3. Motion and Dimensional Sensors

- The four fundamental quantities of the International Measuring System are length, time, mass, and temperature.
- Units and standards for all other quantities are derived from these.
- Motion and dimension are based on two of the fundamental quantities, length and time.
- Many other quantities; such as force, pressure, temperature, etc., are often measured by converting them to motion and then measuring this resulting motion.
- The standard of length, meter, was defined as 1/10,000,000 of the distance from the Equator to North pole passing through Paris.
- In 1960, meter was defined in terms of the wavelength of a krypton-86 lamp as the length equal to 1,650,763.73 wavelengths in vacuum.
- The fundamental unit of time, second, was defined as 1/86,400 of a day.
- With higher accuracy, second was defined as the interval of time corresponding to 9,192,631,770 cycles of the atomic resonant frequency of cesium 133.

- Static calibration of translational devices

  - Micrometers: 0.01 mm.
  - Lever arrangements (about a 10:1 ratio) or wedge-type mechanisms (about 100:1 ratio)
  - Gage blocks: more accuracy, small blocks of hard, dimensionally stable steel or other material, made up in sets which can be stacked up over a wide range and in small steps

- Rotational or angular displacement is not itself a fundamental quantity since it is based on length,
  - Angle blocks: steel blocks with a specified angle between the two contact surfaces. These angle blocks can be stacked to build up any desired angle accurately and in small increments.

3.1 Relative Displacement, Translational and Rotational

3.1.1 Resistive Potentiometers

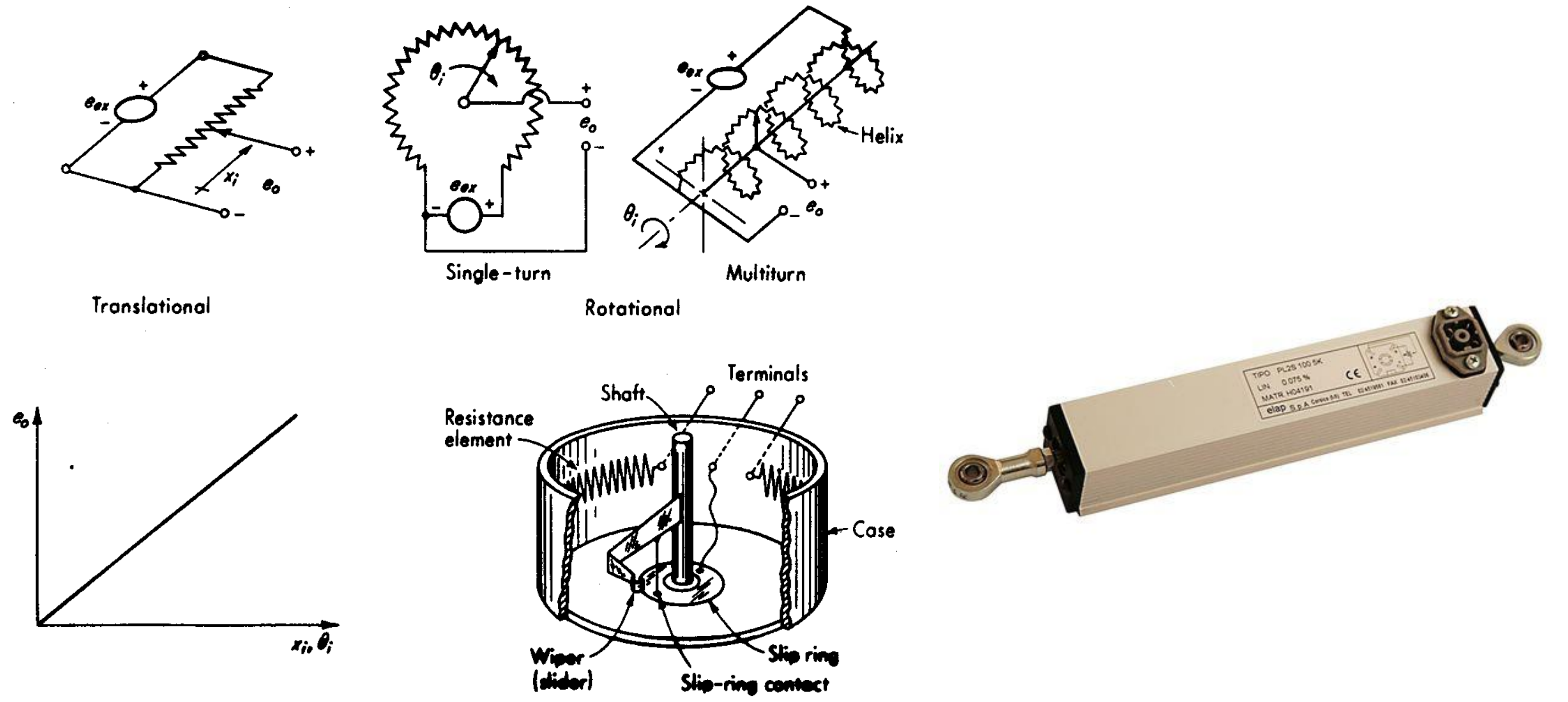


Figure 3.1.1-1 Potentiometer Displacement Transducer

- A resistive potentiometer consists of a resistance element provided with a movable contact.
- The contact motion can be translation, rotation, or a combination of the two, thus allowing measurement of rotary and translatory displacements.
- The resistance element is excited with either dc or ac voltage, and the output voltage is (ideally) a linear function of the input displacement.
- Resistance elements in common use may be classified as wire-wound, conductive plastic, hybrid, or cermet.

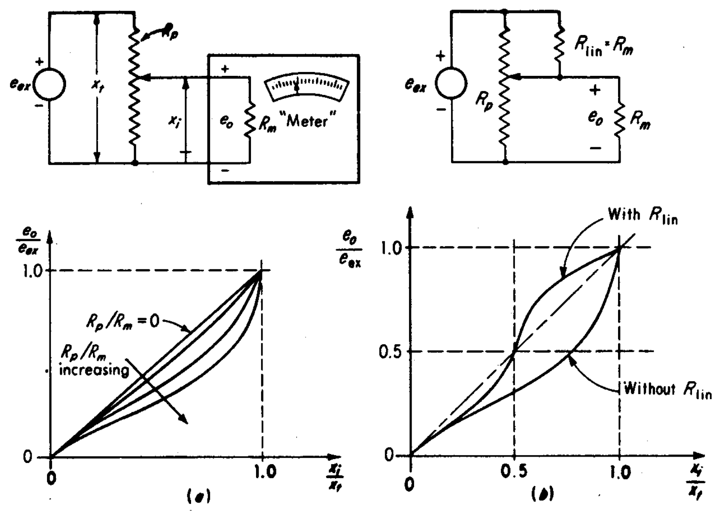


Figure 3.1.1-2 Potentiometer Loading Effect

$$\frac{e_o}{e_{ex}} = \frac{\dfrac{(x_i/x_t)R_pR_m}{(x_i/x_t)R_p + R_m}}{\dfrac{(x_i/x_t)R_pR_m}{(x_i/x_t)R_p + R_m} + ((x_t - x_i)/x_t)R_p} = \frac{1}{1/(x_i/x_t) + (R_p/R_m)(1 - x_i/x_t)} \tag{3.1.1-1}$$

For ideal conditions, $R_p/R_m = 0$ for an open circuit,

$$\frac{e_o}{e_{ex}} = \frac{x_i}{x_t} \tag{3.1.1-2}$$

- In actual practice, $R_m \neq \infty$, the position of maximum error occurs in the neighborhood of $x_i/x_t = 0.67$, and the maximum error is approximately $15R_p/R_m$ percent of full scale.

If the heat dissipation is limited to P watts, the maximum allowable excitation voltage,

$$e_{ex\_max} = \sqrt{PR_p} \tag{3.1.1-3}$$

- Wirewound resistance element has larger resistance, thus, improves the sensitivity.
- The variation of wirewound resistance is not a linear continuous change.

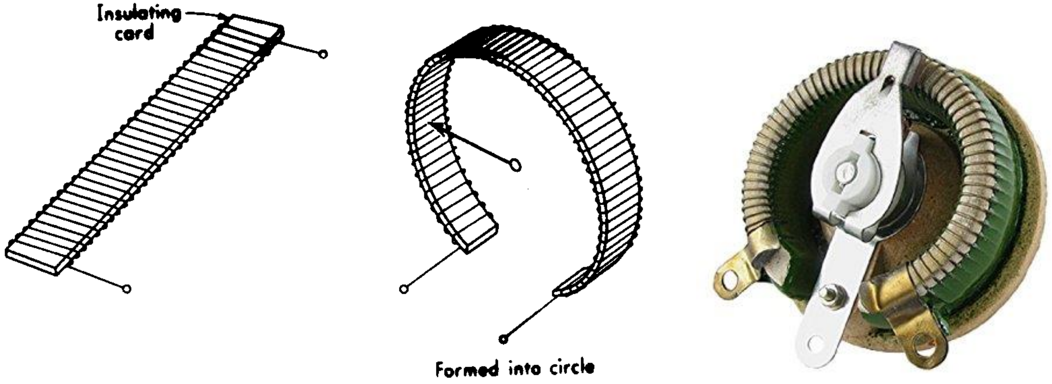


Figure 3.1.1-3 Construction of Wirewound Resistance Element

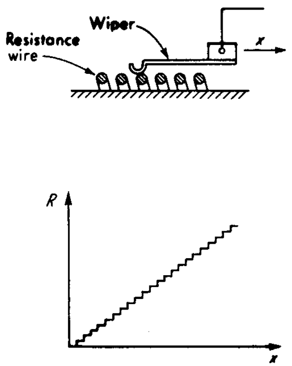


Figure 3.1.1-4 Resolution of Wirewound Potentiometers

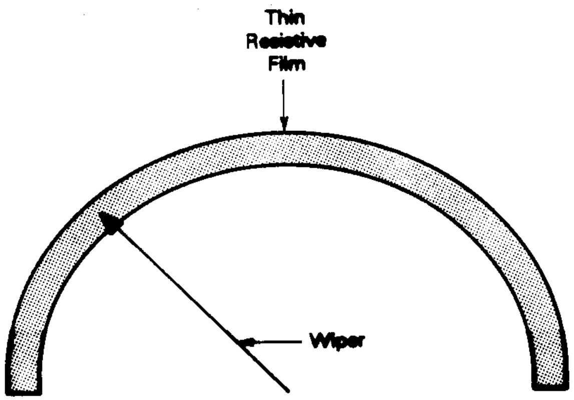


Figure 3.1.1-5 Thin Film Potentiometer

- Thin film potentiometer, having a smooth surface to the wiper, improves resolution and life; however, it is more temperature-sensitive, has a high (and variable) wiper contact resistance, and can tolerate only moderate wiper currents.
- Thin film elements
    - Cermet: combination of ceramic and metallic materials)
    - Conductive plastic: mixture of plastic resin with proprietary conductive powders

- Multiturn potentiometer in form of helix is used to increase the resolution. The wiper travels along a lead screw.
- Potentiometer is a zero-order instrument since the impedance of the winding is almost purely resistive.

3.1.2 Resistance Strain Gages

The resistance *R* of a conductor of uniform cross-sectional area *A* and length *L*, made of a material with resistivity ρ,

$$R = \frac{\rho L}{A} \tag{3.1.2-1}$$

If this conductor is stretched or compressed,

$$dR = \frac{A(\rho dL + L d\rho) - \rho L dA}{A^2} \tag{3.1.2-2}$$

$$dV = L(1+\varepsilon)A(1-\varepsilon\nu)^2 - AL \tag{3.1.2-3}$$

ε: strain, υ: Poisson's ratio, $\varepsilon \ll 1$, $(1-\nu\varepsilon)^2 \approx 1-2\nu\varepsilon$, $V = AL$, $dV = AdL + LdA$

$$dV \approx AL\varepsilon(1-2\nu) = AdL + LdA \tag{3.1.2-4}$$

$\varepsilon = dL/L$,

$$AdL(1-2\nu) = AdL + LdA \tag{3.1.2-5}$$

$$-2\nu AdL = LdA \tag{3.1.2-6}$$

$$dR = \frac{\rho AdL + LAd\rho + 2\nu\rho AdL}{A^2} \tag{3.1.2-7}$$

$$dR = \frac{\rho dL(1+2\nu)}{A} + \frac{Ld\rho}{A} \tag{3.1.2-8}$$

$$\frac{dR}{R} = \frac{dL}{L}(1+2\nu) + \frac{d\rho}{\rho} \tag{3.1.2-9}$$

$$\text{Gage factor} = \frac{dR/R}{dL/L} = 1 + 2\nu + \frac{d\rho/\rho}{dL/L} \tag{3.1.2-10}$$

- If the gage factor is known, measurement of dR/R allows measurement of the strain $dL/L = \varepsilon$.
- The term $(d\rho/\rho)/(dL/L)$ can also be expressed as $\pi_1 E$, where π_1 is longitudinal piezoresistance coefficient and E is modulus of elasticity.
- Poisson's ratio is always between 0 and 0.5 for all materials.
- Strain gage is basically represented by zero-order dynamic model.
- Several types of strain gages: unbonded metal-wire gage, bonded metal-wire gage, and bonded metal-foil gage, etc.
- Typical gage resistances are 120, 350, and 1,000 Ω, with the allowable gage current determined by heat-transfer conditions but typically 5 to 40 mA; gage factors are 2 to 4.
- Gage combinations called rosettes, are available in many configurations for specific stress-analysis or transducer applications. Precise relative orientation of the several gages is manufactured.

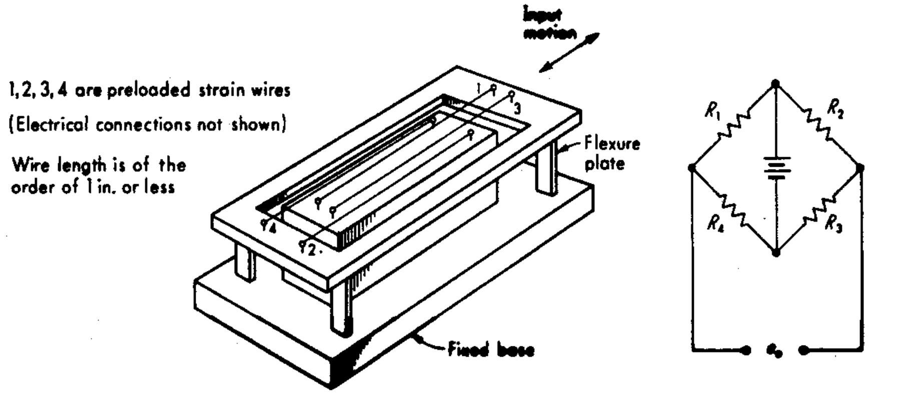


Figure 3.1.2-1 Unbonded Strain Gage

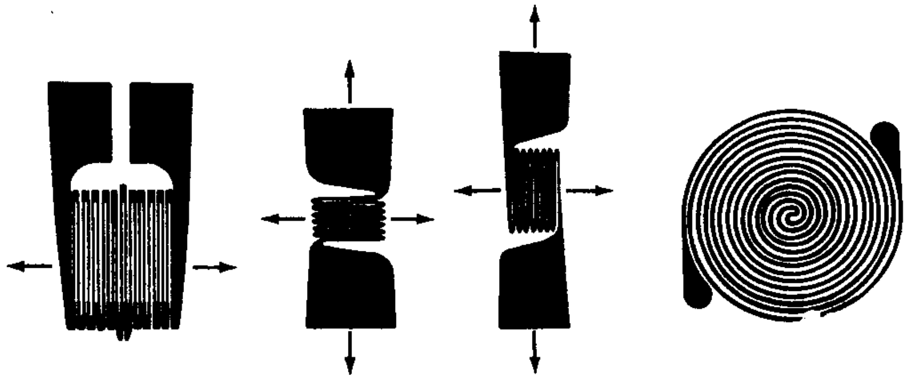


Figure 3.1.2-2 Foil Strain Gage

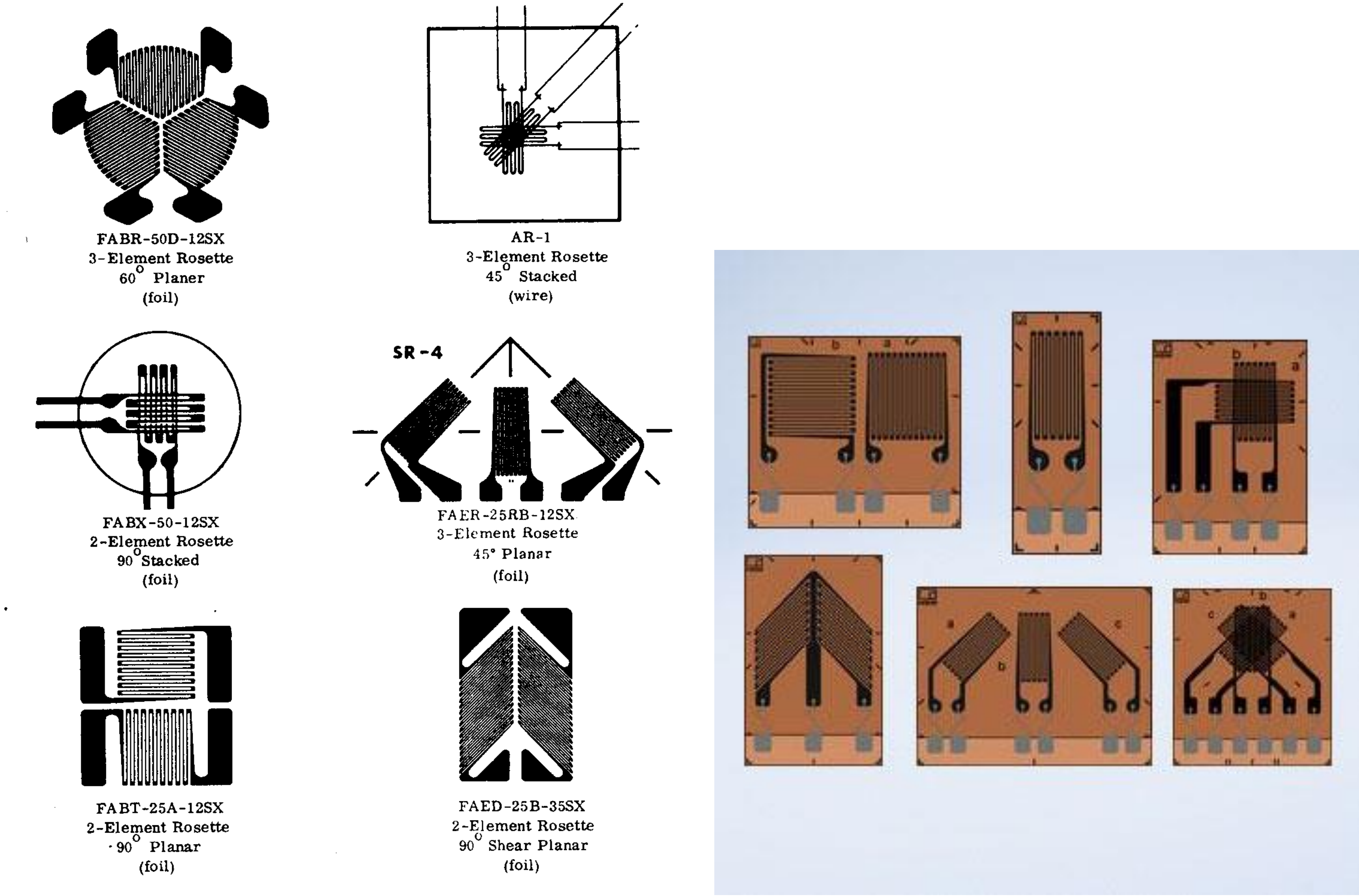


Figure 3.1.2-3 Strain-Gage Rosettes

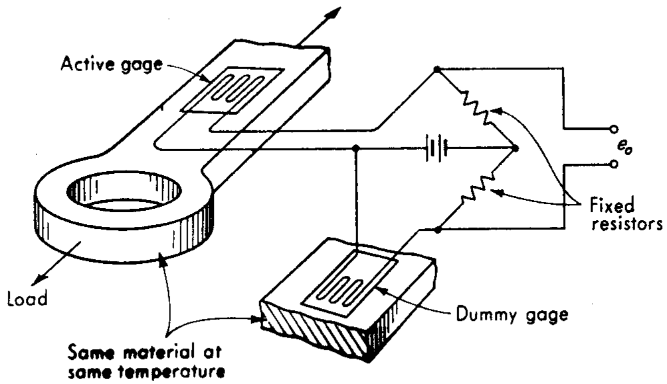


Figure 3.1.2-4 Strain-Gage Temperature Compensation

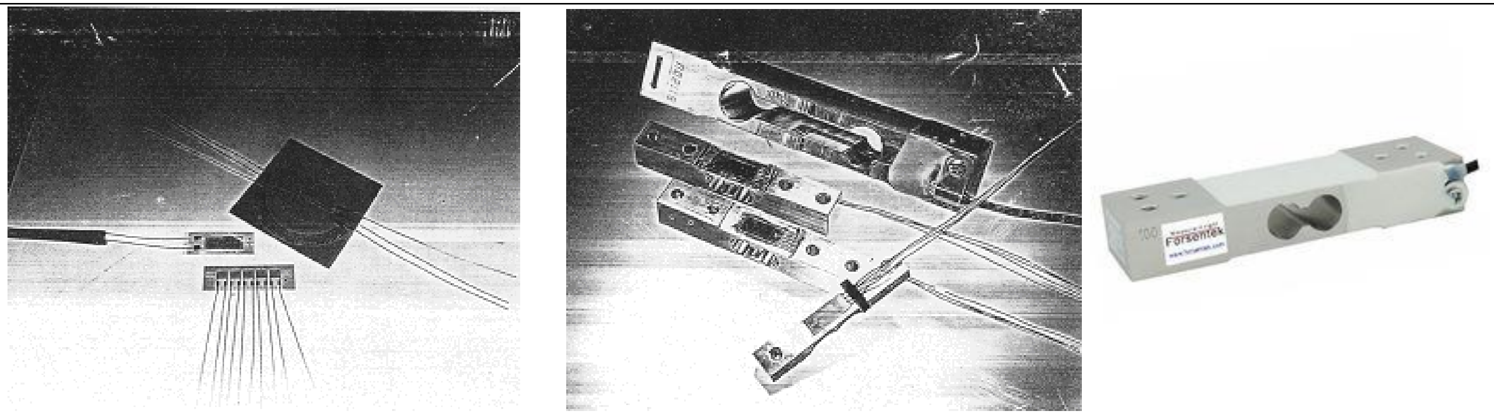


Figure 3.1.2-5 Strain-Gage and Load Cell

3.1.3 Differential Transformers

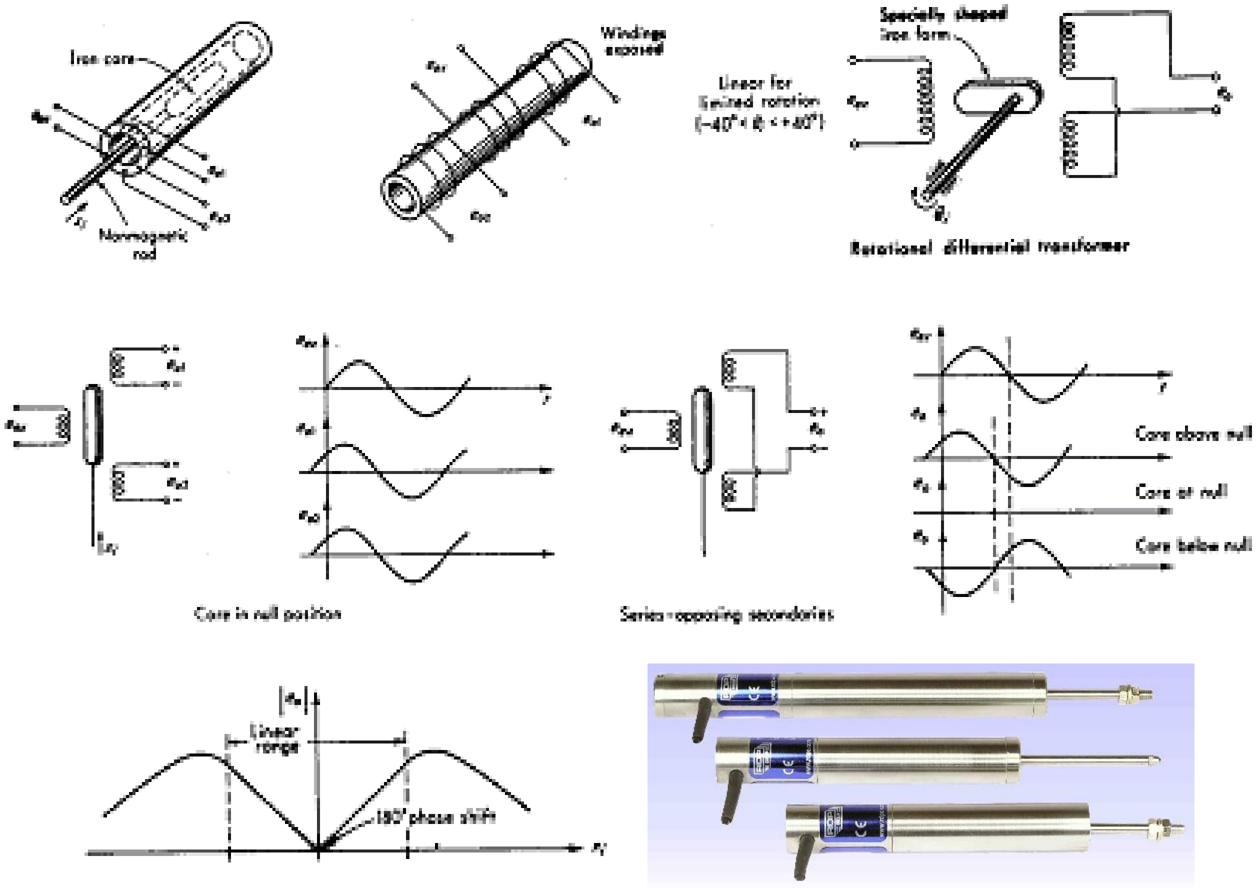


Figure 3.1.3-1 Differential Transformer

- The excitation of linear variable differential transformer (LVDT) is normally a sinusoidal voltage of 3 to 15 V rms amplitude and frequency of 60 to 20,000 Hz.
- The two identical secondary coils have induced in them sinusoidal voltages of the same frequency as the excitation; however, the amplitude varies with the position of the iron core.
- When the secondaries are connected in series opposition, a null position exists ($x_i = 0$) at which the net output e_o is essentially zero.
- Motion of the core from null then causes a larger mutual inductance (coupling) for one coil and a smaller mutual inductance for the other, and the amplitude of e_o becomes a nearly linear function of core position for a considerable range either side of null.
- The output e_o is generally out of phase with the excitation e_{ex}; however, this varies with the frequency of e_{ex}, and for each differential transformer there exists a particular frequency at which this phase shift is zero.
- If the differential transformer is used with some readout system that requires a small phase shift between e_o and e_{ex}, excitation at the correct frequency can solve this problem.
- If the output voltage is applied directly to an ac meter or an oscilloscope, this phase shift is not a problem.
- The dynamic response of LVDT is limited mainly by the excitation frequency, since it must be much higher than the core-motion frequencies. For adequate demodulation and filtering, a frequency ratio of higher than 10:1 is desired.

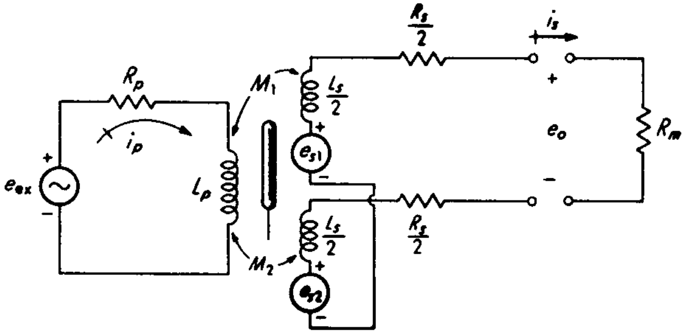


Figure 3.1.3-2 Circuit Analysis

Applying Kirchhofs's voltage-loop law, if the output is an open circuit,

$$i_p R_p + L_p \frac{di_p}{dt} - e_{ex} = 0 \qquad (3.1.3\text{-}1)$$

$$e_{s1} = M_1 \frac{di_p}{dt} \qquad (3.1.3\text{-}2)$$

$$e_{s2} = M_2 \frac{di_p}{dt} \qquad (3.1.3\text{-}3)$$

M_1 and M_2: the respective mutual inductances.

$$e_s = e_{s1} - e_{s2} = (M_1 - M_2)\frac{di_p}{dt} \qquad (3.1.3\text{-}4)$$

- The net mutual inductance M_1 - M_2 is the quantity that varies linearly with core motion.

$$e_o(s) = e_s(s) = (M_1 - M_2)\frac{s}{L_p s + R_p} e_{ex}(s) \tag{3.1.3-5}$$

$$\frac{e_o}{e_{ex}}(s) = \frac{\left[(M_1 - M_2)/R_p\right]s}{\tau_p s + 1} \tag{3.1.3-6}$$

where $\tau_p = L_p/R_p$.

$$\frac{e_o}{e_{ex}}(i\omega) = \frac{\omega(M_1 - M_2)/R_p}{\sqrt{(\omega\tau_p)^2 + 1}}\angle\phi \tag{3.1.3-7}$$

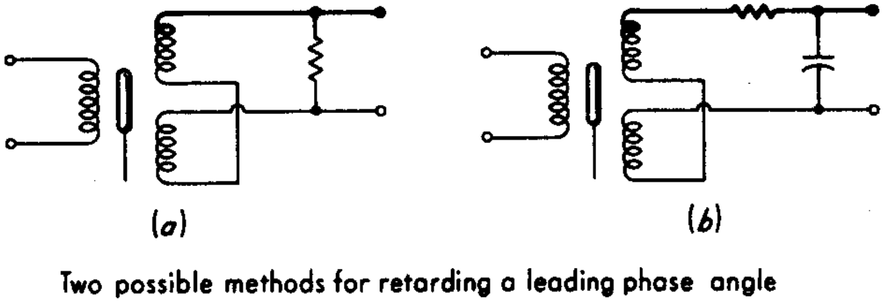
where ϕ: the phase shift between e_o and e_{ex}.

If a voltage-measuring device of input resistance R_m is attached to the output terminals, a current i_s will flow.

$$i_p R_p + L_p i_p s - (M_1 - M_2) i_s s - e_{ex} = 0 \tag{3.1.3-8}$$

$$(M_1 - M_2) i_p s + (R_s + R_m) i_s + L_s i_s s = 0 \tag{3.1.3-9}$$

$$\frac{e_o}{e_{ex}}(s) = \frac{R_m(M_2 - M_1)s}{\left[(M_1 - M_2)^2 + L_p L_s\right]s^2 + \left[L_p(R_s + R_m) + L_s R_p\right]s + (R_s + R_m)R_p} \tag{3.1.3-10}$$



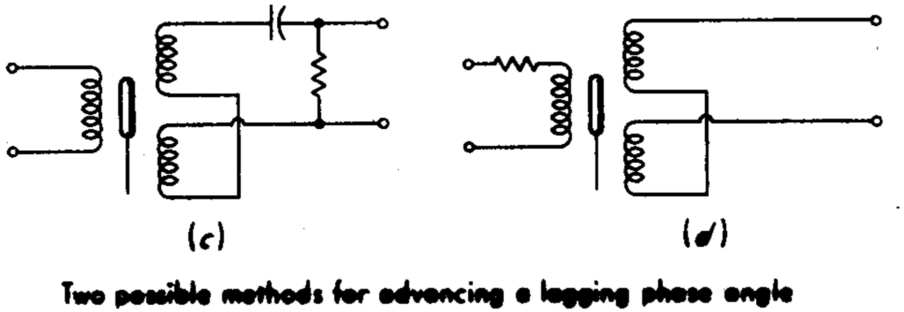


Figure 3.1.3-3 Phase-Angle-Adjustment Circuits

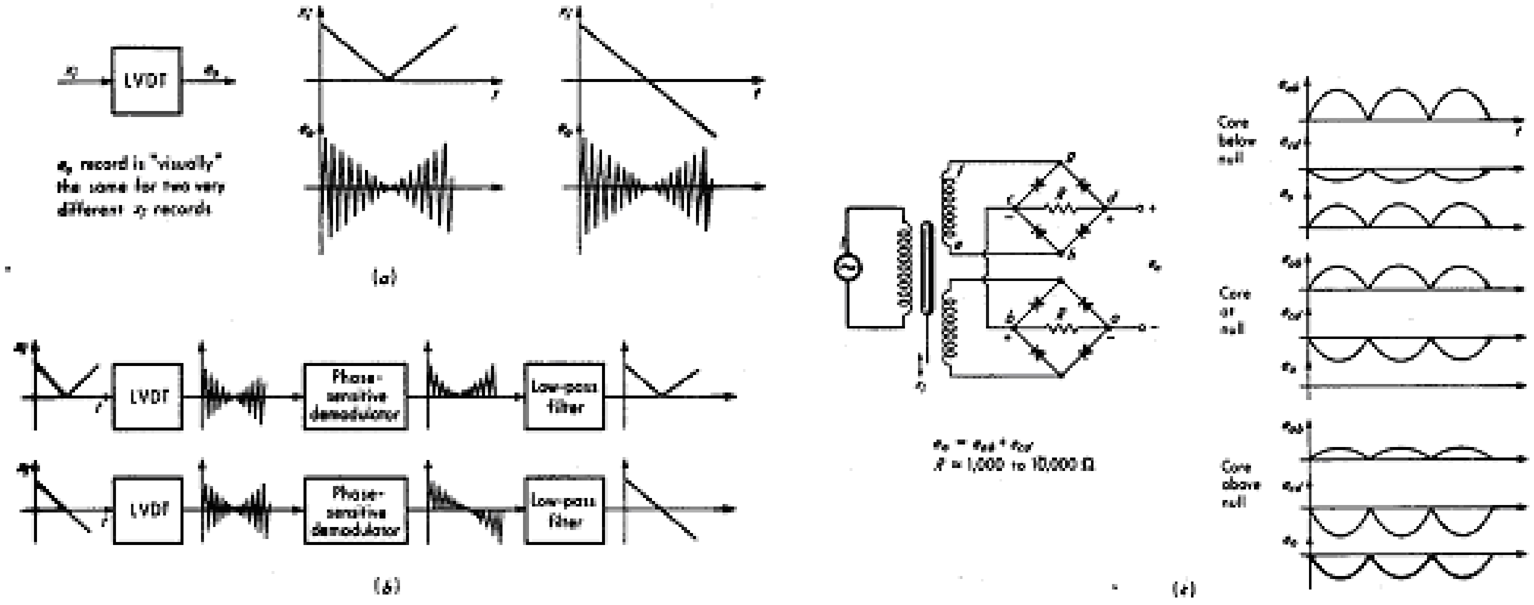


Figure 3.1.3-4 Demodulation and Filtering

3.1.4 Synchros

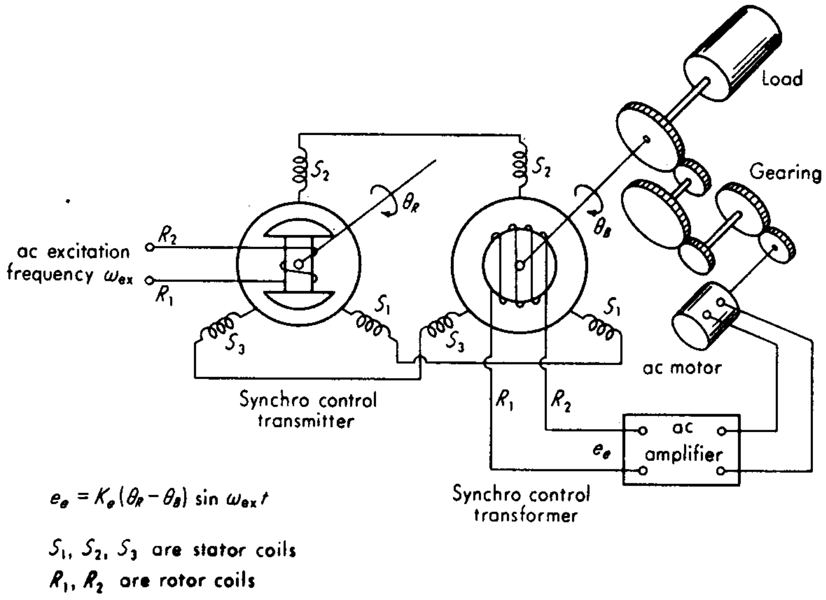


Figure 3.1.4-1 AC Servomechanism

- Synchros are ac electromechanical devices which can perform the function of angle measurement.
- Two different types of synchros, the control transmitter and the control transformer, are used for angle measurement.
- The physical constructions of the control transmitter and control transformer are identical except that the transmitter has a salient-pole (dumbbell) rotor while the transformer has a cylindrical rotor.

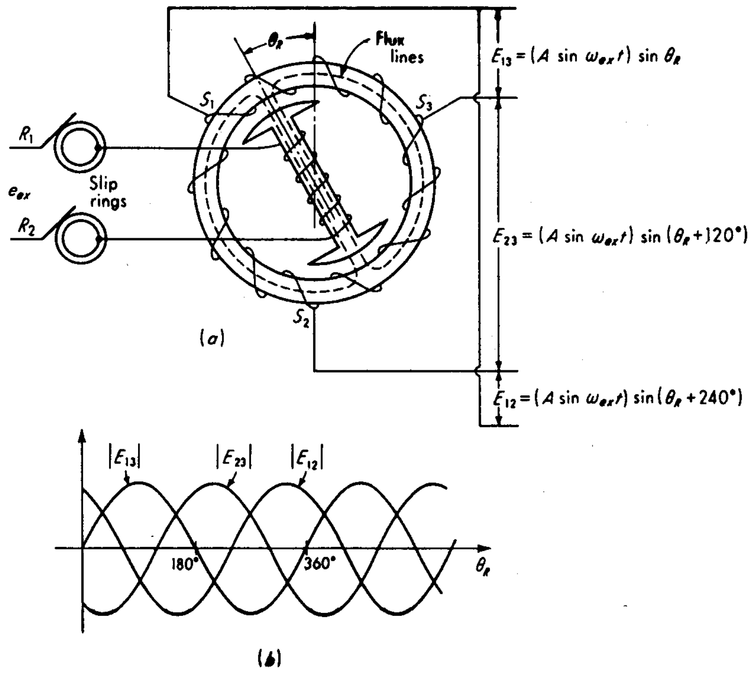


Figure 3.1.4-2 Synchro

- The three voltage signals from the stator coils uniquely define the angular position of the rotor. When these three voltages are applied to the stator coils of a control transformer, they produce a resultant magnetomotive force aligned in the same direction as that of the transmitter rotor.
- The rotor of the transformer acts as a “search coil” in detecting the direction of its stator field.

- If the axis of this coil is aligned with the field, the maximum voltage is induced into the transformer rotor coil.
- If the axis is perpendicular to the field, zero voltage is induced, giving the null position.
- The output-voltage amplitude actually varies sinusoidally with the misalignment angle, but for small angles the sine and the angle are nearly equal, giving a linear output.

For a rotor excitation voltage,

$$e_{ex} = V \sin \omega_{ex} t \tag{3.1.4-1}$$

The voltages induced in the three stator windings,

$$E_{13} = (A \sin \omega_{ex} t) \sin \theta_R \tag{3.1.4-2}$$

$$E_{23} = (A \sin \omega_{ex} t) \sin(\theta_R + 120°) \tag{3.1.4-3}$$

$$E_{12} = (A \sin \omega_{ex} t) \sin(\theta_R + 240°) \tag{3.1.4-4}$$

The error voltage for small misalignment angle,

$$e_e = K_e \sin(\theta_R - \theta_B) \sin \omega_{ex} t \approx K_e (\theta_R - \theta_B) \sin \omega_{ex} t \tag{3.1.4-5}$$

3.1.5 Resolvers

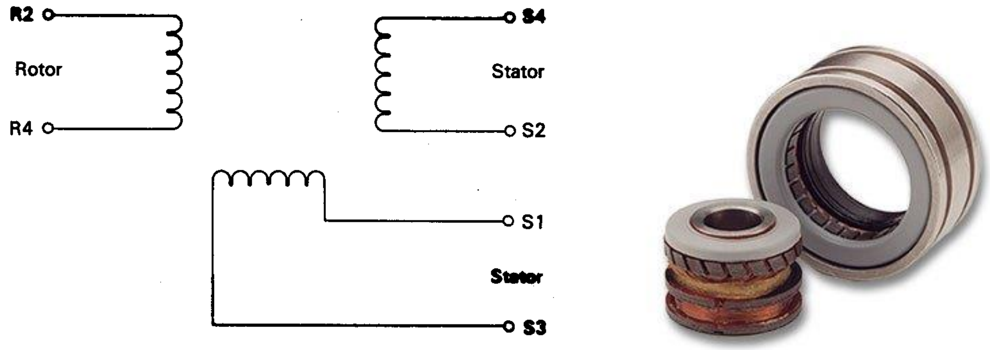


Figure 3.1.5-1 Simple Resolver

- The resolver is actually a form of synchro and is often called a synchro resolver.
- One of the major differences between synchro and resolver is that the stator and rotor windings of the resolver are displaced mechanically 90° to each.
- The most common form of resolver has a single rotor and two stator windings.

For a rotor excitation voltage,

$$e_{ex} = V \sin \omega_{ex} t \qquad (3.1.5\text{-}1)$$

The voltages induced in the two stator windings,

$$E_{13} = (A \sin \omega_{ex} t) \sin \theta_R \quad (3.1.5\text{-}2)$$

$$E_{24} = (A \sin \omega_{ex} t) \cos \theta_R \quad (3.1.5\text{-}3)$$

where θ_r: the resolver shaft angle.

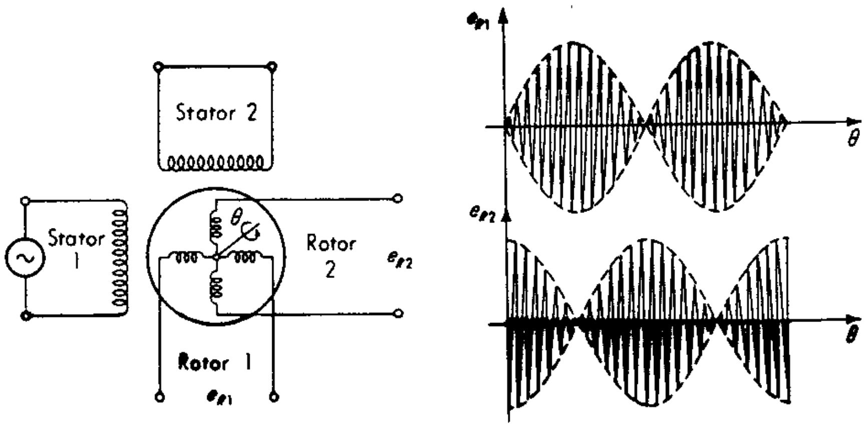


Figure 3.1.5-2 Resolver

The output voltage for small misalignment angle,

$$e_o = K_e \sin(\theta_R - \theta_B) \sin \omega_{ex} t \approx K_e (\theta_R - \theta_B) \sin \omega_{ex} t \quad (3.1.5\text{-}4)$$

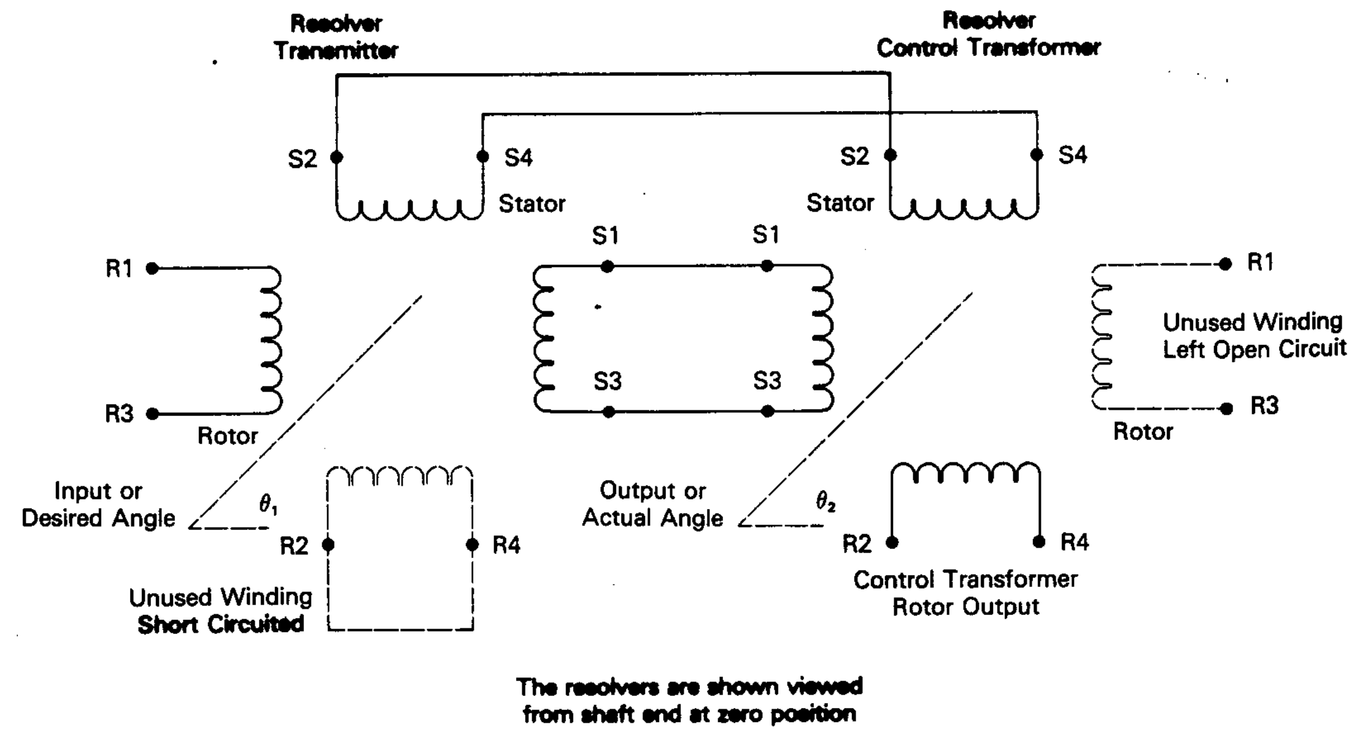


Figure 3.1.5-3 Resolver Transmitter Connected to a Resolver Control Transformer

3.1.6 Variable-Inductance and Variable-Reluctance Pickups

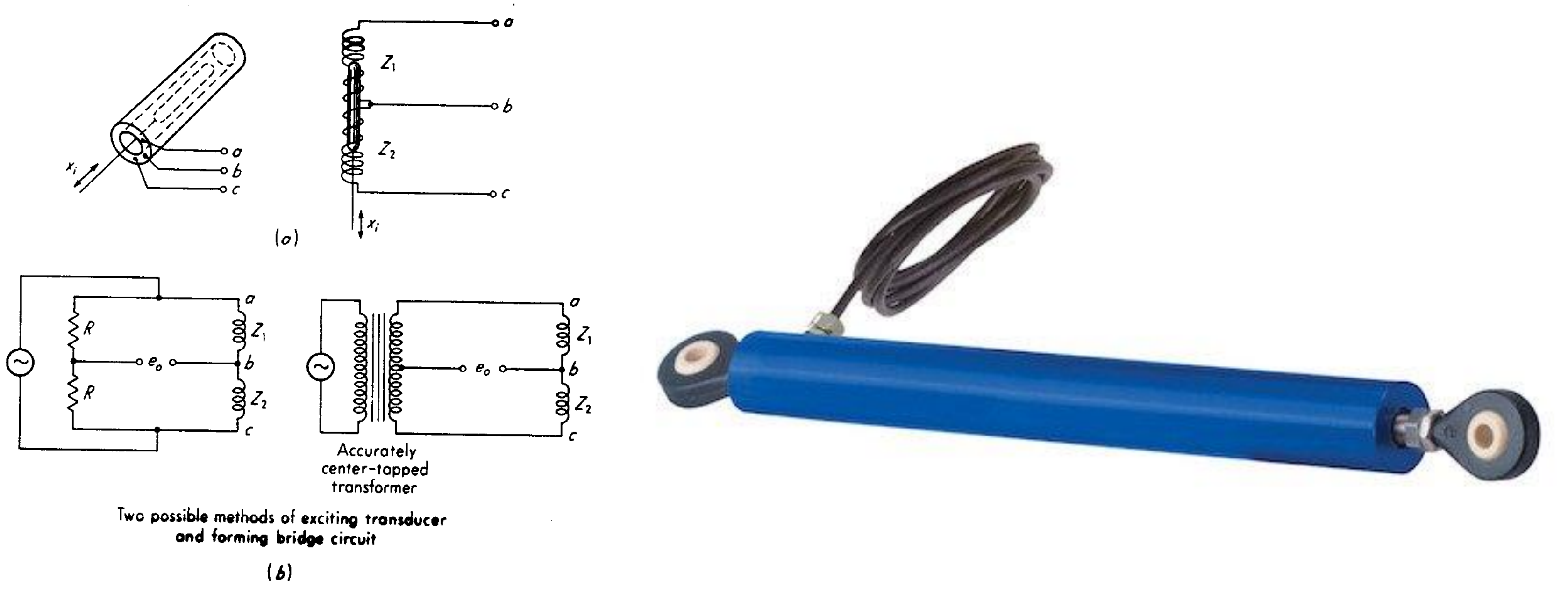


Figure 3.1.6-1 Variable-Inductance Pickup

- In variable-inductance transducer, two inductance coils form two legs of a bridge which is excited with ac signal of 5 to 30 V at 60 to 5,000 Hz.
- With the core at the null position, the inductance of the two coils is equal, the bridge is balanced, and e_o is zero.
- A core motion from null causes a change in the reluctance of the magnetic paths for each of the coils.

- This reluctance change causes a proportional change in inductance for each coil, a bridge unbalance, and thus an output voltage e_o.
- By careful construction, e_o can be made a nearly linear function of x_i over the rated displacement range.
- Two alternative methods of forming the bridge circuit
  - The total transducer impedance (Z_1 plus Z_2) at the excitation frequency is of the order of 100 to 1,000 Ω. The resistors R are usually about the same value as Z_1 and Z_2, and the input impedance of the voltage-measuring device at e_o should be at least $10R$.
  - If the bridge output must be worked into a low-impedance load, R must be quite small. To get high sensitivity, high excitation voltage is needed; this causes a high power loss (heating) in the resistors R. To solve this problem, a center-tapped transformer circuit may be used.

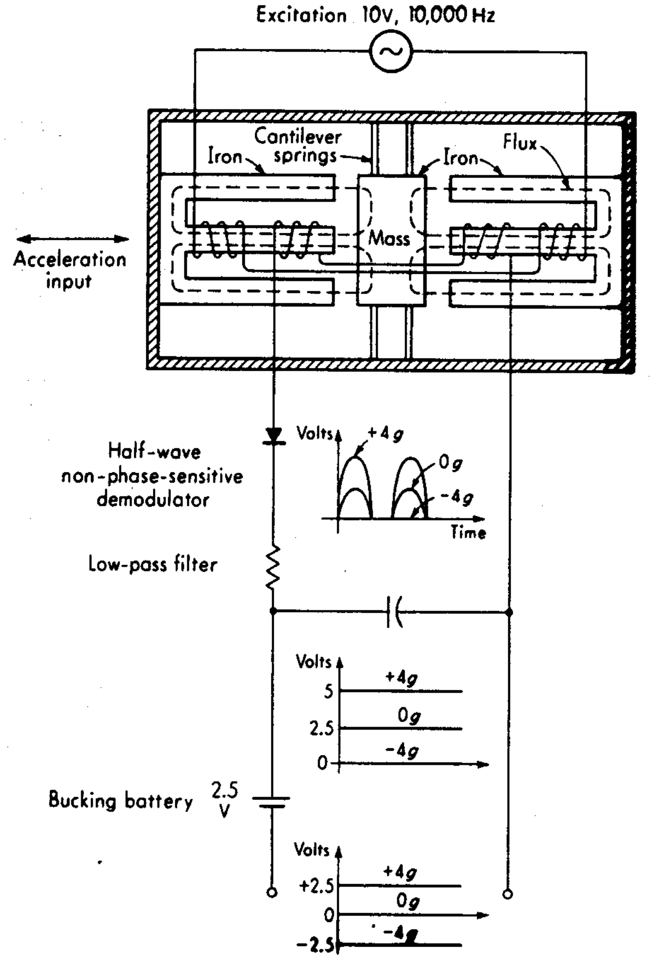


Figure 3.1.6-2 Variable-Reluctance Accelerometer

- In variable-reluctance accelerometer, the mass is an iron and serves as both an inertial element for transducing acceleration to force and a magnetic circuit element for transducing motion to reluctance.
- The primary coils set up a flux dependent on the reluctance of the magnetic path. The main reluctance is the air gap.
- When the core is in the neutral position, the flux is the same for both halves of the secondary coil; and since they are connected in series opposition, the net output voltage is zero.
- A motion of the core increases the reluctance (air gap) on one side and decreases it on the other, causing more voltage to be induced into one half of the secondary coil than the other and thus a net output voltage.
- Motion in the other direction causes the reverse action, with a 180° phase shift occurring at null.
- The output voltage is half-wave, non-phase-sensitive rectified (demodulated) and filtered to produce an output of the same form as the acceleration input.

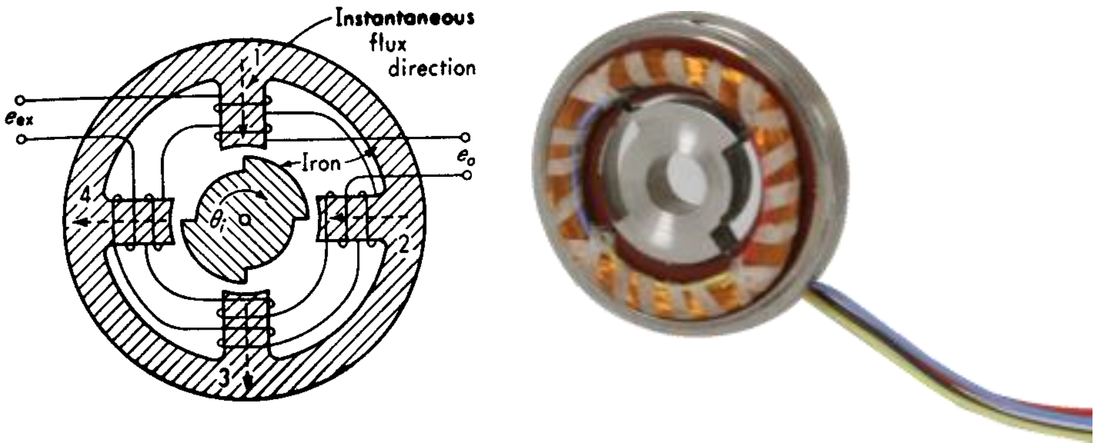


Figure 3.1.6-3 Microsyn

- A variable-reluctance element, Microsyn, is widely used in sensitive gyroscopic instruments.
- At the null position, the voltages induced in coils 1 and 3 (which aid each other) are just balanced by those of coils 2 and 4 (which also aid each other but oppose 1 and 3).
- Motion of the input shaft from the null (say clockwise) increases the reluctance (decreases the induced voltage) of coils 1 and 3 and decreases the reluctance (increases the voltage) of coils 2 and 4, thus giving a net output voltage e_o.
- Motion in the opposite direction causes a similar effect, except the output voltage has a 180° phase shift.
- If a direction-sensitive dc output is required, a phase-sensitive demodulator is necessary.

3.1.7 Eddy-Current Noncontacting Transducers

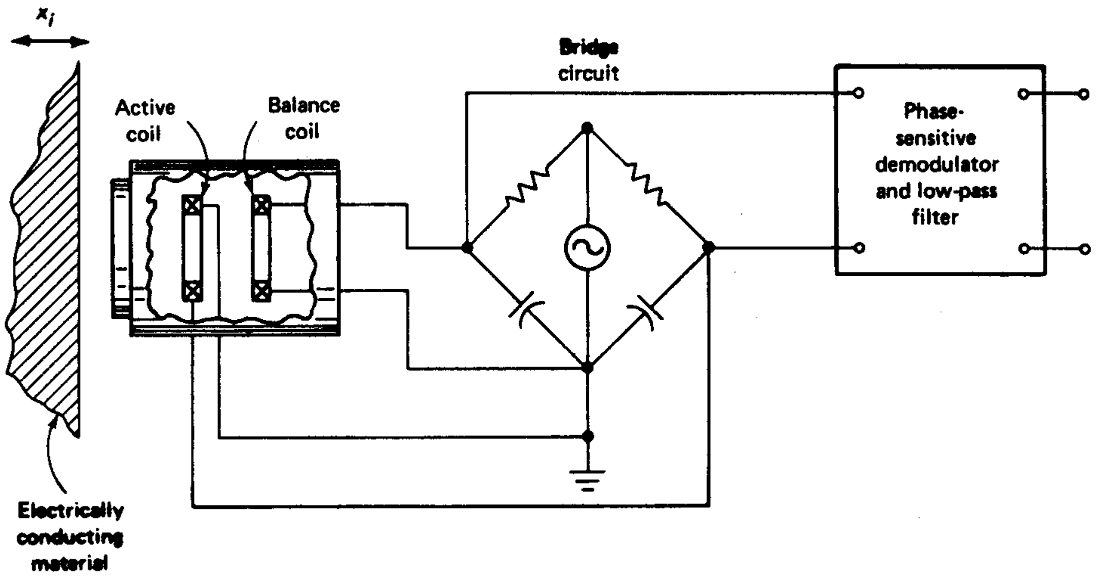


Figure 3.1.7-1 Eddy-Current Noncontacting Transducer

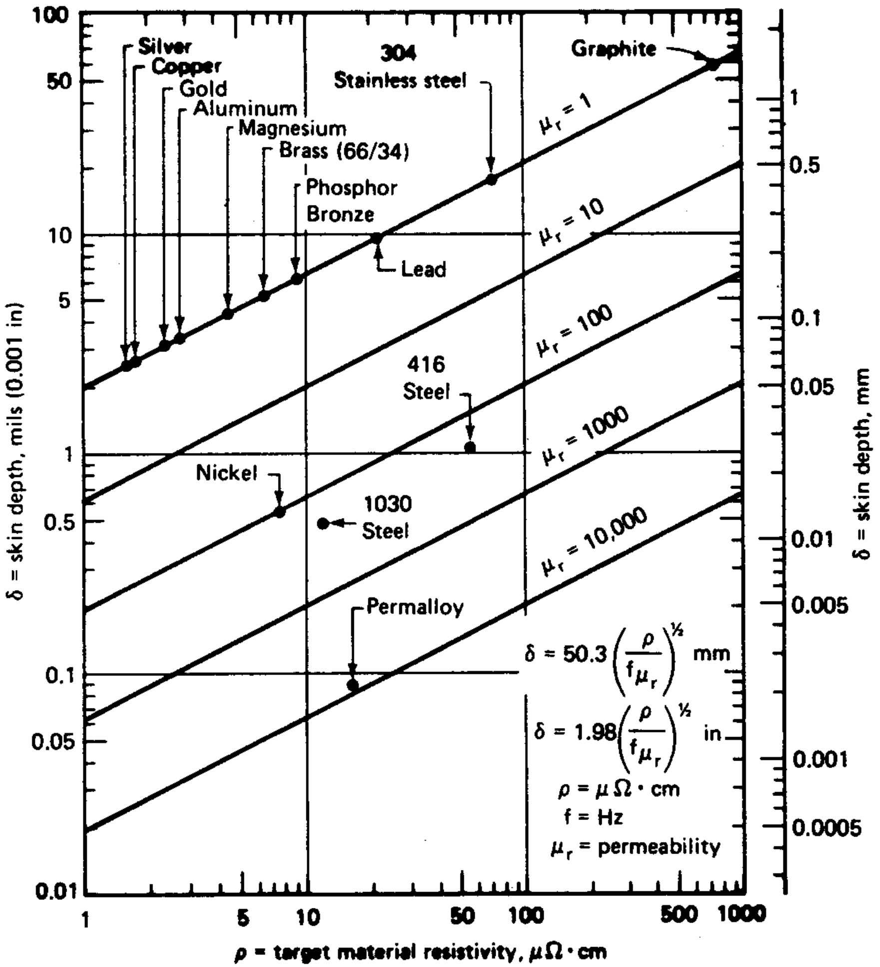


Figure 3.1.7-2 Target-Material Effect on Eddy-Current Transducer at Excitation Frequency of 1 MHz

- The probe of eddy-current transducer usually contains two coils, one (active) which is influenced by the presence of a conducting target and a second (balance) which serves to complete a bridge circuit and provide temperature compensation.
- Bridge excitation is high-frequency (about 1 MHz) ac.
- Magnetic flux lines from the active coil pass into the conductive target surface, producing in the target eddy currents whose density is greatest at the surface and which become negligibly below the surface.
- As the target comes closer to the probe, the eddy currents become stronger, which changes the impedance of the active coil and causes a bridge unbalance related to target position.
- The eddy current at 3 times of skin depth is negligible.
- This unbalance voltage is demodulated, low-pass filtered (and sometimes linearized) to produce a dc output proportional to target displacement.
- The high excitation frequency not only allows the use of thin targets, but also provides good system frequency response (up to 100 kHz).

3.1.8 Capacitance Pickups

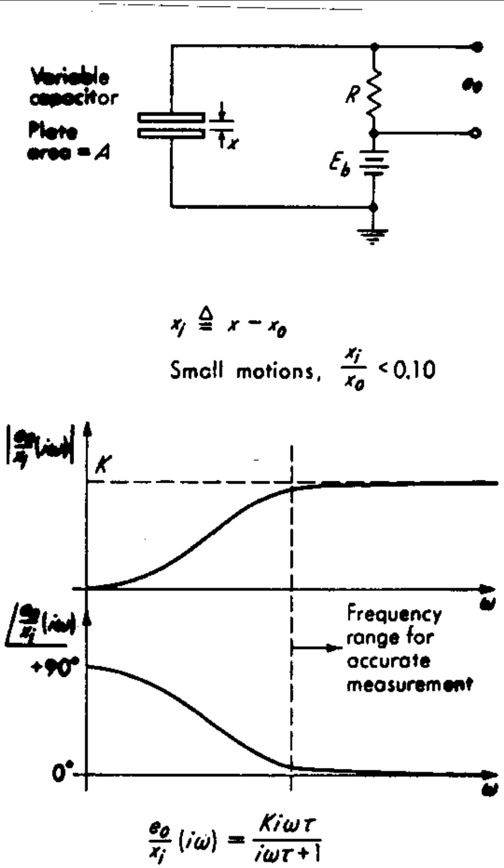


Figure 3.1.8-1 Capacitive Transducer

- The most common form of variable capacitor used in motion transducers is the parallel-plate capacitor with a variable air gap.

$$C = \frac{\varepsilon A}{x} = \frac{0.225A}{x} \qquad (3.1.8\text{-}1)$$

C: capacitance in pF, A: plate area in in^2, and x: plate separation in in., ε: dielectric constant = 0.225 pF/in.

The sensitivity of capacitance to changes in plate separation,

$$\frac{dC}{dx} = -\frac{0.225A}{x^2} \qquad (3.1.8\text{-}2)$$

- The sensitivity increases as x decreases.
- The percentage change in C is equal to the percentage change in x for small changes about any neutral position.

$$\frac{dC}{dx} = -\frac{C}{x} \qquad (3.1.8\text{-}3)$$

$$\frac{dC}{C} = -\frac{dx}{x} \qquad (3.1.8\text{-}4)$$

- When the capacitor plates are stationary with a separation x_o, no current flows and $e_o = 0$. If there is then a relative displacement x_i from the x_o position, a voltage e_o is produced and is related to x_i.

$$\frac{e_o}{x_i}(s) = \frac{K\tau s}{\tau s + 1} \qquad (3.1.8\text{-}5)$$

where $K = E_b/x_o$ V/in, and $\tau = 0.225\text{x}10^{-12} AR/x_o$ s.

- Capacitive transducer does not allow measurement of static displacements since e_o is zero in steady state for any value of x_i.
- For sufficiently rapid variations in x_i, however, the signal e_o will faithfully measure the motion.

$$\frac{e_o}{x_i}(i\omega) = \frac{K\tau i\omega}{i\omega\tau + 1} \tag{3.1.8-6}$$

For $\omega\tau >> 1$,

$$\frac{e_o}{x_i}(i\omega) \approx K \tag{3.1.8-7}$$

- To make $\omega\tau >> 1$ for low frequencies requires a large τ.
- For a given capacitor and x_o, the value of τ can be increased only by increasing R. Typically, R will be $10^6\ \Omega$ or more.
- To prevent loading of the capacitance transducer circuit, the readout device connected to the e_o terminals must have a high ($10^7\ \Omega$ or more) input impedance, such as provided by FET electronics.

3.1.9 Piezoelectric Transducers

- Because of piezoelectric effect, when certain solid materials are deformed, they generate within them an electric charge. This effect is reversible in that if a charge is applied, the material will mechanically deform in response.
- The mechanical-input/electrical-output direction is the basis of many instruments used for measuring acceleration, force, and pressure.
- The electrical-input/mechanical-output direction is applied in small vibration shakers, sonar systems for acoustic ranging and direction detection, industrial ultrasonic nondestructive test equipment, pumps for ink-jet printers, ultrasonic flowmeters, and micromotion actuators.
- Piezoelectric materials
    - Natural crystals (quartz (a), rochelle salt (b)) and synthetic crsystals (lithium sulfate (c), ammonium dihydrogen phosphate(d))
    - Polarized ferroelectric ceramics (barium titanate, etc.)
    - Polymer films

(a) (b) (c) (d) (e)

- Because of their natural asymmetric structure, the crystal materials exhibit the effect without further processing.
- The ferroelectric ceramics must be artificially polarized by applying a strong electric field to the material while it is heated to a temperature above the Curie point of that material and then slowly cooling with the field still applied. (The Curie temperature is the temperature above which a material loses its ferroelectric properties.) When the external field is removed from the cooled material, a polarization is retained and the material exhibits the piezoelectric effect.
- Metal electrodes are plated onto selected faces of the piezoelectric material so that lead wires can be attached for bringing in or leading out the electric charge.
- Since the piezoelectric materials are insulators, the electrodes also become the plates of a capacitor.
- A piezoelectric element used for converting mechanical motion to electric signals may be thought of as a charge generator and a capacitor. Mechanical deformation generates a charge; this charge then results in a definite voltage appearing between the electrodes according to the usual law for capacitors, $E = Q/C$.
- The piezoelectric effect is direction-sensitive in that tension produces a definite voltage polarity while compression produces the opposite.

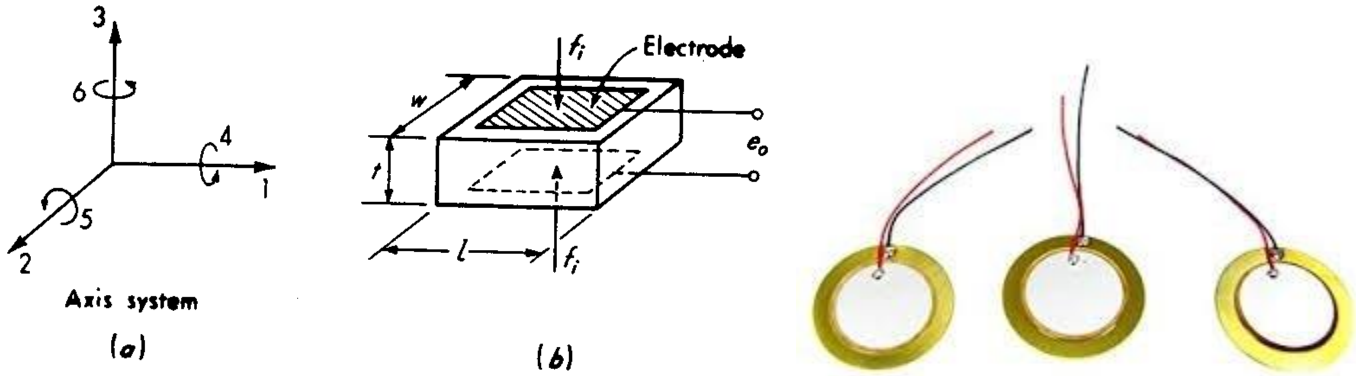


Figure 3.1.9-1 Piezoelectric Transducer

For a barium titanate thickness-expansion device, the pertinent g constant is g_{33}, which is defined as

$$g_{33} = \frac{\text{field produced in direction 3}}{\text{stress applied in direction 3}} = \frac{e_o/t}{f_i/(wl)} \quad (3.1.9\text{-}1)$$

- The first subscript of the constants refers to the direction of the electrical effect and the second to that of the mechanical effect.

To relate applied force to generated charge, the *d* constants can be defined as

$$d_{33} = \frac{\text{charge generated in direction 3}}{\text{force applied in direction 3}} = \frac{Q}{f_i} \qquad (3.1.9\text{-}2)$$

$$C = \frac{\varepsilon w l}{t} \qquad (3.1.9\text{-}3)$$

$$g_{33} = \frac{\text{field}}{\text{stress}} = \frac{e_o w l}{t f_i} = \frac{e_o C}{\varepsilon f_i} = \frac{Q}{\varepsilon f_i} = \frac{d_{33}}{\varepsilon} \text{ or } d_{33} = \varepsilon g_{33} \qquad (3.1.9\text{-}4)$$

- Sometimes it is desired to express the output charge or voltage in terms of deflection (rather than stress or force), since it is really the deformation that causes the charge generation. To do this, we must know the modulus of elasticity.
- The transducer impedance is generally very high; the amplifier is usually a high-impedance type used for buffering purposes rather than voltage gain.
- For the transducer alone, if a static deflection x_i is applied and maintained, a transducer terminal voltage will be developed but the charge will slowly leak off through the leakage resistance of the transducer. Since R_{leak} is generally very large (the order of 10^{11} Ω), this decay would be very slow.
- When an external voltage-measuring device of low input impedance is connected to the transducer, the charge leaks off very rapidly, preventing the measurement of static displacements. Even relatively high-impedance amplifiers generally do not allow static measurements.

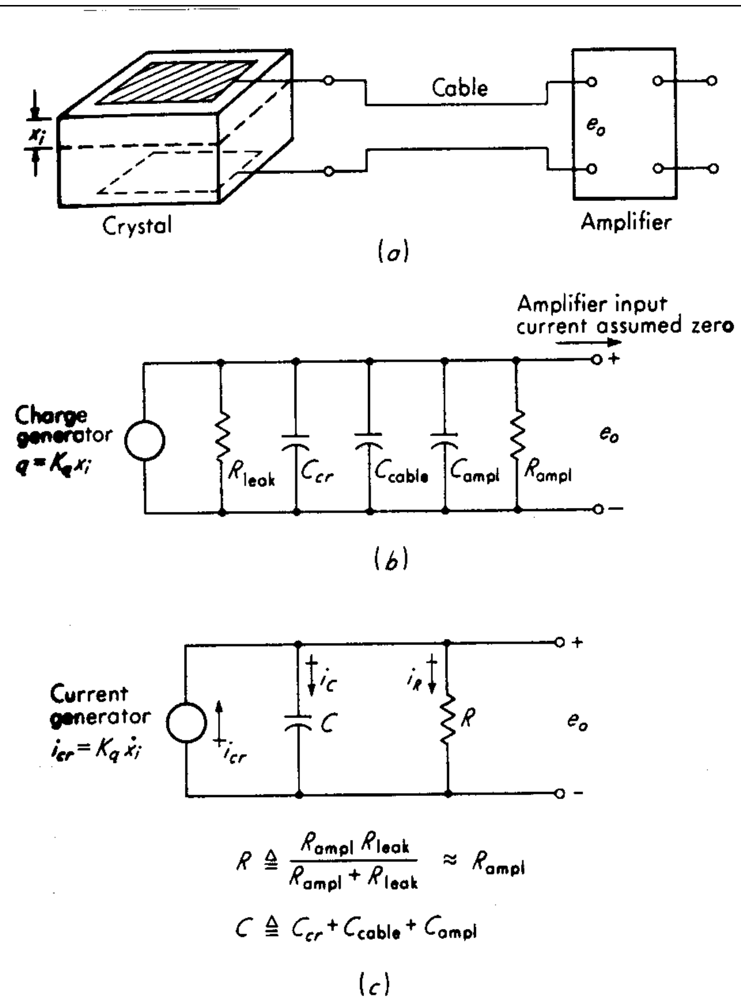


Figure 3.1.9-2 Equivalent Circuit for Piezoelectric Transducer

The charge generated by the crystal,

$$q = K_q x_i \tag{3.1.9-5}$$

where the unit of K_q is C.cm and x_i is deflection in cm.

$$i_{cr} = \frac{dq}{dt} = K_q \frac{dx_i}{dt} \tag{3.1.9-6}$$

$$i_{cr} = i_C + i_R \tag{3.1.9-7}$$

where $C = C_{cr}+C_{cable}+C_{ampl}$, and $R = (R_{ampl}R_{leak})/(R_{ampl}+R_{leak}) \approx R_{ampl}$.

$$e_o = e_C = \frac{\int i_C dt}{C} = \frac{\int (i_{cr} - i_R)dt}{C} \tag{3.1.9-8}$$

$$C\frac{de_o}{dt} = i_{cr} - i_R = K_q \frac{dx_i}{dt} - \frac{e_o}{R} \tag{3.1.9-9}$$

$$\frac{e_o}{x_i}(s) = \frac{K\tau s}{\tau s + 1} \tag{3.1.9-10}$$

K = sensitivity = K_q/C (V/cm), τ = time constant = RC s.

- The steady-state response to a constant x_i is zero; thus we cannot measure static displacements.

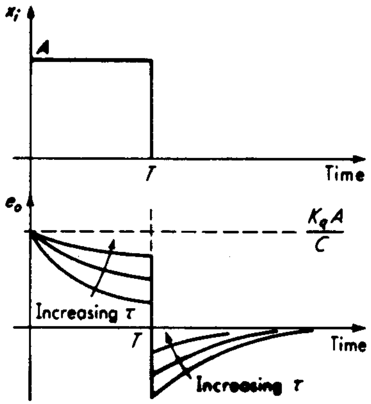


Figure 3.1.9-3 Response of Piezoelectric Transducer

- A large τ is desirable for faithful reproduction of x_i.
- If an increase of τ is required in a specific application, it may be achieved by increasing either or both *R* and *C*.
- An increase in *C* is easily obtained by connecting an external shunt capacitor across the transducer terminals, since shunt capacitors add directly. The price paid for this increase in τ is a loss of sensitivity according to $K = K_q/C$. Often this may be tolerated because of the initial high sensitivity of piezoelectric devices.
- An increase in *R* generally requires an amplifier of greater input resistance. If sensitivity can be sacrificed, a series resistor connected external to the amplifier will increase τ without the need of obtaining a different amplifier.

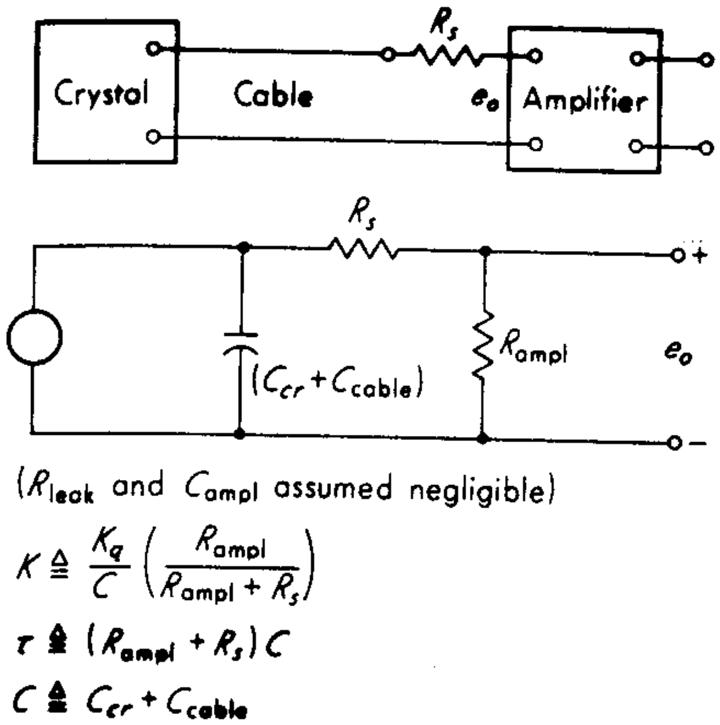


Figure 3.1.9-4 Use of Series Resistor to Increase Time Constant

3.1.10 Ultrasonic Transducers

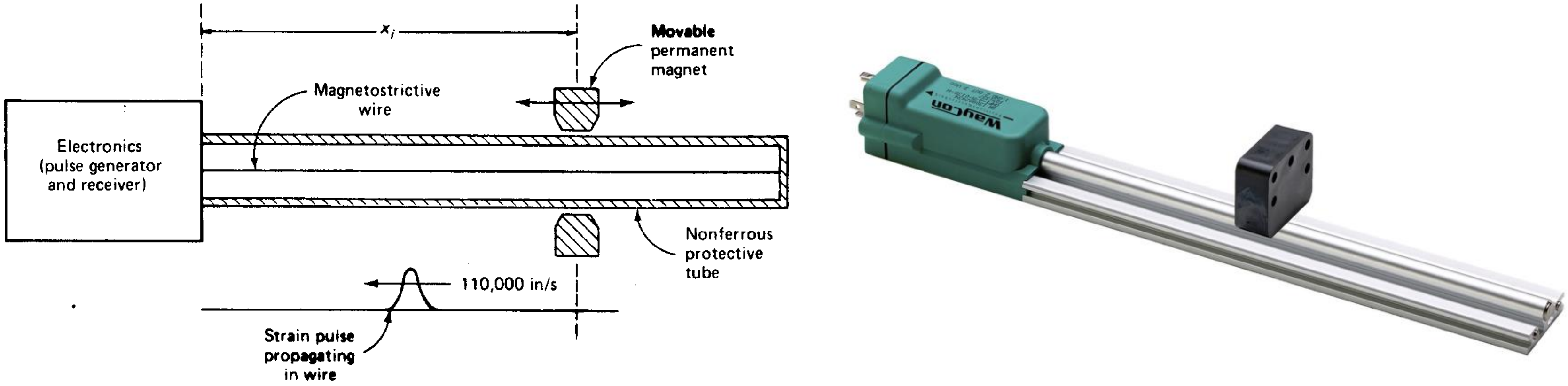


Figure 3.1.10-1 Ultrasonic Displacement Transducer

- Audible range: 20-20 kHz
- Ultrasonic: > 20 kHz
- Sound speed in the air ≈ 330 m/s, in water ≈ 1,550 m/s, in iron ≈ 5,120 m/s
- **Ultrasonic displacement transducer** has full-scale range of 1 to 10 ft or more.
- The transducer utilizes a permanent magnet which moves relative to a magnetostrictive wire enclosed in a nonferrous protective tube. Electronic circuitry drives a current pulse through the wire.

- At the magnet location, magnetostrictive action generates in the wire a stress pulse, which propagates to the receiver location at a fixed speed. At the receiver location, a pickup coil senses the arrival of the pulse. The time interval between the initiating current pulse and the arrival of the sensed stress pulse is proportional to the displacement x_i.
- In automatic focusing system of a camera, the motor drive which focuses the lens gets its information from an **ultrasonic rangefinder** (distance measuring) system.
- An electrostatic transducer is used as a loudspeaker and a microphone. An acoustic signal is sent out and its reflected return timed to allow calculation of the target's distance by use of the known propagation velocity.
- Simpler ultrasonic ranging systems (such as those used for tank liquid-level measurement) might use a single-frequency signal, the camera rangefinder must deal with targets of variable and unpredictable shape and size. This led to use of a multifrequency signal to ensure that a reflected echo would occur reliably.

- In a **digitizer**, used to find dimensions of solid objects out and input the digitized values to various computer-aided design programs, a pen whose tip contains a tiny electric spark source is used to trace the surface of the object. The spark discharge generates a sharp acoustic pulse, which propagates in all directions at known speed. Microphones at the four corners receive the pulses. Timing circuitry allows calculation of the four slant ranges, from which x, y, z coordinates can be calculated by a built-in microprocessor or the user's own general-purpose computer.

3.1.11 Displacement-to-Pressure (Nozzle-Flapper) Transducer

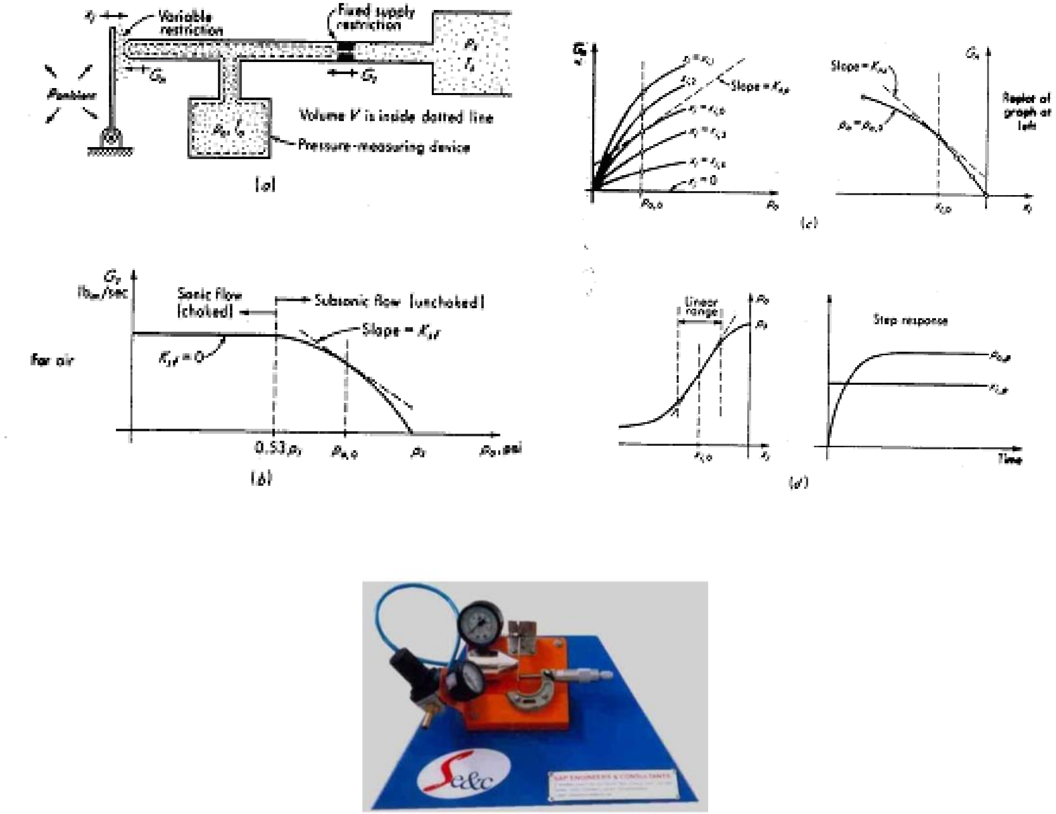


Figure 3.1.11-1Nozzle-Flapper Transducer

- In the nozzle-flapper-transducer, fluid at a regulated pressure is supplied to a fixed-flow restriction and a variable-flow restriction connected in series.
- The variable-flow restriction is varied by moving the flapper to change the distance x_i. This causes a change in output pressure p_o which, for a limited range of motion, is nearly proportional to x_i and extremely sensitive to it.
- A pressure-measuring device connected to p_o can be calibrated to read x_i.
- If the supply pressure p_s and temperature T_s are constant, mass-flow rate G_s depends on p_o only; however, the dependence is nonlinear, and a linearization is applied for small changes from an operating point.

$$G_s = G_s(p_o) \approx G_{s,0} + \left.\frac{dG_s}{dp_o}\right|_{p_o = p_{o,0}} (p_o - p_{o,0}) = G_{s,0} + K_{sp} p_{o,p} \tag{3.1.11-1}$$

where $G_{s,0}$ = value of G_s at equilibrium operating point, $p_{o,0}$ = value of p_o at equilibrium operating point, $p_{o,p}$ = small change in p_o from $p_{o,0}$, and K_{sp} = value of dG_s/dp_o at $p_{o,0}$ (constant).

- The nozzle mass flow rate G_n, depends on only p_o and x_i

$$G_n = G_n(p_o, x_i) \approx G_{n,0} + \left.\frac{\partial G_n}{\partial p_o}\right|_{\substack{p_o = p_{o,0} \\ x_i = x_{i,0}}} (p_o - p_{o,0}) + \left.\frac{\partial G_n}{\partial x_i}\right|_{\substack{p_o = p_{o,0} \\ x_i = x_{i,0}}} (x_i - x_{i,0}) = G_{n,0} + K_{np} p_{o,p} + K_{nx} x_{i,p} \tag{3.1.11-2}$$

- K_{np} and K_{nx} could be found from experimental data.
- The mass storage in volume V can be treated by using the perfect-gas law.

For constant *V*, *R*, and T_o,

$$p_o = \frac{RT_o}{V} M \tag{3.1.11-3}$$

$$p_{o,0} + p_{o,p} = \frac{RT_o}{V}(M_0 + M_p) \tag{3.1.11-4}$$

$$\frac{dp_{o,p}}{dt} = \frac{RT_o}{V}\frac{dM_p}{dt} \tag{3.1.11-5}$$

mass in - mass out = additional mass stored (3.1.11-6)

$$(G_{s,0} + K_{sp} p_{o,p})dt - (G_{n,0} + K_{np} p_{o,p} + K_{nx} x_{i,p})dt = dM_p = \frac{V}{RT_o} dp_{o,p} \tag{3.1.11-7}$$

- If the operating point $p_{o,0}$, $x_{i,0}$ is an equilibrium condition, then $G_{s,0} = G_{n,0}$.

$$\frac{V}{RT_o}\frac{dp_{o,p}}{dt} + (K_{np} - K_{sp}) p_{o,p} = (-K_{nx}) x_{i,p} \tag{3.1.11-8}$$

$$\frac{p_{o,p}}{x_{i,p}}(s) = \frac{K}{\tau s + 1} \tag{3.1.11-9}$$

where, $K = \frac{-K_{nx}}{K_{np} - K_{sp}}$ and $\tau = \frac{V}{RT_o(K_{np} - K_{sp})}$.

- To improve speed of response, the volume *V* should be minimized or $K_{np} - K_{sp}$ should be maximized.
- An increase in $K_{np} - K_{sp}$ will decrease the sensitivity also.

3.1.12 Electro-Optical Devices

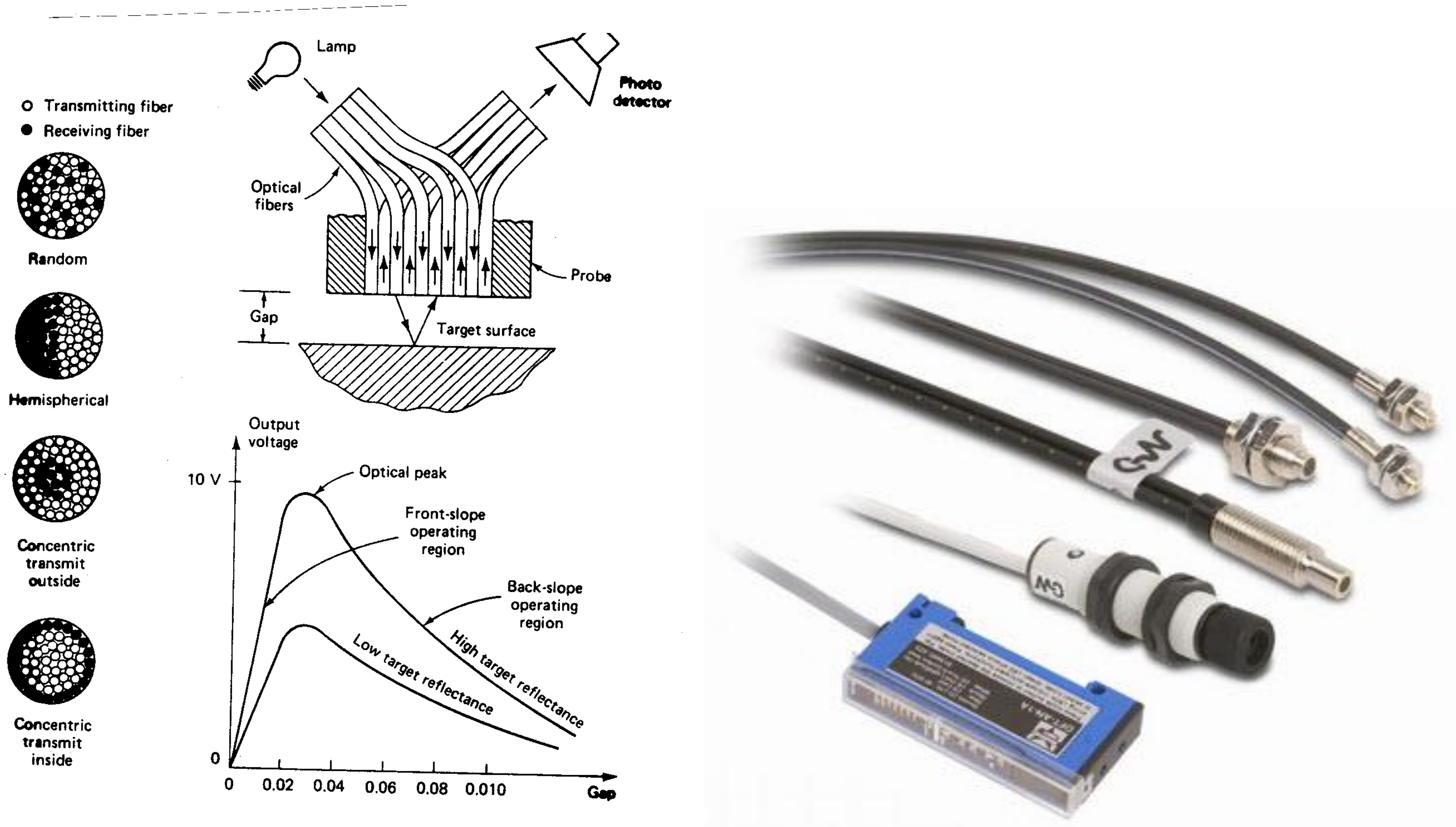


Figure 3.1.12-1 Fiber-Optic Displacement Transducer

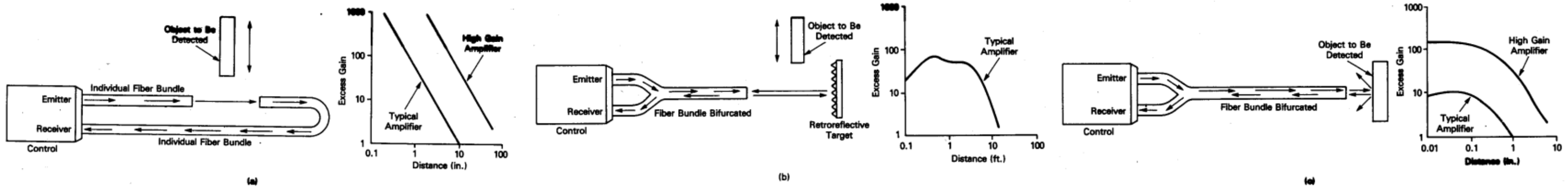


Figure 3.1.12-2 Measurement Techniques

- **Fotonic** or **photo-electric sensor** uses fiber optics to measure the small displacements.
- The optical fiber bundle of the sensor is divided into two groups of fibers.
    - One group (transmitting fibers) is exposed to a light source and thus carries light to the probe tip, where light is emitted and reflected/scattered by the target surface.
    - The reflected light is picked up by the other (receiving) group of fibers, transmitted to the electronics package, and focused on a suitable photodetector whose electronics then produces a dc output related to probe-target gap.

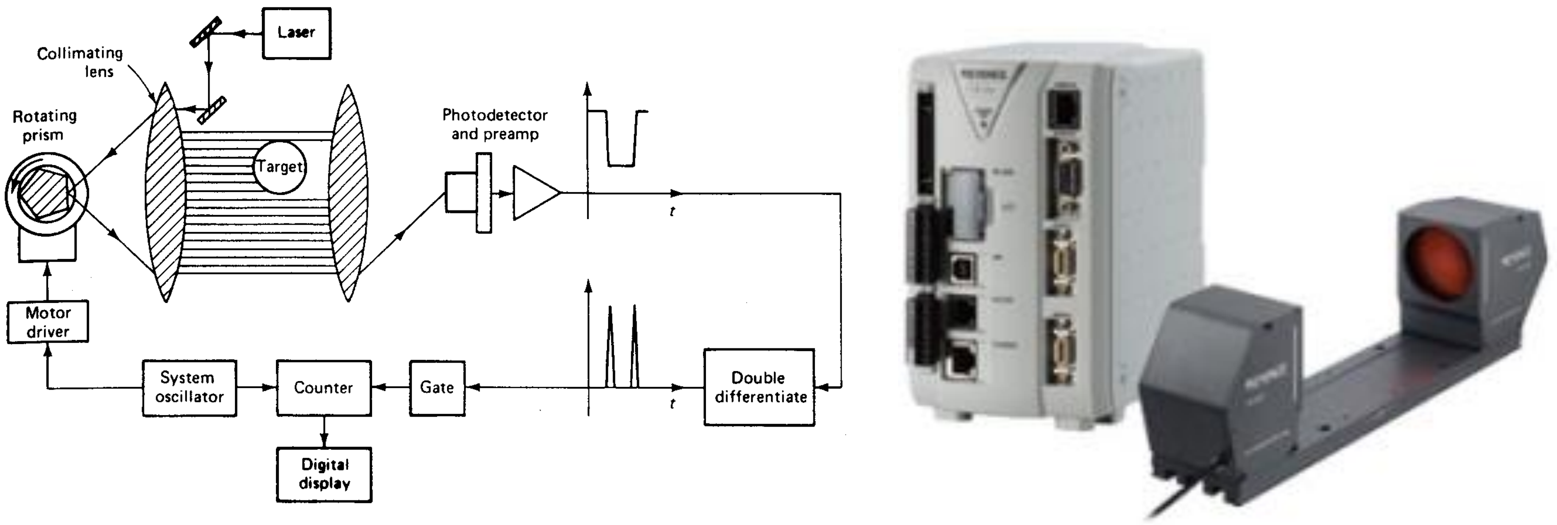
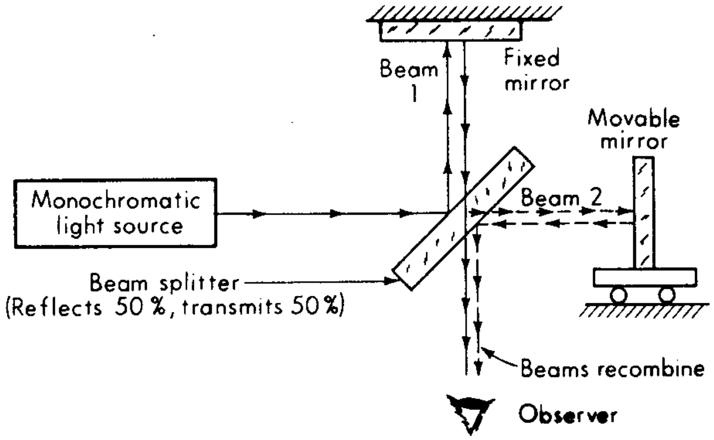


Figure 3.1.12-3 Laser Dimensional Gage

- In **laser dimensional gage**, a single narrow helium-neon laser beam is scanned over a workspace by a five-sided rotating.
- A special collimating lens produces parallel rays, which sweep through the workspace at a linear rate proportional to the prism's rotational speed.
- The position within the workspace of the cylindrical target, whose diameter is to be measured, can be obtained.

- As a result of the shadow cast by the target, the photodetector output voltage exhibits a notch whose width in time is proportional to the target width in space.
- Measurement of this time interval (by gating an electronic counter) is made more precise by electrically double-differentiating the photodetector signal to produce two narrow spikes.



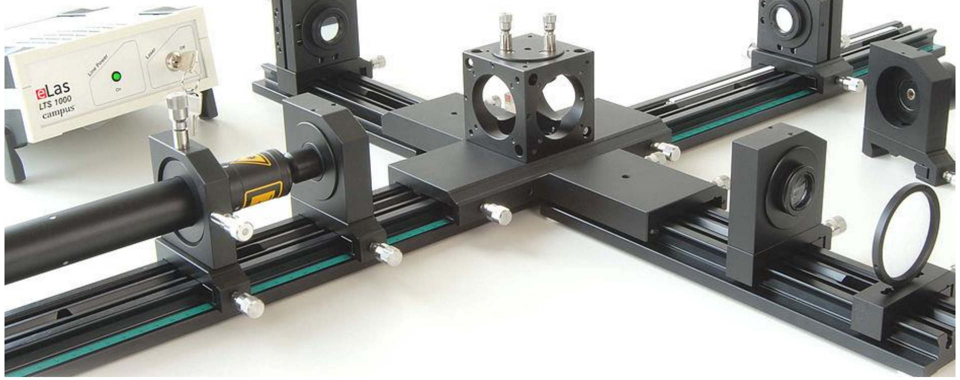


Figure 3.1.12-4 Laser Interferometer

- **Laser interferometer** uses the light-interference principle as a measurement tool.
- Using the wave model of light, the observer will see cycles of light and darkness as the motion of the movable mirror shifted the phase of beam 2 with respect to fixed beam 1, causing alternate reinforcement and interference of the two beams.
- If the light wavelength is known, for instance 0.5×10^{-6} m, then each 0.25×10^{-6} m of mirror movement corresponds to one complete cycle (light to dark to light) of illumination.
- By counting the number of illumination cycles, the distance between any two positions of the movable mirror can be determined.

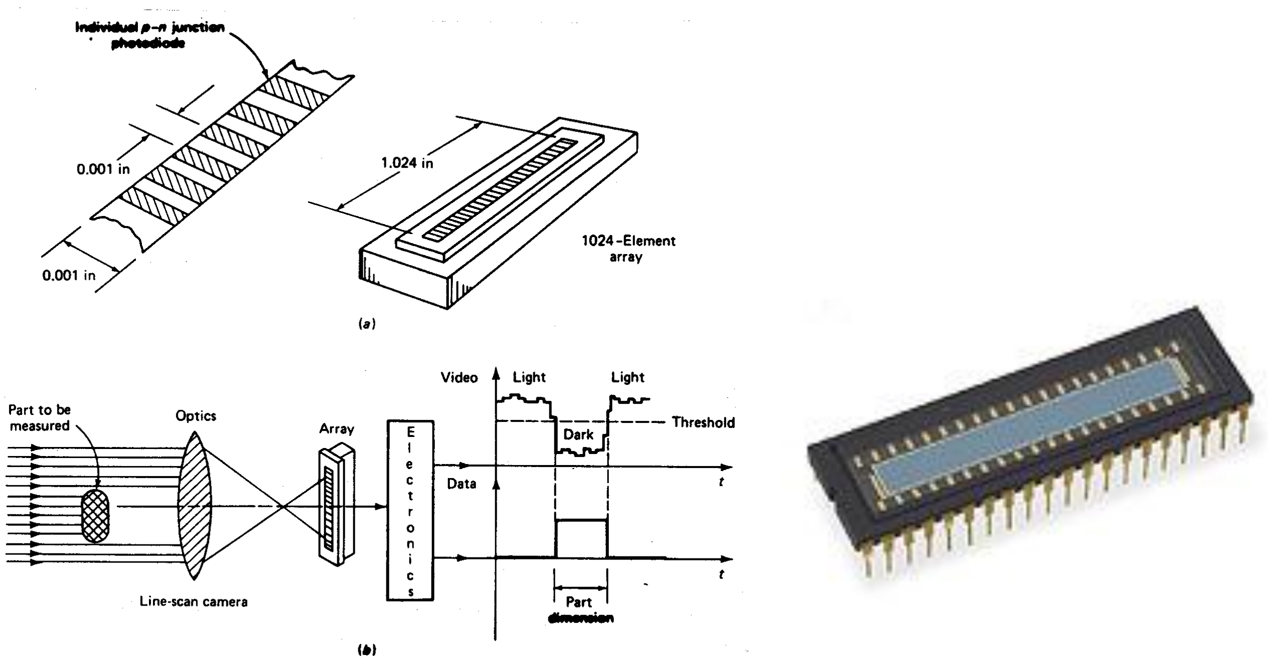


Figure 3.1.12-5 Self-Scanning Diode Arrays and Camera (a) 1024 Element Array (b) Line-Scan Camera

- The development of **self-scanning photodiode arrays** has made possible solid-state cameras for applications in pattern recognition, size and position measurement, etc.
- A linear array on a single silicon chip includes a row (or rows) of individual light-sensitive photodiodes, each with its own charge-storage capacitor and solid-state multiplex switch. Also contained on the chip is a shift register for serial readout of the individual element signals. This device is known as **CCD**, **Charge-Coupled Device**.

- The diodes transduce incident light into charge, which is stored on the capacitor until readout.
- The charge is proportional to the product of light intensity and exposure time; however, a saturation level does exist.
- A linear array can be used to construct a line-scan camera for dimensional measurement. The object to be measured is back-lighted so as to produce a light-dark pattern with transitions at the object's edges. Conventional optics focus an image on the photodiode array.
- Dimensional resolution at the array is limited by the diode spacing.
- The video signal is a boxcar (stepwise-changing) function that shows the time-integrated illumination of each individual picture element (called pixel) over one scan cycle. To get an unambiguous dimension measurement, the video signal is compared with a judiciously chosen threshold level to produce distinct switching in the data signal.

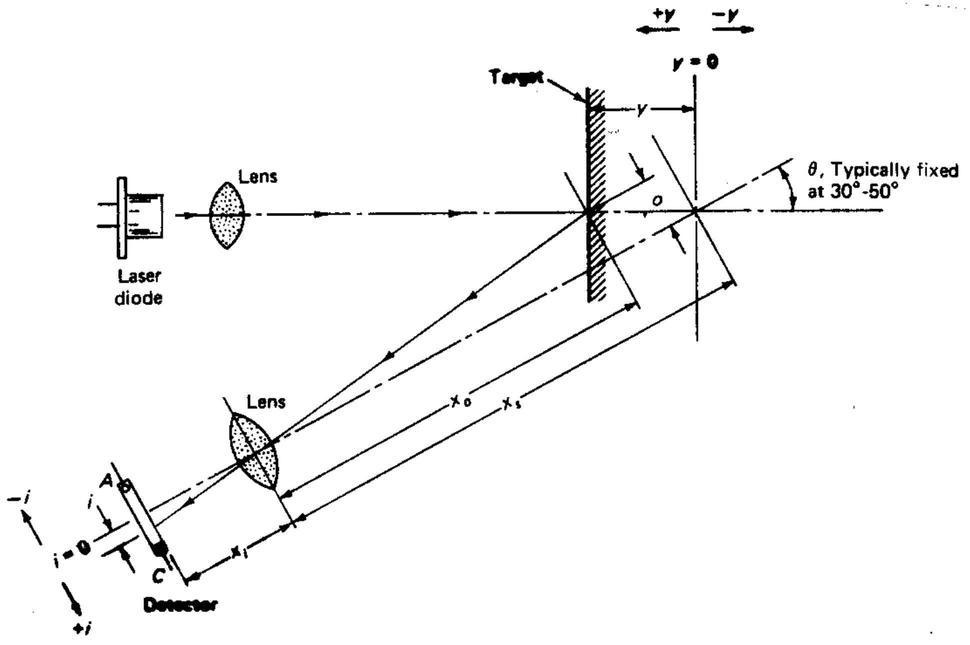


Figure 3.1.12-6 Laser Range Finder

- In **laser range finder**, a laser diode is the light source, projecting a spot onto the surface to be measured. Location of the image of the spot (formed by a suitable lens system) is a function of target displacement which can be determined by the triangulation principle.

3.1.13 Digital Displacement Transducers (Translational and Rotary Encoders)

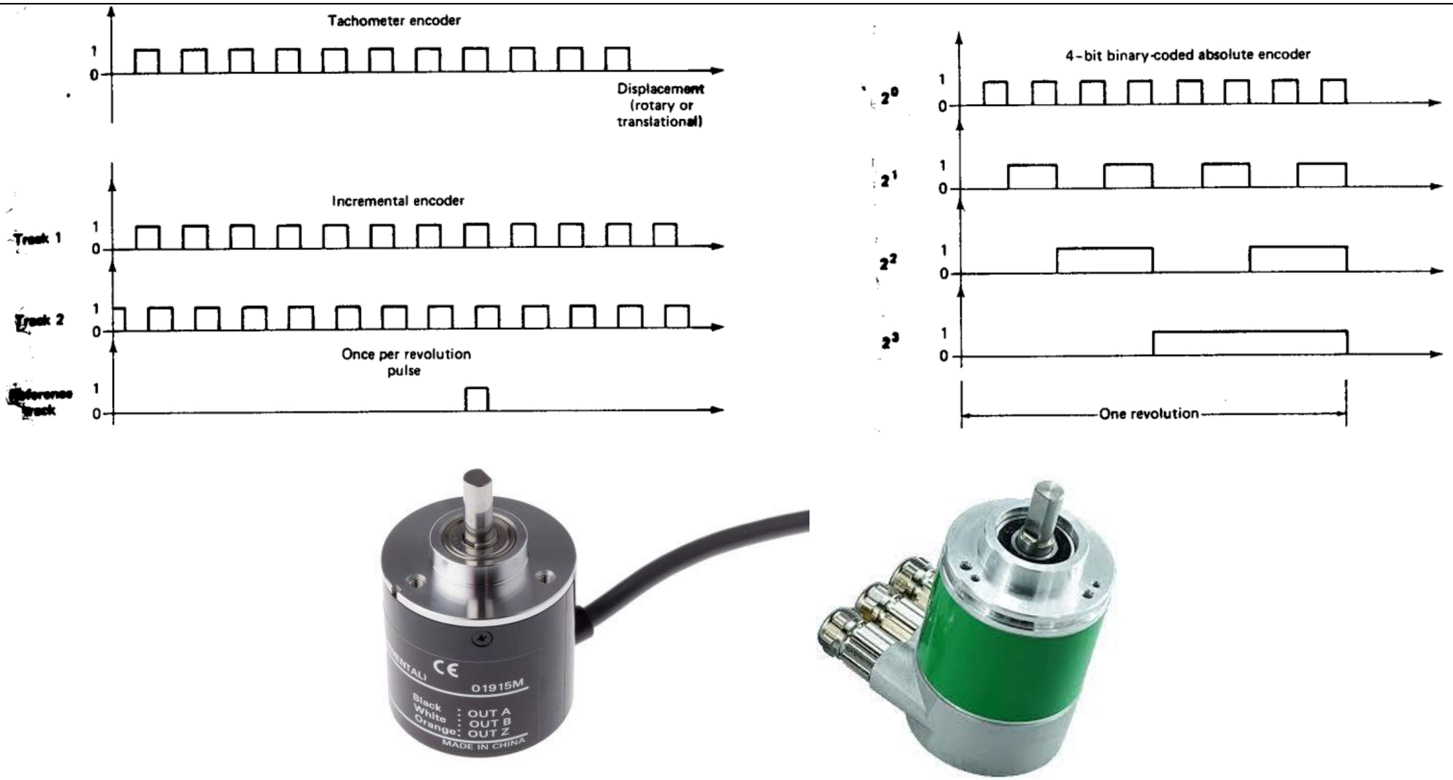


Figure 3.1.13-1 Three Major Classes of Encoders

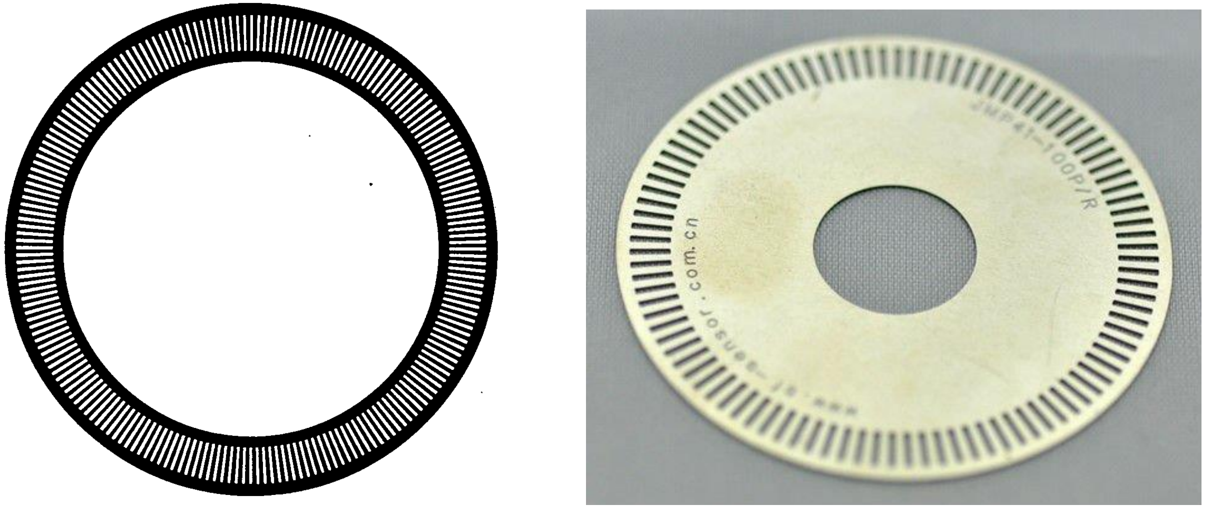


Figure 3.1.13-2 Tachometer-Encoder-Disk Pattern

- A **tachometer encoder** has only a single output signal, which consists of a pulse for each increment of displacement.
- If motion were always in one direction, a digital counter could accumulate these pulses to determine displacement from a known starting point. Any reversed motion would produce identical pulses, causing errors.
- Tachometer encoder usually is used for speed, rather than displacement, measurement in situations where rotation never reverses.

- The **incremental encoder** employs at least two (and sometimes three) signal-generating elements. By mechanically displacing the two tracks, one of the electric signals is shifted 1/4 cycle relative to the other, allowing detection of motion direction by noting which signal rises first.
- A third output, which produces a single pulse per revolution at a distinct point, is sometimes provided as a zero reference.
- An incremental encoder has the advantage of being able to rotate through as many revolutions as the application requires.
- Loss of system power causes total loss of position data with no recovery when power is reapplied.

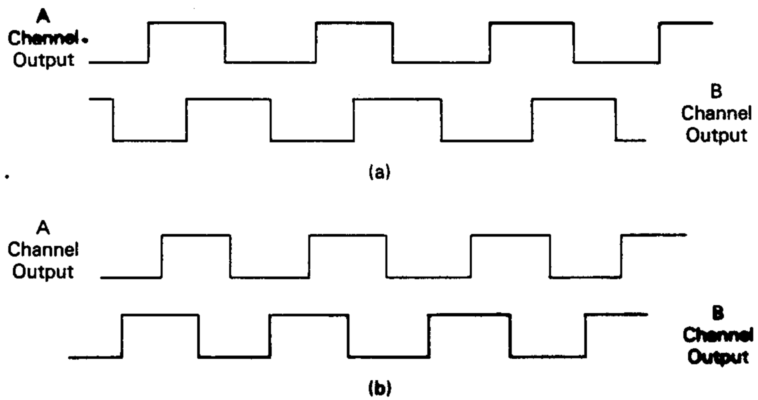


Figure 3.1.13-3 Output from Incremental Encoder

- Absolute encoders generally are limited to a single revolution and utilize multiple tracks and outputs, which are read out in parallel to produce a binary representation of the angular position of the shaft.
- Since there is a one-to-one correspondence between shaft position and binary output, position data are recovered when power is restored after an outage.

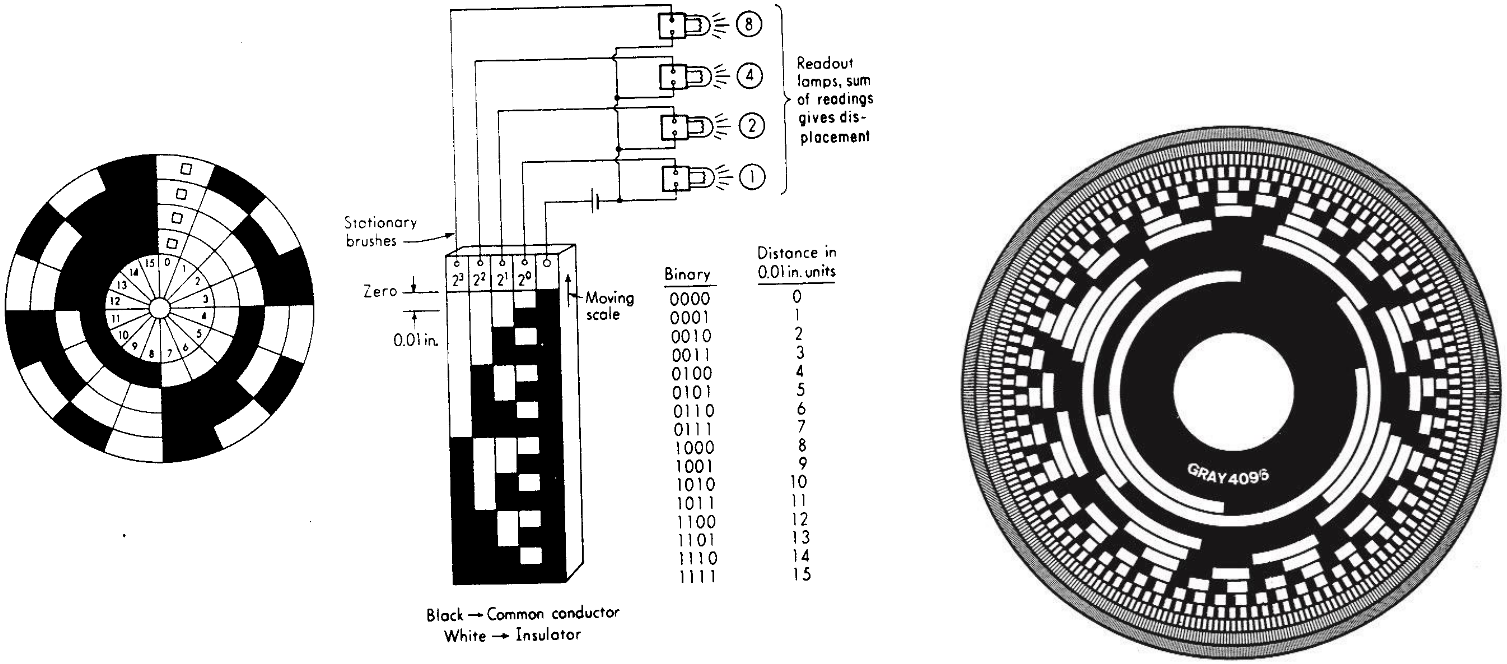


Figure 3.1.13-4 Disk Pattern and of Absolute Encoder

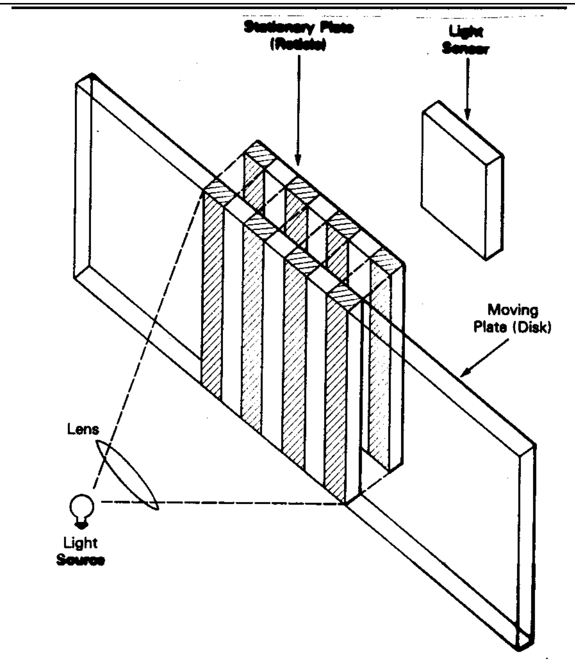


Figure 3.1.13-5 Tachometer or Incremental Encoder

- Tachometer and incremental encoders often employ a grating principle in which two glass disks (one fixed, the other rotating), with identical opaque/clear patterns photographically deposited, are mounted side by side.
- Parallel light is projected through the two disks toward photosensors on the far side.
- When opaque segments are aligned, a minimum (logical 0) signal is produced while alignment of clear segments gives a maximum (logical 1) signal.

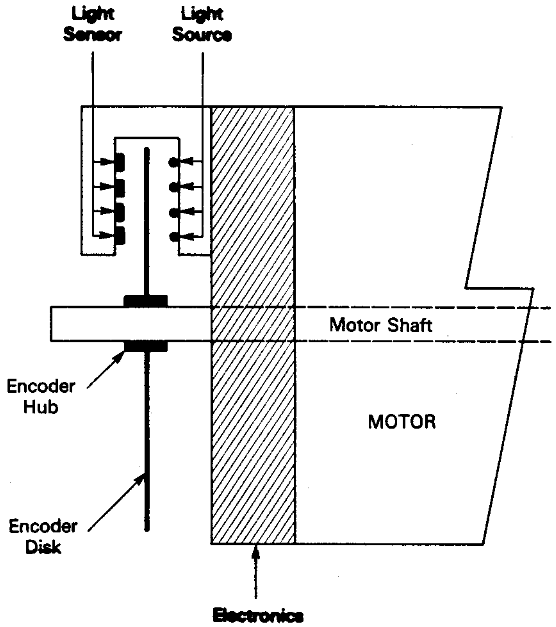


Figure 3.1.13-6 Absolute Encoder

- For absolute encoders, the light is sharply focused, rather than parallel, and only one disk is employed, with the narrow light beam and photosensor acting in the same fashion as the brushes in a contacting encoder.
- Many absolute encoders do not use binary code patterns, but use other patterns as the Gray code to avoid errors resulting from small misalignments possible in any real device.

- The Gray code does not suffer from the interference problem between channels since only one bit changes at each transition.
- Since the Gray-code output may not be compatible with the readout device, conversion from Gray to natural binary (or vice versa) is necessary and is easily accomplished by using standard logic gates.

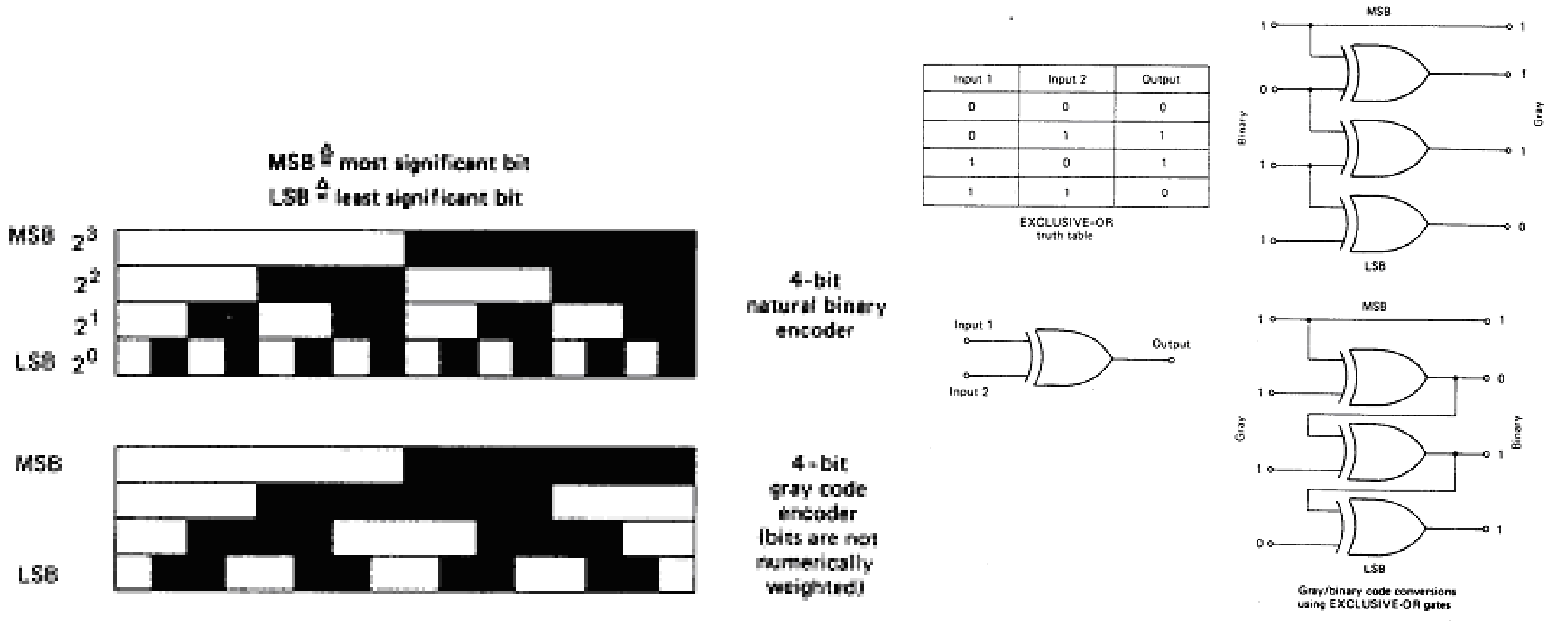


Figure 3.1.13-7 Natural and Gray Binary Codes

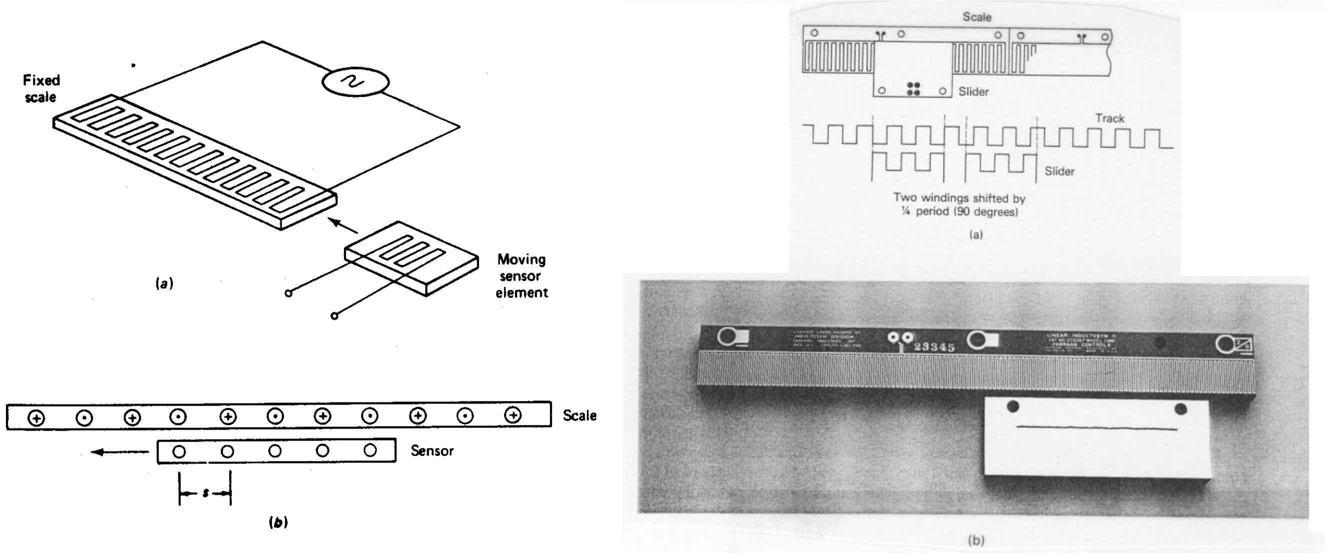


Figure 3.1.13-8 Inductosyn Transducer

- **Inductosyn** is a high-resolution incremental encoder based on the electromagnetic coupling between a fixed scale provided with an ac-excited conductor (produced by printed-circuit techniques) and a similar but smaller sensing winding which travels over the scale.
- When the two patterns are alignned, output is at a positive maximum.
- A displacement of $s/2$ results in minimum output, s gives negative maximum, $3s/2$ gives minimum again, and $2s$ returns the output to positive maximum.
- The output variation over the $2s$ cycle length is essentially cosinusoidal.

- A coarse digital output is obtained by counting the cycles of spacing 2*s*, while fine resolution is obtained by electronically digitizing the analog voltage variation within each cycle.
- To detect direction of motion, the sensor element includes a second winding displaced *s*/2 from the first, providing a sinusoidal signal. With both a sine and cosine output available, the device behaves essentially as a resolver.
- Inductosyns are available in both translational and rotary forms.

3.2 Relative Velocity, Translational and Rotational

3.2.1 Velocity by Electrical Differentiation of Displacement Voltage Signals

- A differentiating circuit to displacement analog information provides a voltage proportional to velocity.
- Differentiation accentuates any low-amplitude, high-frequency noise present in the displacement signal.

3.2.2 Average Velocity from Measured Δx and Δt

- Average velocity is determined from the distance over time interval.
- If the velocity is not constant, time interval should be set to small value.

3.2.3 Mechanical Flyball Angular-Velocity Sensor

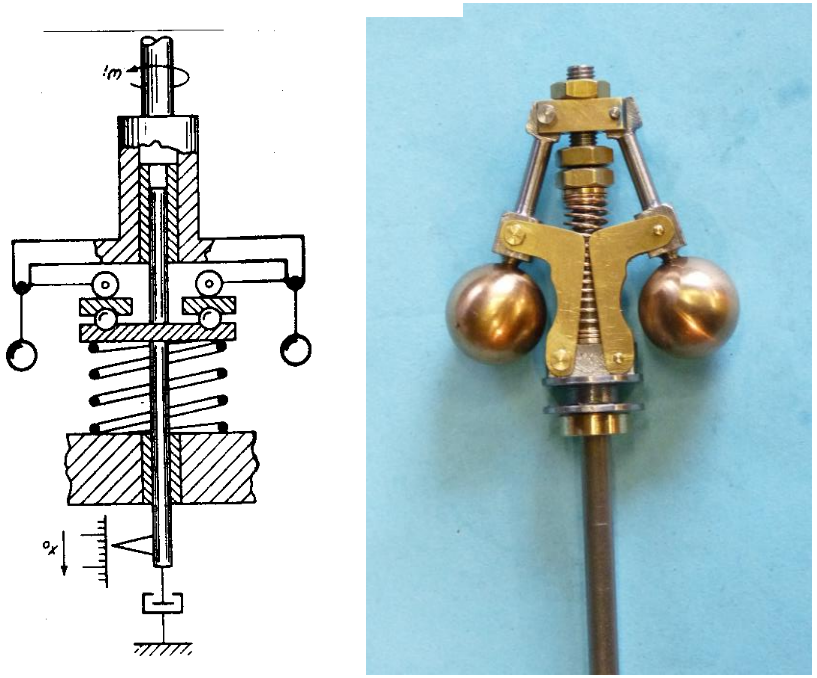


Figure 3.2.3-1 Flyball Velocity Pickup

- In flyball, the centrifugal force varies as the square of input velocity ω_i.

For small changes in ω_i a linearized model, showing the transfer function between ω_i and x_o,

$$\frac{x_o}{\omega_i}(s) = \frac{K}{s^2/\omega_n^2 + 2\zeta s/\omega_n + 1} \tag{3.2.3-1}$$

A nonlinear spring with $F_s = K_s x_o^2$ can be used to get a linear overall characteristic.

$$\text{centrifugal force} = \text{spring force} \qquad (3.2.3\text{-}2)$$

$$F_c = K_c \omega_i^2 = F_s = K_s x_o^2 \qquad (3.2.3\text{-}3)$$

and thus $x_o = \sqrt{K_c / K_s}\,\omega_i$, a linear relationship.

3.2.4 Stroboscopic Methods

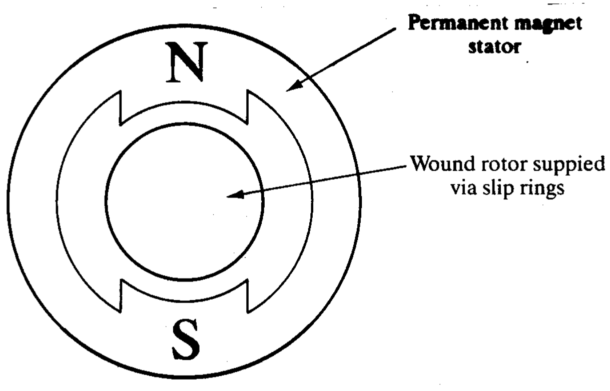
- Rotational velocity may be measured by using electronic stroboscopic lamps which flash at a known and adjustable rate. The frequency of lamp flashing is adjusted until the target appears motionless.
- Synchronism can be achieved at any flashing rate *r* that is an integral submultiple of the speed to be measured, *n*.
- The flashing rate is adjusted until synchronism is achieved at the largest possible flashing rate, say r_1. Then the flashing rate is slowly decreased until synchronism is again achieved at a rate r_2.

$$n = \frac{r_1 r_2}{r_1 - r_2} = \frac{\frac{n}{k} \cdot \frac{n}{(k+1)}}{\frac{n}{k} - \frac{n}{(k+1)}} = \frac{\frac{n^2}{k(k+1)}}{\frac{n}{k(k+1)}} = n \qquad (3.2.4\text{-}1)$$

For *N* times of synchronism (r_1, r_2, r_3, …, r_N),

$$n = \frac{r_1 r_N (N-1)}{r_1 - r_N} = \frac{\frac{n}{k} \cdot \frac{n}{(k+N-1)}(N-1)}{\frac{n}{k} - \frac{n}{(k+N-1)}} = \frac{\frac{n^2}{k(k+N-1)}(N-1)}{\frac{n(N-1)}{k(k+N-1)}} = n \qquad (3.2.4\text{-}2)$$

3.2.5 DC Tachometer Generators for Rotary-Velocity Measurement



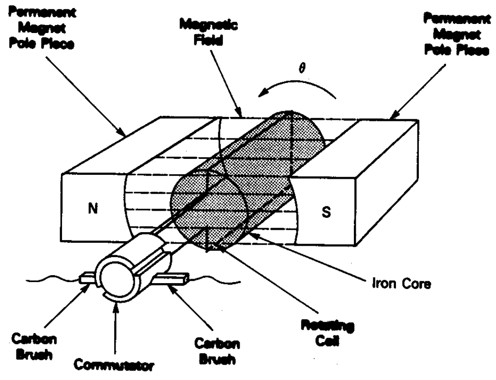


Figure 3.2.5-1 DC Tachometer

- An ordinary dc generator (using either a permanent magnet or separately excited field) produces an output voltage roughly proportional to speed.
- The voltage e_o is a dc voltage proportional to speed which reverses polarity when the angular velocity reverses.
- A small superimposed ripple voltage is present. Low-pass filtering is effective in reducing ripple at high speeds.

3.2.6 AC Tachometer Generators for Rotary-Velocity Measurement

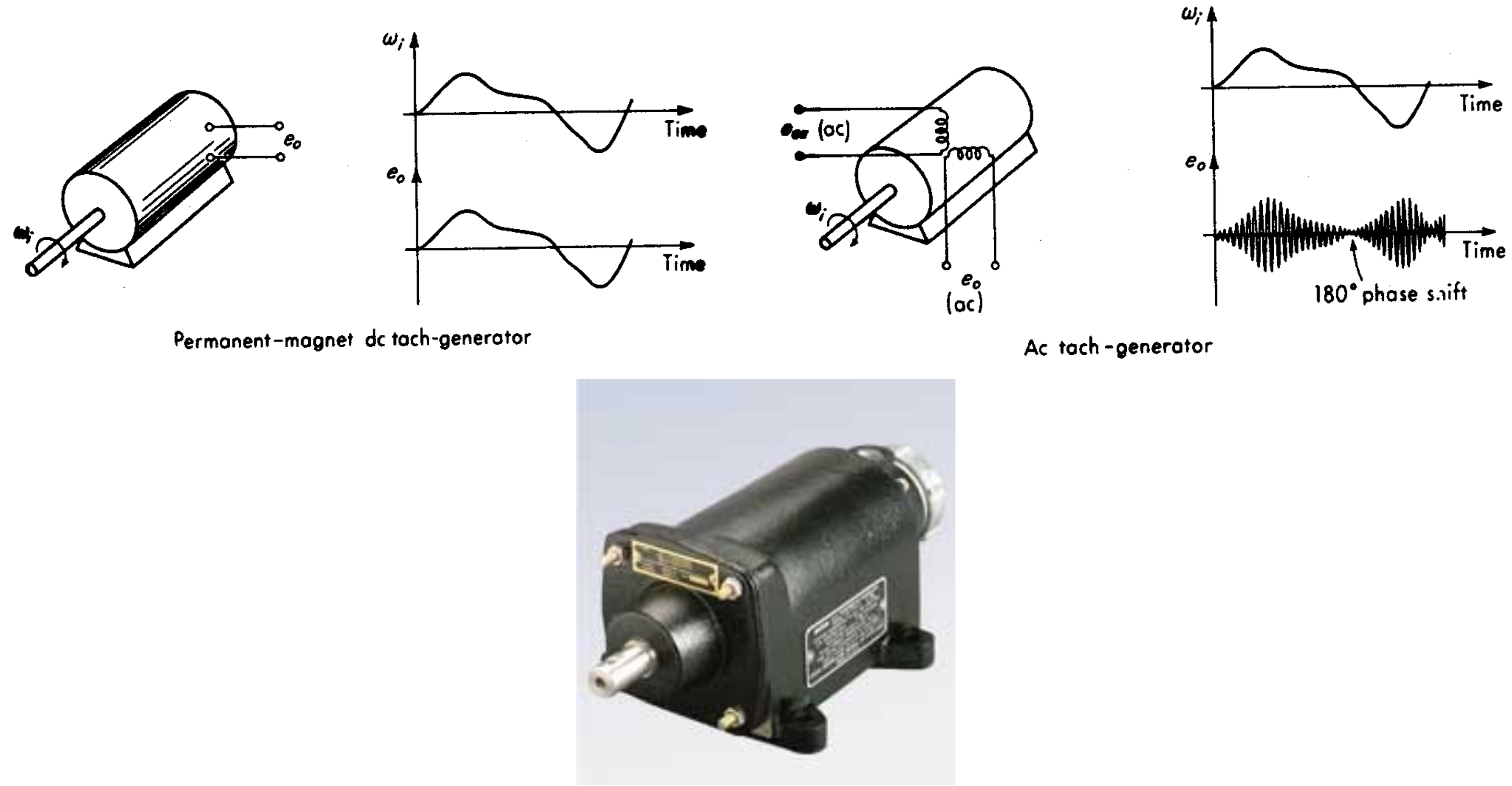


Figure 3.2.6-1 Tachometer Generator

- An ac two-phase squirrel-cage induction motor can be used as a tachometer by exciting one phase with its usual ac voltage and taking the voltage appearing at the second phase as output.
- With the rotor stationary, the output voltage is essentially zero.
- Rotation in one direction causes at the output an ac voltage of the same frequency as the excitation and of an amplitude proportional to the instantaneous speed. This output voltage is in phase with the excitation.
- Reversal of rotation causes the same action, except the phase of the output shifts 180°.

3.2.7 Eddy-Current Drag-Cup Tachometer

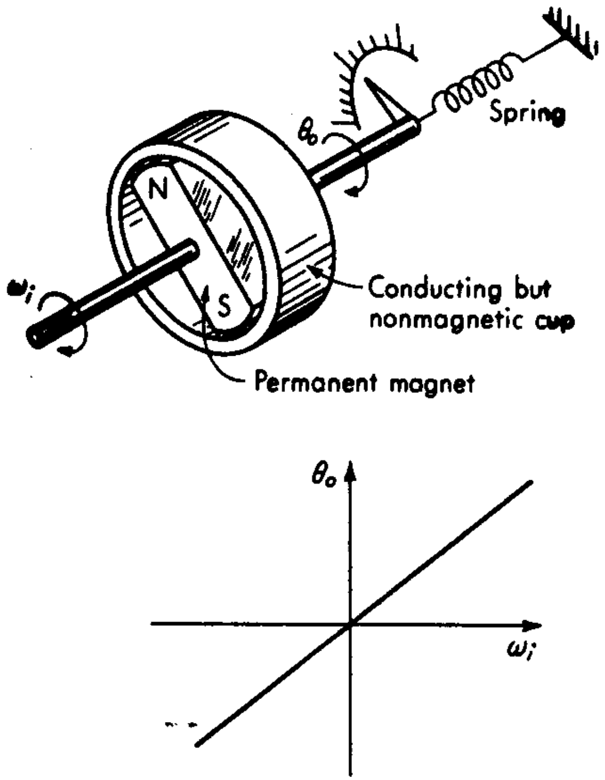


Figure 3.2.7-1 Drag-Cup Velocity Pickup

- Rotation of the magnet induces voltages into the cup, which thereby produces circulating eddy currents in the cup material.
- These eddy currents interact with the magnet field to produce a torque on the cup in proportion to the relative velocity of magnet and cup.
- This causes the cup to turn through an angle θ_o until the linear spring torque just balances the magnetic torque.
- In steady state the angle θ_o is directly proportional to ω_i, the input velocity.
- Dynamic operation is governed by the rotary inertia of parts moving with θ_o, spring stiffness, and the viscous damping effect of the eddy-current coupling between magnet and cup, leading to a second-order response.

$$\frac{\theta_o}{\omega_i}(s) = \frac{K}{s^2/\omega_n^2 + 2\zeta s/\omega_n + 1} \qquad (3.2.7\text{-}1)$$

3.3 Relative Acceleration

3.3.1 Seismic- (Absolute) Displacement Pickups

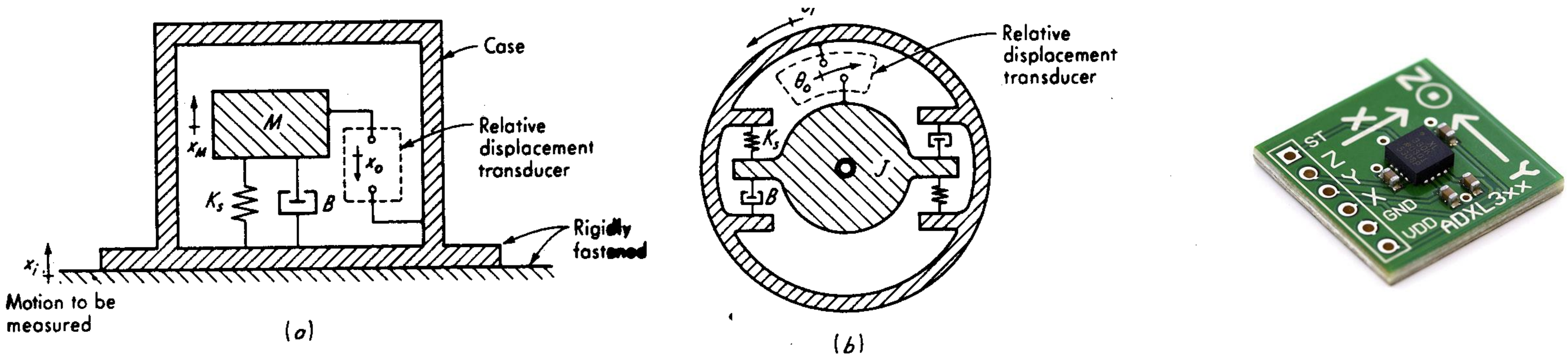


Figure 3.3.1-1 Translational and Rotational Seismic Pickups

- Accelerometers are used for measurement of acceleration and vibratory displacement in the cases where a fixed reference for relative-displacement measurement is not available.

$$K_s x_o + B\dot{x}_o = M\ddot{x}_M = M(\ddot{x}_i - \ddot{x}_o) \quad (3.3.1\text{-}1)$$

where x_i and x_M are the absolute displacements, x_o is reference displacement chosen such that x_o is zero when the gravity force, weight of *M*, is acting along the *x* axis statically.

$$\frac{x_o}{x_i}(s) = \frac{s^2/\omega_n^2}{s^2/\omega_n^2 + 2\zeta s/\omega_n + 1} \tag{3.3.1-2}$$

where $\omega_n = \sqrt{K_s/M}$ and $\zeta = B/(2\sqrt{K_s M})$.

The frequency response,

$$\frac{x_o}{x_i}(i\omega) = \frac{(i\omega/\omega_n)^2}{(i\omega/\omega_n)^2 + 2\zeta i\omega/\omega_n + 1} \tag{3.3.1-3}$$

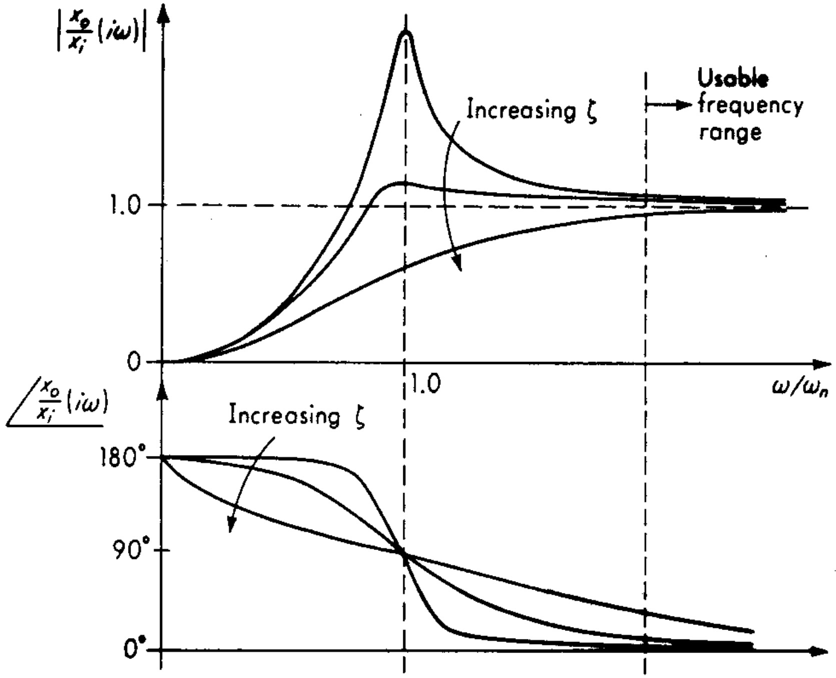


Figure 3.3.1-2 Seismic-Displacement-Pickup Frequency Response

3.3.2 Seismic- (Absolute-) Velocity Pickups

- To measure velocity $\dot{x}_i$ rather than displacement x_i, the relative-displacement transducer is replaced by a relative-velocity transducer which the output is represented by the relation $e_o = K_e \dot{x}_o$.

$$\frac{e_o}{\dot{x}_i}(s) = \frac{K_e s x_o}{s x_i}(s) = \frac{K_e x_o}{x_i}(s) = \frac{K_e s^2/\omega_n^2}{s^2/\omega_n^2 + 2\zeta s/\omega_n + 1} \quad (3.3.2\text{-}1)$$

$$\frac{x_o}{s x_i}(s) = \frac{s/\omega_n^2}{s^2/\omega_n^2 + 2\zeta s/\omega_n + 1} \quad (3.3.2\text{-}2)$$

The frequency response,

$$\frac{x_o}{\dot{x}_i}(s) = \frac{(i\omega)/\omega_n^2}{(i\omega)^2/\omega_n^2 + 2\zeta i\omega/\omega_n + 1} = \frac{1}{2\zeta\omega_n - i\left[\left(\omega_n^2 - \omega^2\right)/\omega\right]} \quad (3.3.2\text{-}3)$$

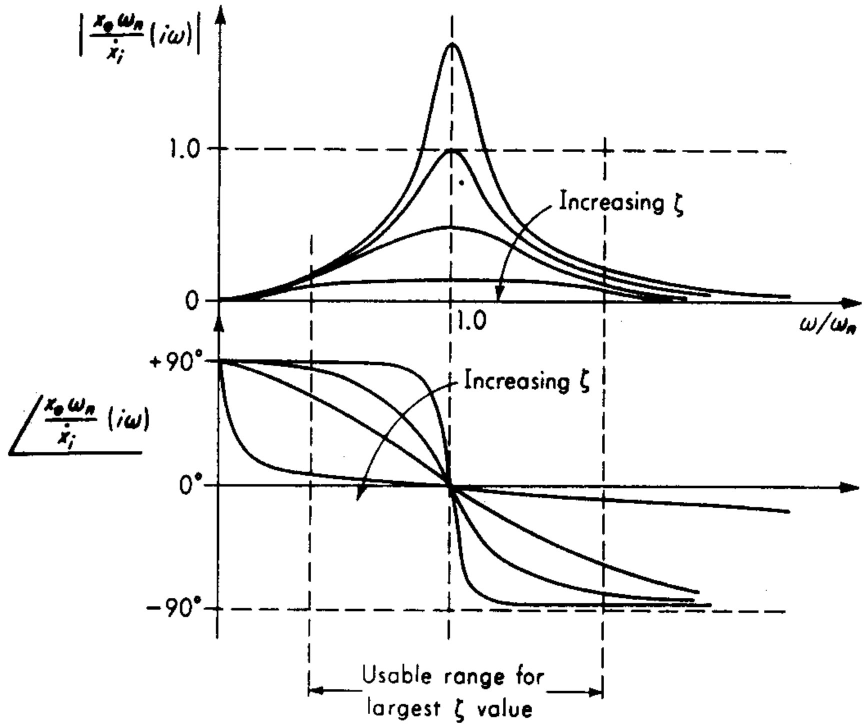


Figure 3.3.2-1 Seismic-Velocity-Pickup Frequency Response

3.3.3 Seismic- (Absolute-) Acceleration Pickups (Accelerometers)

- If the acceleration $\ddot{x}_i$ to be measured is constant. Then, in steady state, the mass *M* will be at rest relative to the case, and thus its absolute acceleration will also be $\ddot{x}_i$.
- If mass *M* is accelerating at $\ddot{x}_i$, there must be some force to cause this acceleration, and if *M* is not moving relative to the case, this force can come only from the spring.
- Since spring deflection x_o is proportional to force, which in turn is proportional to acceleration, x_o is a measure of acceleration $\ddot{x}_i$.
- The majority of accelerometers may be classified as either deflection type or null-balance type.
- The accelerometer used for vibration and shock measurement are usually the deflection type whereas those used for measurement of gross motions of vehicles (submarines, aircraft, spacecraft, etc.) may be either type, with the null-balance being used when extreme accuracy is needed.

3.3.3.1 Deflection-Type Accelerometers

$$\frac{x_o}{s^2 x_i}(s) = \frac{x_o}{\ddot{x}_i}(s) = \frac{1/\omega_n^2}{s^2/\omega_n^2 + 2\zeta s/\omega_n + 1} \quad (3.3.3.1\text{-}1)$$

$$K_s x_o = M\ddot{x}_o = M\ddot{x}_i; \ddot{x}_i = \frac{K_s x_o}{M} \quad (3.3.3.1\text{-}2)$$

$$\omega_n^2 = \frac{K_s}{M} \quad (3.3.3.1\text{-}3)$$

3.3.3.2 Null-Balance- (Servo-) Type Accelerometers

- Servoaccelerometers using the principle of feedback have been developed for applications requiring greater accuracy.
- In the null-balance instruments, the acceleration-sensitive mass is kept very close to the zero-displacement position by sensing this displacement and generating a magnetic force which is proportional to this displacement and which always opposes motion of the mass from neutral.
- This restoring force plays the same role as the mechanical spring force in a conventional accelerometer.
- The advantages derived from this approach are the greater linearity and lack of hysteresis of the electrical spring.

3.4 Gyroscopic (Absolute) Angular-Displacement and Velocity Sensors

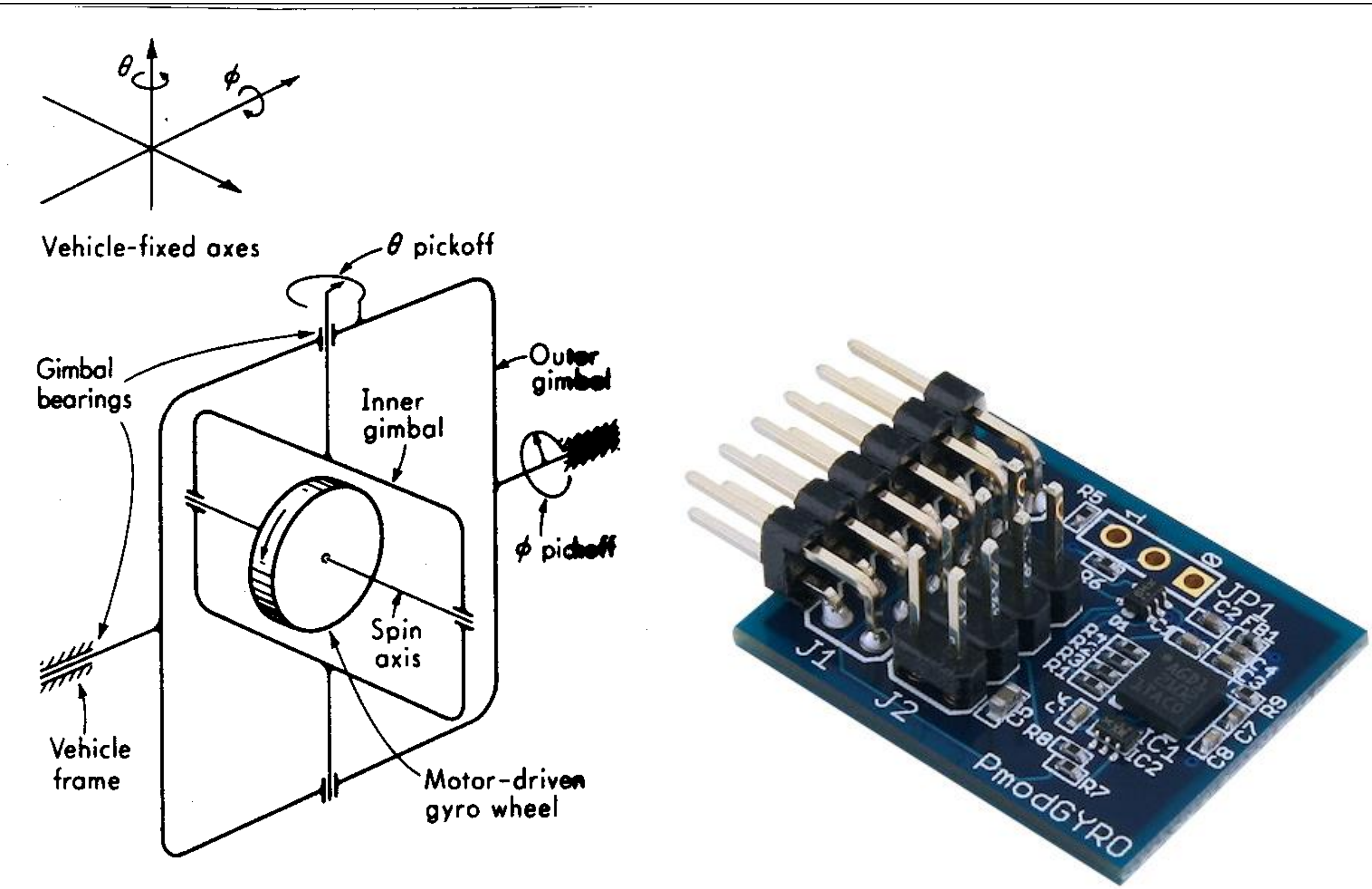


Figure 3.4-1 Free Gyroscope (Two-Axis Position Gyro)

- The free gyro is used to measure the absolute angular displacement of the vehicle to which the instrument frame is attached.
- A single free gyro can measure rotation about two perpendicular axes, the angles θ and ϕ.
- The axis of the spinning gyro wheel remains fixed in space (if the gimbal bearings are frictionless) and thus provides a reference for the relative-motion transducers.
- If the angles to be measured do not exceed about 10°, the readings of the relative-displacement transducers give directly the absolute rotations with good accuracy.
- For larger rotations of both axes, however, there is an interaction effect between the two angular motions, and the transducer readings do not accurately represent the absolute motions of the vehicle.
- The free gyro is also limited to relatively short-time applications (less than about 5 min) since gimbal-bearing friction causes gradual drift (loss of initial reference) of the gyro spin axis.

A constant friction torque T_f causes a drift (precession) of angular velocity ω_d,

$$\omega_d = \frac{T_f}{H_s} \qquad (3.4\text{-}1)$$

H_s: the angular momentum of the spinning wheel.

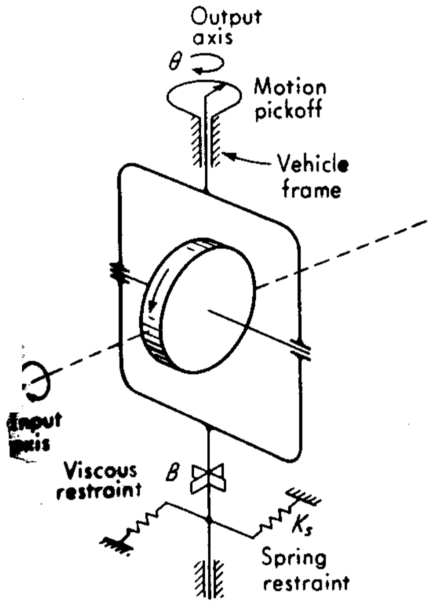


Figure 3.4-2 Single-Axis Restrained Gyro

- A single-axis gyro measures a single angle (or angular rate).
- This approach avoids the coupling or interaction problems of free gyros, and the constrained (rate-integrating) gyros can be constructed with exceedingly small drift.
- Two common types of the constrained gyros: the rate gyro and the rate-integrating gyro.

- The rate gyro measures absolute angular velocity and is widely used to generate stabilizing signals in vehicle control systems.
- The rate-integrating gyro measures absolute angular displacement and thus is utilized as a fixed reference in navigation and attitude-control systems.
- The rate-integrating gyro is functionally identical except that it has no spring restraint.

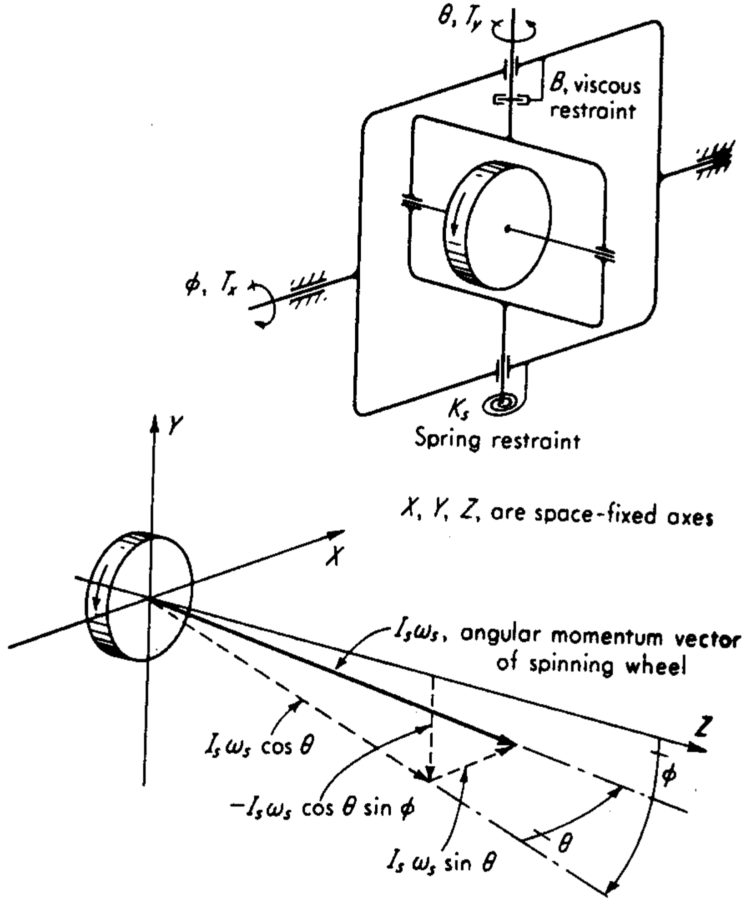


Figure 3.4-3 Gyro Analysis

Newton's law

$$\sum \text{ torques} = \frac{d}{dt}(\text{angular momentum}) \qquad (3.4\text{-}2)$$

For the *x* axis

$$T_x = \frac{d}{dt}\left(H_s \sin\theta + I_x \frac{d\phi}{dt} \right) \qquad (3.4\text{-}3)$$

For the *y* axis

$$T_y - B\frac{d\theta}{dt} - K_s\theta = \frac{d}{dt}\left(-H_s \cos\theta \sin\phi + I_y \frac{d\theta}{dt} \right) \qquad (3.4\text{-}4)$$

I_x : the moment of inertia of everything that rotates when the outer gimbal turns in its bearing

I_y : the moment of inertia of everything that rotates when the inner gimbal turns in its bearing

T_x and T_y : the external applied torques

H_s : the constant angular momentum from gyro wheel (the gyro wheel is driven by a constant-speed motor)

For small rotation, cos θ ≈ 1, sin θ ≈ θ, sin ϕ ≈ ϕ,

$$T_x = H_s \frac{d\theta}{dt} + I_x \frac{d^2\phi}{dt^2} \qquad (3.4\text{-}5)$$

$$T_y - B\frac{d\theta}{dt} - K_s\theta = -H_s \frac{d\phi}{dt} + I_y \frac{d^2\theta}{dt^2} \qquad (3.4\text{-}6)$$

$$\phi = \frac{\left(I_y s^2 + BD + K_s\right)T_x - \left(H_s s\right)T_y}{s^2\left(I_x I_y s^2 + BI_x s + H_s^2 + I_x K_s\right)} \tag{3.4-7}$$

$$\frac{\phi}{T_x}(s) = \frac{I_y s^2 + BD + K_s}{s^2\left(I_x I_y s^2 + BI_x s + H_s^2 + I_x K_s\right)} = G_1(s) \tag{3.4-8}$$

$$\frac{\phi}{T_y}(s) = -\frac{H_s}{s\left(I_x I_y s^2 + BI_x s + H_s^2 + I_x K_s\right)} = G_2(s) \tag{3.4-9}$$

$$\frac{\theta}{T_x}(s) = \frac{H_s}{s\left(I_x I_y s^2 + BI_x s + H_s^2 + I_x K_s\right)} = G_3(s) = -G_2(s) \tag{3.4-10}$$

$$\frac{\theta}{T_y}(s) = \frac{I_x}{I_x I_y s^2 + BI_x s + H_s^2 + I_x K_s} = G_4(s) \tag{3.4-11}$$

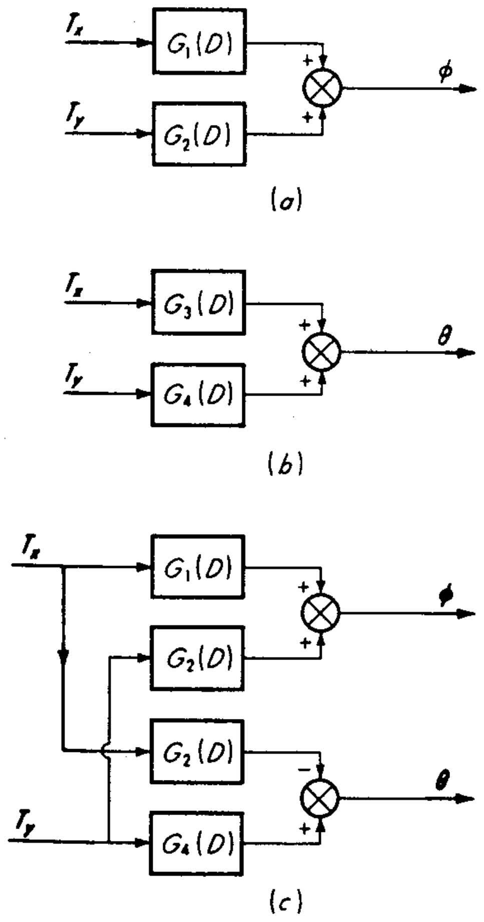


Figure 3.4-4 Gyro Block Diagrams

- For single-axis rate and rate-integrating gyros, the input is the motion ϕ, the torque T_x also exists and would be felt by the vehicle, the angle θ is an indication of the angle ϕ (rate-integrating gyro) or angular velocity $\dot{\phi}$ (rate gyro), the torque T_y (neglecting bearing friction) is zero.

$$T_y - Bs\theta - K_s\theta = -H_s s\phi + I_y s^2\theta; T_y = 0 \tag{3.4-12}$$

$$\frac{\theta}{\phi}(s) = \frac{H_s s}{I_y s^2 + Bs + K_s} \tag{3.4-13}$$

For a rate gyro,

$$\frac{\theta}{s\phi}(s) = \frac{\theta}{\dot{\phi}}(s) = \frac{K}{s^2/\omega_n^2 + 2\zeta s/\omega_n + 1} \tag{3.4-14}$$

where $K = H_s / K_s$, $\omega_n = \sqrt{K_s / I_y}$, and $\zeta = B / 2\sqrt{I_y K_s}$.

- A high sensitivity is achieved by large angular momentum H_s and soft spring K_s.
- Large angular momentum is obtained in small size by using high-speed motors to spin the gyro wheel.
- To measure all three components (roll, pitch, and yaw) of angular velocity in a vehicle, an arrangement of three rate gyros may be employed.

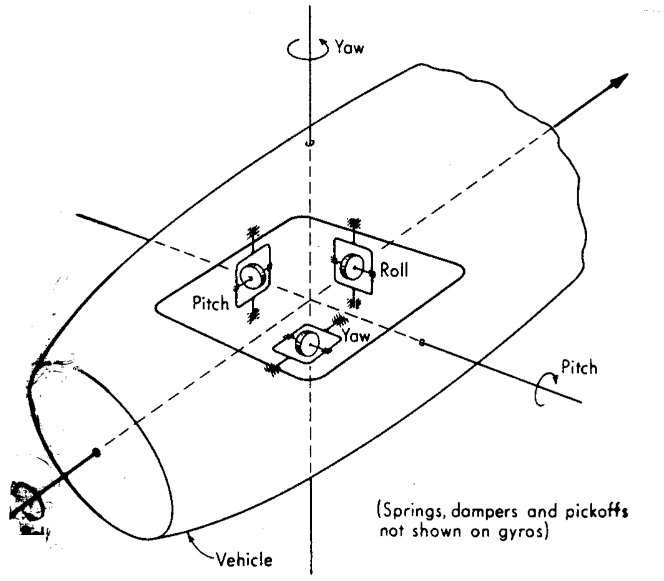


Figure 3.4-5 Three-Axis Rate-Gyro Package

To obtain a rate-integrating gyro, the spring restraint is removed.

$$\frac{\theta}{\phi}(s) = \frac{K}{\tau s + 1} \qquad (3.4\text{-}14)$$

where $K = H_s / B$ and $\tau = I_y / B$.

- The output angle θ is a direct indication of the input angle ϕ according to a standard first-order response form.
- High sensitivity again requires high H_s.

4. Force, Torque, and Power Sensors

- Force is defined by the equation $F = MA$.
- Mass is a fundamental quantity, and its standard is a cylinder of platinum-iridium, called the International Kilogram, kept in a vault at Sevres, France.
- Acceleration is not a fundamental quantity, derived from length and time.
- The acceleration of gravity, g, is a convenient standard which can be determined by measuring the period and effective length of a pendulum or by determining the change with time of the speed of a freely falling body.
- The actual value of g varies with location and also slightly with time (in a periodic predictable fashion) at a given location. It also may change (slightly) unpredictably because of local geological activity.
- The standard value of g refers to the value at sea level and 45° latitude and is numerically 980.665 cm/s^2.

The value at any latitude ϕ degrees,

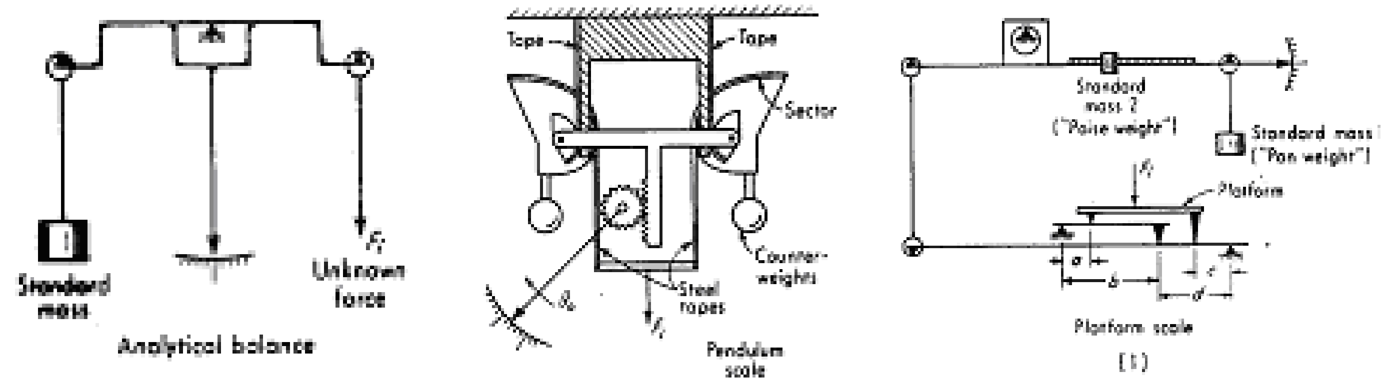
$$g = 978.049(1 + 0.0052884\sin^2\phi - 0.0000059\sin^2 2\phi)\ \text{cm/s}^2 \tag{4-1}$$

The correction for altitude h in meters above sea level,

$$\text{Correction} = -(0.00030855 + 0.00000022\cos 2\phi)h + 0.000072\left(\frac{h}{1000}\right)\ \text{cm/s}^2 \tag{4-2}$$

4.1 Basic Methods of Force Measurement

1. Balancing it against the known gravitational force on a standard mass, $F = mg$
2. Measuring the acceleration of a body of known mass to which the unknown force is applied, $F = ma$
3. Balancing it against a magnetic force developed by interaction of a current-carrying coil and a magnet, $F = iLB$
4. Transducing the force to a fluid pressure and then measuring the pressure, $F = PA$
5. Applying the force to some elastic member and measuring the resulting deflection, $F = kx$
6. Measuring the change in precession of a gyroscope caused by an applied torque related to the measured force, $T_x = H_s\dot{\theta}$
7. Measuring the change in natural frequency of a wire tensioned by the force, $F = 4m(\omega L)^2$



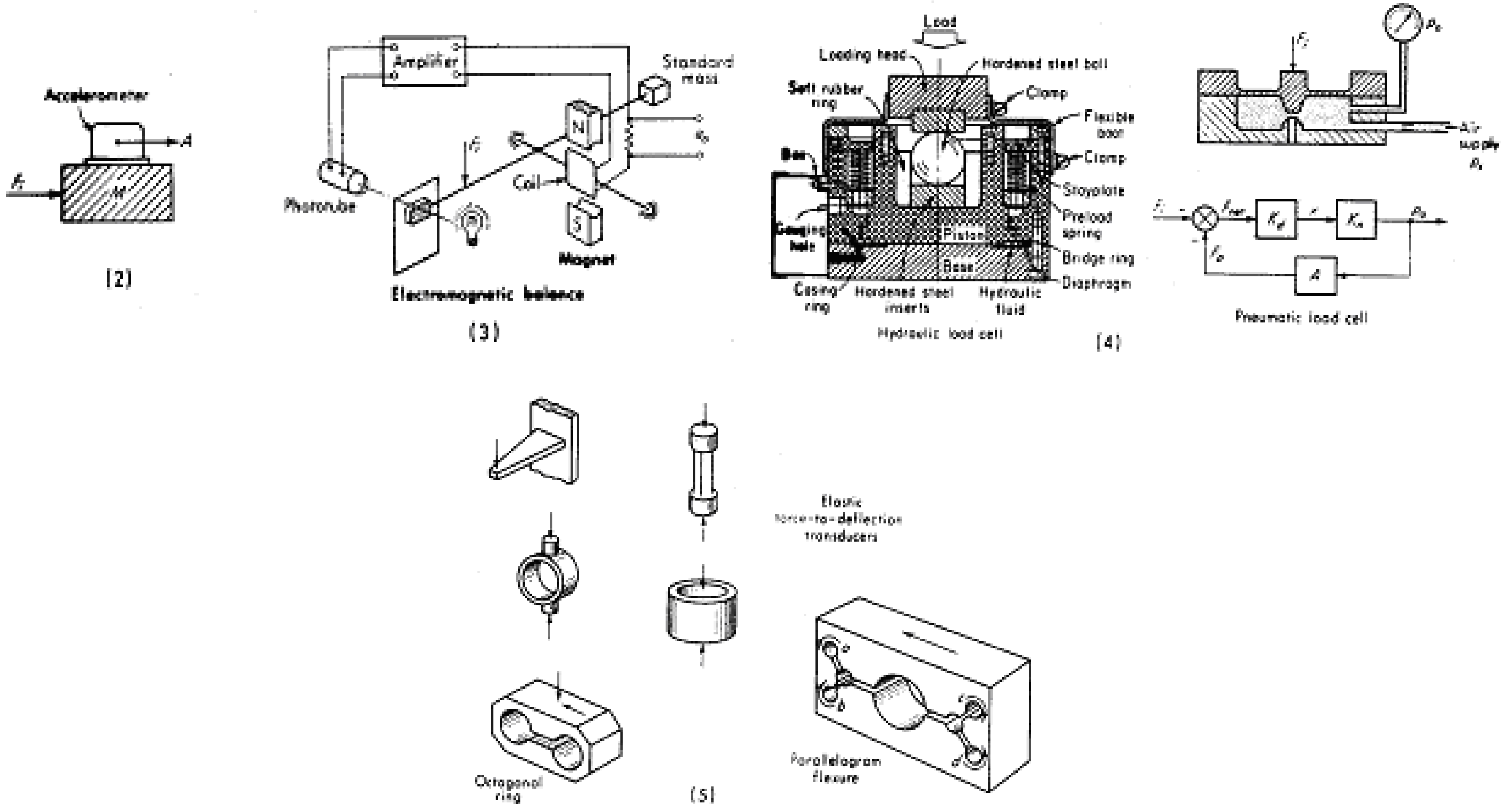


Figure 4.1-1 Basic Force-Measurement Methods

4.2 Characteristics of Elastic Force Transducers

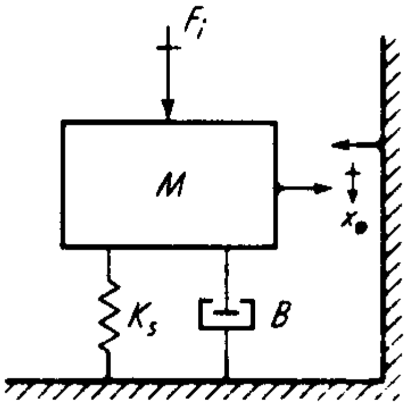


Figure 4.2-1 Elastic Force Transducer

$$F_i - K_s x_o - B\dot{x}_o = M\ddot{x}_o \quad (4.2\text{-}1)$$

$$\frac{x_o}{F_i}(s) = \frac{K}{s^2/\omega_n^2 + 2\zeta s/\omega_n + 1} \quad (4.2\text{-}2)$$

where $\omega_n = \sqrt{K_s / M}$, $\zeta = B/(2\sqrt{K_s M})$, and $K = 1/K_s$.

4.2.1 Bonded-Strain-Gage Transducers

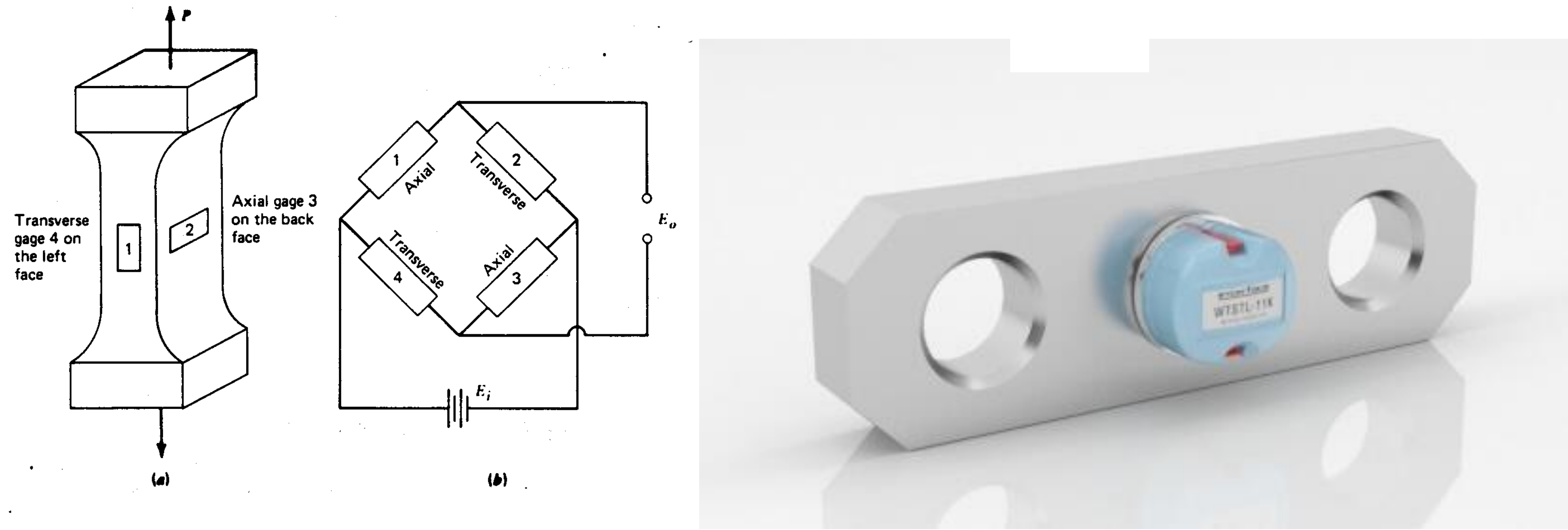


Figure 4.2.1-1 Link-Type Load Cell (a) Elastic Element with Strain Gages (b) Gage Positions in the Wheatstone Bridge

When the load *P* is applied to the link, axial and transverse strains ε_a and ε_t develop in the link.

$$\varepsilon_a = \frac{P}{AE} \quad (4.2.1\text{-}1)$$

$$\varepsilon_t = -\frac{vP}{AE} \quad (4.2.1\text{-}2)$$

where A = the cross-sectional area of the link, E = the modulus of elasticity of the link material, and v = Poisson's ratio of the link material.

The response of the gages to the applied load,

$$\frac{\Delta R_1}{R_1} = \frac{\Delta R_3}{R_3} = S_g \varepsilon_a = \frac{S_g P}{AE} \tag{4.2.1-3}$$

$$\frac{\Delta R_2}{R_2} = \frac{\Delta R_4}{R_4} = S_g \varepsilon_t = -\frac{v S_g P}{AE} \tag{4.2.1-4}$$

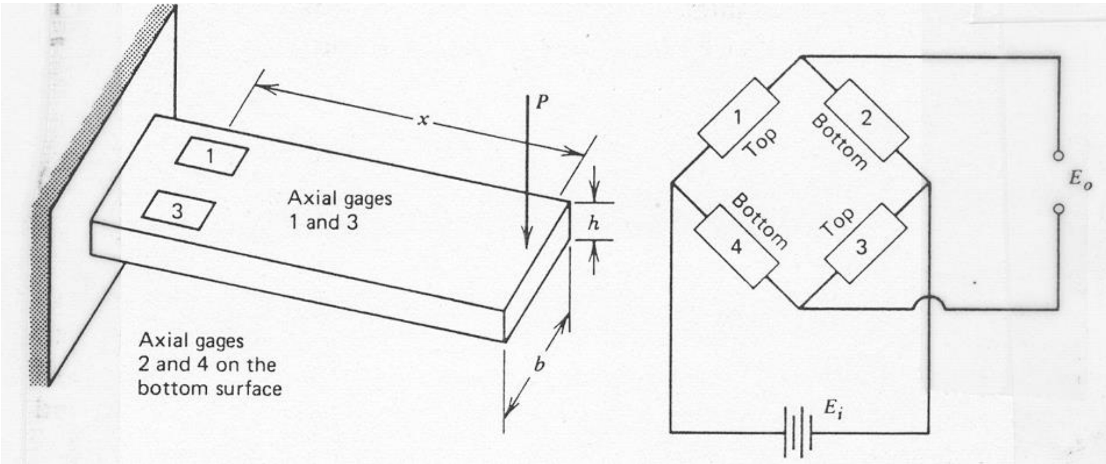
where s_g = gage factor.

If the four strain gages are assumed identical, the output voltage E_o from the Wheatstone bridge

$$E_o = \frac{S_g P(1+v)E_i}{2AE} \tag{4.2.1-5}$$

The sensitivity S of the load cell,

$$S = \frac{E_o}{P} = \frac{S_g (1+v)E_i}{2AE} \tag{4.2.1-6}$$



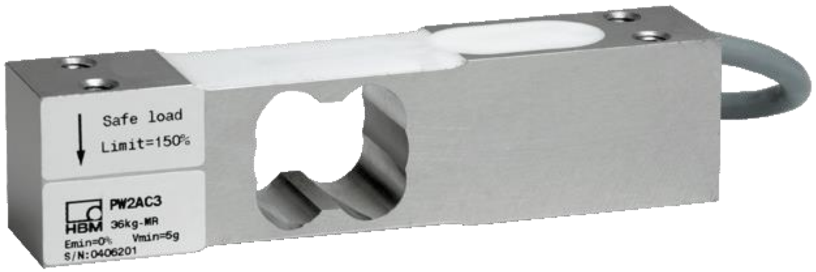


Figure 4.2.1-2 Beam-Type Load Cells (a) A Selection of Beam-Type Load Cells

(b) Elastic Element with Strain Gages (c) Gage Positions in the Wheatstone Bridge

The load P produces a moment $M = Px$ at the gage location x.

$$\varepsilon_1 = -\varepsilon_2 = \varepsilon_3 = -\varepsilon_4 = \frac{6M}{Ebh^2} = \frac{6Px}{Ebh^2} \tag{4.2.1-7}$$

where b = the width of the cross section of the beam, h = the height of the cross section of the beam, and E = modulus of elasticity of the beam.

The response of the strain gages,

$$\frac{\Delta R_1}{R_1} = -\frac{\Delta R_2}{R_2} = \frac{\Delta R_3}{R_3} = -\frac{\Delta R_4}{R_4} = \frac{6S_g Px}{Ebh^2} \tag{4.2.1-8}$$

where s_g = gage factor.

If the four strain gages are assumed identical, the output voltage E_o from the Wheatstone bridge,

$$E_o = \frac{6S_g PxE_i}{Ebh^2} \tag{4.2.1-9}$$

The sensitivity S of the load cell,

$$S = \frac{E_o}{P} = \frac{6S_g xE_i}{Ebh^2} \tag{4.2.1-10}$$

4.3 Resolution of Vector Forces and Moments into Rectangular Components

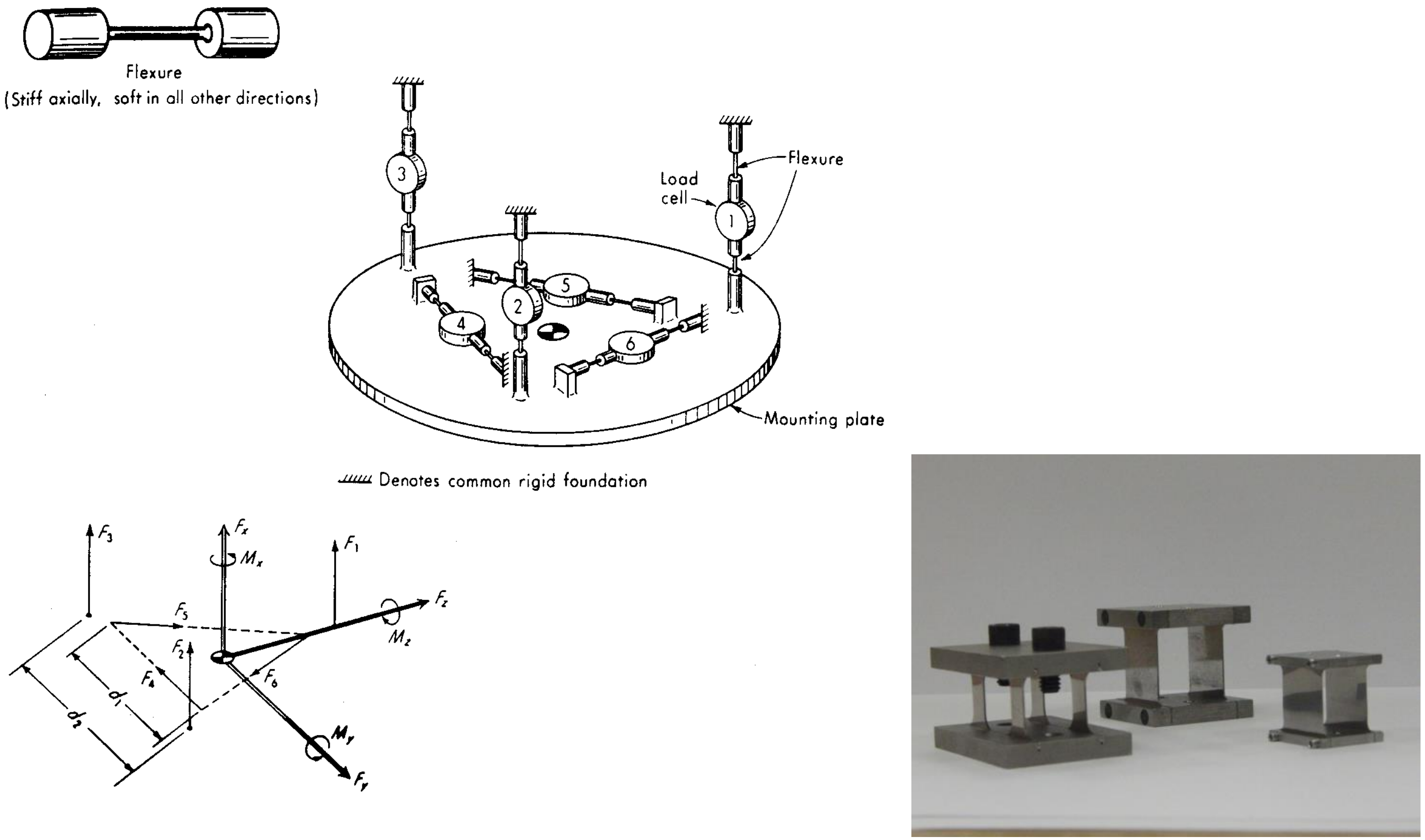


Figure 4.3-1 Six-Component Load Frame and Flexure

$$F_x = F_1 + F_2 + F_3 \tag{4.3-1}$$

$$F_y = \frac{F_5 - 2F_4 + F_6}{2} \tag{4.3-2}$$

$$F_z = \frac{\sqrt{3}(F_5 - F_6)}{2} \tag{4.3-3}$$

$$M_x = -d_1 \frac{F_4 + F_5 + F_6}{2\sqrt{3}} \tag{4.3-4}$$

$$M_y = d_2 \frac{2F_1 - F_2 - F_3}{2\sqrt{3}} \tag{4.3-5}$$

$$M_z = d_2 \frac{F_3 - F_2}{2} \tag{4.3-6}$$

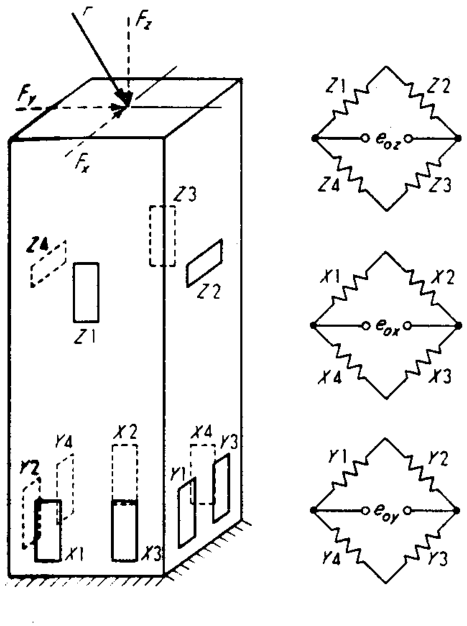


Figure 4.3-2 Resolution of Vector Forces

4.4 Torque Measurement on Rotating Shafts

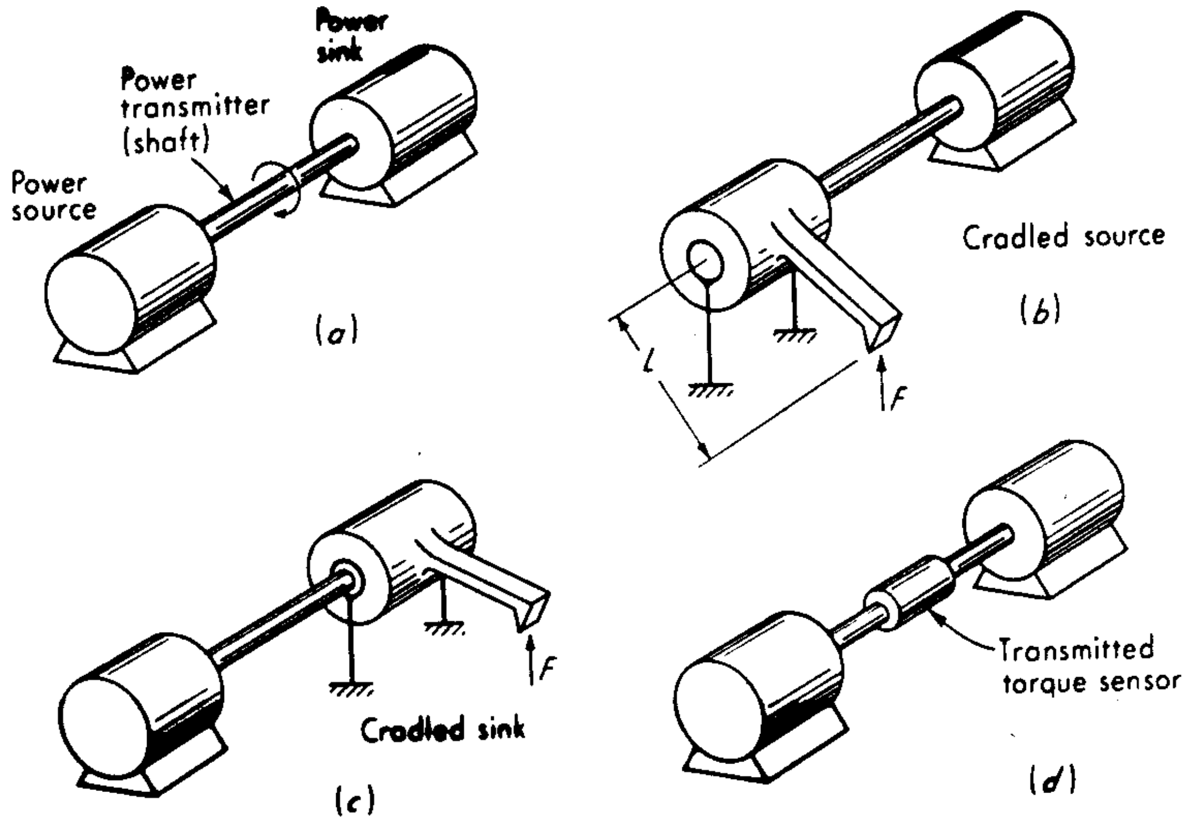


Figure 4.4-1 Torque Measurement of Rotating Machines

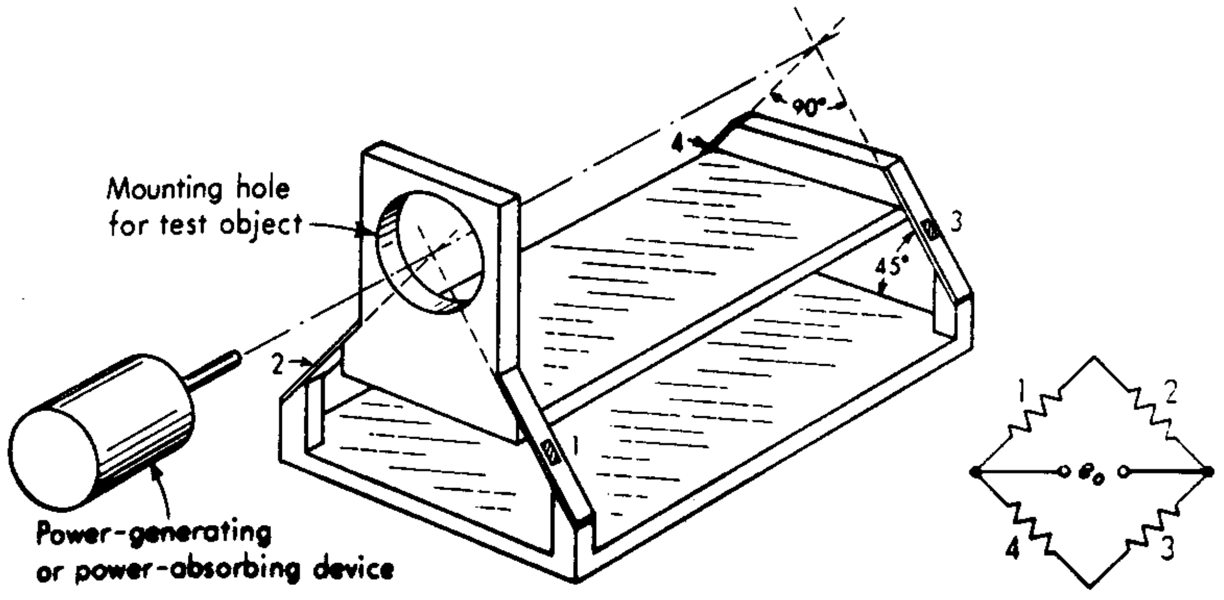


Figure 4.4-2 Strain-Gage Torque Table

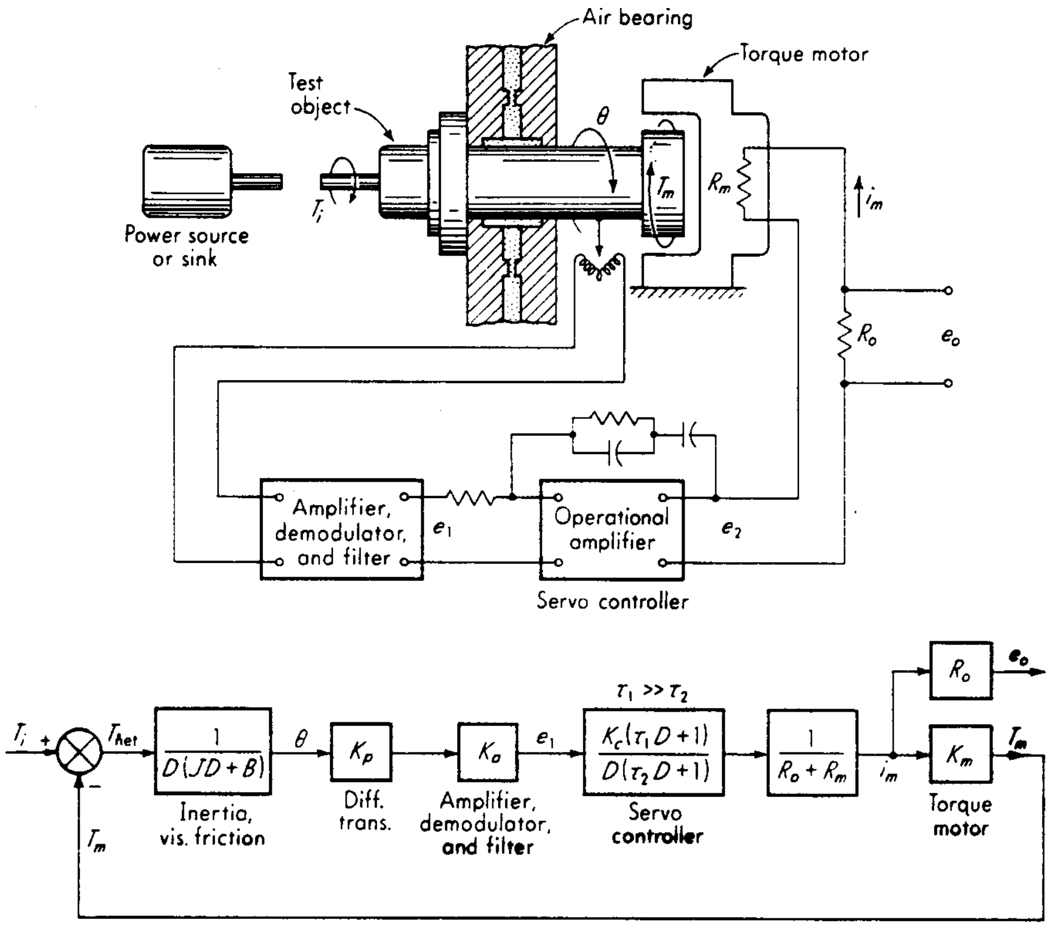


Figure 4.4-3 Feedback Torque Sensor, Null-Balance Torque Meter

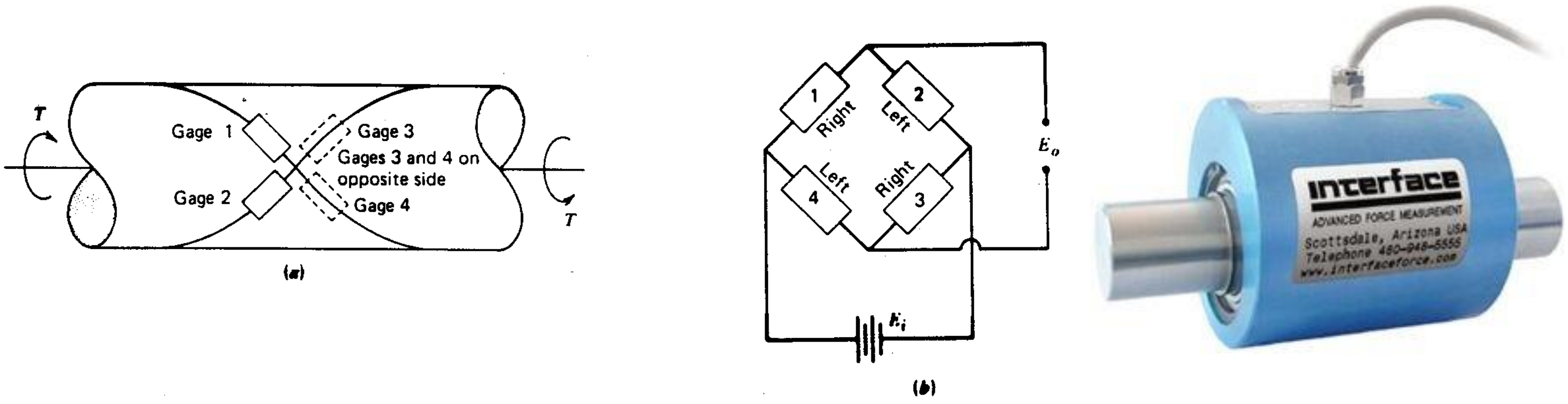


Figure 4.4-4 Torque Cell (a) Elastic Element with Strain-Gage Sensors (b) Gage Positions in the Wheatstone Bridge

The shearing stress τ in the circular shaft is related to the applied torque *T*.

$$\tau_{xz} = \frac{TD}{2J} = \frac{16T}{\pi D^3} \tag{4.4-1}$$

where *D* = the diameter of the shaft and *J* = the polar moment of inertia of the circular cross section.

Since the normal stresses $\sigma_x = \sigma_y = \sigma_z = 0$ for a circular shaft subjected to pure torsion, from Mohr's circle,

$$\sigma_1 = -\sigma_2 = \tau_{xz} = \frac{16T}{\pi D^3} \tag{4.4-2}$$

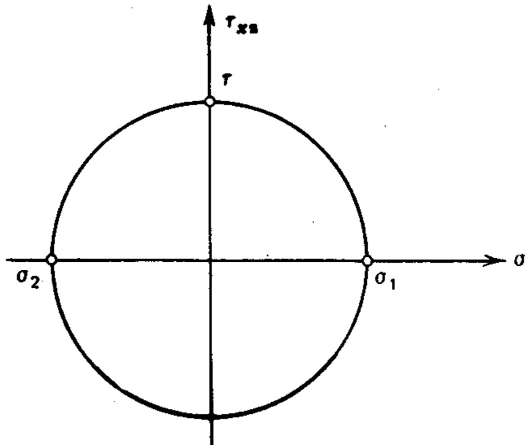


Figure 4.4-5 Mohr's Circle for the Stage of Stress in a Circular Shaft Subjected to a Pure Torque

Principle strains ε_1 and ε_2,

$$\varepsilon_1 = \frac{1}{E}(\sigma_1 - v\sigma_2) = \frac{16T}{\pi D^3}\left(\frac{1+v}{E}\right) \tag{4.4-3}$$

$$\varepsilon_2 = \frac{1}{E}(\sigma_2 - v\sigma_1) = -\frac{16T}{\pi D^3}\left(\frac{1+v}{E}\right) \tag{4.4-4}$$

The response of the strain gages,

$$\frac{\Delta R_1}{R_1} = -\frac{\Delta R_2}{R_2} = \frac{\Delta R_3}{R_3} = -\frac{\Delta R_4}{R_4} = \frac{16T}{\pi D^3}\left(\frac{1+v}{E}\right)S_g \tag{4.4-5}$$

where S_g = gage factor.

If the four strain gages are assumed identical, the output voltage E_o from the Wheatstone bridge,

$$E_o = \frac{16T}{\pi D^3}\left(\frac{1+\nu}{E}\right)S_g E_i \tag{4.4-6}$$

The sensitivity S of the load cell,

$$S = \frac{E_o}{T} = \frac{16}{\pi D^3}\left(\frac{1+\nu}{E}\right)S_g E_i \tag{4.4-7}$$

4.5 Shaft Power Measurement (Dynamometers)

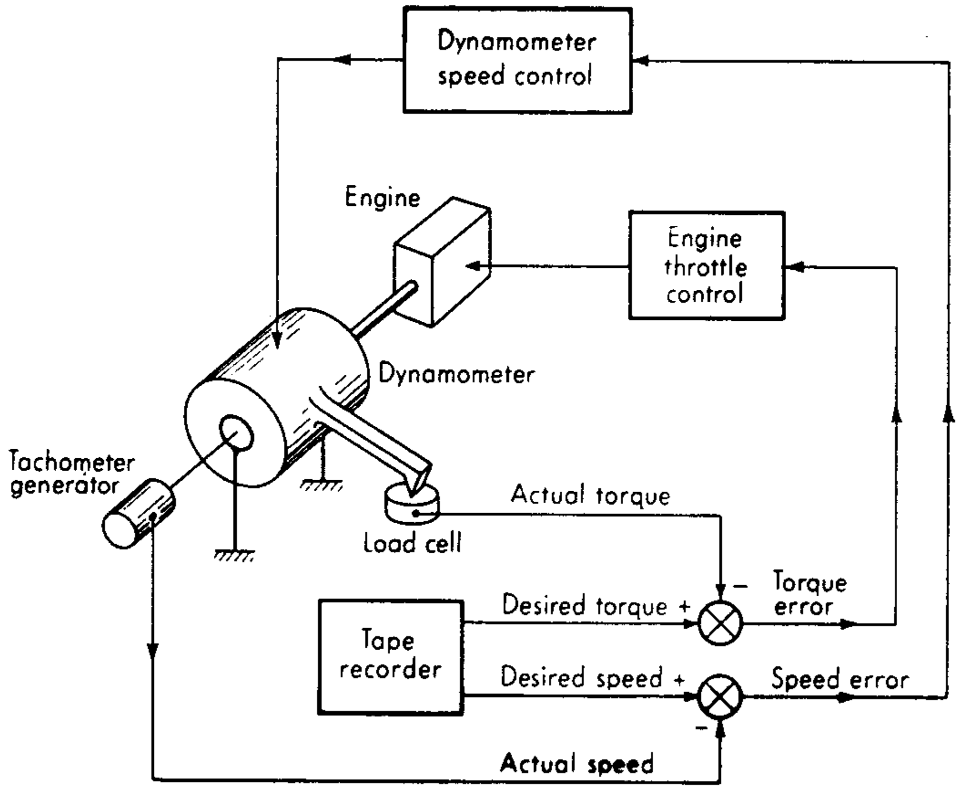


Figure 4.5-1 Servo-controlled Dynamometer

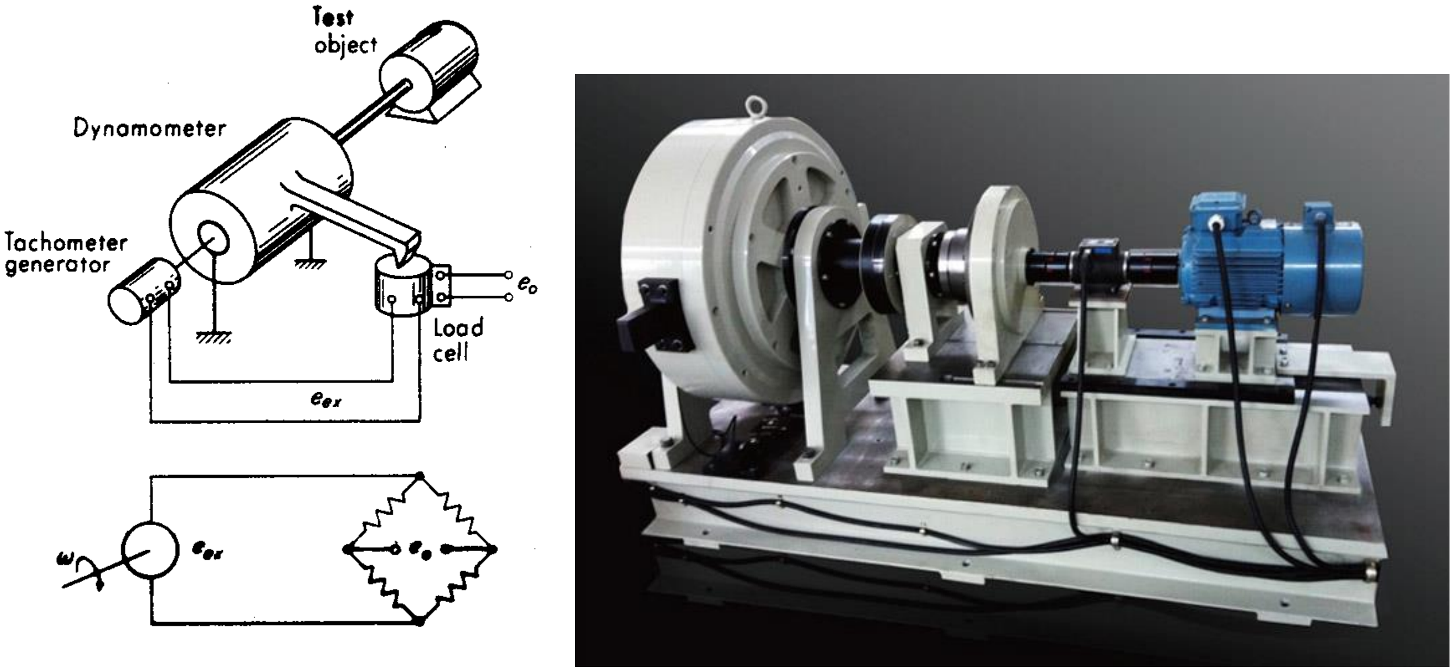


Figure 4.5-2 Instantaneous Power Measurement

- Speed is measured with a dc tachometer generator, and this voltage is applied as the excitation of a strain-gage load cell used to measure torque. Since bridge output is directly proportional to excitation voltage and directly proportional to torque, the voltage e_o is actually an instantaneous power signal.

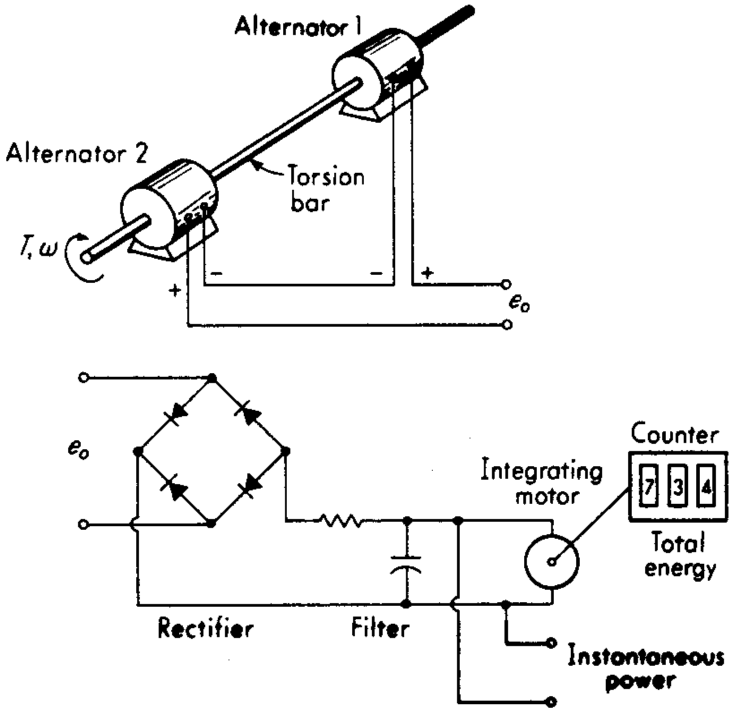


Figure 4.5-3 Alternator Power Measurement

$$\text{Alternator 1 output} = K_\omega \omega \sin \omega t \tag{4.5-1}$$

$$\text{Alternator 2 output} = K_\omega \omega \sin(\omega t + \phi) \tag{4.5-2}$$

where K_ω = amplitude of peak voltage, $\phi = K_t T$, K_t = angular variation coefficient, T = torque.

The net output of the series-connected alternators,

$$\text{Net output} = e_o = K_\omega \omega[\sin \omega t - \sin(\omega t + K_t T)] \quad (4.5\text{-}3)$$

$$e_o = K_\omega \omega[\sin \omega t - (\sin \omega t \cos K_t T + \cos \omega t \sin K_t T)] \quad (4.5\text{-}4)$$

The twist angle $\phi = K_t T$ is very small, and so $\cos(K_t T) \approx 1$ and $\sin(K_t T) \approx K_t T$.

$$e_o = -K_\omega \omega K_t T \cos \omega t \quad (4.5\text{-}5)$$

- e_o is a sine wave of amplitude proportional to ωT and thus to power.
- The ac voltage is rectified and filtered to produce a proportional dc value.
- If total energy over a time period is desired, an integrator is available to integrate the dc voltage. Total revolutions (read by an ordinary mechanical counter) give total energy.

4.6 Gyroscopic Force and Torque Measurement

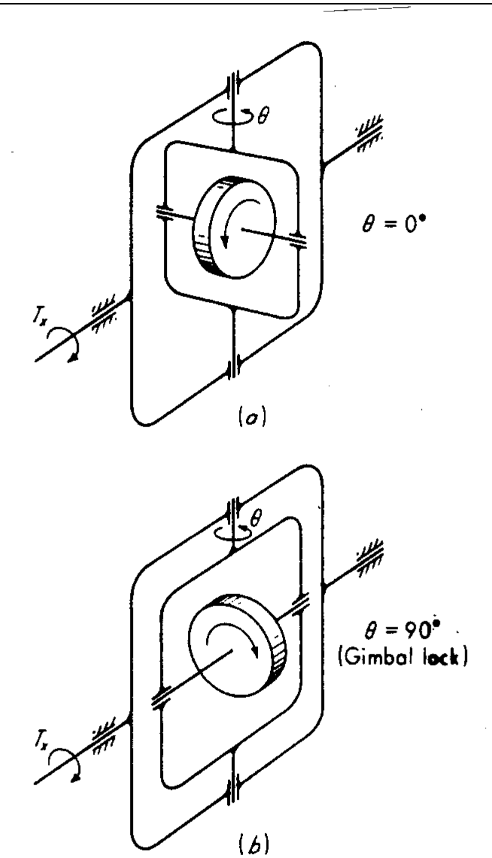


Figure 4.6-1 Gyroscopic Torque Measurement

$$\frac{\theta}{T_x}(s) = \frac{H_s / I_x}{s(I_y s^2 + Bs + H_s^2 / I_x + K_s)} \tag{4.6-1}$$

For a free gyro, *B* and *K* are effectively zero.

$$\frac{\dot{\theta}}{T_x}(s) = \frac{K}{s^2 / \omega_n^2 + 1} \tag{4.6-2}$$

where $K = 1/H_s$ and $\omega_n = \sqrt{H_s^2 / I_x I_y}$.

- A constant torque T_x will produce a precessional angular velocity $\dot{\theta}$ in direct proportion according to $\dot{\theta} = KT_x$.
- When θ reaches 90°, the gyro is gimbal locked, a torque T_x produces no precession at all. The inner and outer gimbals both rotate together about the *x* axis.
- The torque vector and spin angular-momentum vector must be perpendicular to prevent gimbal lock.

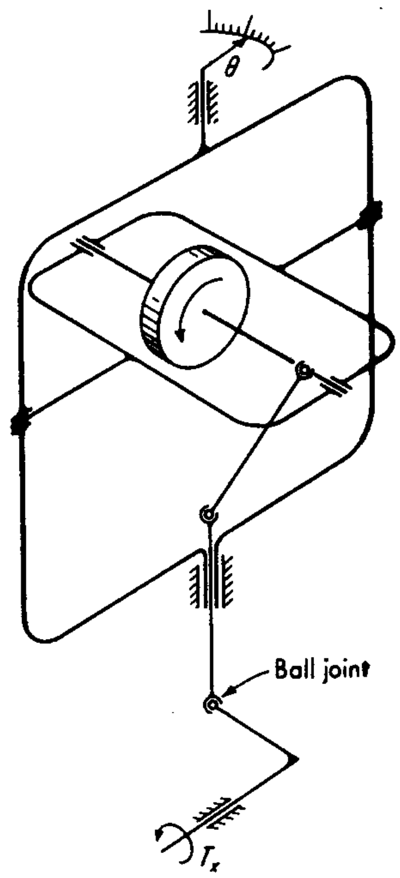


Figure 4.6-2 Solution of Gimbal-Lock Problem

4.7 Vibrating-Wire Force Transducers

The first natural frequency ω of a string of length L and mass per unit length m_1, which is tensioned by the force F,

$$\omega = \frac{1}{2L}\sqrt{\frac{F}{m_1}} \tag{4.7-1}$$

- Since ω varies with F, the measuring principle is analog; however, the frequency is easily measured with conventional digital counters, so the transducer is sometimes described as a digital device.

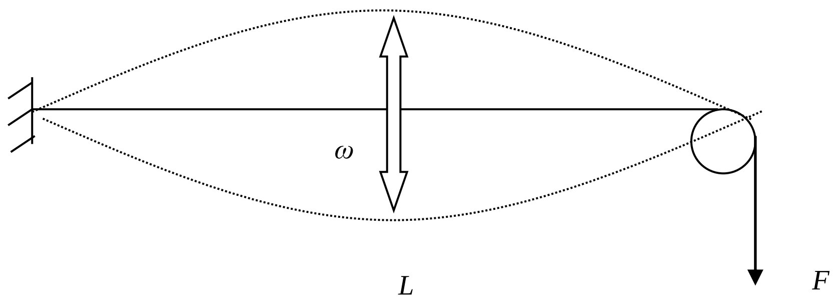


Figure 4.6-2 Vibrating-Wire Force Transducer