omega L_u \quad (2.2.4\text{-}5)$$

then, Z_v must consist of a variable-resistance box and a variable-inductance box.

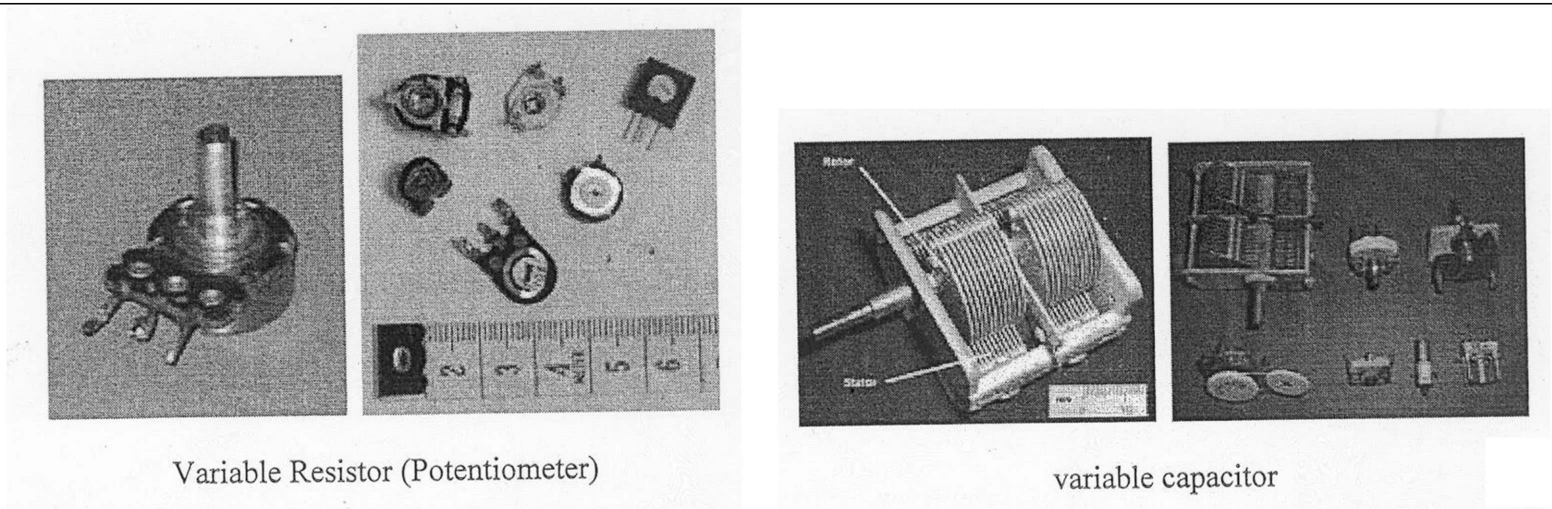


Figure 2.2.4-2 Variable Resistor and Capacitor

2.2.5 Maxwell Bridge

- In a Maxwell bridge the requirement for a variable-inductance box is avoided by introducing instead a second variable resistance.
- The circuit requires one standard fixed-value capacitor, two variable-resistance boxes and one standard fixed-value resistor.

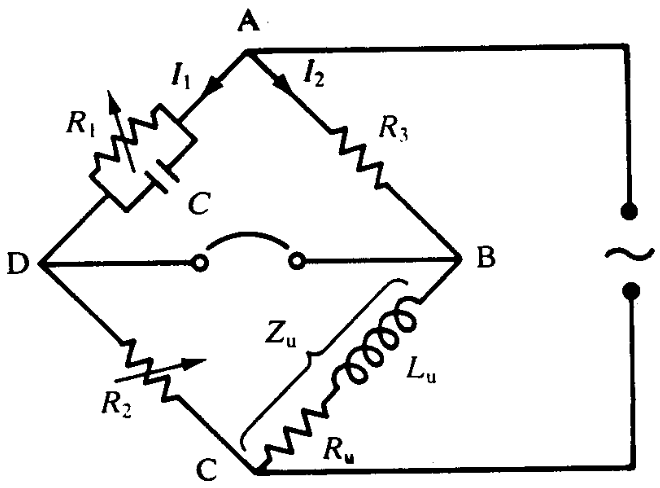


Figure 2.2.5-1 Maxwell Bridge

At the null point,

$$I_1 Z_{AD} = I_2 Z_{AB} \tag{2.2.5-1}$$

$$I_1 Z_{DC} = I_2 Z_{BC} \tag{2.2.5-2}$$

$$Z_{BC} = \frac{Z_{DC} Z_{AB}}{Z_{AD}} \tag{2.2.5-3}$$

$$Z_{AD} = \frac{R_1}{1 + j\omega C R_1},\ Z_{AB} = R_3,\ Z_{BC} = R_u + j\omega L_u, \text{ and } Z_{DC} = R_2 \tag{2.2.5-4}$$

$$R_u + j\omega L_u = \frac{R_2 R_3 (1 + j\omega C R_1)}{R_1} \tag{2.2.5-5}$$

$$R_u = \frac{R_2 R_3}{R_1},\ L_u = R_2 R_3 C \tag{2.2.5-6}$$

2.2.6 Deflection-Type A.C. Bridge

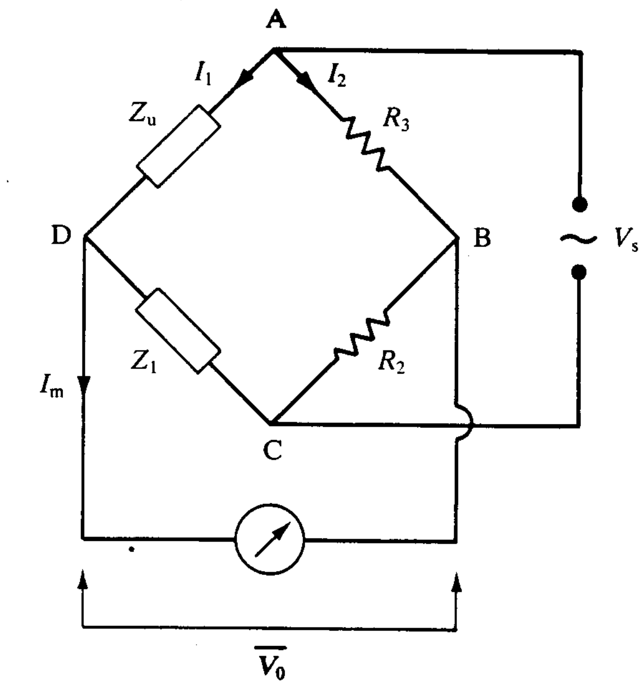


Figure 2.2.6-1 Deflection-Type A.C. Bridge

For capacitance measurement:

$$Z_u = \frac{1}{j\omega C_u},\ Z_1 = \frac{1}{j\omega C_1} \tag{2.2.6-1}$$

For inductance measurement:

$$Z_u = j\omega L_u\,,\; Z_1 = j\omega L_1 \tag{2.2.6-2}$$

$$I_1 = \frac{V_s}{Z_1 + Z_u}\,,\; I_2 = \frac{V_s}{R_2 + R_3} \tag{2.2.6-3}$$

$$V_{AD} = I_1 Z_u \;,\; V_{AB} = I_2 R_3 \tag{2.2.6-4}$$

$$V_o = V_{BD} = V_{AD} - V_{AB} = V_s\left(\frac{Z_u}{Z_1 + Z_u} - \frac{R_3}{R_2 + R_3}\right) \tag{2.2.6-5}$$

For capacitances:

$$V_o = V_s\left(\frac{1/C_u}{1/C_1 + 1/C_u} - \frac{R_3}{R_2 + R_3}\right) = V_s\left(\frac{C_1}{C_1 + C_u} - \frac{R_3}{R_2 + R_3}\right) \tag{2.2.6-6}$$

For inductances:

$$V_o = V_s\left(\frac{L_u}{L_1 + L_u} - \frac{R_3}{R_2 + R_3}\right) \tag{2.2.6-7}$$

- This relationship for inductance is only approximation since inductive impedances are never pure inductances as assumed but always contain a finite resistance (Z_u = jωL_u + R).

2.3 Amplifiers

- An amplifier is used to increase low-level signals to a higher level.

$$E_o = GE_i \tag{2.3-1}$$

E_i : the voltage input to the amplifier, E_o : the voltage output, $E_o/E_i = G$: the gain of the amplifier.

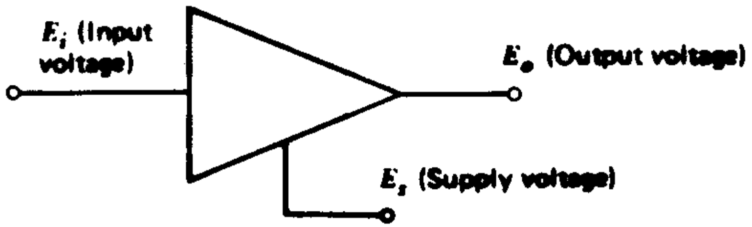


Figure 2.3-1 Symbol for an Amplifier

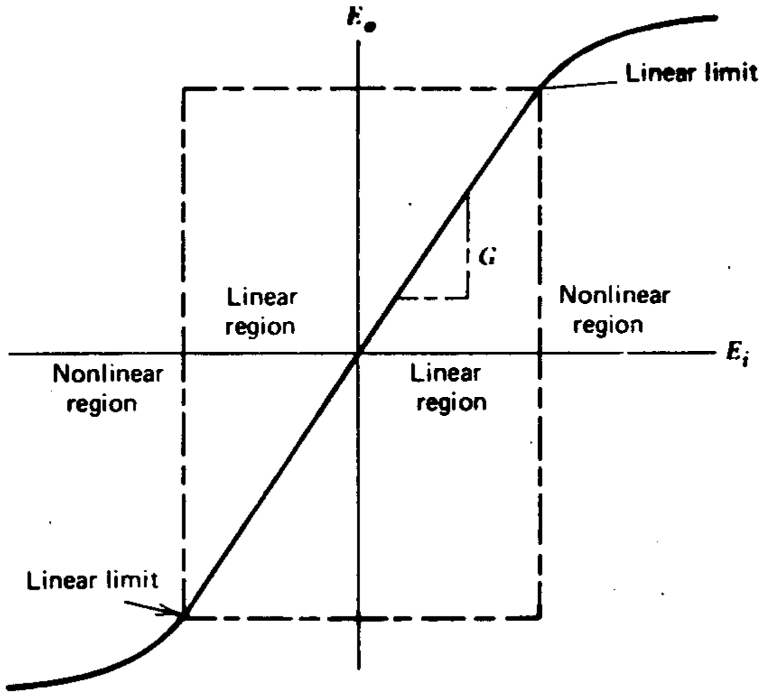


Figure 2.3-2 A Typical Voltage-Input/Voltage-Output Curve for an Amplifier

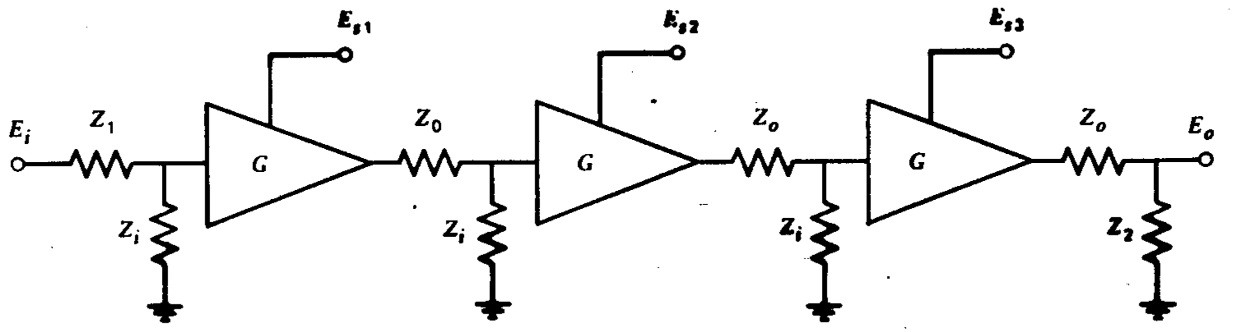


Figure 2.3-3 A High-Gain Amplifier System Consisting of 3 Amplifiers Connected in Series

- If the gain from a single amplifier is not sufficient, two or more amplifiers can be series connected (cascaded).
- Three amplifiers system has an output voltage E_o

$$E_o = G^3 \left(\frac{Z_i}{Z_i + Z_1} \right) \left(\frac{Z_i}{Z_i + Z_o} \right)^2 \left(\frac{Z_2}{Z_o + Z_2} \right) E_i \qquad (2.3\text{-}2)$$

where

Z_i : the amplifier input impedance.

Z_o : the amplifier output impedance.

Z_1 : the internal impedance of the source.

Z_2 : the input impedance of the voltage recorder.

- In properly designed voltage amplifiers $Z_i >> Z_1$ and $Z_i >> Z_o$;

$$E_o = G^3\left(\frac{Z_2}{Z_o + Z_2}\right)E_i \tag{2.3-3}$$

- The term $Z_2/(Z_o + Z_2)$ represents the voltage attenuation due to the current required to drive the voltage recorder.
- By maintaining $Z_2 >> Z_o$ (using a recorder with a high input impedance), this attenuation term approaches unity.

$$E_o = G^3 E_i \tag{2.3-4}$$

- The output voltage of an amplifier for a step input can be approximated by an exponential function.

$$E_o = G(1 - e^{-t/\tau})E_i \tag{2.3-5}$$

τ : the time constant for the amplifier.

- The frequency response of an amplifier is flat between the lower and upper frequency limits f_l and f_u.
- An Amplifier can be designed with coupling circuit that maintains a constant gain down to zero frequency. These amplifiers are known as dc or dc-coupled amplifiers.
- If a capacitor is placed in series with the input to the amplifier to block the dc components of the input signal, this ac-coupled amplifier exhibits a gain G = 0 when the frequency of the input signal drops to zero.

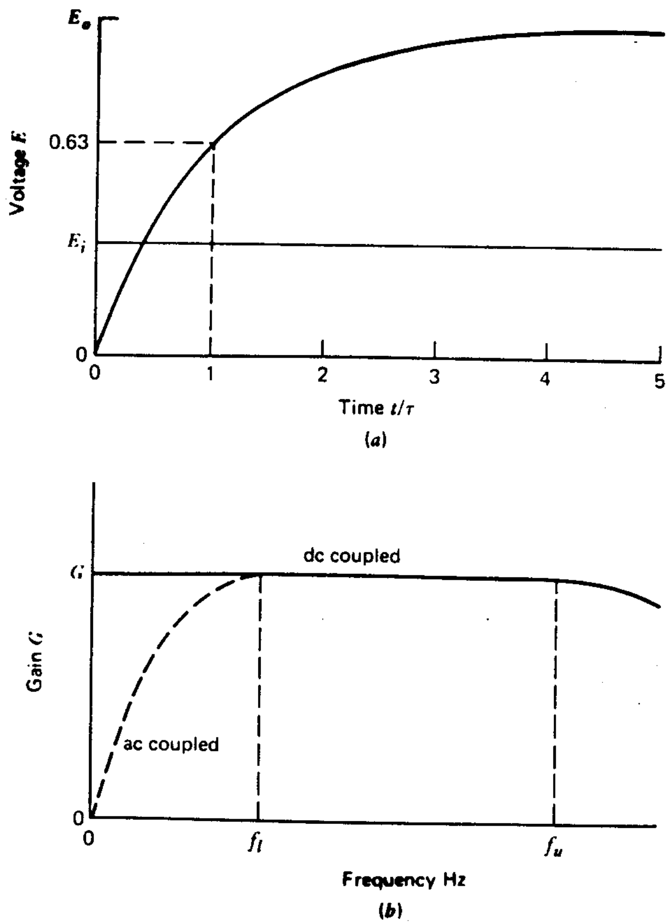


Figure 2.3-4 Frequency Response of an Amplifier-Recorder System (a) Amplifier Response to a Step-Input Voltage (b) Gain as a Function of Frequency of the Input Voltage

- The amplifiers can be classified as single-ended amplifiers if both the input and output voltages are referenced to ground.
- Single-ended amplifiers can be used only when the output from the signal conditioning circuit is referenced to ground, as is the case for the potentiometer circuit.
- Since the output from a Wheatstone bridge is not referenced to ground, differential amplifiers must be employed.
- In a differential amplifier, two separate voltages, each referenced to ground, are connected to the input. The output is single-ended and referenced to ground.
- The output voltage from the differential amplifier,

$$E_o = G(E_{i1} - E_{i2}) \tag{2.3-6}$$

- The differential amplifier rejects common-mode signals (those voltages that are identical on both inputs).
- Common-mode signals include spurious pickup (noise), temperature-induced drift, and power supply ripple.

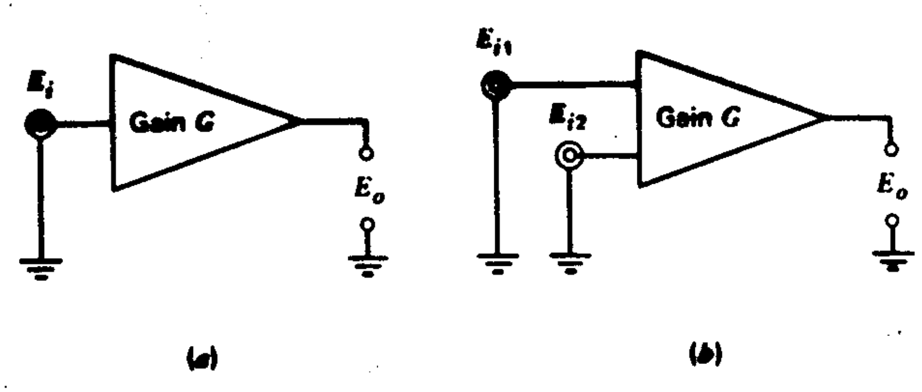


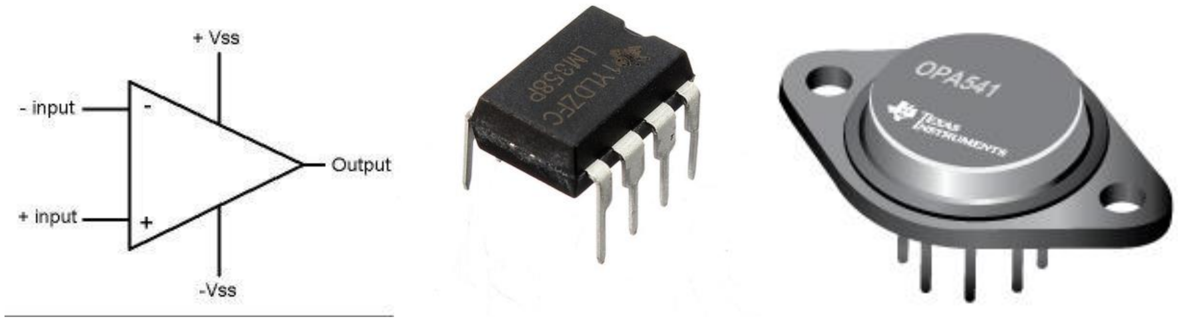
Figure 2.3-5 Single-Ended and Differential Amplifiers (a) Single-Ended Input and Output (b) Double-Ended Input and Single-Ended Output.

2.4 Operational Amplifiers

- An operational amplifier (op-amp) is a complete amplifier circuit (an integrated circuit where components such as transistors, diodes, resistors, etc. have been miniaturized into a single element) that can be employed in a number of different ways by adding a small number of external passive components such as resistors or capacitors.
- Operational amplifiers have an extremely high gain ($G = 10^5$ is a typical value), which can be considered infinite for the purpose of analysis and design of circuits containing the op-amp.
- The input impedance (typically R = 4 MΩ and C = 8 pF) is so high that circuit loading usually is not a consideration.
- Output resistance (of the order of 100 Ω) is sufficiently low to be considered negligible in most applications.
- The two input terminals of op-amp are identified as the inverting (-) terminal and the noninverting (+) terminal.
- The output voltage E_o of an op-amp,

$$E_o = G(E_{i2} - E_{i1}) \tag{2.4-1}$$

- Op-amp is a differential amplifier; however, it is not used as a conventional differential amplifier because of its high gain and poor stability.
- The op-amp can be used effectively, however as a part of a larger circuit (with more accurate and more stable passive elements) for many applications, including inverting amplifiers, voltage followers, summing amplifiers, integrating amplifiers, and differentiating amplifiers.



(a) (b) (c)

Figure 2.4-1 (a) Operational Amplifier Symbol (b) LM358 Signal Operational Amplifier (c) OPA541 Power Operational Amplifier

2.4.1 Inverting Amplifier

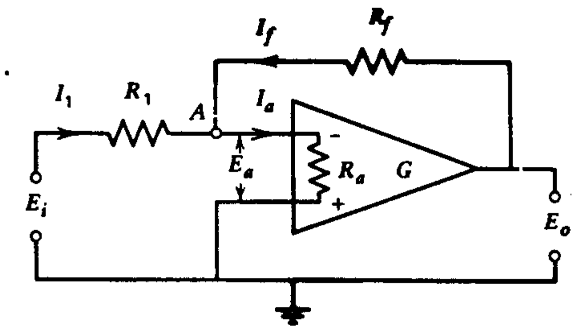


Figure 2.4.1-1 An Inverting Amplifier with Single-Ended Input and Output

$$I_1 + I_f = I_a \quad (2.4.1\text{-}1)$$

$$I_1 = \frac{E_i - E_a}{R_1} \quad (2.4.1\text{-}2)$$

$$I_f = \frac{E_o - E_a}{R_f} \quad (2.4.1\text{-}3)$$

$$I_a = \frac{E_a}{R_a} \quad (2.4.1\text{-}4)$$

where E_a : the voltage drop across the op-amp

$$E_a = -\frac{E_o}{G} \tag{2.4.1-5}$$

$$\frac{E_o}{E_i} = -\frac{R_f}{R_1}\left[\frac{1}{1+\frac{1}{G}\left(1+\frac{R_f}{R_1}+\frac{R_f}{R_a}\right)}\right] \tag{2.4.1-6}$$

- Because the op-amp gain G is very high, the gain of the inverting circuit G_c can be accurately approximated by

$$G_c = \frac{E_o}{E_i} \approx -\frac{R_f}{R_1} \tag{2.4.1-7}$$

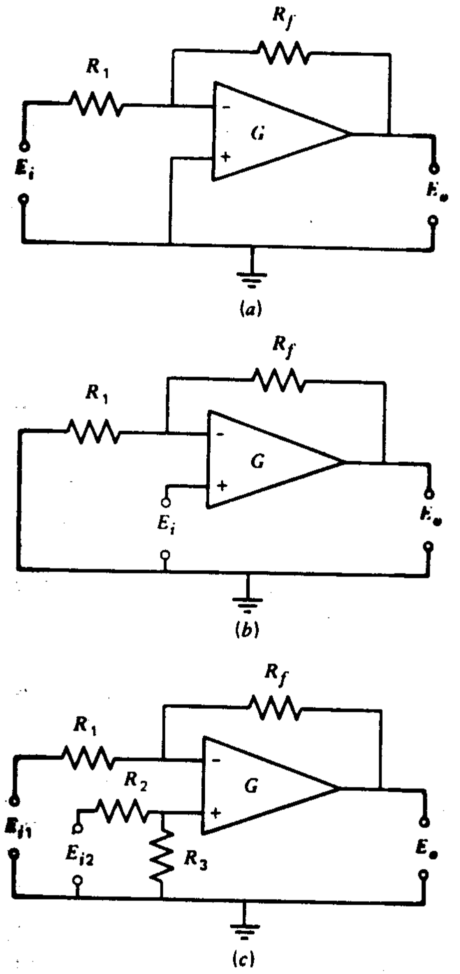


Figure 2.4.1-2 (a) Inverting Amplifier (b) Noninverting Amplifier (c) Differential Amplifier

Inverting Amplifier:

$$G_c = \frac{E_o}{E_i} = -\frac{R_f}{R_1}\left[\frac{1}{1+\frac{1}{G}\left(1+\frac{R_f}{R_1}+\frac{R_f}{R_a}\right)}\right] \approx -\frac{R_f}{R_1} \tag{2.4.1-8}$$

Noninverting Amplifier:

$$G_c = \frac{E_o}{E_i} = \frac{G}{1+\frac{GR_1}{R_1+R_f}} \approx 1+\frac{R_f}{R_1} \tag{2.4.1-9}$$

Differential Amplifier:

$$E_o \approx \frac{R_3}{R_2}\left(\frac{1+\frac{R_f}{R_1}}{1+\frac{R_3}{R_2}}\right)E_{i2} - \left(\frac{R_f}{R_1}\right)E_{i1} \tag{2.4.1-10}$$

If $R_f/R_1 = R_3/R_2$:

$$G_c \approx \frac{E_o}{E_{i2}-E_{i1}} \approx \frac{R_f}{R_1} \tag{2.4.1-11}$$

- A biasing circuit is used to calibrate an op-amp so that the output voltage is zero when the two terminals have the same input voltages.
- For an inverting amplifier with single-ended input and output, $R_1 = R_3$, $R_2 = 10\ \Omega$ and $R_4 = 25\ \text{k}\Omega$. A voltage $E_1 = \pm 15$ V is often used, since the zero-offset voltage of the op-amp can be either positive or negative.

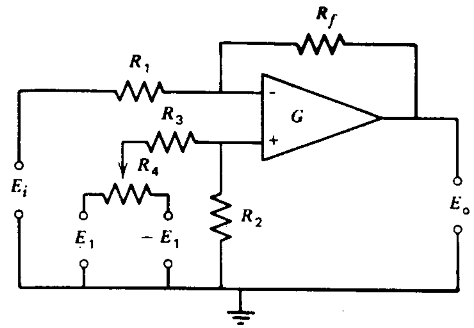


Figure 2.4.2-1 Biasing Circuit for a Single-Ended Input and Output Amplifier

- The frequency response of instrument amplifiers constructed with op-amps depends upon the frequency response of the op-amp and the feedback fraction. Since the gain of an op-amp depends on frequency (decreases with increasing frequency), the gain of the circuit also decreases with increasing frequency.
- The frequency responses of op-amps vary appreciably (depending upon design characteristics); however, a frequency response of 10 kHz or above is common. This is sufficient frequency response for most mechanical measurements.

2.4.2 Voltage Follower

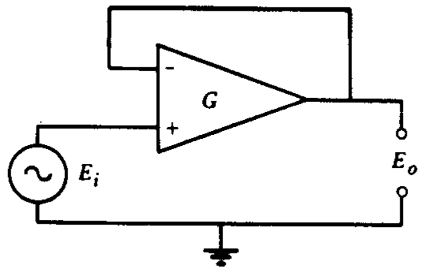


Figure 2.4.2-1 A High-Impedance Voltage Follower Circuit

- The purpose of the voltage follower is to serve as an insulator between the transducer and the voltage recording instrument.
- The voltage follower is also known as a unity-gain buffer amplifier.

$$E_o = G(E_{i2} - E_{i1}) = G(E_i - E_o) \quad (2.4.2\text{-}1)$$

$$G_c = \frac{E_o}{E_i} = \frac{G}{1+G} \quad (2.4.2\text{-}2)$$

- The input resistance of the voltage follower circuit is given by Ohm's law.

$$R_{ci} = \frac{E_i}{I_a} \quad (2.4.2\text{-}3)$$

- The input current I_a can be expressed in terms of the input and output voltages of the op-amp and the input resistance of the op-amp.

$$I_a = \frac{E_i - E_o}{R_i} \tag{2.4.2-4}$$

$$R_{ci} = \frac{E_i R_i}{E_i - E_o} = \frac{E_i R_i}{E_i - \frac{GE_i}{1+G}} = (1+G)R_i \tag{2.4.2-5}$$

- Since both G and R_i are very large for op-amps (i.e., $G = 10^5$ and $R_i = 1$ to 10 MΩ), the input impedance of the voltage follower circuit is of the order of 10^{11} to 10^{12} Ω.
- This input impedance is sufficient to minimize any drain of charge from a piezoelectric transducer during a readout period of short duration.
- The output resistance R_{co} of the voltage follower circuit is extremely low.

$$R_{co} = \frac{R_o}{1+G} \tag{2.4.2-6}$$

R_o : the output resistance of the op-amp.

2.4.3 Summing Amplifiers

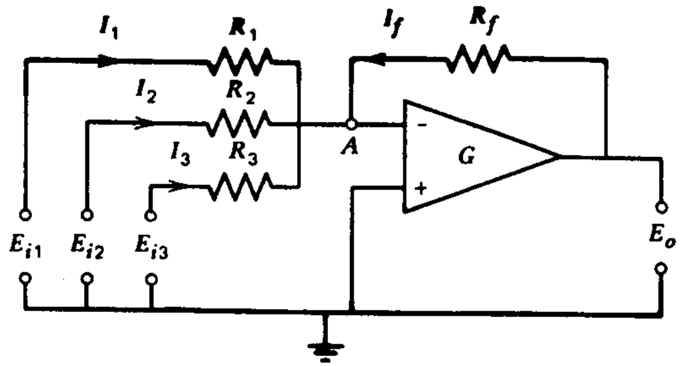


Figure 2.4.3-1 A Summing Amplifier Incorporating an Op-Amp Circuit

$$I_1 + I_2 + I_3 + I_f = 0 \tag{2.4.3-1}$$

$$\frac{E_{i1}}{R_1} + \frac{E_{i2}}{R_2} + \frac{E_{i3}}{R_3} + \frac{E_o}{R_f} = 0 \tag{2.4.3-2}$$

$$E_o = -R_f\left(\frac{E_{i1}}{R_1} + \frac{E_{i2}}{R_2} + \frac{E_{i3}}{R_3}\right) \tag{2.4.3-3}$$

If $R_1 = R_2 = R_3 = R_f$,

$$E_o = -\left(E_{i1} + E_{i2} + E_{i3}\right) \tag{2.4.3-4}$$

If the gain G and the input impedance R_i are finite,

$$E_o = -\frac{\dfrac{E_{i1}}{R_1} + \dfrac{E_{i2}}{R_2} + \dfrac{E_{i3}}{R_3}}{\dfrac{1}{R_f} + \dfrac{1}{G}\left(\dfrac{1}{R_1} + \dfrac{1}{R_2} + \dfrac{1}{R_3} + \dfrac{1}{R_f} + \dfrac{1}{R_i}\right)} \tag{2.4.3-5}$$

2.4.4 Integrating Amplifiers

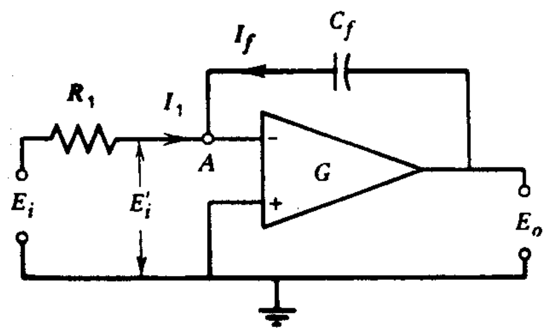


Figure 2.4.4-1 An Integrating Amplifier Incorporating an Op-Amp Circuit

$$I_1 + I_f = 0 \tag{2.4.4-1}$$

$$\frac{E_i - E_i'}{R_1} + I_f = 0 \tag{2.4.4-2}$$

$$E_i' = -\frac{E_o}{G} \tag{2.4.4-3}$$

- The voltage $E_i' \to 0$ when the gain G is large.

$$\frac{E_{ii}}{R_1} + I_f = 0 \tag{2.4.4-4}$$

The charge q on the capacitor,

$$q = \int_0^t I_f \, dt = C_f E_o \qquad (2.4.4\text{-}5)$$

$$E_o = -\frac{1}{R_1 C_f} \int_0^t E_i \, dt \qquad (2.4.4\text{-}6)$$

2.4.5 Differentiating Amplifier

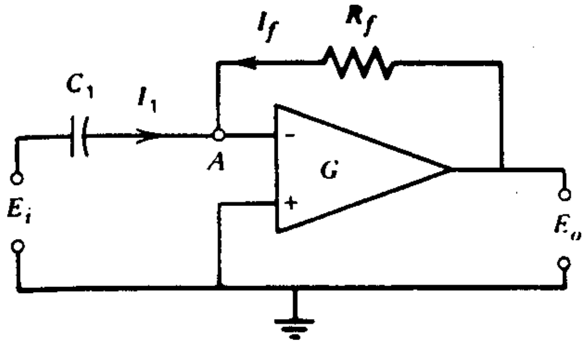


Figure 2.4.5-1 A Differentiating Amplifier Incorporating an Op-Amp Circuit

$$I_1 + I_f = 0 \quad (2.4.5\text{-}1)$$

$$I_1 + \frac{E_o - E_i'}{R_f} = 0 \quad (2.4.5\text{-}2)$$

$$E_i' = -\frac{E_o}{G} \quad (2.4.5\text{-}3)$$

- The voltage $E_i' \to 0$ when the gain G is large.

$$I_1 + \frac{E_o}{R_f} = 0 \quad (2.4.5\text{-}4)$$

The charge q on the capacitor,

$$q = \int_0^t I_1 dt = C_1 E_i \tag{2.4.5-5}$$

$$E_o = -R_f C_1 \frac{dE_i}{dt} \tag{2.4.5-6}$$

- Considerable care must be exercised to minimize noise when the differentiating amplifier is used, since noise is simply superimposed on the input voltage and differentiated in such a way that it contributes to the output voltage and produces error.
- The effects of high-frequency noise can be suppressed by placing a capacitor across resistance R_f; however, the presence of this capacitor affects the differentiating process.

2.5 Filters

- In many instrumentation applications, the signal from the transducer is combined with noise or some other undesirable voltage.
- These parasitic voltages can often be eliminated with a filter that is designed to attenuate the undesirable signals, but transmit the transducer signal without significant attenuation or distortion.
- Filtering of the signal is possible if the frequencies of the parasitic and transducer signals are different.
- Filters are classified into passive and active filters.

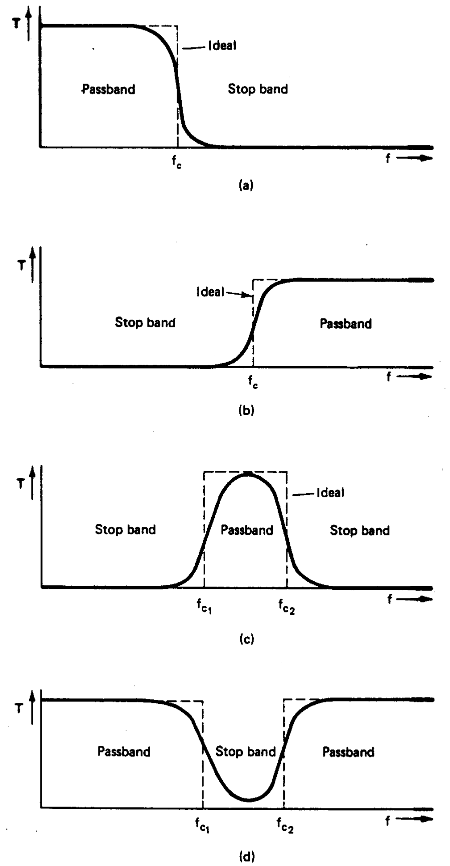


Figure 2.5-1 Frequency Selective Filter Characteristics: (a) Low-Pass Filter (b) High-Pass Filter (c) Band-Pass Filter (d) Band-Reject Filter

2.5.1 Low-Pass RL Filter

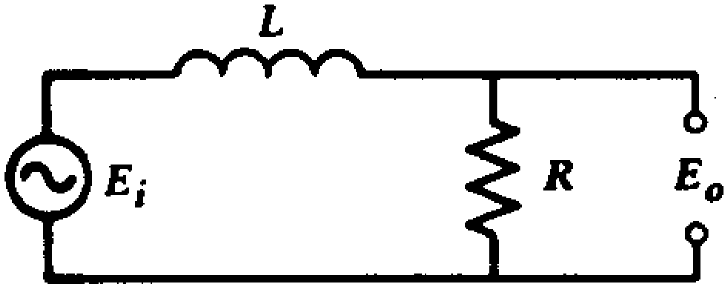


Figure 2.5.1-1 Low-Pass RL Filter

$$L\frac{dI}{dt} + RI = E_i \quad (2.5.1\text{-}1)$$

$$I = I_0 \sin \omega t \quad (2.5.1\text{-}2)$$

$$E_i = I_0(R\sin \omega t + \omega L\cos \omega t) = I_0\sqrt{R^2 + (\omega L)^2}\left(\frac{R}{\sqrt{R^2 + (\omega L)^2}}\sin \omega t + \frac{\omega L}{\sqrt{R^2 + (\omega L)^2}}\cos \omega t\right) \quad (2.5.1\text{-}3)$$

$$E_i = I_0\sqrt{R^2 + (\omega L)^2}\sin(\omega t + \phi) \quad (2.5.1\text{-}4)$$

$$\phi = \tan^{-1}\frac{\omega L}{R} \quad (2.5.1\text{-}5)$$

$$E_o = IR = I_0 R\sin \omega t \quad (2.5.1\text{-}6)$$

$$\frac{E_o}{E_i} = \frac{1}{\sqrt{1+\left(\frac{\omega L}{R}\right)^2}} \tag{2.5.1-7}$$

- As $\omega \to 0$, $E_o/E_i \to 1$, $\phi \to 0°$.
- As $\omega \to \infty$, $E_o/E_i \to 0$, $\phi \to 90°$.
- The transfer function decreases from 0.995 for $\omega L/R = 0.10$, to 0.01 for $\omega L/\mathrm{R} = 100$.
- A 2 percent attenuation of the transducer signal occurs when $\omega_t L/R = 0.203$. To avoid errors greater than 2 percent, L and R must be selected when designing the filter such that $\omega_t L/R \leq 0.203$.
- A reduction of 90 percent in the noise signal can be achieved if $\omega_p L/R = 10$.
- It is not always possible to simultaneously limit the attenuation of the transducer signal to 2 percent while reducing the parasitic voltages by 90 percent, since this requires that $\omega_p/\omega_t \geq 50$. If $\omega_p/\omega_t < 50$, it will be necessary to accept a higher ratio of parasitic signal.

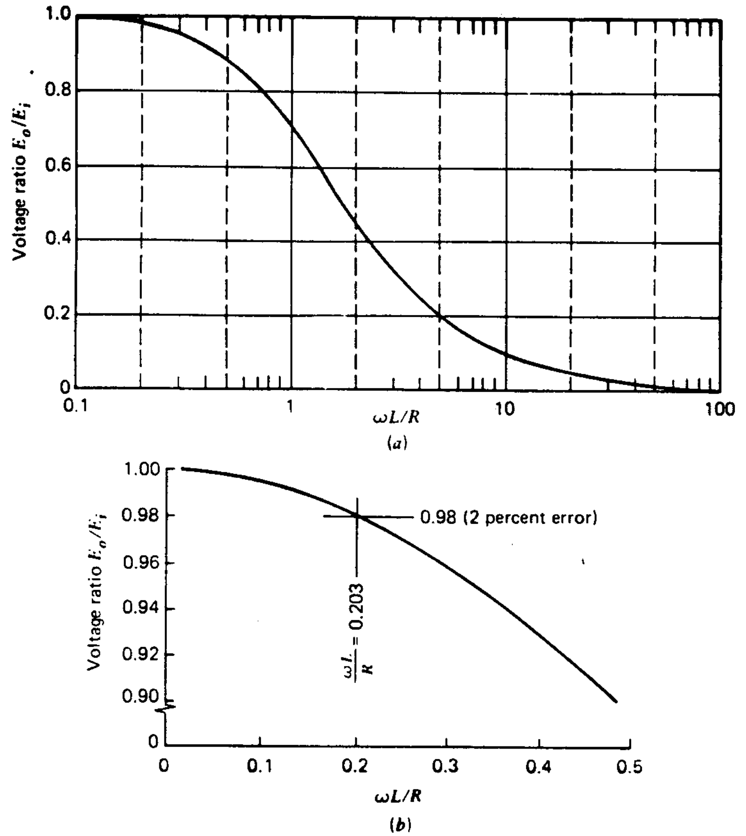


Figure 2.5.1-1 Response Curve for a Low-Pass RL Filter (a) Complete Response Curve

(b) Low-Frequency End of Response Curve

2.5.2 High-Pass RC Filter

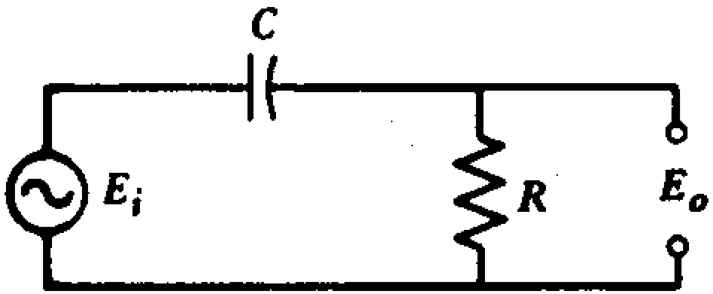


Figure 2.5.2-1 High-Pass RC Filter

$$\frac{\int Idt}{C} + RI = E_i \tag{2.5.2-1}$$

$$I = I_0 \sin \omega t \tag{2.5.2-2}$$

$$E_i = \frac{I_0}{\omega C}(\omega CR \sin \omega t - \cos \omega t) = \frac{I_0}{\omega C}\sqrt{(\omega CR)^2 + 1}\left(\frac{\omega CR}{\sqrt{(\omega CR)^2 + 1}}\sin \omega t - \frac{1}{\sqrt{(\omega CR)^2 + 1}}\cos \omega t\right) \tag{2.5.2-3}$$

$$E_i = \frac{I_0}{\omega C}\sqrt{(\omega CR)^2 + 1}\sin(\omega t - \phi) \tag{2.5.2-4}$$

$$\phi = \tan^{-1}\frac{1}{\omega CR} \tag{2.5.2-5}$$

$$E_o = IR = I_0 R \sin \omega t \tag{2.5.2-6}$$

$$\frac{E_o}{E_i} = \frac{1}{\sqrt{1+\left(\frac{1}{\omega CR}\right)^2}} \qquad (2.5.2\text{-}7)$$

- As $\omega \to 0$, $E_o/E_i \to 0$, $\phi \to 90°$.
- As $\omega \to \infty$, $E_o/E_i \to 1$, $\phi \to 0°$.

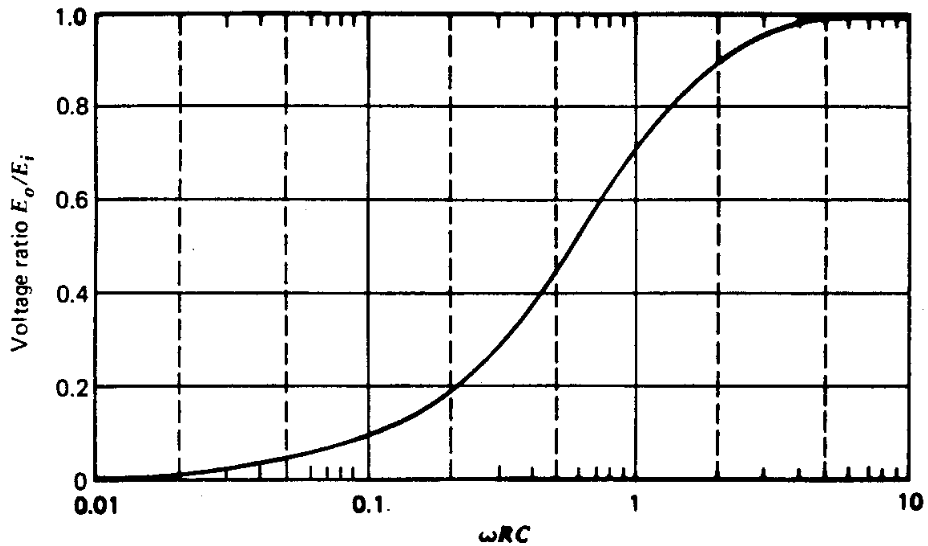


Figure 2.5.2-1 Response Curve for a High-Pass RC Filter

2.5.3 Low-Pass RC Filter

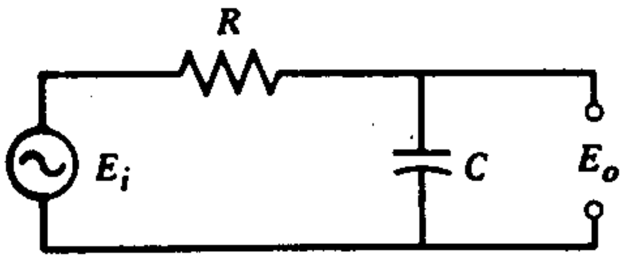


Figure 2.5.3-1 Low-Pass RC Filter

$$\frac{\int I dt}{C} + RI = E_i \tag{2.5.3-1}$$

$$I = I_0 \sin \omega t \tag{2.5.3-2}$$

$$E_i = \frac{I_0}{\omega C}(\omega CR \sin \omega t - \cos \omega t) = \frac{I_0}{\omega C}\sqrt{(\omega CR)^2 + 1}\left(\frac{\omega CR}{\sqrt{(\omega CR)^2 + 1}} \sin \omega t - \frac{1}{\sqrt{(\omega CR)^2 + 1}} \cos \omega t\right) \tag{2.5.3-3}$$

$$E_i = \frac{I_0}{\omega C}\sqrt{(\omega CR)^2 + 1} \sin(\omega t - \phi) \tag{2.5.3-4}$$

$$\phi = \tan^{-1}\frac{1}{\omega CR} \tag{2.5.3-5}$$

$$E_o = -\frac{I_0}{\omega C}\cos \omega t = \frac{I_0}{\omega C}\sin(\omega t - 90°) \tag{2.5.3-6}$$

$$\frac{E_o}{E_i} = \frac{1}{\sqrt{1+(\omega CR)^2}} \tag{2.5.3-7}$$

- As $\omega \to 0$, $E_o/E_i \to 1$, $\phi \to 90°$, output has the same phase as input.
- As $\omega \to \infty$, $E_o/E_i \to 0$, $\phi \to 0°$, output lags input for 90°.
- A 2 percent attenuation of the transducer signal occurs when $\omega RC = 0.203$. To avoid errors greater than 2 percent, C and R must be selected when designing the filter such that $\omega RC \leq 0.203$.
- A reduction of 90 percent in the noise signal can be achieved if $\omega RC = 10$.

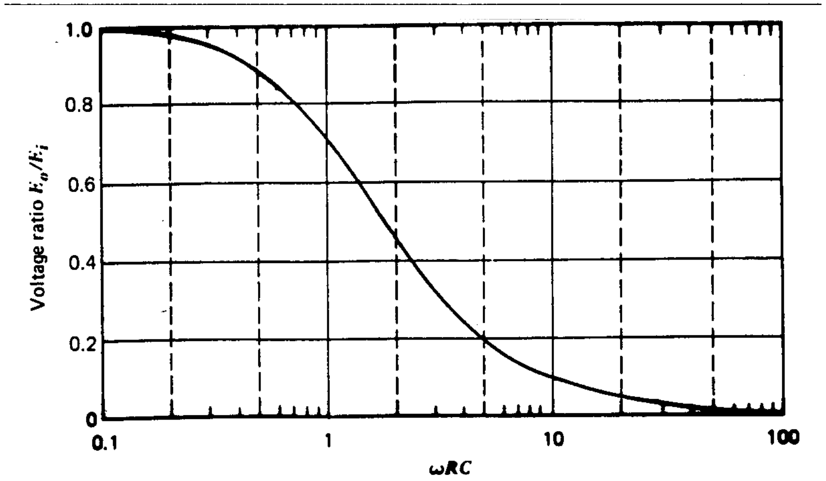


Figure 2.5.3-1 Response Curve for a Low-Pass RC Filter

2.5.4 Wein-Bridge Notched Filter

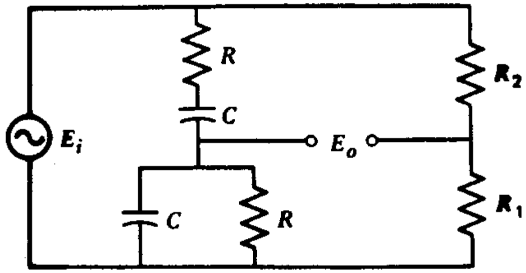


Figure 2.5.4-1 Wein-Bridge Notched Filter

- The Wein-bridge filter, notched filter, produces high attenuation at a selected filter frequency ω_f.

$$\frac{E_o}{E_i} = \left[\frac{1}{3 + i\left(\frac{\omega}{\omega_f} - \frac{\omega_f}{\omega}\right)} - \frac{1}{1 + \frac{R_2}{R_1}} \right] \quad (2.5.4\text{-}1)$$

$$\omega_f = \frac{1}{RC} \quad (2.5.4\text{-}2)$$

- If the Wein-bridge filter is tuned to the frequency of a parasitic signal ($\omega_f = \omega_p$), the reactive term vanishes.

$$\frac{E_o}{E_i} = \left[\frac{1}{3} - \frac{R_1}{R_1 + R_2} \right] \quad (2.5.4\text{-}3)$$

- For the special case where $R_2 = 2R_1$, the voltage ratio $E_o/E_i = 0$, which indicates that the noise or parasitic signal is completely eliminated at the critical frequency of the filter.
- This high attenuation at a selected filter frequency can be used very effectively to eliminate 50-Hz noise.
- The Wein-bridge filter will always attenuate the transducer signal; therefore, a correction must be introduced to compensate for the attenuation.
- The Wein-bridge filter should be used only when the transducer signal is a pure sinusoid with a known frequency ω.

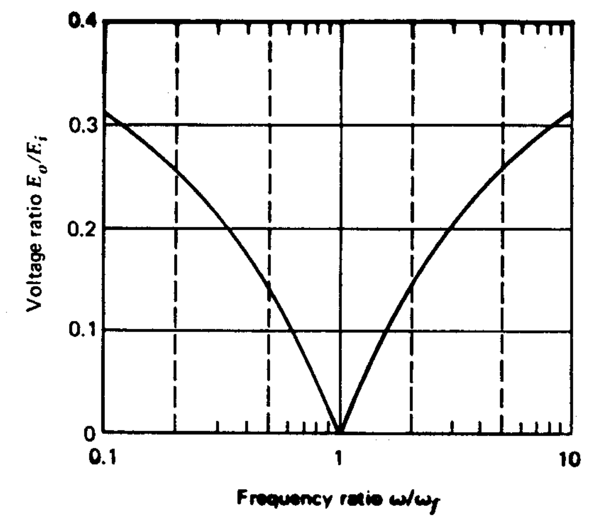


Figure 2.5.4-1 Response Curve for a Wein-Bridge Filter

2.5.5 LC Filters

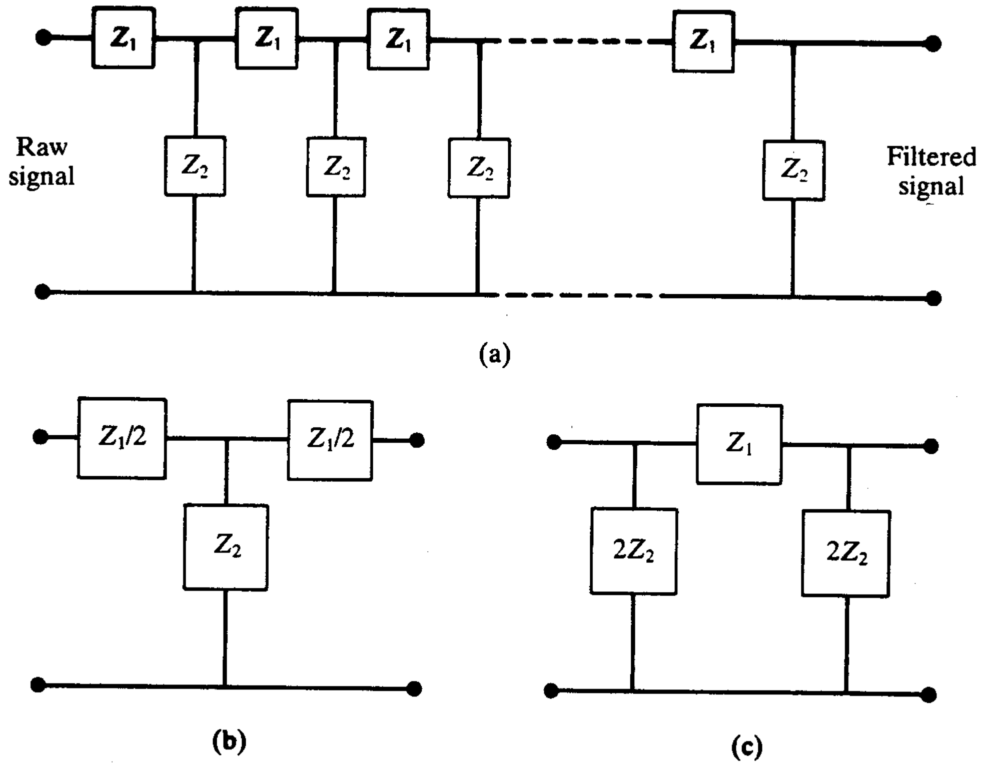


Figure 2.5.5-1 (a) LC-Filter Network (b) T-Section (c) π-Section

- Each element of the LC-filter network can be represented by either a T-section or π-section.
- To obtain proper matching between filter sections, it is necessary for the input impedance of each section to be equal to the load impedance for that section. This value of impedance is known as the characteristic impedance (Z_0).

For a T-section of filter, the characteristic impedance,

$$Z_0 = \sqrt{Z_1 Z_2 \left(1 + \frac{Z_1}{4Z_2}\right)} \tag{2.5.5-1}$$

- Frequency values for which Z_0 is real lie in the pass band and frequencies for which Z_0 is imaginary lie in the stop band.

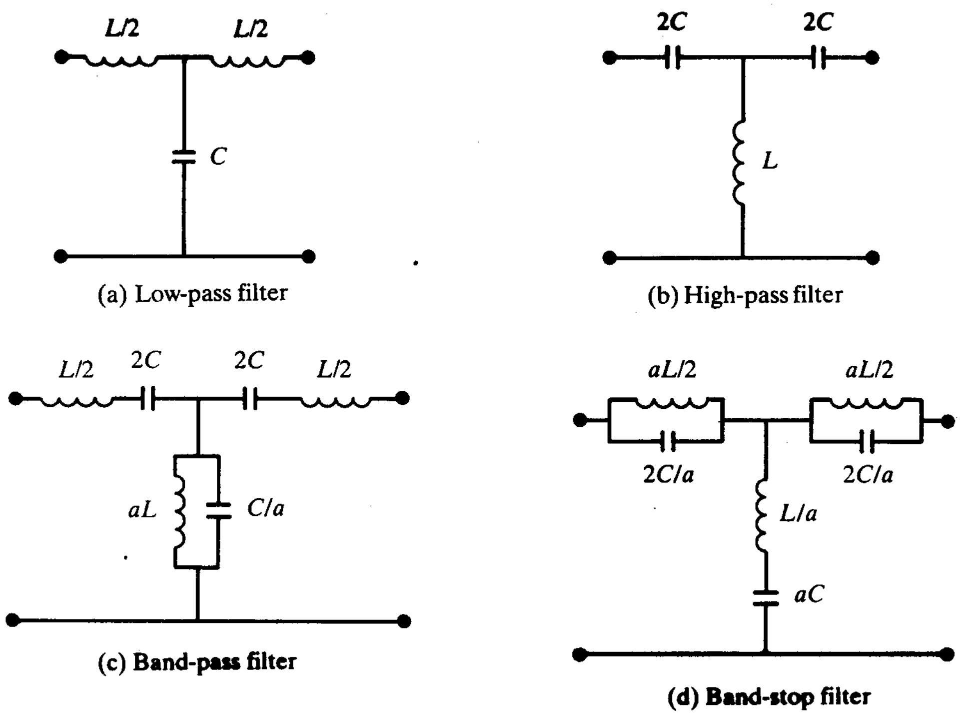


Figure 2.5.5-2 Circuit Components for LC Filters (T-Sections)

- For low-pass filter where $Z_1 = j\omega L$ and $Z_2 = 1/j\omega C$,

$$Z_0 = \sqrt{\frac{L}{C}\left(1 - \frac{\omega^2 LC}{4}\right)} \tag{2.5.5-2}$$

- For frequencies where $\omega < (4/LC)^{1/2}$, Z_0 is real, and for higher frequencies, Z_0 is imaginary.

3 db cut-off frequency, voltage ration of $1/\sqrt{2} = 0.707$,

$$f_c = \frac{\omega_c}{2\pi} = \frac{1}{\pi\sqrt{LC}} \tag{2.5.5-3}$$

- A high-pass filter can be synthesized with exactly the same cut-off frequency if the impedance values chosen are:

$$Z_1 = 1/j\omega C \text{ and } Z_2 = j\omega L \tag{2.5.5-4}$$

- The product Z_1Z_2 could be represented by a constant k which is independent of frequency. Because of this, such filters are known by the name of constant-k filters.
- A constant-k band-pass filter can be realized with the following choice of impedance values:

$$Z_1 = j\omega L + \frac{1}{j\omega C} \text{ and } Z_2 = \frac{(j\omega La)(a/j\omega C)}{j\omega La + a/j\omega C} \tag{2.5.5-5}$$

- The frequencies f_1 and f_2 defining the end of the pass band are most easily expressed in terms of a frequency f_0 in the center of the pass band.

$$f_0 = \frac{1}{2\pi\sqrt{LC}} \qquad f_1 = f_0\left\lfloor\sqrt{1+a} - \sqrt{a}\right\rfloor \qquad f_2 = f_0\left\lfloor\sqrt{1+a} + \sqrt{a}\right\rfloor \tag{2.5.5-6}$$

- For a constant-k band-stop filter, the appropriate impedance values are:

$$Z_1 = \frac{(j\omega La)(a/j\omega C)}{j\omega La + a/j\omega C} \text{ and } Z_2 = \frac{1}{a}\left(j\omega L + \frac{1}{j\omega C}\right) \tag{2.5.5-7}$$

- The frequencies defining the ends of the stop band are normally defined in terms of the frequency f_0 in the center of the stop band:

$$f_0 = \frac{1}{2\pi\sqrt{LC}} \qquad f_1 = f_0(1 - a/4) \qquad f_2 = f_0(1 + a/4) \qquad (2.5.5\text{-}8)$$

- A practical filter does not eliminate frequencies in the stop band but merely attenuates them by a certain amount. Improved attenuation characteristics can be obtained by putting several T-sections in cascade. If perfect matching is assumed then two T-sections give twice the attenuation of one section.

2.5.6 Active Filters

- Passive filters use only passive elements, resistors, capacitors, and inductors. An active filter uses one or more active devices, usually an operational amplifier, in the filter circuit.
- Some advantages of active filters over their passive counterparts are;

1. Gain: With active filters, transfer functions with maximum values greater than unity are possible.
2. Minimal loading effects: The transfer characteristics of an active filter can be substantially independent of the load driven by the filter or of the source that drives the filter.
3. Inductorless filters: With active filters, only resistive and capacitative elements are required; no inductors are needed. This can be an especially useful feature for operation at relatively low frequencies (<10 kHz) where otherwise large inductors would be required.

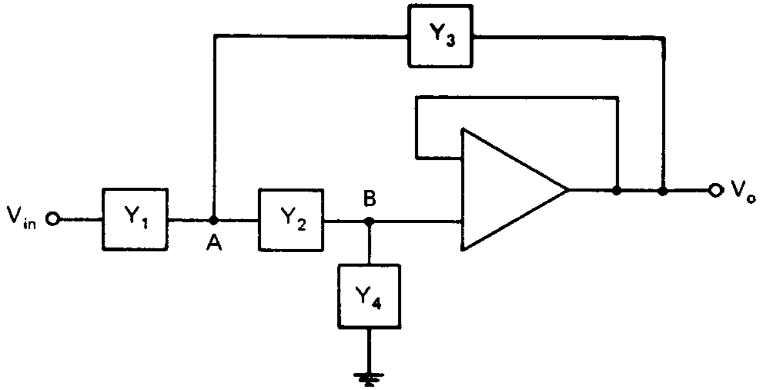


Figure 2.5.6-1 General Two-Pole Active Filter Circuit

At node *A*,

$$V_{in}Y_1 + V_oY_2 + V_oY_3 = V_A(Y_1 + Y_2 + Y_3) \tag{2.5.6-1}$$

At nobe *B*,

$$V_AY_2 = V_o(Y_2 + Y_4) \tag{2.5.6-2}$$

$$V_{in}Y_1 + V_o(Y_2 + Y_3) = \frac{V_o(Y_2 + Y_4)(Y_1 + Y_2 + Y_3)}{Y_2} \tag{2.5.6-3}$$

$$T = \frac{V_o}{V_{in}} = \frac{Y_1Y_2}{Y_1Y_2 + Y_1Y_4 + Y_2Y_4 + Y_3Y_4} = \frac{Y_1Y_2}{Y_1Y_2 + Y_4(Y_1 + Y_2 + Y_3)} \tag{2.5.6-4}$$

Two-Pole Low-Pass Active Filter

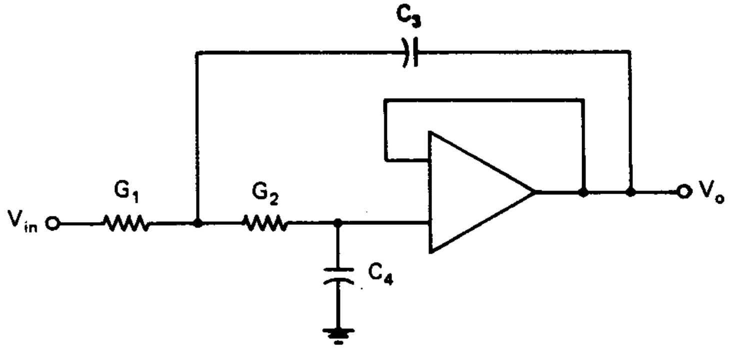


Figure 2.5.6-2 Two-Pole Active Low-Pass Filter

$Y_1 = G_1$, $Y_2 = G_2$, $Y_3 = sC_3$, and $Y_4 = sC_4$,

$$T = \frac{G_1 G_2}{G_1 G_2 + sC_4(G_1 + G_2 + sC_3)} \quad (2.5.6\text{-}5)$$

- As $\omega \to 0$, $T \to 1$.
- As $\omega \to \infty$, $T \to 0$.

If $G_1 = G_2 = G = 1/R$, $\tau_4 = RC_4$ and $\tau_3 = RC_3$

$$T = \frac{1}{1 + s\tau_4(2 + s\tau_3)} \quad (2.5.6\text{-}6)$$

$s = j\omega$,

$$T = \frac{1}{1 + j\omega\tau_4(2 + j\omega\tau_3)} = \frac{1}{1 - \omega^2\tau_3\tau_4 + j2\omega\tau_4} \tag{2.5.6-7}$$

$$|T|^2 = \frac{1}{1 + \omega^2(4\tau_4^2 - 2\tau_3\tau_4) + \omega^4\tau_3^2\tau_4^2} \tag{2.5.6-8}$$

- For a maximally flat filter frequency response that falls off monotonically with frequency, $d|T|^2/d\omega = 0$ at $\omega = 0$.

$$2\tau_4^2 - \tau_3\tau_4 + \omega^2\tau_3^2\tau_4^2 = 0 \tag{2.5.6-9}$$

For the zero slope to occur only at $\omega = 0$, $2\tau_4 = \tau_3$,

$$C_3 = 2C_4 \tag{2.5.6-10}$$

$$|T|^2 = \frac{1}{1 + 4(\omega\tau_4)^4} \tag{2.5.6-11}$$

The 3-dB point will occur when $|T|^2 = 1/2$, so $\omega\tau_4 = 1/\sqrt{2}$, and the 3-dB bandwidth or cutoff frequency is given by

$$\omega_{3db} = \frac{1}{\sqrt{2}\tau_4} = \frac{0.7071}{RC_4} \tag{2.5.6-12}$$

- The filter with a maximally flat transfer characteristic in the passband is called a Butterworth filter.

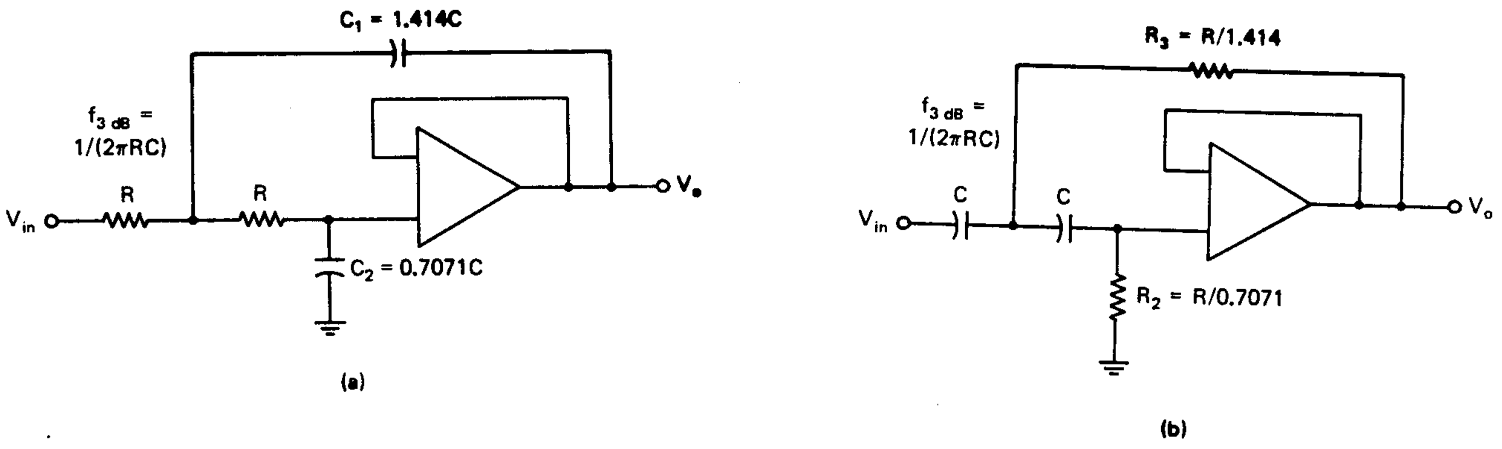


Figure 2.5.6-3 (a) Two-Pole Butterworth Low-Pass Active Filter, (b) Two-Pole Butterworth High-Pass Active Filter

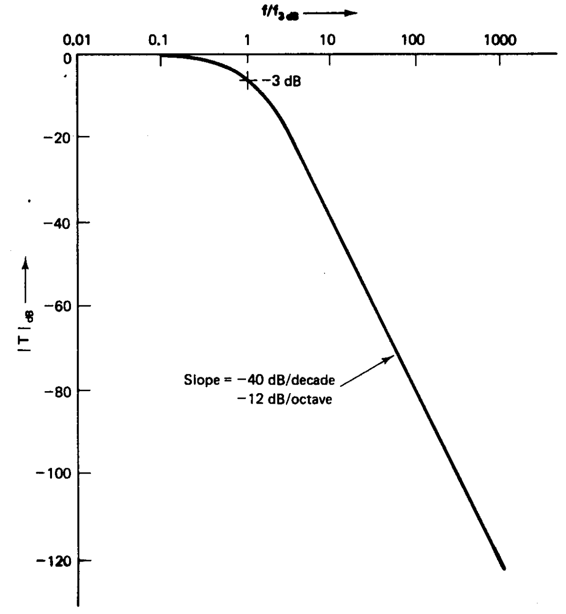


Figure 2.5.6-4 Frequency Response to Two-Pole Butterworth Filter

Higher-Order Butterworth Active Filters

- The filter order is the number of poles.
- An *N*-pole active low-pass filter will have a high-frequency asymptotic response that falls off at a rate of *N* x 20 dB/decade.
- An *N*-pole high-pass filter will have a low-frequency stop-band response that increases at a rate of *N* x 20 dB/decade.

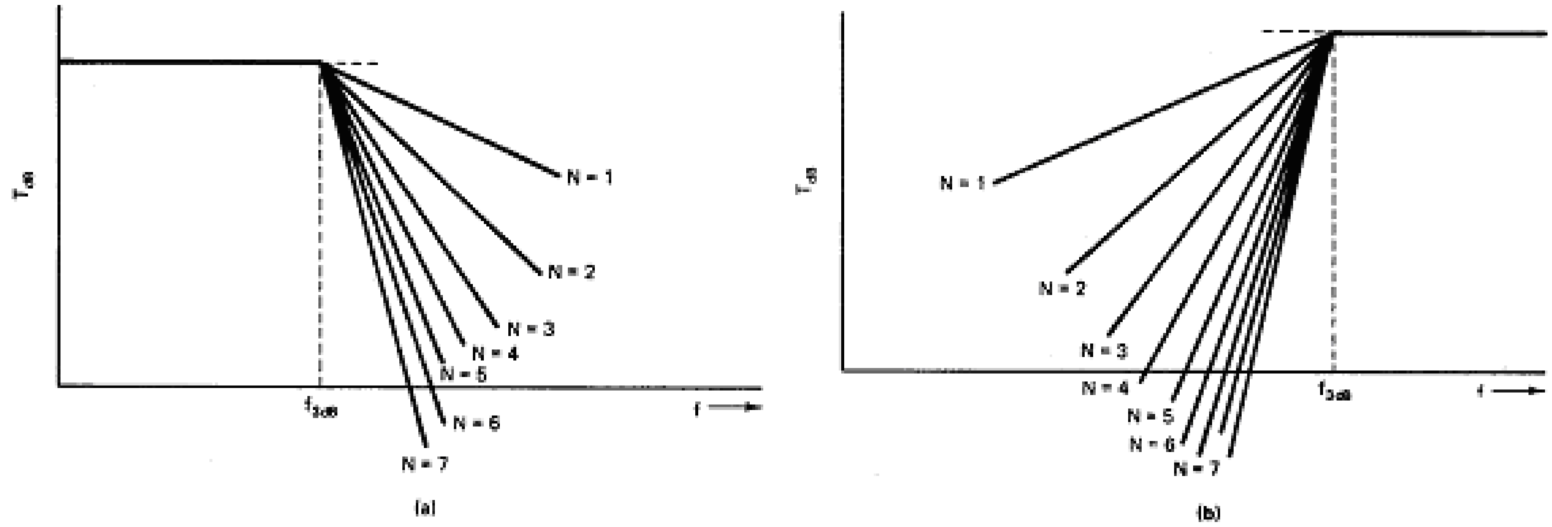


Figure 2.5.6-5 Frequency Response Characteristics of *N*-Pole Filters, (a) Low-Pass Filters (b) High-Pass Filters

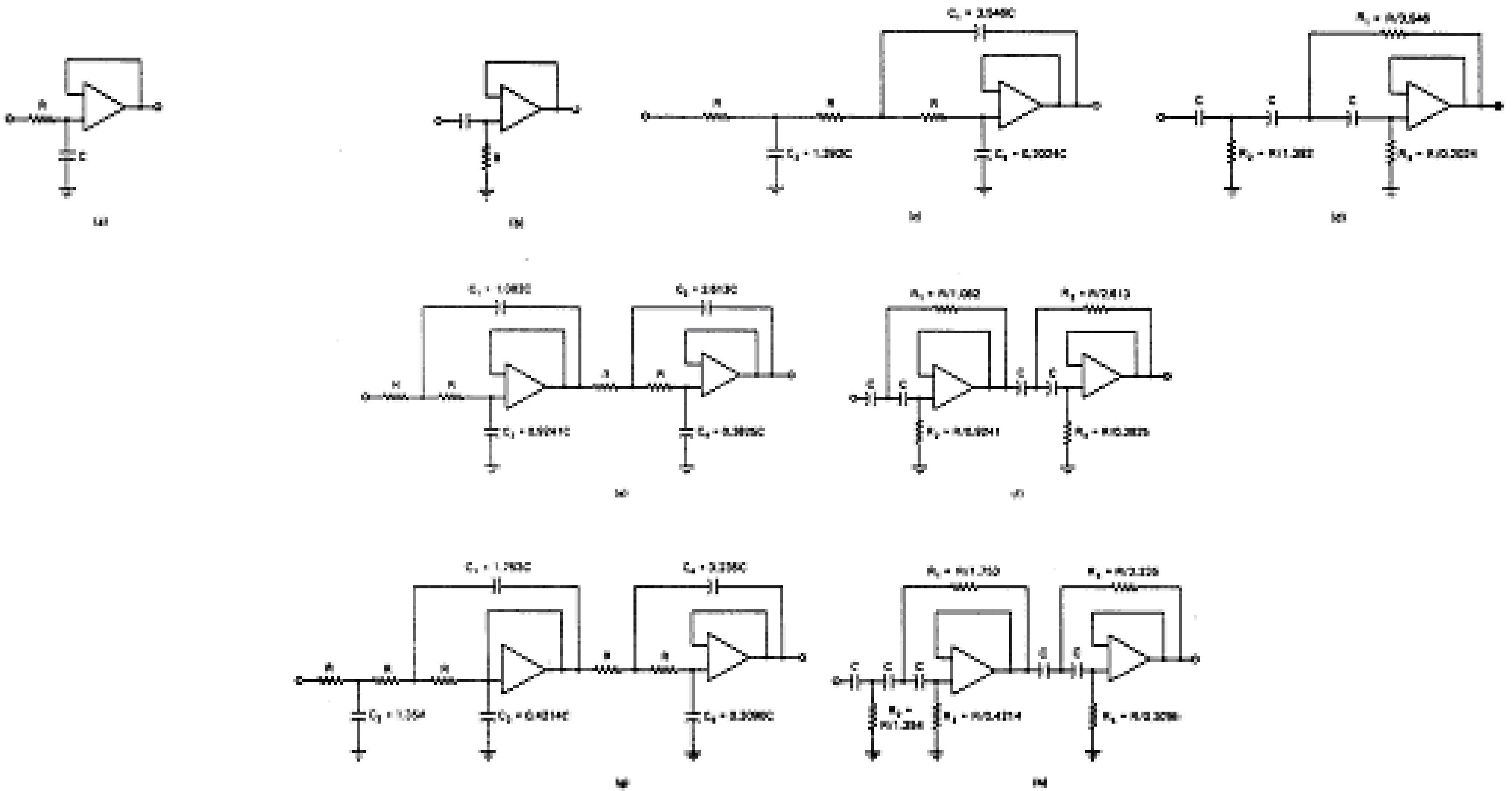


Figure 2.5.6-6 Active Butterwotth Filter Circuits (a) Single-Pole Low-Pass Filter (b) Single-Pole High-Pass Filter (c) Three-Pole Low-Pass Filter (d) Three-Pole High-Pass Filter (e) Four-Pole Low-Pass Filter (f) Four-Pole High-Pass Filter (g) Five-Pole Low-Pass Filter (h) Five-Pole High-Pass filter, for every case the 3-dB frequency $f = 1/(2\pi RC)$

Band-Pass Active Filters

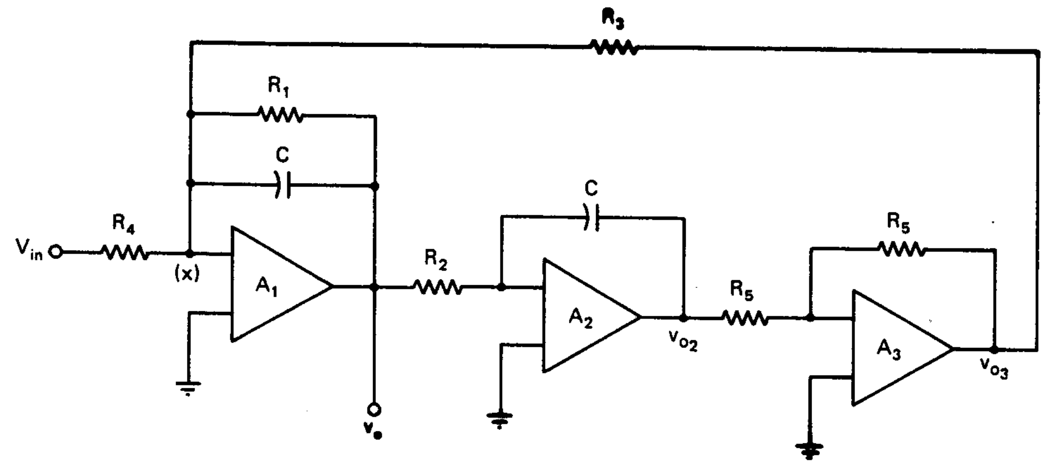


Figure 2.5.6-7 Band-Pass Filter

$$V_{o2} = -\frac{V_o(1/sC)}{R_2} = -\frac{V_o}{sCR_2} \tag{2.5.6-13}$$

$$V_{o3} = -V_{o2} = \frac{V_o}{sCR_2} \tag{2.5.6-14}$$

At node (x),

$$\frac{V_{in}}{R_4} + \frac{V_o}{R_1} + sCV_o + \frac{V_o}{sCR_2R_3} = 0 \tag{2.5.6-15}$$

$$T = \frac{V_o}{V_{in}} = \frac{-1/R_4}{1/R_1 + sC + 1/(sCR_2R_3)} \qquad (2.5.6\text{-}16)$$

$s = j\omega$,

$$T = \frac{-1/R_4}{1/R_1 + j\omega C + 1/(j\omega CR_2R_3)} \qquad (2.5.6\text{-}17)$$

The gain at resonance,

$$T_r = -\frac{R_1}{R_4} \qquad (2.5.6\text{-}18)$$

The resonant frequency,

$$\omega_r C = \frac{1}{\omega_r CR_2R_3} \text{ or } \omega_r = \frac{1}{C\sqrt{R_2R_3}} \qquad (2.5.6\text{-}19)$$

The 3-dB bandwidth,

$$\omega_{3dB} = \frac{1}{CR_1} \qquad (2.5.6\text{-}20)$$

Band-Pass Amplifier Using Just One Operational Amplifier

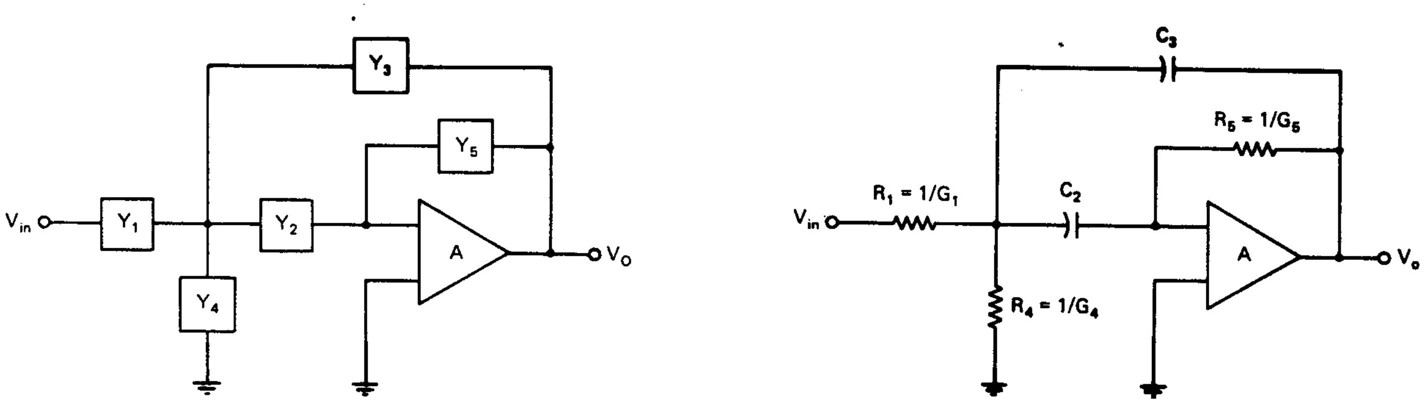


Figure 2.5.6-8 (a) Active Filter Circuit (b) Band-Pass Active Filter

$$T = \frac{V_o}{V_{in}} = \frac{-Y_1Y_2}{Y_2Y_3 + Y_5(Y_1 + Y_2 + Y_3 + Y_4)} \qquad (2.5.6\text{-}21)$$

For $Y_1 = G_1$, $Y_2 = sC_2$, $Y_3 = sC_3$, $Y_4 = G_4$, and $Y_5 = G_5$.

$$T = \frac{-sG_1C_2}{s^2C_2C_3 + G_5(G_1 + sC_2 + sC_3 + G_4)} = \frac{-G_1}{sC_3 + (G_1 + G_4)G_5/(sC_2) + G_5(C_2 + C_3)/C_2} \qquad (2.5.6\text{-}22)$$

The gain at resonance,

$$T_r = -\frac{(R_5/R_1)C_2}{C_2 + C_3} \qquad (2.5.6\text{-}23)$$

The resonant frequency,

$$\omega_r^2 = \frac{G_5(G_1 + G_4)}{C_2 C_3} \tag{2.5.6-24}$$

The 3-dB bandwidth,

$$\omega_{3dB} = \frac{G_5(C_2 + C_3)}{(C_2 + C_3)} \tag{2.5.6-25}$$

If $C_2 = C_3 = C$,

$$T_r = -\frac{R_5}{2R_1} \tag{2.5.6-26}$$

$$\omega_r^2 = \frac{G_5(G_1 + G_4)}{C^2} \tag{2.5.6-27}$$

$$\omega_{3dB} = \frac{2}{R_5 C} \tag{2.5.6-28}$$

Band-Reject Active Filters

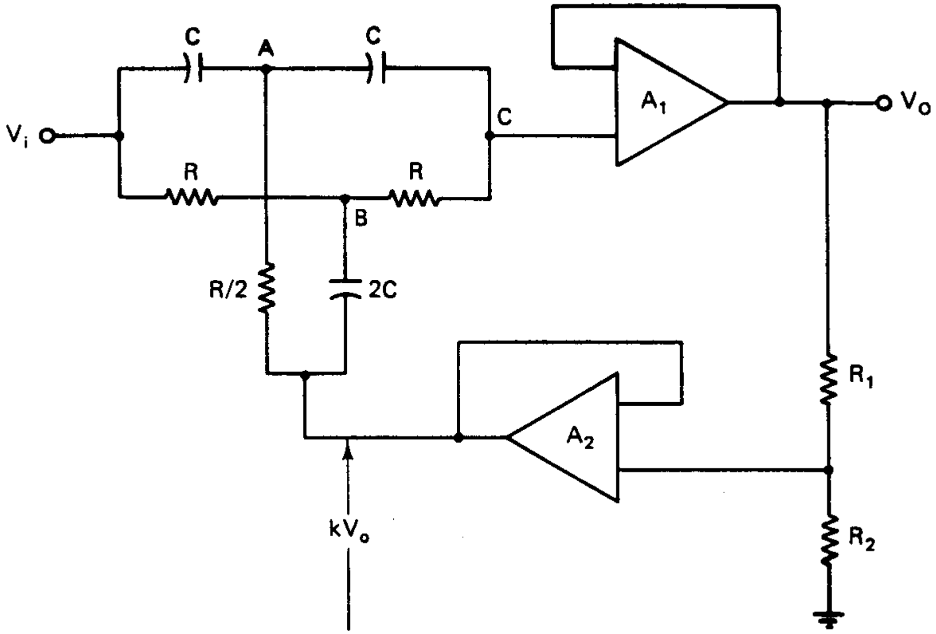


Figure 2.5.6-9 Band-Reject Active Filter

Node *A*: $$sCV_{in} + sCV_o + 2GV_oR_2/(R_1 + R_2) = 2(sC + G)V_A \quad (2.5.6\text{-}29)$$

Node *B*: $$GV_{in} + GV_o + 2sCV_oR_2/(R_1 + R_2) = 2(sC + G)V_B \quad (2.5.6\text{-}30)$$

Node *C*: $$sCV_A + GV_B = (G + sC)V_o \quad (2.5.6\text{-}31)$$

$$T = \frac{V_o}{V_{in}} = \frac{s^2 + (G/C)^2}{s^2 + (G/C)^2 + 4(1 - R_2/(R_1 + R_2))s(G/C)} \qquad (2.5.6\text{-}32)$$

The notch frequency ω_o,

$$\omega_o = \frac{G}{C} = \frac{1}{RC} \qquad (2.5.6\text{-}33)$$

$s = j\omega$,

$$T = \frac{\omega^2 - \omega_o^2}{\omega^2 - \omega_o^2 - 4(1 - R_2/(R_1 + R_2))\, j\omega\omega_o} \qquad (2.5.6\text{-}34)$$

- At $\omega = \omega_o$ the transfer function will be zero.
- As ω goes to zero, the transfer function will approach unity.
- As ω goes to infinity, the transfer function will also asymptotically approach unity.
- In actual practice, the high-frequency response will be limited by the frequency response of the operational amplifier.

The 3-dB bandwidth,

$$\omega_{3dB} = \frac{4(1 - R_2/(R_1 + R_2))}{RC} \qquad (2.5.6\text{-}35)$$

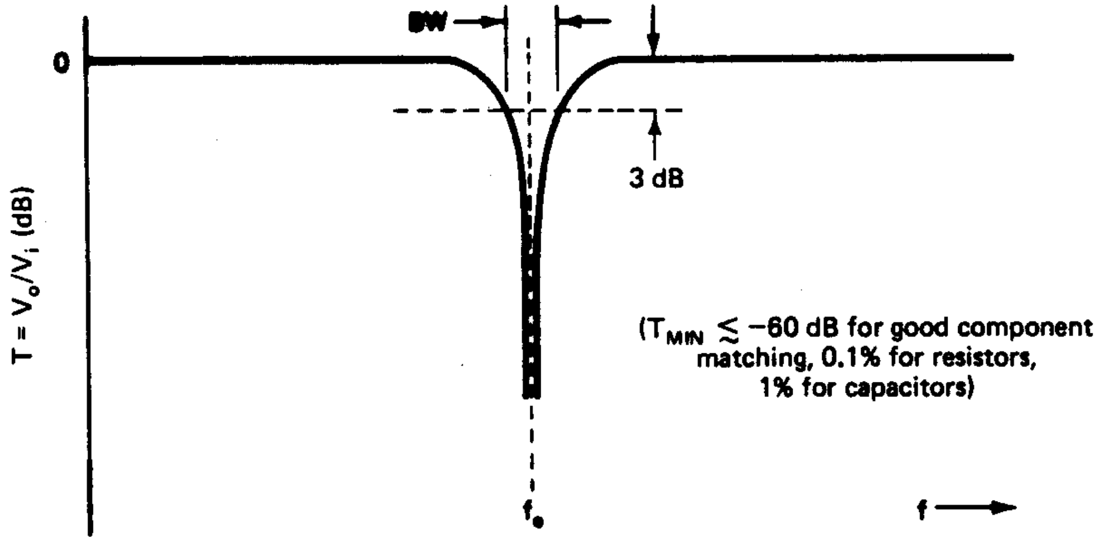


Figure 2.5.6-10 Frequency Response Characteristics of the Band-Reject (Notch) Filter

2.6 A/D and D/A Converters

2.6.1 A/D Converter

- A/D converter is used to convert analog signal from monitored information to digital signal, which is the form that computer can understand.
- Normally, A/D converter divides domain of the input voltage into equal range then assigns value which locates equivalently the closest point to each input. Because the assigned value is not continuous, the error between the input and the equivalent value is called quantized error.
- Resolution of A/D converter depends upon the number of expressed bits. For example, with 8 bits, the converter can detects the difference of 256 steps.
- The most well-known converters are parallel comparator A/D converter and successive approximation A/D converter.

1. Parallel Comparator A/D Converter. In general, for N bits A/D converter, 2^N-1 comparators are required. The reference voltage divider sets up the reference levels for the comparators. The divided voltages are then compared with the signal, being converted to digital. Speed of conversion by parallel comparator A/D converter is very fast.

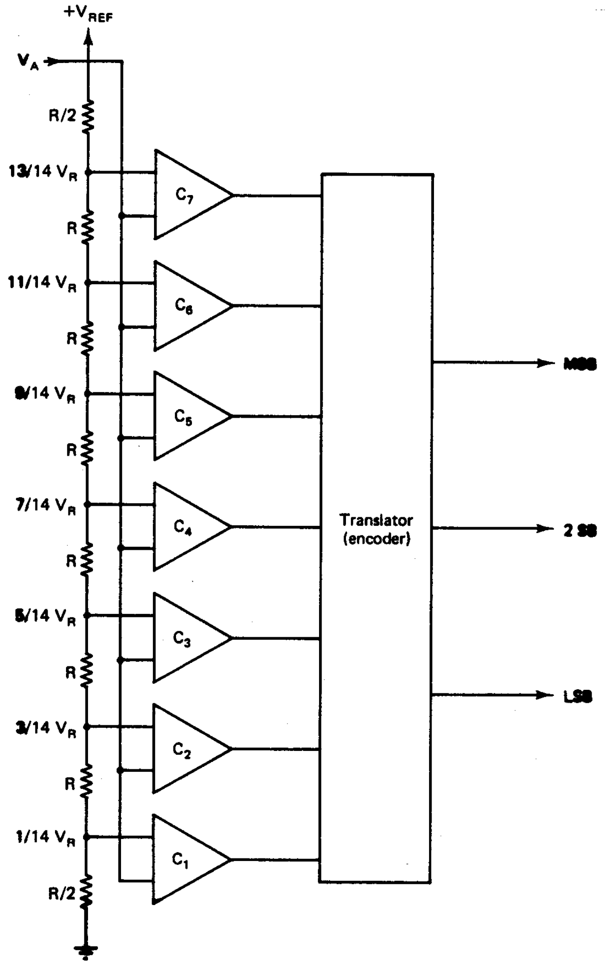


Figure 2.6.1-1 Structure of Parallel Comparator A/D Converter

2. Successive Approximation A/D Converter. The successive approximation register for an *N*-bit converter contains *N* flip-flops that are set to the high state, one at a time, to produce the bit inputs to the D/A converter. The output voltage of the D/A converter is compared to the analog input voltage. The result of comparison will adjust a bit of switching circuit from the upper bit. Adjustment occurs until all the bits of the D/A converter are adjusted. Speed of conversion is slower than in parallel comparator A/D converter.

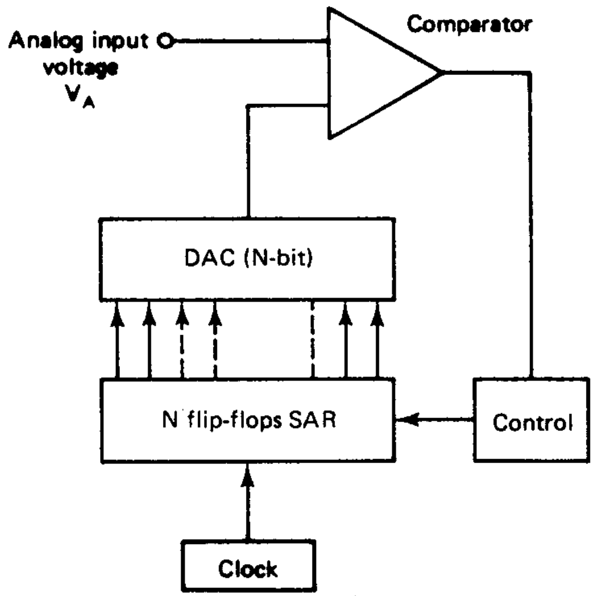


Figure 2.6.1-2 Structure of Successive Approximation A/D Converter

2.6.2 D/A Converter

D/A converter converts digital signal to analog signal.

1. Resistance Weighting. By switching each bit to reference voltage or to ground, output voltage will express the analog value by the following calculation.

$$V_o = -\left(\frac{R_f}{R}D_{N-1} + \frac{R_f}{2R}D_{N-2} + \cdots + \frac{R_f}{2^{N-1}R}D_0\right)V_{ref} = -\frac{R_f V_{ref}}{2^{N-1}R}(2^{N-1}D_{N-1} + 2^{N-2}D_{N-2} + \cdots + 2^0 D_0) \tag{2.6.2-1}$$

The structure of resistance weighting D/A converter is simple but in order to obtain high resolution wide range of the resistance is necessary. Furthermore, precision of the resistance effects directly to the output voltage.

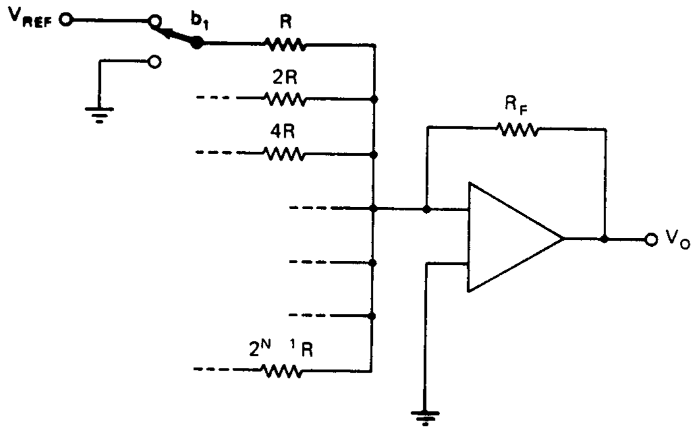


Figure 2.6.2-1 Structure of Resistance Weighting D/A Converter

2. R-2R Resistor Ladder Networks. Except the feedback resistance, 3R, this circuit composes of only 2 values of resistance, R and 2R. For consideration, let bit *k* connects to the reference voltage while the remained bits connect to ground. The voltage at point A_k, v_k, will become $-v_{\text{ref}}/3$ while the voltage at the connected point A_{k+1}, v_{k+1}, will become $v_k/2$. After we repeat the calculation, the voltage at point A_N, v_N, will become $-v_{\text{ref}}/(2^{N-k}3)$. By superposition method of the effect of each bit, the result voltage at point A_N will be

$$v_N = -\frac{v_{ref}}{3}\left(\frac{D_0}{2^{N-0}} + \frac{D_1}{2^{N-1}} + \cdots + \frac{D_N}{2^{N-N}}\right) \tag{2.6.2-2}$$

Output voltage, v_o, will be

$$v_o = \frac{v_{ref}}{2}\left(\frac{D_0}{2^{N-0}} + \frac{D_1}{2^{N-1}} + \cdots + \frac{D_N}{2^{N-N}}\right) \tag{2.6.2-3}$$

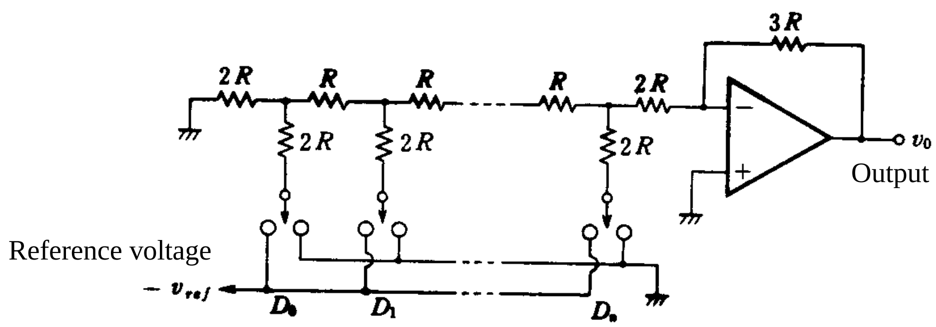


Figure 2.6.2-2 Structure of R-2R Ladder Circuit D/A Converter

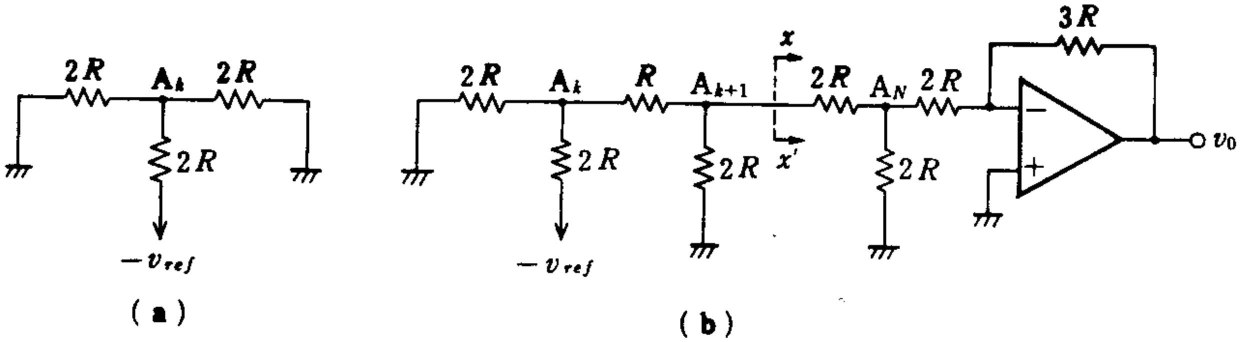


Figure 2.6.2-3 Equivalent Circuit When Bit *k* connects to Reference Voltage