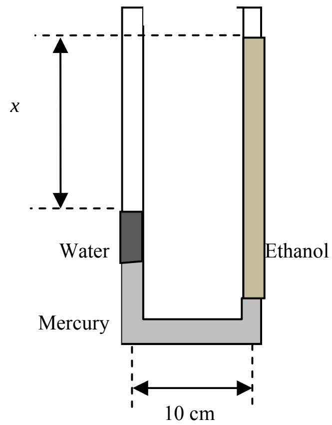
Final Examination   Sensing and Actuation AT74.03   November 23, 2013

Time: 10:00-11:30 h.   Open Book
Marks: 100

Attempt all questions.

Q.1 200 cm^3 of mercury with density of 13,550 kg/m^3, 50 cm^3 of water with density of 1,000 kg/m^3, and 300 cm^3 of ethanol with density of 790 kg/m^3 are filled into a U-tube manometer with uniform cross section area of 10 cm^2 as shown in the below figure. Determine the level difference, *x*, between both ends which are opened to atmospheric pressure. (20)



Solution

The total length of the fluid is the total volume divided by tube cross section area.

$$L_{mercury} = \frac{200}{10} = 20\ cm \quad (1)$$

$$L_{water} = \frac{50}{10} = 5\ cm \quad (2)$$

$$L_{ethanol} = \frac{300}{10} = 30\ cm \quad (3)$$

The pressures at the bottom of U-tube manometer from both ends are the same.

$$\rho_{water} g h_{water} + \rho_{mercury} g h_{mercury} = \rho_{ethanol} g h_{ethanol} + \rho_{mercury} g h_{mercury} \quad (4)$$

$$(1000)g(0.05) + (13550)g(y) = (790)g(0.30) + (13550)g(0.10 - y) \quad (5)$$

$$y = 0.0569\ m = 5.69\ cm \quad (6)$$

Total length of the left end of U-tube manometer

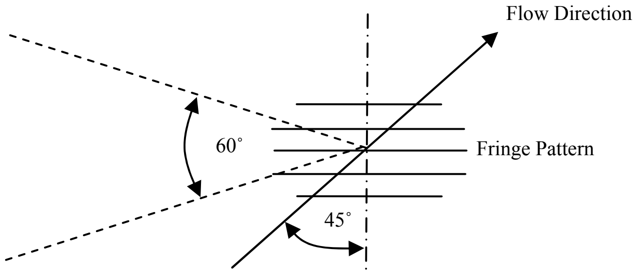
$$5 + 5.69 = 10.69\ cm \quad (7)$$

Total length of the right end of U-tube manometer

$$30 + (10 - 5.69) = 34.31\ cm \quad (8)$$

$$x = 34.31 - 10.69 = 23.62\ cm \quad (9)$$

Q.2 Determine the fluid flow speed, V, when the frequency from laser doppler velocimeter, f, indicates 1 MHz during the fluid flows 45˚ respect to the fringe pattern and red light from the laser has 740 nm wavelength, λ, and the fringe pattern is the result of two laser beams which are projected 60˚ relatively. (20)



Solution

The frequency from laser Doppler velocimeter,

$$f = \frac{2V\sin(\theta/2)}{\lambda}\sin(\alpha) \quad (1)$$

$$1 \times 10^6 = \frac{2V\sin(60^\circ/2)}{740\times10^{-9}}\sin(45^\circ) \quad (2)$$

$$V = 1.046 \text{ m/s} \quad \underline{(3)}$$

Q.3 An RTD made of Nickel has relation between change of resistance and change of temperature following quadratic equation, $\frac{\Delta R}{R_0} = \gamma_1 \Delta T + \gamma_2 \Delta T^2$. The RTD resistance is measured as 100 Ω at 0˚C, as 101.6 Ω at 100˚C, and as 105.2 Ω at 200˚C. Determine the unknown temperature when the resistance is read at 110 Ω. (20)

Solution

$$\frac{\Delta R}{R_0} = \gamma_1 \Delta T + \gamma_2 \Delta T^2 \quad (1)$$

$$\Delta R = R_0\gamma_1 \Delta T + R_0\gamma_2 \Delta T^2 \quad (2)$$

$$101.6 - 100 = R_0\gamma_1(100-0) + R_0\gamma_2(100-0)^2 \quad (3)$$

$$1.6 = 100R_0\gamma_1 + 10000R_0\gamma_2 \quad (4)$$

$$105.2 - 100 = R_0\gamma_1(200-0) + R_0\gamma_2(200-0)^2 \quad (5)$$

$$5.2 = 200R_0\gamma_1 + 40000R_0\gamma_2 \quad (6)$$

$$R_0\gamma_1 = 0.006 \quad (7)$$

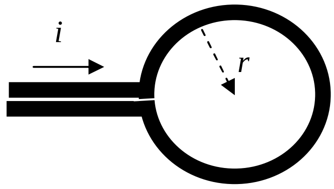
$$R_0\gamma_2 = 0.0001 \quad (8)$$

$$110 - 100 = 0.006(T-0) + 0.0001(T-0)^2 \quad (9)$$

$$10 = 0.006T + 0.0001T^2 \quad (10)$$

$$T = 287.65\ ^{\circ}C \tag{11}$$

Q.4 From the Biot-Savart law, prove that the magnetic field at the center of a single loop wire is $B = \frac{\mu i}{2r}$. (20)



Solution

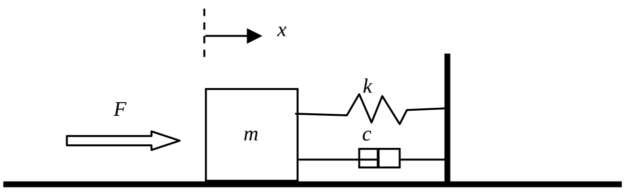
$$dB = \frac{\mu i ds \times r}{4\pi r^3} \tag{1}$$

$$B = \int dB = \int_0^{2\pi r} \frac{\mu i ds \times r}{4\pi r^3} \tag{2}$$

$$B = \frac{\mu i}{4\pi} \int_0^{2\pi r} \frac{ds}{r^2} \tag{3}$$

$$B = \frac{\mu i}{4\pi} \cdot \frac{2\pi r}{r^2} = \frac{\mu i}{2r} \tag{4}$$

Q.5 A pneumatic actuator generates force, F, to move a mass-spring-damper system on a frictionless floor as shown in the below figure. Determine the power from the actuator as a function of time, $P(t)$, in order to move the mass following the function $x = Asin(2\pi f t)$ from the rest position. Determine the magnitude of the maximum power, P_{max}, required from the actuator. A is the amplitude of motion, f is the oscillation frequency. (20)



Solution

$$x = Asin(2\pi f t) \tag{1}$$

$$\dot{x} = 2\pi f Acos(2\pi f t) \tag{2}$$

$$\ddot{x} = -4\pi^2 f^2 Asin(2\pi f t) \tag{3}$$

$$F - kx - c\dot{x} = m\ddot{x} \tag{4}$$

$$F = -m4\pi^2 f^2 Asin(2\pi f t) + kAsin(2\pi f t) + c2\pi f Acos(2\pi f t) \tag{5}$$

$$P(t) = F\dot{x} = (2\pi f k A^2 - 8\pi^3 m f^3 A^2) sin(2\pi f t) cos(2\pi f t) + 4\pi^2 c f^2 A^2 cos^2(2\pi f t) \tag{6}$$

$$P(t) = (\pi fkA^2 - 4\pi^3 mf^3 A^2)sin(4\pi ft) + 2\pi^2 cf^2 A^2 cos(4\pi ft) + 2\pi^2 cf^2 A^2 \quad (7)$$

$$P(t) = \sqrt{(\pi fkA^2 - 4\pi^3 mf^3 A^2)^2 + (2\pi^2 cf^2 A^2)^2} sin(4\pi ft + \phi) + 2\pi^2 cf^2 A^2$$

$$\text{when } \phi = atan\left(\frac{2\pi cf}{(k - 4\pi^2 mf^2)}\right) \quad \underline{(8)}$$

$$P_{max} = \sqrt{(\pi fkA^2 - 4\pi^3 mf^3 A^2)^2 + (2\pi^2 cf^2 A^2)^2} + 2\pi^2 cf^2 A^2 \quad \underline{(9)}$$