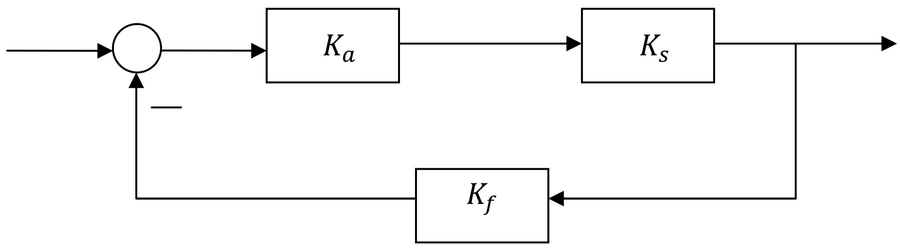
Midterm Examination Sensing and Actuation AT74.03 October 1, 2015

Time: 10:00-11:30 h. Open Book
Marks: 100

Attempt all questions.

Q.1 A system is represented by the transfer function K_s. This system is disturbed by environment variation which makes change in the transfer function with maximum error of 20%. In order to solve this problem, the system is connected with a high gain amplifier, K_a, and a feedback system, K_f, as shown in the below figure.



(a) If the amplifier gain, K_a, is very high, determine the feedback system transfer function that still makes the new system have to same transfer function as the original system. (10)

(b) If the feedback system is disturbed by environment variation with the maximum error of 2%, determine the maximum error of the new system. (10)

Solution

(a) The closed loop transfer function,

$$H_c = \frac{K_a K_s}{1 + K_a K_s K_f}$$

When K_a, is very high,

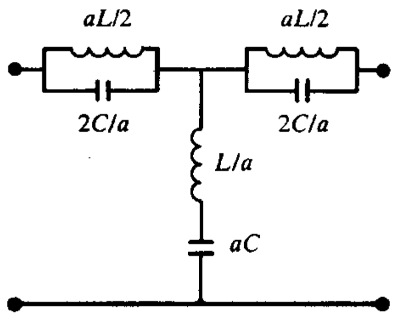
$$H_c \approx \frac{1}{K_f} = K_s$$

Thus,

$$K_f = \frac{1}{K_s}$$

(b) The maximum error of the new system is the same as the maximum error of the feedback system of 2%.

Q.2 Determine range of stop-band in rad/sec of the LC band-stop filter. (20)



Solution

The characteristic impedance,

$$Z_0 = \sqrt{Z_1 Z_2 \left(1 + \frac{Z_1}{4Z_2}\right)}$$

When

$$Z_1 = \frac{j\omega La \cdot \frac{a}{j\omega C}}{j\omega La + \frac{a}{j\omega C}} = \frac{jaL\omega}{1 - LC\omega^2}$$

$$Z_2 = \frac{j\omega L}{a} + \frac{1}{jaC\omega} = \frac{1 - LC\omega^2}{jaC\omega}$$

Thus

$$Z_0 = \sqrt{\frac{jaL\omega}{1 - LC\omega^2} \cdot \frac{1 - LC\omega^2}{jaC\omega} \left(1 + \frac{\frac{jaL\omega}{1-LC\omega^2}}{4\frac{1-LC\omega^2}{jaC\omega}}\right)}$$

$$Z_0 = \sqrt{\frac{L}{C}\left(1 - \frac{a^2 LC\omega^2}{4(1 - LC\omega^2)^2}\right)}$$

The characteristic impedance is real number when,

$$1 \geq \frac{a^2 LC\omega^2}{4(1 - LC\omega^2)^2}$$

$$4L^2C^2\omega^4 - (8 + a^2)LC\omega^2 + 4 \geq 0$$

$$\left(\omega^2 - \frac{8 + a^2 - a\sqrt{a^2 + 16}}{8LC}\right)\left(\omega^2 - \frac{8 + a^2 + a\sqrt{a^2 + 16}}{8LC}\right) \geq 0$$

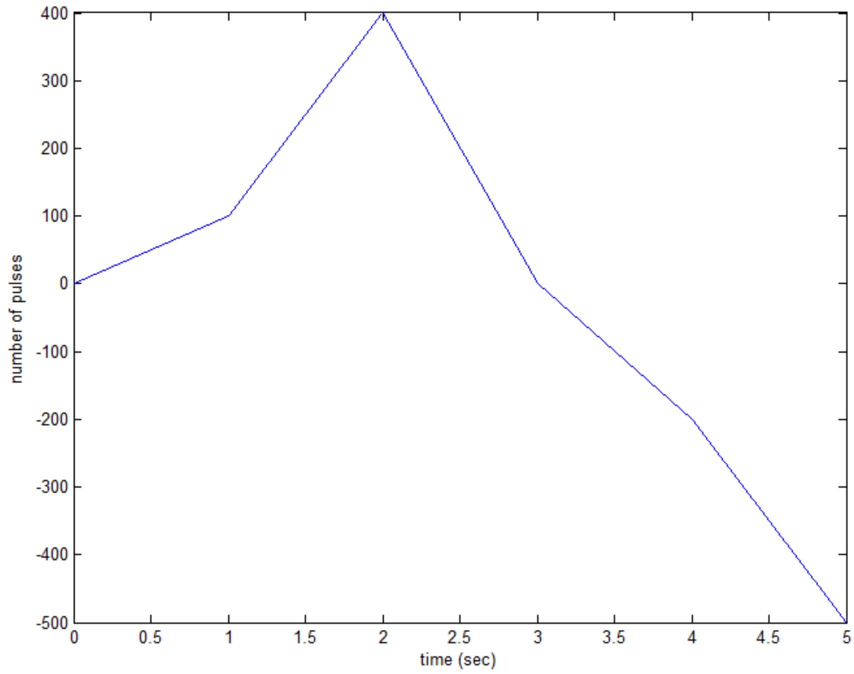
When we consider the positive frequency only,

$$\left(\omega - \sqrt{\frac{8 + a^2 - a\sqrt{a^2 + 16}}{8LC}}\right)\left(\omega - \sqrt{\frac{8 + a^2 + a\sqrt{a^2 + 16}}{8LC}}\right) \geq 0$$

Thus the stop-band frequency,

$$\left(\sqrt{\frac{8+a^2-a\sqrt{a^2+16}}{8LC}},\sqrt{\frac{8+a^2+a\sqrt{a^2+16}}{8LC}}\right)$$

Q.3 If cumulative number of pulses from an incremental encoder attached directly with a motor shaft is shown in the graph below. Determine the motor angular displacement in degree and angular velocity in deg/sec as functions of time at each time interval during 0-5 sec. Assume the encoder has 300 pulses per revolution. (20)



Solution

During 0-1 sec,

$$angular\ displacement = \frac{100}{300} \times 360t = 120t\ degree$$

$$angular\ velocity = \frac{100}{300} \times 360 = 120\ degree/sec$$

During 1-2 sec,

$$angular\ displacement = 120 + \frac{300}{300} \times 360(t-1) = 120 + 360(t-1)\ degree$$

$$angular\ velocity = \frac{300}{300} \times 360 = 360\ degree/sec$$

During 2-3 sec,

$$angular\ displacement = 480 - \frac{400}{300} \times 360(t-2) = 480 - 480(t-2)\ degree$$

$$angular\ velocity = \frac{-400}{300} \times 360 = -480\ degree/sec$$

During 3-4 sec,

$$angular\ displacement = -\frac{200}{300} \times 360(t-3) = -240(t-3)\ degree$$

$$angular\ velocity = \frac{-200}{300} \times 360 = -240\ degree/sec$$

During 4-5 sec,

$$angular\ displacement = -240 - \frac{300}{300} \times 360(t-4) = -240 - 360(t-4)\ degree$$

$$angular\ velocity = \frac{-300}{300} \times 360 = -360\ degree/sec$$

Q.4 An angular accelerometer has the seismic moment of inertia of 0.004 kg.m^2, damping coefficient of 0.0005 N.m.s/degree, spring stiffness of 0.001 N.m/degree. Determine the input angular acceleration in deg/sec^2 when the spring deflects for $\pi/90$ rad at steady state. Determine also the oscillation frequency of the deflection in Hz before the steady state. (20)

Solution

At the steady state,

$$k\theta = J\alpha$$

$$\left(0.001\ N.\frac{m}{deg}\right)\left(\frac{\pi}{90} \cdot \frac{180}{\pi}\ deg\right) = (0.004kg.m^2)\alpha$$

$$\alpha = 0.5\ rad/sec^2 = 0.5 \cdot \frac{180}{\pi} = 28.65\ deg/sec^2$$

The characteristic equation of the accelerometer,

$$Js^2 + Bs + K = 0$$

$$(0.004kg.m^2)s^2 + \left(0.0005N.m.s/deg.\frac{180\ deg}{\pi\ rad}\right)s + \left(0.001\ N.m/deg.\frac{180\ deg}{\pi\ rad}\right) = 0$$

$$0.004s^2 + 0.0287s + 0.0573 = 0$$

$$s^2 + 7.1620s + 14.3239 = (s + 3.5810 + 1.2249j)(s + 3.5810 - 1.2249j) = 0$$

The oscillation frequency,

$$\omega = 1.2249\ rad/sec = 0.1949\ Hz$$

Q.5 Torque is measured by using a gimbal-lock-free gyroscope with flywheel angular momentum of 5 kg.m^2.rad/sec and moment of inertia of everything in one axis of 2 kg.m^2 and in the other axis of 3 kg.m^2. Determine the input torque in Nm when the gyroscope axis rotates around the angular velocity of 5 rad/sec. Then determine the oscillation frequency of the gyroscope axis angular velocity in Hz. (20)

Solution

The transfer function from torque input to angular velocity output of gyroscope,

$$\frac{\omega}{T_x} = \frac{1/H_s}{\left(\frac{I_x I_y}{H_s{}^2}\right)s^2 + 1}$$

$$\frac{5}{T_x} = \frac{1}{5}$$

$$T_x = 25\ Nm$$

Characteristic equation,

$$\left(\frac{I_x I_y}{H_s{}^2}\right)s^2 + 1 = 0$$

$$s^2 + \left(\frac{H_s{}^2}{I_x I_y}\right) = \left(s + j\frac{H_s}{\sqrt{I_x I_y}}\right)\left(s - j\frac{H_s}{\sqrt{I_x I_y}}\right) = 0$$

Oscillation frequency,

$$\frac{H_s}{\sqrt{I_x I_y}} = \frac{5}{\sqrt{2 \times 3}} = 2.0412\ rad/\sec = 0.3249\ Hz$$