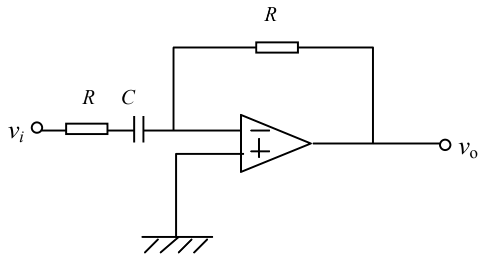
Midterm Examination   Sensing and Actuation AT74.03   October 10, 2013

Time: 10:00-11:30 h.   Open Book
Marks: 100

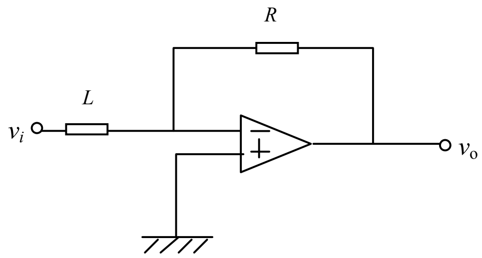
Attempt all questions.

Q.1 Determine transfer function and magnitude ratio from input, v_i, to output, v_o, at 0 Hz and ∞ Hz of the following three circuits. (30)

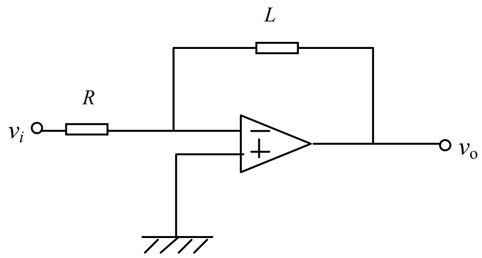
(a)



(b)



(c)



Solution

(a)

$$i = \frac{v_i}{R + \frac{1}{sC}} = \frac{sCv_i}{sRC + 1}$$

$$-v_o = iR = \frac{sRCv_i}{sRC + 1}$$

$$\frac{v_o}{v_i} = -\frac{sRC}{sRC+1}$$

$$\left|\frac{v_o}{v_i}\right|_\omega = \left|-\frac{j\omega RC}{j\omega RC+1}\right| = \frac{\omega RC}{\sqrt{(\omega RC)^2+1}}$$

$$\left|\frac{v_o}{v_i}\right|_0 = \frac{0}{\sqrt{0+1}} = 0$$

$$\left|\frac{v_o}{v_i}\right|_\infty = \frac{1}{\sqrt{(1)^2+0}} = 1$$

(b)

$$i = \frac{v_i}{sL}$$

$$-v_o = iR = \frac{Rv_i}{sL}$$

$$\frac{v_o}{v_i} = -\frac{R}{sL}$$

$$\left|\frac{v_o}{v_i}\right|_\omega = \left|-\frac{R}{j\omega L}\right| = \frac{R}{\omega L}$$

$$\left|\frac{v_o}{v_i}\right|_0 = \infty$$

$$\left|\frac{v_o}{v_i}\right|_\infty = 0$$

(c)

$$i = \frac{v_i}{R}$$

$$-v_o = isL = \frac{sLv_i}{R}$$

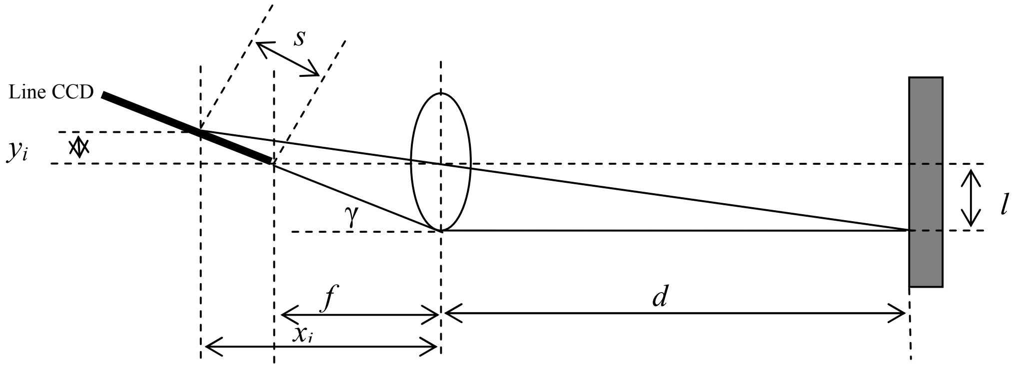
$$\frac{v_o}{v_i} = -\frac{sL}{R}$$

$$\left|\frac{v_o}{v_i}\right|_\omega = \left|-\frac{j\omega L}{R}\right| = \frac{\omega L}{R}$$

$$\left|\frac{v_o}{v_i}\right|_0 = 0$$

$$\left|\frac{v_o}{v_i}\right|_\infty = \infty$$

Q.2 Laser range finder is used to determine the range, *d*, to an object as shown in the below figure. If the focused point, *s*, on the line CCD shows 0.1 mm, determine the range to the object. The focal length, *f*, is 5 mm, the lens radius, *l*, is 20 mm. (15)



Solution

Lens relation,

$$\frac{1}{f} = \frac{1}{x_i} + \frac{1}{d}$$

$$x_i = \frac{fd}{d-f}$$

Straight line relation,

$$y_i = \frac{l}{f}x_i - l$$

$$y_i = \frac{ld}{d-f} - l$$

$$s = \sqrt{(x_i - f)^2 + y_i^2}$$

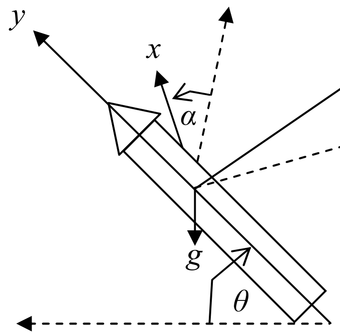
$$s = \sqrt{(\frac{fd}{d-f} - f)^2 + (\frac{ld}{d-f} - l)^2}$$

$$s = \frac{f}{d-f}\sqrt{f^2 + l^2}$$

$$d = \frac{f}{s}\sqrt{f^2 + l^2} + f$$

$$d = \frac{5}{0.1}\sqrt{(5)^2 + (20)^2} + 5 = 1035.8\,mm = 1.0358\,m$$

Q.3 A 3-axis accelerometer attached with an airplane is applied to determine attitude of the airplane respect to gravity force. If x-axis points to the right, y-axis points to the front, and z-axis points to the top of the airplane as shown in the below figure. Determine the airplane attitude (roll, α, and pitch, θ, angles) when the output from the accelerometer shows 0.866g along x-axis, and -0.3535g along z-axis when g represents gravitational acceleration. What is the output from the accelerometer along y-axis? (15)



Solution

Acceleration along x-axis,

$$a_x = gcos(\theta)sin(\alpha) = 0.866g$$

Acceleration along z-axis,

$$a_z = -gcos(\theta)\cos(\alpha) = -0.3535g$$

$$\alpha = \text{atan}\,(-\frac{a_x}{a_z}) = \text{atan}\,(\frac{0.866g}{0.3535g}) = 67.79°$$

$$\theta = acos\left(\frac{0.866}{sin(67.79°)}\right) = 20.71°$$

Acceleration along y-axis,

$$a_y = -gsin(\theta) = -gsin(20.71°) = -0.3536g$$

Q.4 A link-type load cell is applied to measure axial force. When a force of100 N is applied to this load cell, the voltage output from the DC Wheatstone bridge circuit of the load cell shows 50 mV at the supplied voltage to the bridge circuit of 12 V. Determine the unknown force if the voltage output from the bridge circuit indicates 60 mV at the supplied voltage to the bridge circuit of 5 V. (15)

Solution

The voltage output from DC Wheatstone bridge circuit of link-type load cell varies with the multiplication of the supplied voltage and the force.

$$E_o = KE_iF$$

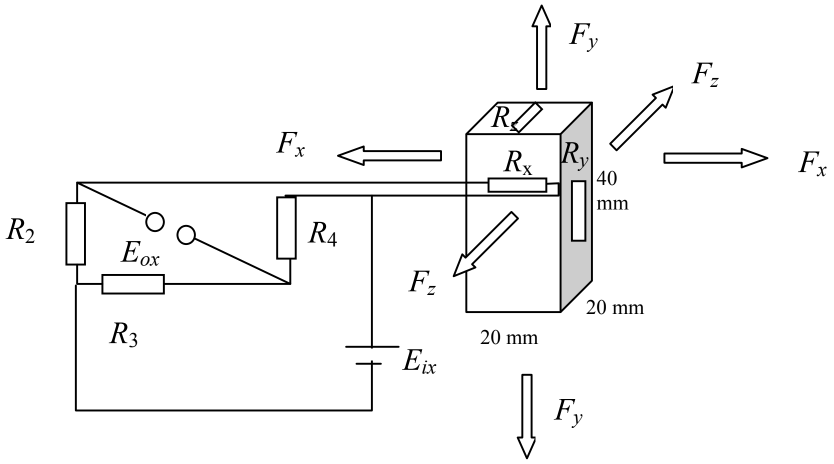
$$50 \times 10^{-3}V = K(12V)(100N)$$

$$K = \frac{50 \times 10^{-3}}{1200} = 4.17 \times 10^{-5} N^{-1}$$

$$F = \frac{E_o}{KE_i}$$

$$F = \frac{60 \times 10^{-3}}{4.17 \times 10^{-5}(5)} = 287.77N$$

Q.5 3 identical strain gages are used to determine forces in 3 axis acting on a bar as shown in the below figure. Each strain gage is installed in each separated DC Wheatstone bridge circuit having the remaining resistors with the same nominal resistance value to ensure balancing of the bridge circuit during no load. Determine forces in 3 axis when the output readings indicate 3 mv from E_{ox}, 4 mv from E_{oy}, and 5 mv from E_{oz}. Only the bridge circuit for R_x is shown in the figure, the bridge circuits for R_y and R_z are not shown in the figure. The bar is made of material with Young's modulus, E, of 120 GPa with Poisson's ratio, γ, of 0.4. Each strain gage has nominal resistance, R, of 120 Ω with gage factor, S_g, of 4. The supply voltage, E_i, is 5 V for all the bridge circuits. (15)



<u>**Solution**</u>

$$\sigma_x = \frac{F_x}{A_x}$$

$$\varepsilon_x = \frac{(\sigma_x - \gamma\sigma_y)}{E} = \frac{1}{E}\left(\frac{F_x}{A_x} - \gamma\frac{F_y}{A_y} - \gamma\frac{F_z}{A_z}\right)$$

$$\frac{\Delta R_1}{R_1} = S_g\varepsilon_x = \frac{S_g}{E}\left(\frac{F_x}{A_x} - \gamma\frac{F_y}{A_y} - \gamma\frac{F_z}{A_z}\right)$$

$$E_{ox} = \frac{r}{(1+r)^2}\left[\frac{\Delta R_1}{R_1}\right]E_{ix} = \frac{r}{(1+r)^2}\left[\frac{S_g}{E}\left(\frac{F_x}{A_x} - \gamma\frac{F_y}{A_y} - \gamma\frac{F_z}{A_z}\right)\right]E_{ix}$$

$$E_{oy} = \frac{r}{(1+r)^2}\left[\frac{S_g}{E}\left(\frac{F_y}{A_y} - \gamma\frac{F_z}{A_z} - \gamma\frac{F_x}{A_x}\right)\right]E_{iy}$$

$$E_{oz} = \frac{r}{(1+r)^2}\left[\frac{S_g}{E}\left(\frac{F_z}{A_z} - \gamma\frac{F_x}{A_x} - \gamma\frac{F_y}{A_y}\right)\right]E_{iy}$$

$$E_{ox} = 3\times10^{-3}$$

$$= \frac{1}{4}\left[\frac{4}{120\times10^9}\left(\frac{F_x}{20\times40\times10^{-6}} - 0.4\frac{F_y}{20\times20\times10^{-6}} - 0.4\frac{F_z}{20\times40\times10^{-6}}\right)\right]5$$

$$72 = \frac{F_x}{800} - 0.4\frac{F_y}{400} - 0.4\frac{F_z}{800}$$

$$E_{oy} = 4\times10^{-3}$$

$$= \frac{1}{4}\left[\frac{4}{120\times10^9}\left(\frac{F_y}{20\times20\times10^{-6}} - 0.4\frac{F_z}{20\times40\times10^{-6}} - 0.4\frac{F_z}{20\times40\times10^{-6}}\right)\right]5$$

$$96 = \frac{F_y}{400} - 0.4\frac{F_z}{800} - 0.4\frac{F_x}{800}$$

$$E_{oz} = 5\times10^{-3}$$

$$= \frac{1}{4}\left[\frac{4}{120\times10^9}\left(\frac{F_z}{20\times40\times10^{-6}} - 0.4\frac{F_x}{20\times40\times10^{-6}} - 0.4\frac{F_y}{20\times20\times10^{-6}}\right)\right]5$$

$$120 = \frac{F_z}{800} - 0.4\frac{F_x}{800} - 0.4\frac{F_y}{400}$$

$$F_x = x\times10^5 N$$

$$F_y = y\times10^5 N$$

$$F_z = z\times10^5 N$$

Q.6 Design a non-contact torque sensor used to determine torque from a motor acting on a joint of a robot arm. (10)