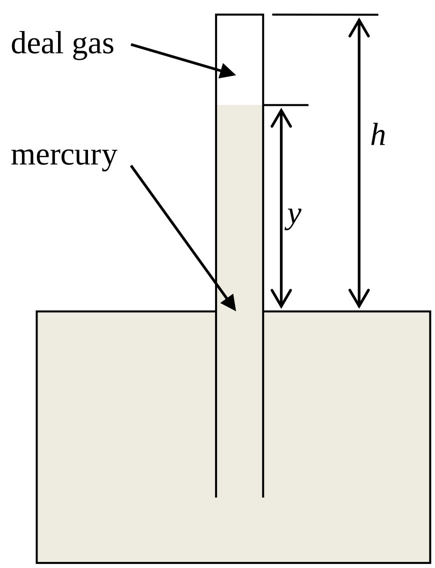
Final Examination Sensing and Actuation AT74.03 November 19, 2015

Time: 10:00-11:30 h. Open Book
Marks: 100

Attempt all questions.

Q.1 A barometer made of a glass tube with 1 cm^2 cross section area is filled with mercury (density of 13,550 kg/m^3). At the atmospheric pressure of 101.325 kP, the barometer contains 1 cm^3 ideal gas when the total height of glass tube, h, is pulled to 10 cm. Determine the height of mercury, y, as a function of total height of the glass tube, h. Then determine the height of mercury when the glass tube is pulled up to 20 cm. Use the gravitational acceleration of 10 m/s^2. (20)



Solution

At the height, h, of 10 cm, the pressure of ideal gas is determined.

$$P_{10} + 13550 \times 10 \times 0.09 = 101325 \quad (1)$$

$$P_{10} = 89130 \ \text{Pa} \quad (2)$$

From ideal gas law,

$$PV = 89130 \times 0.000001 = 0.08913 = P_h V_h \quad (3)$$

$$P_h = \frac{0.08913}{V_h} = \frac{0.08913}{(h-y)0.0001} = \frac{891.3}{(h-y)} \quad (4)$$

$$P_h + 13550 \times 10 \times y = 101325 \quad (5)$$

$$P_h = 101325 - 135500y \quad (6)$$

(4)=(6),

$$\frac{891.3}{(h-y)} = 101325 - 135500y \quad (7)$$

$$135500y^2 - (135500h + 101325)y + 101325h - 891.3 = 0 \quad (8)$$

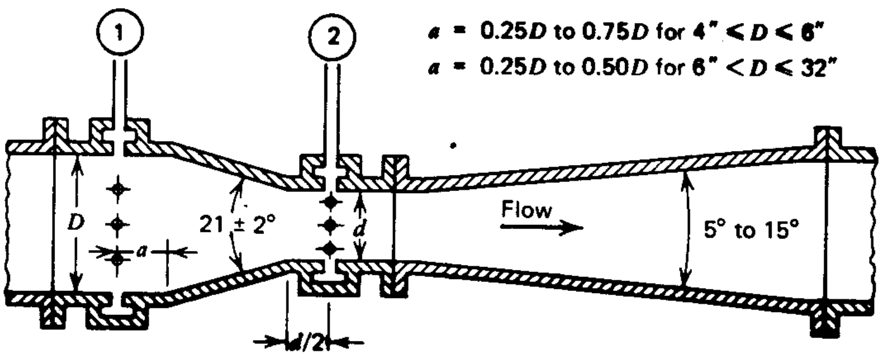
$$y = \frac{135500h + 101325 - \sqrt{(135500h + 101325)^2 - 4(135500)(101325h - 891.3)}}{2(135500)} \quad \underline{(9)}$$

When the glass is pulled to 20 cm,

$$y = \frac{135500(0.2) + 101325 - \sqrt{(135500(0.2) + 101325)^2 - 4(135500)(101325(0.2) - 891.3)}}{2(135500)}$$

$$= 0.1882\ m = 18.82\ cm \quad \underline{(10)}$$

Q.2 A venturi meter is used to determine gross volume flow rate of an uncompressible liquid with the mass density of 5,000 kg/m^3 as shown in the below figure. If the pressure difference between point 1 and point 2 of the meter is 20 kPa when the liquid flows at the rate of 30 m^3/s. Determine the pressure at point 1 when the liquid with the flow rate of 150 m^3/s flows through this meter and the pressure at point 2 indicates 200 kPa. (20)



Solution

The relation between volume flow rate and pressure difference follows,

$$Q = \frac{C_v A_2}{\sqrt{1-(A_2/A_1)^2}}\sqrt{2\left(\frac{p_1}{\rho} + gz_1 - \frac{p_2}{\rho} - gz_2\right)} \quad (1)$$

$$30 = k\sqrt{2\left(\frac{20000}{5000}\right)} \quad (2)$$

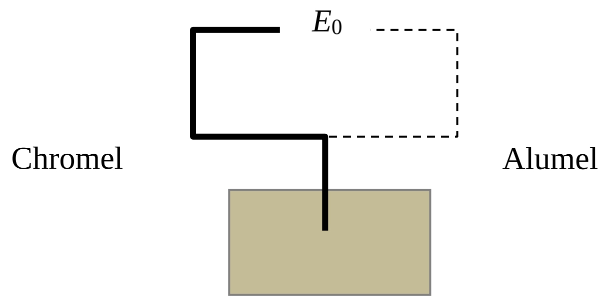
$$k = 10.61 \quad (3)$$

At 150 m^3/s flow,

$$150 = 10.61\sqrt{2\left(\frac{P_1 - 200000}{5000}\right)} \quad (4)$$

$$P_1 = 699679.55\ \text{Pa} \quad \underline{(5)}$$

Q.3 A K-type thermocouple made of Chromel-Alumel having sensitivity of 40 μV/ºC is used to measure unknown temperature. If the voltage output, E_0, indicates 2 V when the thermocouple is immersed into 50 ºC liquid. Determine temperature of the liquid when the voltage output indicates 2.005 V. (20)



<u>Solution</u>

The voltage output from thermocouple,

$$E_0 = ST + k = 40T + k \quad (1)$$

With 2 V output,

$$2000000 = 40(50) + k \quad (2)$$

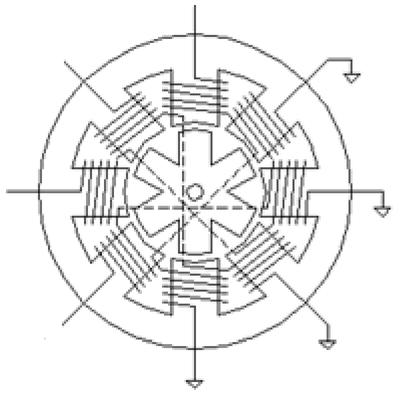
$$k = 1998000 \quad (3)$$

With 2.005 V output,

$$2005000 = 40(T) + 1998000 \quad (4)$$

$$T = 175\ ^{\circ}\mathrm{C} \quad \underline{(5)}$$

Q.4 Determine the speed in RPM of the variable reluctance stepper motor as shown in the below figure when the motor driver circuit receives input frequency of 4 Hz. (10)



Solution

Step angle,

$$\theta_s = \frac{360}{6} - \frac{360}{8} = 15^\circ \quad (1)$$

Resolution,

$$S = \frac{360}{15} = 24 \quad (2)$$

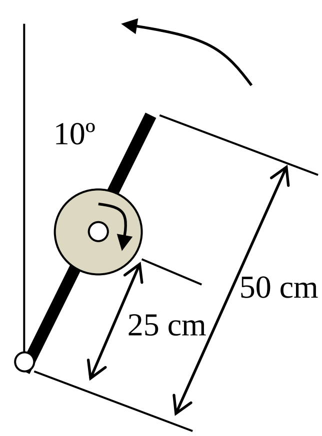
Speed,

$$RPM = \frac{60f}{S} = \frac{60(4)}{24} = 10 \quad \underline{(3)}$$

Q.5 A flywheel with mass, m_f, of 2 kg and moment of inertia, I_f, of 0.8 kg.m^2 attaching at the center of mass of a stick is used to keep a stick with mass, m_s, 5 kg and moment of inertia, I_s, of 1.5 kg.m^2 at the position of 10 degree as shown in the below figure. Use gravitational acceleration of 10 m/s^2.

(a) Determine the required torque from a DC motor used to keep the stick at this position. Then determine the angular acceleration at the flywheel. (15)

(b) If the maximum speed of the motor is 3000 rpm, determine the maximum power obtained from the DC motor when the motor applies constant torque as obtained in (a). Then determine time duration that the flywheel can keep the stick at this position before falling down assume the flywheel starts from zero rpm at the beginning. (15)



Solution

(a) The moment around pivot point at 10 degree is the torque used to keep the stick.

$$M = m_s g(l_s/2) sin(10^\circ) + m_f g l_f sin(10^\circ) \text{ N}$$

$$= 5 \times 10 \times 0.25 sin(10^\circ) + 2 \times 10 \times 0.25 sin(10^\circ) = 3.04\ Nm \quad \underline{(1)}$$

The angular acceleration at flywheel is determined.

$$M = I_f \alpha \quad (2)$$

$$3.04 = 0.8\alpha \quad (3)$$

$$\alpha = 3.8\, rad/sec^2 \quad \underline{(4)}$$

(b) The maximum power obtained at constant torque in (a),

$$P = M\omega = 3.04 \times \frac{3000\times 2\pi}{60} = 955.04\, W \quad \underline{(5)}$$

Based on the angular acceleration,

$$\omega_f = \alpha t \quad (6)$$

$$\frac{3000\times 2\pi}{60} = 3.8t \quad (7)$$

$$t = 82.67\, sec \quad \underline{(8)}$$