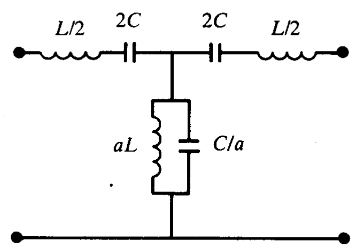
Midterm Examination   Sensing and Actuation AT74.03   October 11, 2012

Time: 10:00-11:30 h.   Open Book

Marks: 100

Attempt all questions.

Q.1 Prove that range of the pass band of an LC band-pass filter is between $f_1 = f_0\left[\sqrt{1+a} - \sqrt{a}\right]$ and $f_2 = f_0\left[\sqrt{1+a} + \sqrt{a}\right]$ when $f_0 = \dfrac{1}{2\pi\sqrt{LC}}$. (20)



Solution

The characteristic impedance,

$$Z_0 = \sqrt{Z_1 Z_2\left(1 + \frac{Z_1}{4Z_2}\right)}$$

When $Z_1 = j\omega L + \dfrac{1}{j\omega C}$ and $Z_2 = \dfrac{(j\omega La)(a/j\omega C)}{j\omega La + a/j\omega C}$,

$$Z_0 = \sqrt{\left(j\omega L + \frac{1}{j\omega C}\right)\left(\frac{(j\omega La)(a/j\omega C)}{j\omega La + a/j\omega C}\right)\left(1 + \frac{j\omega L + \frac{1}{j\omega C}}{4\frac{(j\omega La)(a/j\omega C)}{j\omega La + a/j\omega C}}\right)}$$

$$Z_0 = \sqrt{\left(\frac{1-\omega^2 LC}{j\omega C}\right)\left(\frac{j\omega La^2}{a - \omega^2 LCa}\right)\left(1 + \frac{\frac{1-\omega^2 LC}{j\omega C}}{4\frac{j\omega La^2}{a-\omega^2 LCa}}\right)}$$

$$Z_0 = \sqrt{\frac{La}{C}\left(1 - \frac{(1-\omega^2 LC)^2}{4\omega^2 LCa}\right)}$$

The pass band is the frequency range that makes the characteristic impedance become real number.

$$1-\frac{\left(1-\omega^2 LC\right)^2}{4\omega^2 LCa}>0$$

$$4\omega^2 LCa-1+2\omega^2 LC-\left(\omega^2 LC\right)^2>0$$

$$-\omega^4 L^2C^2+\omega^2 LC(4a+2)-1>0$$

$$\omega^4 L^2C^2-\omega^2 LC(4a+2)+1<0$$

$$\left(\omega^2-\frac{LC(4a+2)-\sqrt{L^2C^2(16a^2+16a+4)-4L^2C^2}}{2L^2C^2}\right)\left(\omega^2-\frac{LC(4a+2)+\sqrt{L^2C^2(16a^2+16a+4)-4L^2C^2}}{2L^2C^2}\right)<0$$

$$\left(\omega^2-\frac{(2a+1)-2\sqrt{(a^2+a)}}{LC}\right)\left(\omega^2-\frac{(2a+1)+2\sqrt{(a^2+a)}}{LC}\right)<0$$

$$\left(\omega-\sqrt{\frac{(2a+1)-2\sqrt{(a^2+a)}}{LC}}\right)\left(\omega+\sqrt{\frac{(2a+1)-2\sqrt{(a^2+a)}}{LC}}\right)\left(\omega-\sqrt{\frac{(2a+1)+2\sqrt{(a^2+a)}}{LC}}\right)\left(\omega+\sqrt{\frac{(2a+1)+2\sqrt{(a^2+a)}}{LC}}\right)<0$$

Consider only the positive frequency,

$$\left(\omega-\sqrt{\frac{(2a+1)-2\sqrt{(a^2+a)}}{LC}}\right)\left(\omega-\sqrt{\frac{(2a+1)+2\sqrt{(a^2+a)}}{LC}}\right)<0$$

$$\left(\omega^2-\omega\left[\frac{\sqrt{(2a+1)-2\sqrt{(a^2+a)}}+\sqrt{(2a+1)+2\sqrt{(a^2+a)}}}{\sqrt{LC}}\right]+\frac{1}{LC}\right)<0$$

$$\left(\omega^2-\omega\left[\frac{\sqrt{(2a+1)-2\sqrt{(a^2+a)}+(2a+1)+2\sqrt{(a^2+a)}+2\sqrt{\left[(2a+1)-2\sqrt{(a^2+a)}\right]\left[(2a+1)+2\sqrt{(a^2+a)}\right]}}}{\sqrt{LC}}\right]+\frac{1}{LC}\right)<0$$

$$\left(\omega^2-\omega\left[\frac{\sqrt{(4a+2)+2\sqrt{4a^2+4a+1-4a^2-4a}}}{\sqrt{LC}}\right]+\frac{1}{LC}\right)<0$$

$$\left(\omega^2-\omega\left[\frac{2\sqrt{(a+1)}}{\sqrt{LC}}\right]+\frac{1}{LC}\right)<0$$

$$\left(\omega-\sqrt{\frac{a+1}{LC}}+\sqrt{\frac{a}{LC}}\right)\left(\omega-\sqrt{\frac{a+1}{LC}}-\sqrt{\frac{a}{LC}}\right)<0$$

$$\sqrt{\frac{a+1}{LC}}-\sqrt{\frac{a}{LC}}<\omega<\sqrt{\frac{a+1}{LC}}+\sqrt{\frac{a}{LC}}$$

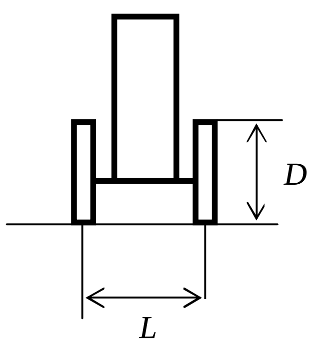
$$\frac{1}{2\pi\sqrt{LC}}(\sqrt{a+1}-\sqrt{a})<f<\frac{1}{2\pi\sqrt{LC}}(\sqrt{a+1}+\sqrt{a})$$

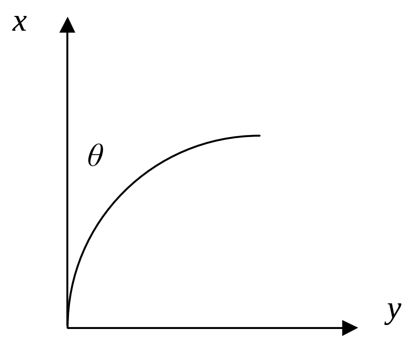
Q.2 Two incremental encoders each with N_r pulse-per-revolution are equipped on the left and right wheels of a mobile robot. Diameter of each wheel is D and distance between these two wheels is L.

(a) If the frequency of pulses of the left encoder is measured as $f_l(t)$ while the frequency of pulses of the right encoder is measured as $f_r(t)$, determine as functions of time of the speed of the center of the mobile robot and the angular velocity of the robot heading. (6)

(b) Determine as functions of time of coordinate of the robot and its heading, assume the robot is at coordinate (0, 0) and heads 0 degree at t = 0 second. (6)

(c) If N_r = 300, D = 30 cm, L = 50 cm, and the frequencies of pulses are measured and found constantly at $f_l(t)$ = 20 Hz, $f_r(t)$ = -5 Hz, determine the robot coordinate and its heading at t = 10 seconds. (8)





<u>Solution</u>

(a)

Speed of the left wheel,

$$v_l = \frac{f_l \pi D}{N_r} \text{ m/s} \tag{1}$$

Speed of the right wheel,

$$v_r = \frac{f_r \pi D}{N_r} \text{ m/s} \tag{2}$$

Speed of the center of the mobile robot,

$$v = \frac{v_l + v_r}{2} = \frac{(f_l + f_r)\pi D}{2N_r} \text{ m/s} \tag{3}$$

Angular velocity of the heading of mobile robot,

$$\omega = \frac{v_l - v_r}{L} = \frac{(f_l - f_r)\pi D}{LN_r} \text{ rad/s} = \frac{180(f_l - f_r)D}{LN_r} \text{ degree/s} \tag{4}$$

(b) Heading of the mobile robot,

$$\theta(t) = \int_0^t \frac{(f_l - f_r)\pi D}{LN_r} d\tau \text{ rad} = \int_0^t \frac{180(f_l - f_r)D}{LN_r} d\tau \text{ degree} \tag{5}$$

Coordinate of the mobile robot,

$$x(t) = \int_0^t \frac{(f_l + f_r)\pi D}{2N_r} \cos\left(\int_0^t \frac{(f_l - f_r)\pi D}{LN_r} d\tau \right) d\tau \text{ m} \tag{6}$$

$$y(t) = \int_0^t \frac{(f_l + f_r)\pi D}{2N_r} \sin\left(\int_0^t \frac{(f_l - f_r)\pi D}{LN_r} d\tau \right) d\tau \text{ m} \tag{7}$$

(c) Heading of the mobile robot,

$$\theta(t) = \int_0^{10} \frac{(20+5)\pi(0.3)}{0.5(300)} d\tau = 1.57 \text{ rad} = \int_0^{10} \frac{(20+5)180(0.3)}{0.5(300)} d\tau = 90 \text{ degree} \tag{8}$$

Coordinate of the mobile robot,

$$x(t) = \int_0^{10} \frac{(20-5)\pi(0.3)}{2(300)} \cos(0.157\tau) d\tau = 0.15 \sin(0.157\tau)\Big|_0^{10} = 0.15 \text{ m} \tag{9}$$

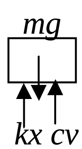
$$y(t) = \int_0^{10} \frac{(20-5)\pi(0.3)}{2(300)} \sin(0.157\tau) d\tau = -0.15 \cos(0.157\tau)\Big|_0^{10} = 0.15 \text{ m} \tag{10}$$

Q.3 Determine the deflection of a seismic mass in an accelerometer from gravitational acceleration when the accelerometer is placed on a table as shown in figure (a). Then determine the deflection again when the accelerometer falls freely from the table as shown in figure (b). Assume the accelerometer axis is always aligned with the gravitational acceleration. The seismic mass is 200 g, spring stiffness is 10 N/mm, the damping coefficient is 5 Ns/m, and gravitational acceleration is 10 m/s^2. (10)

(a) (b)

Solution

Consider FBD of case (a),

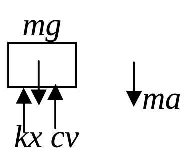


Since the accelerometer is placed on a table, thus at the steady state there is no damping force,

$$mg = kx$$

$$x = \frac{mg}{k} = \frac{0.2(10)}{10000} = 0.0002\, m = 0.2\, mm$$

Consider FBD of case (b),



Since the accelerometer falls freely, $a = g$, thus there is no spring force and damping force.

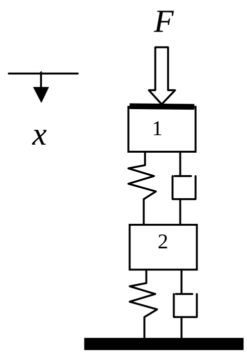
$$mg - kx - cv = mg$$

$$x = 0\, mm$$

Q.4 A force sensor is made from two sections of materials with different stiffness.

(a) If the two sections are bonded in series as shown in figure (1), determine the magnitude of force, F, if the deflection, x, at the steady state is read at 2 mm. Assume the deflection is 0mm at no force. (10)

(a) If the two sections are bonded in parallel as shown in figure (2), determine the magnitude of force, F, if the deflection, x, at the steady state is read at 2 mm. Assume the deflection is 0mm at no force. (10)



$m_1 = 50$ g
$k_1 = 10000$ N/m
$c_1 = 5000$ Ns/m
$m_2 = 30$ g
$k_2 = 8000$ N/m
$c_2 = 2000$ Ns/m

Figure (1)

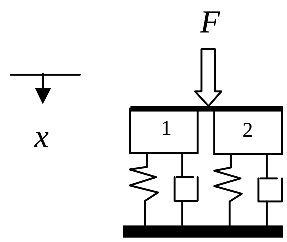


Figure (2)

Solution

(a) At steady state,

$$F = k_1 x_1 = k_2 x_2$$

$$x = x_1 + x_2 = \frac{F}{k_1} + \frac{F}{k_2} = \frac{k_1 + k_2}{k_1 k_2} F$$

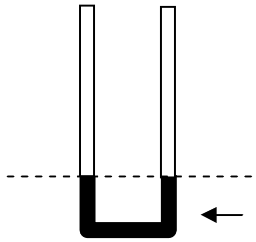
$$F = \frac{k_1 k_2}{k_1 + k_2} x = \frac{10000(8000)}{10000 + 8000} 0.002 = 8.89\ N$$

(b) At steady state,

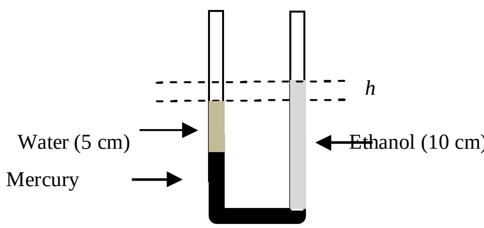
$$F = k_1 x + k_2 x$$

$$F = (k_1 + k_2)x = (10000 + 8000)0.002 = 36\ N$$

Q.5 A uniform U-tube manometer which both ends are exposed to atmospheric pressure is filled with mercury with mass density of 13,550 kg/m^3 as shown in the below figure.



If the left end is added with water with mass density of 1,000 kg/m^3 for 5 cm, the right end is added with ethanol with mass density of 790 kg/m^3 for 10 cm, determine the level difference, *h*, of both ends while the two ends are still exposed to the atmospheric pressure. Apply g = 9.8 m/s^2. (20)



Solution

Since both end are exposed to atmospheric pressure,

$$1atm + 1{,}000(9.8)(0.05) + 13{,}550(9.8)(x) - 790(9.8)(0.1) = 1atm$$

$$x = 0.0021$$

When *x* is level difference of mercury.

Thus,

$$h = 0.1 - 0.0021 - 0.05 = 0.0479\, m = 4.79\, cm$$

Q.6 Design a sensor used to determine speed of a running car on an express way in real time. The sensor must be installed at the left side of the express way. (10)