Final Examination Sensing and Actuation AT74.03 April 27, 2023

Time: 13:00-14:30 h. Open Book

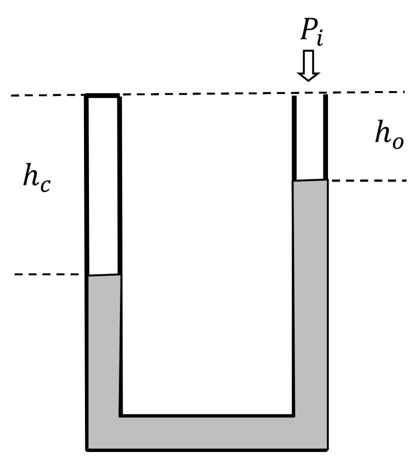
Marks: 100

Attempt all questions.

Q.1 An unknown pressure, P_i, is supplied to the opened end of a uniform diameter U-tube which has a closed end on the other side as shown in the below figure. Both sides of the U-tube have the same length of L. The tube is then filled with mercury with mass density of ρ_m from the opened end gradually. Use gravitational acceleration of g.

(a) Determine the unknown pressure as a function of relation between the height of the mercury on the closed end side, h_c, and the opened end side, h_o and the given parameters. (15)

(b) Determine the height at the closed end side (h_c) when the atmospheric pressure of 101,325 Pa is supplied and L = 50 cm, h_o = 0 cm, use ρ_m = 13,550 kg/m^3 and g = 9.8 m/s^2. (5)



__Solution__

(a)

Equate the gas law between h_c of L m with h_c of any height,

$$P_i(LA) = P(h_c A) \tag{1}$$

Thus

$$P = \frac{LP_i}{h_c} \tag{2}$$

Determine the height of the opened end side,

$$P = P_i + \rho_m(g)(h_c - h_o) \tag{3}$$

Equate (2) with (3),

$$\frac{LP_i}{h_c} = P_i + \rho_m(g)(h_c - h_o) \tag{4}$$

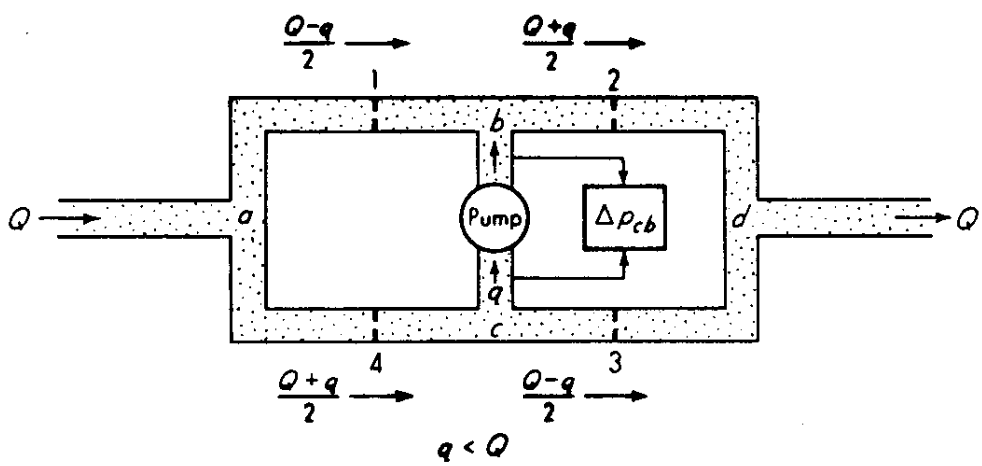
$$P_i = \frac{\rho_m(g)(h_c^2 - h_c h_o)}{L - h_c} \tag{5}$$

(b)

$$101325 = \frac{13{,}550(9.8)(h_c^2 - 0)}{0.5 - h_c} \tag{6}$$

$$h_c = 0.34 \text{ m} \tag{7}$$

Q.2 A bridge circuit flow meter, as shown in the below figure is used to determine the mass flow rate of water which is assumed incompressible with mass density of 1,000 kg/m^3 and frictionless. A small pump creates a volume flowrate of q. When the mass flow rate, G, is 400 kg/s, the pressure difference, ΔP, is detected at 2,000 Pa. Determine the volume flow rate, Q, when the pressure difference is detected at 4,000 Pa. (20)



Solution

From the relation,

$$\Delta P = KG \tag{1}$$

$$2{,}000\ Pa = K(400\ kg/s) \tag{2}$$

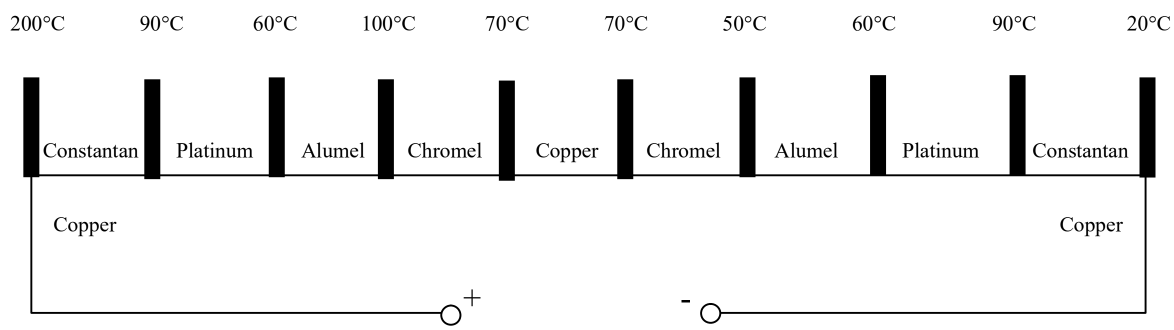
$$K = 5\ Pa \cdot s/kg \tag{3}$$

When the pressure difference is 4,000 Pa,

$$4{,}000\ Pa = (5\ Pa \cdot s/kg)(Q\ m^3/s)(1{,}000\ kg/m^3) \tag{4}$$

$$Q = 0.8\ m^3/s \tag{5}$$

Q.3 A multi-junction thermocouple is used to measure the temperature. Determine the voltage output of the thermocouple of the condition as shown in the below figure. Thermoelectric sensitivity in combination with platinum of several materials are given as follows: Copper is 6.5 µV/°C, Chromel is 25.8 µV/°C, Alumel is –13.6 µV/°C, and Constantan is -35 µV/°C. (20)



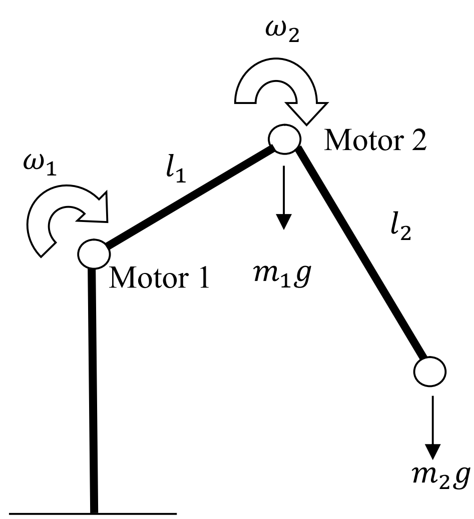
Solution

$$V_0 = (S_{Copper} - S_{Constantan})(200-20) + (S_{Constantan} - S_{Platinum})(90-90)$$
$$+(S_{Platinum} - S_{Alumel})(60-60) + (S_{Alumel} - S_{Chromel})(100-50)$$
$$+(S_{Chromel} - S_{Copper})(70-70) \quad (1)$$

$$V_0 = (6.5+35)\times 180 - 35\times 0 + 13.6\times 0 - (13.6+25.8)\times 50 + (25.8-6.5)\times 0 \quad (2)$$

$$V_0 = (6.5+35)\times 180 - (13.6+25.8)\times 50 = 5500\ \mu V = 5.5\ mV \quad \underline{(3)}$$

Q.4 A robot consisting of two links is moving in the vertical plane as shown in the below figure.



(a) When link 1 of length l_1 is rotating at a constant speed of ω_1 and link 2 of length l_2 is rotating at a constant speed of ω_2 and the links are considered as point masses as shown in the figure, determine the required power of both motor 1 and motor 2. Assume there is no safety factor is considered. (15)

(b) Determine the required power of both motors when $m_1 = 20\ kg$, $m_2 = 15\ kg$, $l_1 = 0.5\ m$, $l_2 = 0.4\ m$, $\omega_1 = 15\ rpm$, $\omega_2 = 60\ rpm$, $g = 9.8\ m/s^2$. (5)

Solution

(a)

Consider the maximum torque on link 2,

$$T_2 - m_2 g l_2 = 0 \quad (1)$$

$$T_2 = m_2 g l_2 \quad (2)$$

$$P_2 = m_2 g l_2 \omega_2 \quad \underline{(3)}$$

Consider the maximum torque on link 1,

$$T_1 - T_2 - m_1 g l_1 = 0 \quad (4)$$

$$T_1 = m_1 g l_1 + m_2 g l_2 \quad (5)$$

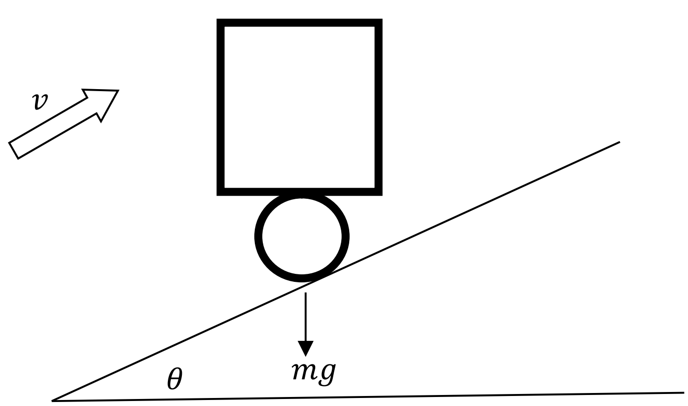
$$P_1 = (m_1 g l_1 + m_2 g l_2)\omega_1 \quad \underline{(6)}$$

(b)

$$P_2 = m_2 g l_2 \omega_2 = 15 \times 9.8 \times 0.4 \times 60 \times \frac{2\pi}{60} = 369.45\ W \quad \underline{(7)}$$

$$P_1 = (m_1 g l_1 + m_2 g l_2)\omega_1 = (20 \times 9.8 \times 0.5 + 15 \times 9.8 \times 0.4) \times 15 \times \frac{2\pi}{60} = 246.30\ W \quad \underline{(8)}$$

Q.5 A differential wheeled mobile robot is moving at a constant speed uphill as shown in the below figure.



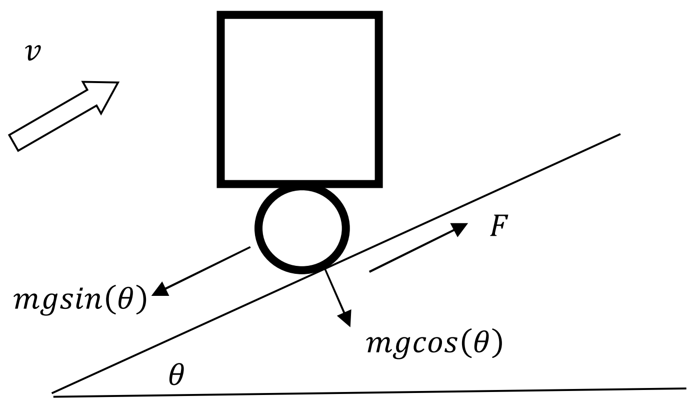
(a) When the robot has a total mass of m is running at a constant speed of v on a hill of slope θ. The static friction coefficient between the wheel and the hill surface is considered very large and makes no slip, determine the required power from each wheel motor. Assume there is no safety factor is considered. (15)

(b) Determine the required power from each wheel motor when $m = 50\ kg$, $v = 0.25\ m/s$, $\theta = 30°$, $g = 9.8\ m/s^2$. (5)

Solution

(a)

Consider the normal force and friction force,



Since there is no slip, the friction force, F, will be the same as the tangential force from gravity,

$$F - mgsin(\theta) = 0 \quad (1)$$

$$F = mgsin(\theta) \quad (2)$$

$$P = mgvsin(\theta) \quad (3)$$

Since there are two driving wheels,

$$P = 0.5mgvsin(\theta) \quad \underline{(4)}$$

(b)

Consider the required torque on each wheel motor,

$$P = 0.5mgvsin(\theta) = 0.5 \times 50 \times 9.8 \times 0.25 \times 0.5 = 30.625\ W \quad \underline{(5)}$$