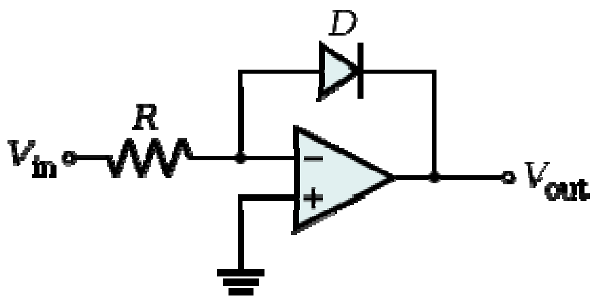
Midterm Examination Sensing and Actuation AT74.03 October 2, 2014

Time: 10:00-11:30 h. Open Book
Marks: 100

Attempt all questions.

Q.1 Log amplifier and exponential amplifier are the basic circuits for multiplication and division circuits.

(a) Circuit of a log amplifier is shown in the below circuit. Determine output voltage, v_{out}, as a function of input voltage, v_{in}. The relation between current and voltage for PN junction diode actually follows $I_d = I_s(e^{\frac{v_d}{V_t}} - 1)$, which can be estimated by $I_d \approx I_s e^{\frac{v_d}{V_t}}$ since V_t is relatively small. I_d is the current passing the diode, v_d is the voltage across the diode, V_t is a thermal voltage constant, I_s is a saturation current constant, and R is a resistance as shown in the below circuit (10)



Solution

$$I_r = I_d = \frac{v_{in}}{R}$$

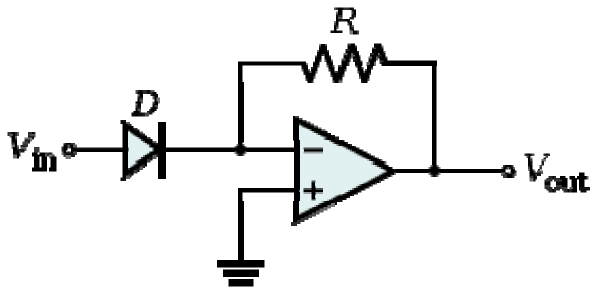
$$I_d = I_s(e^{\frac{v_d}{V_t}} - 1) \approx I_s e^{\frac{v_d}{V_t}}$$

$$\frac{v_{in}}{R} \approx I_s e^{\frac{-v_{out}}{V_t}}$$

$$\ln\left(\frac{v_{in}}{I_s R}\right) \approx \frac{-v_{out}}{V_t}$$

$$v_{out} \approx -V_t \ln\left(\frac{v_{in}}{I_s R}\right)$$

(b) Circuit of an exponential amplifier is shown in the below circuit. Determine output voltage, v_{out}, as a function of input voltage, v_{in}. Use the relations and notations as given in (a). (10)



Solution

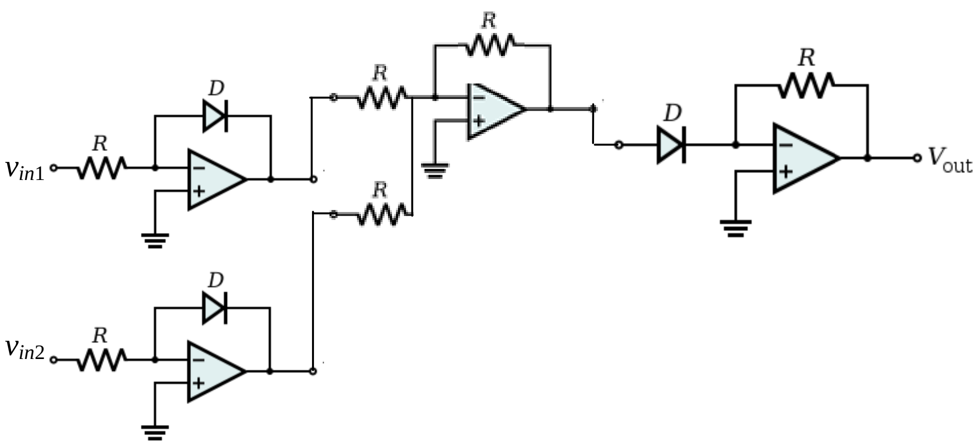
$$I_r = I_d = \frac{-v_{out}}{R}$$

$$I_d = I_s(e^{\frac{v_d}{V_t}} - 1) \approx I_s e^{\frac{v_d}{V_t}}$$

$$\frac{-v_{out}}{R} \approx I_s e^{\frac{v_{in}}{V_t}}$$

$$v_{out} \approx -I_s R e^{\frac{v_{in}}{V_t}}$$

(c) The multiplication circuit is shown below. Determine the relation between input voltages, v_{in1} and v_{in2}, and the output voltage, v_{out}. (10)



Solution

$$v_1 \approx -V_t \ln\left(\frac{v_{in1}}{I_s R}\right)$$

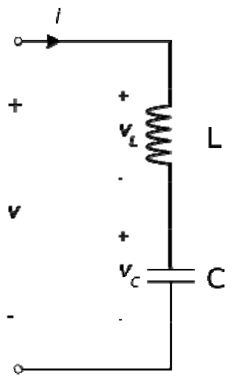
$$v_2 \approx -V_t \ln\left(\frac{v_{in2}}{I_s R}\right)$$

$$v_3 \approx V_t \ln\left(\frac{v_{in1}}{I_s R}\right) + V_t \ln\left(\frac{v_{in2}}{I_s R}\right) = V_t \ln\left(\frac{v_{in1} \times v_{in2}}{I_s^2 R^2}\right)$$

$$v_{out} \approx -I_s R e^{\frac{V_t \ln\left(\frac{v_{in1} \times v_{in2}}{I_s^2 R^2}\right)}{V_t}} = -\frac{v_{in1} \times v_{in2}}{I_s R}$$

Q.2 Determine transfer functions from input voltage to output current of the following LC circuits. Then roughly plot the Bode diagrams of the circuits, identify zeros, poles, slopes in the Bode magnitude plot, and phases in the Bode phase plot.

(a) Series LC circuit as shown in the below figure. (10)



Solution

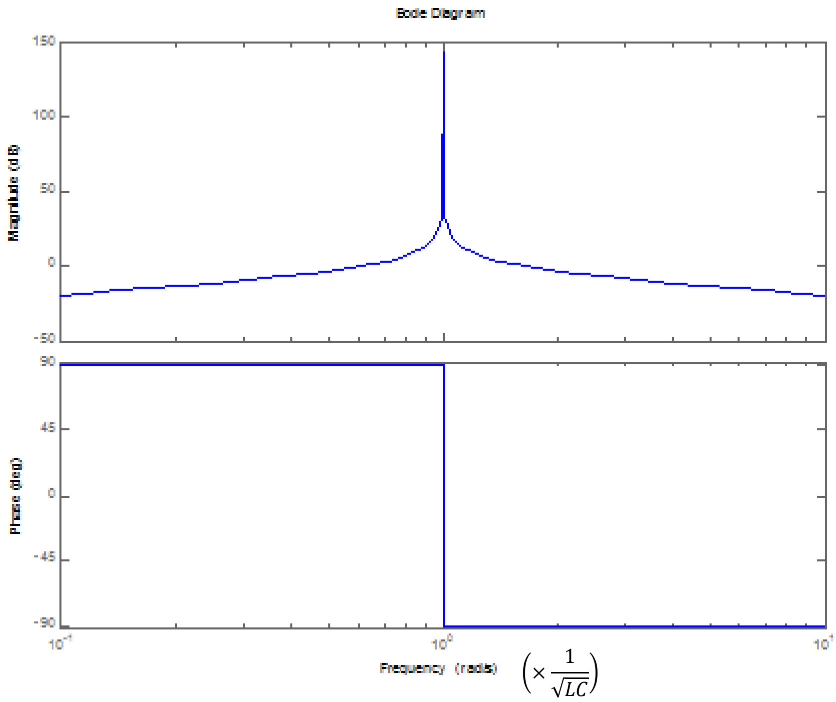
$$v = L\frac{di}{dt} + \frac{\int i dt}{C}$$

$$V = \left(Ls + \frac{1}{Cs}\right) I$$

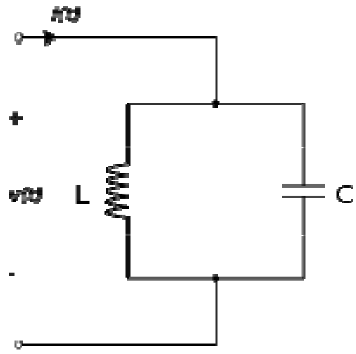
$$\frac{I}{V} = \frac{Cs}{LCs^2 + 1}$$

Zero = 0

Poles = $\pm \frac{j}{\sqrt{LC}}$



(b) Parallel LC circuit as shown in the below figure. (10)



Solution

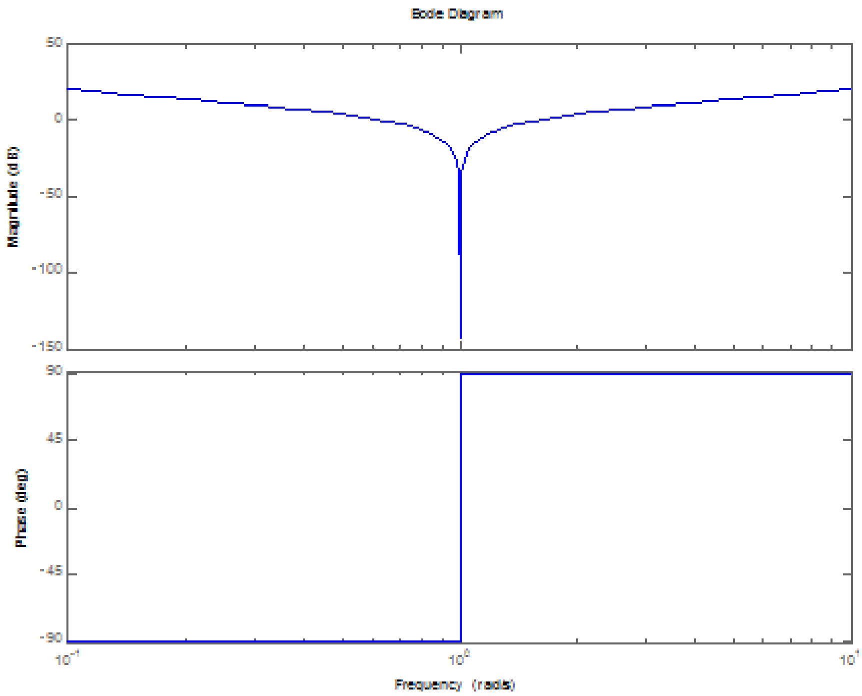
$$i = C\frac{dv}{dt} + \frac{\int v dt}{L}$$

$$I = \left(Cs + \frac{1}{Ls}\right)V$$

$$\frac{I}{V} = \frac{LCs^2 + 1}{Ls}$$

$$\text{Zero} = \pm \frac{j}{\sqrt{LC}}$$

$$\text{Poles} = 0$$



Q.3 An Eddy current drag-cup tachometer is used to measure speed of an engine. The dead space of the tachometer is between the engine speed of -120 and 120 rpm. Outside the dead space, the rate of change of voltage output from the tachometer has linear relationship with the rate of change of engine speed. From the experiment when the engine speed is 2,000 rpm the voltage output is 2 V. Determine the engine speed when the tachometer indicates voltage output of 3.5 V. (10)

Solution

Outside the dead space, the relation between engine speed and voltage output follows

$$V = m\omega + c \tag{1}$$

At 120 rpm, the voltage output is 0 V.

$$0 = m120 + c \tag{2}$$

At 2,000 rpm, the voltage output is 2 V.

$$2 = m2000 + c \tag{3}$$

(3)-(2),

$$2 = m1880 \tag{4}$$

$$m = \frac{1}{940} \tag{5}$$

$$c = -\frac{6}{47} \tag{6}$$

When the output voltage is 3.5 V,

$$3.5 = \frac{1}{940}\omega - \frac{6}{47} \quad (7)$$

$$\omega = 3410 \text{ rpm} \quad \underline{(8)}$$

Q.4 A rate-integrating gyro installed in a car is used to measure pitch angle of a hill. The hill height, h, is a function of hill horizontal distance, x, as expressed by the equation $h = 200e^{-\frac{x^2}{400}}$ when all the units are in meters. Static sensitivity of the rate-integrating gyro is 0.1. Determine the output deflection, θ, from the gyro as a function of hill horizontal distance. Determine the hill pitch angle and the output deflection when the hill horizontal distance is 50 meters. (20)

<u>Solution</u>

Slope of the hill is determined from,

$$\tan\phi = \frac{dh}{dx} = -xe^{-\frac{x^2}{400}}$$

$$\phi = \text{atan}\left(-xe^{-\frac{x^2}{400}}\right)$$

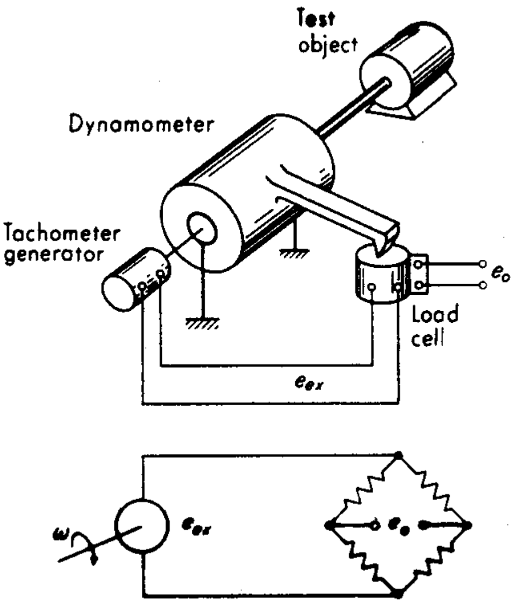
$$\theta = K\phi = 0.1\text{atan}\left(-xe^{-\frac{x^2}{400}}\right)$$

When x = 50 m,

$$\phi = \text{atan}\left(-50e^{-\frac{50^2}{400}}\right) = -0.0962 \text{ rad} = -5.51 \text{ degree}$$

$$\theta = 0.1\text{atan}\left(-50e^{-\frac{50^2}{400}}\right) = -0.0096 \text{ rad} = -0.55 \text{ degree}$$

Q.5 A dynamometer is used to measure power of a vehicle engine. When the engine speed is 2500 rpm and the engine torque is detected at 100 Nm make the amplitude of the voltage output from the Wheatstone bridge circuit become 0.4 V. Determine the power when the voltage output indicates 1.5 V. (10)



Solution

The voltage output is proportional to power.

$$e_o = KP = KT\omega$$

$$0.4 = K100 \times 2500\frac{2\pi}{60}$$

$$K = 1.53 \times 10^{-5}\text{Volts/Watt}$$

$$1.5 = 1.53 \times 10^{-5}P$$

$$P = 98{,}175 \text{ Watts}$$

Q.6 Design a sensor used to determine distance from the sensor to a flying airplane. (10)