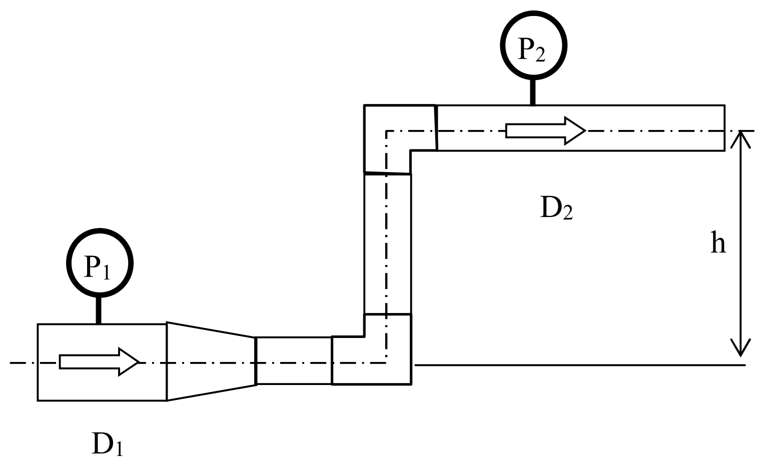
Final Examination Sensing and Actuation AT74.03 May 7, 2021

Time: 9:00-10:30 h. Open Book
Marks: 100

Attempt all questions.

Q.1 Water flows from a pipe with 5 cm diameter (D_1) through a pipe fitting adapter to another pipe with 3 cm diameter (D_2). Two pressure sensors are used to measure the water pressure at two locations locating 2 m height difference (h) as shown in the below figure. Determine the volume flow rate of water (Q) when the pressure difference (P_1-P_2) indicates 50 kPa. Assume water is incompressible and its flow is frictionless. Use water density (ρ) of 1000 kg/m^3 and gravitational acceleration (g) of 9.8 m/s^2. (20)



Solution

From Bernoulli's equation,

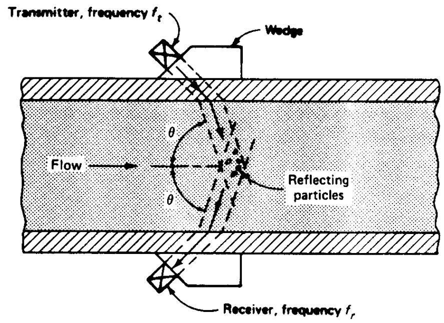
$$\frac{P_1}{\rho} + \frac{V_1^2}{2} + gz_1 = \frac{P_2}{\rho} + \frac{V_2^2}{2} + gz_2 \quad (1)$$

$$Q = \frac{A_2}{\sqrt{1-(A_2/A_1)^2}}\sqrt{2\left(\frac{P_1-P_2}{\rho} + g(z_1 - z_2)\right)} \quad (2)$$

$$Q = \frac{\pi \times 0.03^2/4}{\sqrt{1-(0.03/0.05)^4}}\sqrt{2\left(\frac{50000}{1000} + 9.8 \times 2\right)} \quad (3)$$

$$Q = 0.0097\ m^3/s \quad (4)$$

Q.2 Doppler ultrasonic flowmeter is used to determine water flow speed. When water flows to the right direction at 25 m/s and the frequency at the transmitter is 40 kHz, the frequency at the receiver is measured at 39.5 kHz. Determine the water flow direction and its speed when the frequency at the transmitter is 60 kHz and the frequency at the receiver is measured at 60.2 kHz. (20)



Solution

The frequency difference of Doppler ultrasonic flowmeter,

$$f_t - f_r = \frac{2f_t cos(\theta)}{c} V = kf_t V \quad (1)$$

$$40 - 39.5 = k \times 40 \times 25 \quad (2)$$

$$k = 0.0005 \quad (3)$$

$$60 - 60.2 = 0.0005 \times 60 \times V \quad (4)$$

$$V = -6.67\ m/s \quad \underline{(5)}$$

The water flows to the left with the speed of 6.67 m/s.

Q.3 A thermistor is used to measure an unknown temperature. During the calibration experiments, at temperature 0°C, the thermistor resistance is measured as 7476.98 Ω; at temperature 50°C, the thermistor resistance is measured as 773.92 Ω. What is the thermistor resistance if the temperature is at temperature 100°C. (20)

Solution

The relation between thermistor resistance and temperature is expressed by

$$R = R_0 e^{\beta\left(\frac{1}{T} - \frac{1}{T_0}\right)} \quad (1)$$

At 0°C,

$$7476.98 = R_0 e^{\beta\left(\frac{1}{273} - \frac{1}{T_0}\right)} \quad (2)$$

At 50°C,

$$773.92 = R_0 e^{\beta\left(\frac{1}{323} - \frac{1}{T_0}\right)} \quad (3)$$

(2)/(3),

$$\frac{7476.98}{773.92} = e^{\beta\left(\frac{1}{273} - \frac{1}{323}\right)} \quad (4)$$

$$\beta = 4000 \quad (5)$$

At 100°C,

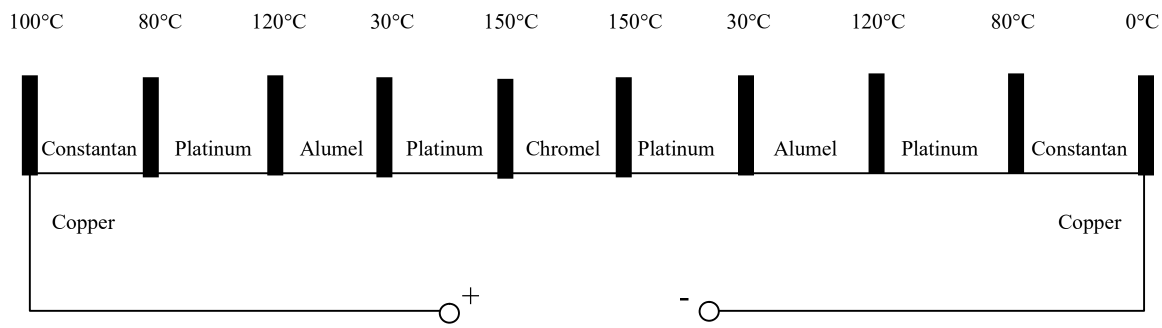
$$R = R_0 e^{4000\left(\frac{1}{373}-\frac{1}{T_0}\right)} \tag{6}$$

(2)/(6),

$$\frac{7476.98}{R} = e^{4000\left(\frac{1}{273}-\frac{1}{373}\right)} \tag{7}$$

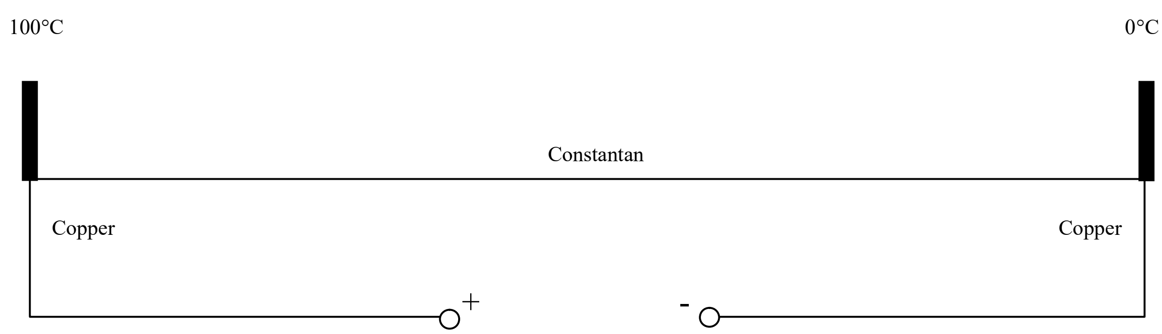
$$R = 147.15\,\Omega \tag{8}$$

Q.4 A multi-junction thermocouple is used to measure the temperature. Determine the voltage output of the thermocouple of the condition as shown in the below figure. Thermoelectric sensitivity in combination with platinum of several materials are given as follows: Copper is 6.5 μV/°C, Chromel is 25.8 μV/°C, Alumel is –13.6 μV/°C, and Constantan is -35 μV/°C. (20)



Solution

Based on thermocouple properties, the thermocouple at the given condition can be simplified as



Solution

$$V_0 = \left(S_{Copper} - S_{Constantan}\right)100 + C + \left(S_{Constantan} - S_{Copper}\right)0 - C \tag{1}$$

$$V_0 = (6.5 + 35)100 = 4150\ \mu V = 4.15\ mV \tag{2}$$

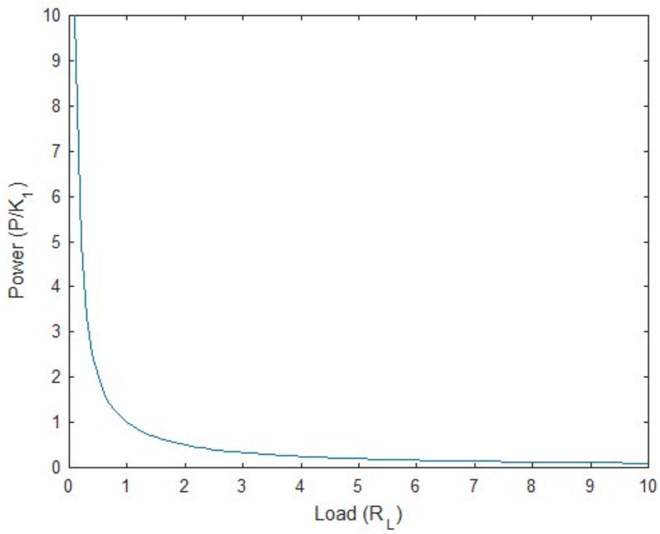
Q.5 Determine the power equations obtained from (a) voltage feedback (b) current feedback and (c) current and voltage feedback servo amplifiers as functions of load, R_L when the amplifier input voltage, V_{in}, is fixed. Roughly plot the powers as functions of the load. (20)

<u>Solution</u>

(a) Voltage feedback servo amplifier

$$V_{out} = -\frac{R_{vf}}{R_{in}} V_{in} \tag{1}$$

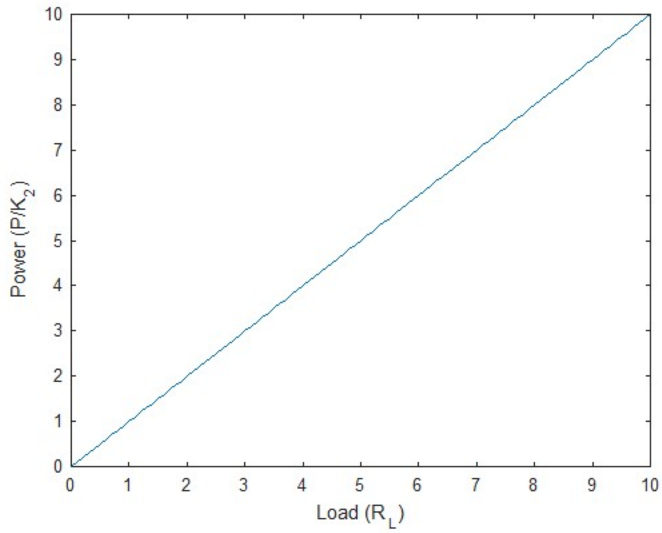
$$P = \frac{V_{out}^2}{R_L} = \frac{R_{vf}^2 V_{in}^2}{R_{in}^2 R_L} = \frac{K_1}{R_L} \tag{2}$$



(b) Current feedback servo amplifier

$$I_L = \frac{R_B}{R_{in} R_s} V_{in} \tag{3}$$

$$P = I_L^2 R_L = \frac{R_B^2 V_{in}^2 R_L}{R_{in}^2 R_s^2} = K_2 R_L \tag{4}$$



(c) Current and voltage feedback servo amplifier

$$V_{out} = -\frac{R_{vf}}{R_{in}}V_{in} - I_L R_s \frac{R_{vf}}{R_{cf}} \tag{5}$$

$$P = V_{out} I_L = -\frac{R_{vf}V_{in}}{R_{in}} I_L - I_L^2 \frac{R_s R_{vf}}{R_{cf}} \tag{6}$$

$$I_L = \sqrt{\frac{P}{R_L}} \tag{7}$$

$$P = -\frac{R_{vf}V_{in}}{R_{in}}\sqrt{\frac{P}{R_L}} - \frac{R_s R_{vf}}{R_{cf}}\frac{P}{R_L} \tag{8}$$

$$P\left(1 + \frac{R_s R_{vf}}{R_{cf} R_L}\right) = -\frac{R_{vf}V_{in}}{R_{in}}\sqrt{\frac{P}{R_L}} \tag{9}$$

$$P^2\left(1 + \frac{R_s R_{vf}}{R_{cf} R_L}\right)^2 = \left(\frac{R_{vf}V_{in}}{R_{in}}\right)^2 \frac{P}{R_L} \tag{10}$$

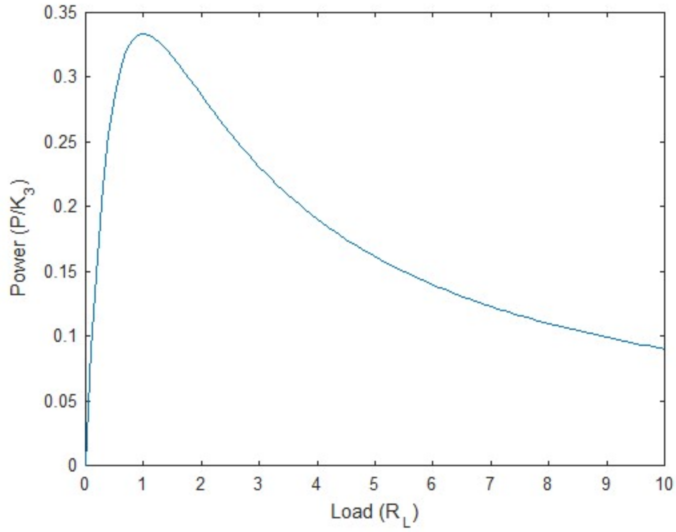
$$P\left(1 + \frac{R_s R_{vf}}{R_{cf} R_L}\right)^2 = \left(\frac{R_{vf}V_{in}}{R_{in}}\right)^2 \frac{1}{R_L} \tag{11}$$

$$P\left(\frac{R_{cf}^2 R_L^2 + 2R_s R_{vf} R_{cf} R_L + R_s^2 R_{vf}^2}{R_{cf}^2 R_L^2}\right) = \frac{R_{vf}^2 V_{in}^2}{R_{in}^2 R_L} \tag{12}$$

$$P = \frac{R_{vf}^2 V_{in}^2 R_{cf}^2 R_L}{R_{in}^2 R_{cf}^2 R_L^2 + 2R_s R_{vf} R_{cf} R_L R_{in}^2 + R_s^2 R_{vf}^2 R_{in}^2} \tag{13}$$

$$P = \frac{K_3 R_L}{K_4 R_L^2 + K_5 R_L + K_6} \tag{14}$$

$$P = \frac{K_3}{K_4 R_L + K_5 + K_6/R_L} \tag{15}$$


Power (P/K3)
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
0 1 2 3 4 5 6 7 8 9 10
Load (RL)