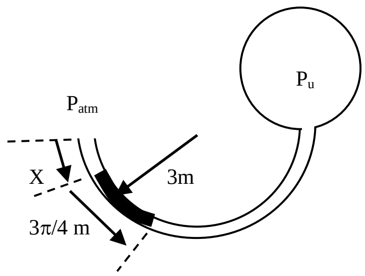
**Final Examination Sensing and Actuation AT74.03 November 26, 2019**

Time: 9:00-10:30 h. Open Book

Marks: 100

Attempt all questions.

**Q.1** A uniform-cross-section-area half circle tube is used to measure unknown pressure, $P_u$, as shown in the figure below.




One end of the tube is exposed to atmosphere, the other end is connected with a container of gas with unknown pressure. If mercury is filled in the tube for the amount that makes the length of 3π/4 m. Use atmospheric pressure, $P_{atm}$, of 101.325 kPa, mercury density of 13550 kg/m$^3$, gravitational acceleration of 9.8 m/s$^2$ and the cross-section-area of the tube is very small compared to the length of the mercury.

(a) Determine the unknow pressure if x = π/8 m. (10)

(b) Determine the unknow pressure if x = 15π/16 m. (10)

**Solution**

(a) At x = π/8 m, the other end of the mercury, y, is calculated,

$$y = \frac{\pi}{8} + \frac{3\pi}{4} = \frac{7\pi}{8} \quad (1)$$

The mercury is on the left side of the tube only.

$$P_u = 101325 + 13550 \times 9.8 \times 3 \times \left( sin\left(\frac{7\pi}{3\times8}\right) - sin\left(\frac{\pi}{3\times8}\right)\right) = 365375.5 \text{ Pa} \quad \underline{(2)}$$

(b) At x = 15π/16 m, the other end of the mercury, y, is calculated,

$$y = \frac{15\pi}{16} + \frac{3\pi}{4} = \frac{27\pi}{16} \quad (3)$$

The mercury is on both sides of the tube.

$$P_u = 101325 + 13550 \times 9.8 \times 3 \times \left( sin\left(\frac{27\pi}{3\times16}\right) - sin\left(\frac{15\pi}{3\times16}\right)\right) = 160807.9 \text{ Pa} \quad \underline{(4)}$$

**Q.2** A rotameter is used to measure volume flow rate of an unknown fluid. When the float rises to 5 cm level, the fluid flows at 0.8125 m$^3$/s. When the float rises to 10 cm level, the fluid flows at 1.5 $m^3$/s. When the float rises to 20 cm level, the fluid flows at 3.25 $m^3$/s. Determine the fluid flow rates, when the float rises to 30 cm level. (20)

**Solution**

The relation between the float position and the volume flow rate is expressed by quadratic equation.

$$Q = V(\pi(R + mx)^2 - A) \tag{1}$$

$$Q = ax^2 + bx + c \tag{2}$$

When the float rises to 5 cm level,

$$0.8125 = 0.05^2 a + 0.05b + c = 0.0025a + 0.05b + c \tag{3}$$

When the float rises to 10 cm level,

$$1.5 = 0.1^2 a + 0.1b + c = 0.01a + 0.1b + c \tag{4}$$

When the float rises to 20 cm level,

$$3.25 = 0.2^2 a + 0.2b + c = 0.04a + 0.2b + c \tag{5}$$

$$\begin{bmatrix} 0.8125 \\ 1.5 \\ 3.25 \end{bmatrix} = \begin{bmatrix} 0.0025 & 0.05 & 1 \\ 0.01 & 0.1 & 1 \\ 0.04 & 0.2 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} \tag{6}$$

Thus,

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 0.0025 & 0.05 & 1 \\ 0.01 & 0.1 & 1 \\ 0.04 & 0.2 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 0.8125 \\ 1.5 \\ 3.25 \end{bmatrix} = \begin{bmatrix} 25 \\ 10 \\ 0.25 \end{bmatrix} \tag{7}$$

When the float rises to 30 cm level,

$$Q = 25 \times 0.3^2 + 10 \times 0.3 + 0.25 = 5.5 \text{ m}^3/\text{s} \tag{8}$$

**Q.3** An RTD made of Nickel has the relation between temperature and resistance expressed by $R = 50 + \gamma_1 T + \gamma_2 T^2$ Ω when $T$ is temperature in °C.

(a) Determine the sensitivity of this Nickel RTD (in Ω/°C) and the sensitivity of this Nickel RTD when it is shunted with a constant resistor of 150 Ω (in Ω/°C) as functions of $\gamma_1, \gamma_2, T$. (10)

(b) Determine the sensitivities from both Nickel RTD and Nickel RTD with 150 Ω shunting resistor if $\gamma_1 = 0.125$ Ω/°C, $\gamma_2 = 0.0025$ Ω/°C$^2$ at 0 and 200 °C. (10)

**Solution**

(a)

The sensitivity of Nickel RTD is determined,

$$S_{Ni} = \frac{\boldsymbol{dR}}{\boldsymbol{dT}} = \gamma_1 + 2\gamma_2 T \tag{1}$$

Nickel RTD with 150 Ω shunting,

$$R = \frac{150(50+\gamma_1 T+\gamma_2 T^2)}{200+\gamma_1 T+\gamma_2 T^2} \quad (2)$$

The sensitivity of Nickel RTD with 150 Ω shunting is determined,

$$S_{Ni/150} = \frac{\boldsymbol{dR}}{\boldsymbol{dT}} = \frac{150(\gamma_1+2\gamma_2 T)(200+\gamma_1 T+\gamma_2 T^2)-150(50+\gamma_1 T+\gamma_2 T^2)(\gamma_1+2\gamma_2 T)}{(200+\gamma_1 T+\gamma_2 T^2)^2} \quad (3)$$

$$S_{Ni-150} = \frac{\boldsymbol{dR}}{\boldsymbol{dT}} = \frac{22500(\gamma_1+2\gamma_2 T)}{(200+\gamma_1 T+\gamma_2 T^2)^2} \quad \underline{(4)}$$

(b)

At 0°C,

$$S_{Ni\_0} = \gamma_1 + 2\gamma_2 T = 0.125 \quad \underline{(5)}$$

$$S_{Ni/150\_0} = \frac{22500(\gamma_1+2\gamma_2 T)}{(200+\gamma_1 T+\gamma_2 T^2)^2} = \frac{22500(0.125)}{40000} = 0.0703 \quad \underline{(6)}$$

At 200°C,

$$S_{Ni\_200} = \gamma_1 + 2\gamma_2 T = 0.125 + 2 \times 0.0025 \times 200 = 1.1250 \quad \underline{(7)}$$

$$S_{Ni/150\_200} = \frac{22500(\gamma_1+2\gamma_2 T)}{(200+\gamma_1 T+\gamma_2 T^2)^2} = \frac{22500(0.125+2\times0.0025\times200)}{(200+0.125\times200+0.0025\times200^2)^2} = 0.2396 \quad \underline{(8)}$$

**Q.4** A linear stepping motor is used to carry parts from station A to station B which are 20 meters apart. If platen of the linear stepping motor has 20 teeth per meter and forcer has 16 teeth per meter. Determine the full-step drive frequency and number of steps in order to carry parts from station A to station B at constant speed for 5 seconds. (20)

**<u>Solution</u>**

Distance between platen teeth,

$$d_p = \frac{1}{20} = 0.05 \text{ m} \quad (1)$$

Distance between forcer teeth,

$$d_f = \frac{1}{16} = 0.0625 \text{ m} \quad (2)$$

Distance per step,

$$d_f - d_p = 0.0125 \text{ m} \quad (3)$$

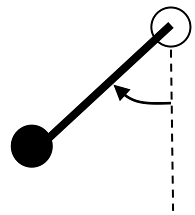
Number of steps for the distance of 20 m,

$$N = \frac{20}{0.0125} = 1600 \text{ steps} \quad \underline{(4)}$$

Driving frequency,

$$f = \frac{1600}{5} = 320 \text{ Hz} \quad \underline{(5)}$$

**Q.5** A motor is used to rotate a point mass load in vertical plane at a constant angular velocity as shown in the figure below.




(a) Determine the required power from the motor as a function of angular position of the arm. Use notation $m, l, \omega, g$ to represent mass of load, arm length, angular velocity, and gravitational acceleration respectively. (10)

(b) If $m = 2\ kg, l = 75\ cm, \omega = 30\ RPM, g = 9.8\ m/s^2$, Select the motor power by using 2 as the safety factor and select gear transmission ratio. Assume without gear the motor speed is 2400 RPM. (10)

**Solution**

(a)

Torque depends on angular position,

$$T = mglsin(\theta) \quad (1)$$

Thus, the power also depends on angular position.

$$P = T\omega = mgl\omega sin(\theta) \quad \underline{(2)}$$

(b)

The motor is selected at the maximum power.

$$P_{max} = 2 \times 9.8 \times 0.75 \times 30 \left(\frac{2\pi}{60}\right) = 46.18\ W \quad (3)$$

Use 2 as safety factor, thus, the selected motor should have the minimum power

$$P = 46.18 \times 2 = 92.36\ W \quad \underline{(4)}$$

The required gear ratio should be

$$N = \frac{2400}{30} = 80{:}1 \text{ ratio} \quad \underline{(5)}$$