Final Examination   Sensing and Actuation AT74.03   November 22, 2012

Time: 10:00-11:30 h.   Open Book

Marks: 100

Attempt all questions.

Q.1 From the modified Planck's law, $W = \frac{C_1}{\lambda^5(e^{C_2/(\lambda T)}-1)}$, prove that the peak radiation intensity occurs at the wavelength about $C_2/(5T)$. (20)

Solution

The peak radiation intensity is determined by solving,

$$\frac{dW}{d\lambda} = 0 \quad (1)$$

$$\frac{dW}{d\lambda} = \frac{dW}{d(\lambda^5(e^{C_2/(\lambda T)}-1))} \cdot \frac{d(\lambda^5(e^{C_2/(\lambda T)}-1))}{d\lambda} = 0 \quad (2)$$

$$\frac{dW}{d\lambda} = -\frac{C_1}{(\lambda^5(e^{C_2/(\lambda T)}-1))^2} \cdot \left[\left(e^{C_2/(\lambda T)} - 1\right)\frac{d(\lambda^5)}{d\lambda} + \lambda^5 \frac{d((e^{C_2/(\lambda T)}-1))}{d\lambda}\right] = 0 \quad (3)$$

$$\frac{dW}{d\lambda} = -\frac{C_1}{(\lambda^5(e^{C_2/(\lambda T)}-1))^2} \cdot \left[5\lambda^4\left(e^{C_2/(\lambda T)} - 1\right) - \lambda^3 e^{C_2/(\lambda T)} C_2/T\right] = 0 \quad (4)$$

$$5\lambda^4\left(e^{C_2/(\lambda T)} - 1\right) - \lambda^3 e^{C_2/(\lambda T)} C_2/T = 0 \quad (5)$$

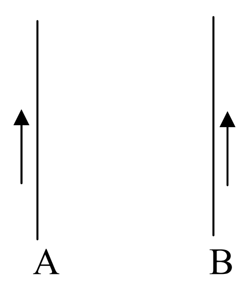
Since $e^{C_2/(\lambda T)} \gg 1, e^{C_2/(\lambda T)} - 1 \approx e^{C_2/(\lambda T)}$,

$$5\lambda^4 e^{C_2/(\lambda T)} - \lambda^3 e^{C_2/(\lambda T)} C_2/T \approx 0 \quad (6)$$

$$5\lambda - C_2/T \approx 0 \quad (7)$$

$$5\lambda \approx C_2/(5T) \quad \underline{(8)}$$

Q.2 Two extremely long straight wires, A and B, are placed in parallel in the air with the distance between them of 5 cm. Determine magnitude of magnetic force per unit length acting on each wire when the current of 1 A flows in wire A and the current of 4 A flows in wire B in the same direction. Is the magnetic force attraction or repulsive force? Apply air permeability of $4\pi x10^{-7}$ Tm/A. (20)



Solution

The magnetic field generated by wire A at B,

$$B = \frac{\boldsymbol{\mu i_A}}{\boldsymbol{2\pi r}} \quad (1)$$

The magnetic force at wire B is the attraction force when the currents flow in the same direction.

$$F = i_B L B = \frac{\boldsymbol{\mu L i_A i_B}}{\boldsymbol{2\pi r}} \quad (2)$$

Force per unit length,

$$\frac{F}{L} = \frac{\boldsymbol{\mu i_A i_B}}{\boldsymbol{2\pi r}} = \frac{4\pi\times10^{-7}(1)(4)}{2\pi(0.05)} = 16\mu N/m \quad \underline{(3)}$$

Q.3 Determine number of slots surrounding the stator housing of a 3-phase induction motor if the motor runs at 1,194 rpm speed when 50 Hz 3-phase electrical power is supplied. Assume the motor has 0.5% slip rate and number of slots per individual pole of each phase is 3. (20)

Solution

$$slip = \frac{rpm_{theory} - rpm_{actual}}{rpm_{theory}} 100\% \quad (1)$$

$$0.5\% = \frac{rpm_{theory} - 1194}{rpm_{theory}} 100\% \quad (2)$$

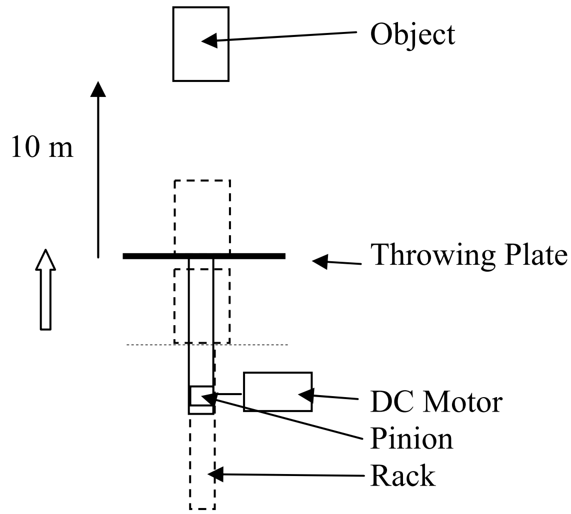
$$rpm_{theory} = 1200 = \frac{120(50)}{p} \quad (3)$$

$$p = 5 \quad (4)$$

Total number of slots,

$$slot\ number = phase \times pole \times \frac{slot}{pole} = 3\ phase \times 5\ pole \times 3\frac{slot}{pole} = 45\ slots \quad \underline{(5)}$$

Q.4 Determine the minimum output power of a DC motor used to throw a 5 kg object from no speed to the height of 20 meters as shown in the figure. The duration that the motor exerts force is 2 second. Use gravitational acceleration of 10 m/s^2 in the calculation. Assume mass of the throwing plate, mass of the rack and moment of inertia of the pinion are very small and negligible in the calculation. (20)



Solution

Maximum power of the DC motor is obtained during the maximum force at the maximum speed which is at the instant when the object just leaves from the throwing plate.

Determine the initial speed when the object just leaves from the throwing plate and move to the height of 10 m.

$$v^2 = u^2 + 2as \quad (1)$$

$$0 = u^2 - 2(10)(20) \quad (2)$$

$$u = 40\ m/s \quad (3)$$

Determine the required acceleration of the object,

$$v = u + at \quad (4)$$

$$40 = 0 + a(2) \quad (5)$$

$$a = 20\ m/s^2 \quad (6)$$

Determine the required force from the motor,

$$F - mg = ma \quad (7)$$

$$F - 5(10) = 5(20) \quad (8)$$

$$F = 150\ N \quad (9)$$

The required output power from the motor,

$$P = Fv \quad (10)$$

$$P = 150(40) = 6000\ W \quad (11)$$

Q.5 Design a Cartesian robot whose its end effector can be controlled to stop at any position in the working volume and the exerting force between the end effector and object can be controlled. Select the actuators and sensors of the system. (20)