

Time: 10:00-11:30 h.

Open Book

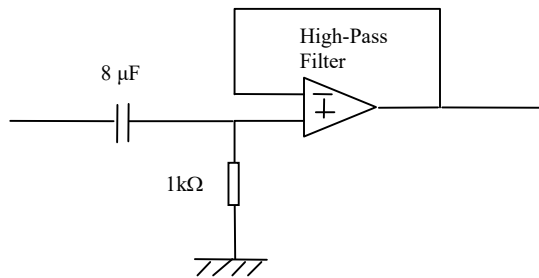
Marks: 100

Attempt all questions.

Q.1 Design a circuit that converts input sinusoidal signal expressed by $v_i = \sin(2\pi f)$ into output DC signal with magnitude of 0 Volt when the input signal is at 20 Hz and magnitude of 5 Volts when the input signal is at 20 kHz. Linear conversion is not required. (25)

Solution

1. A high-pass filter can be used to vary the amplitude of the sinusoidal signal. Use a high-pass filter with cut-off frequency at 20 Hz.



$$f = \frac{1}{2\pi RC}$$

$$RC = \frac{1}{2\pi(20)} = 0.008$$

Select $R = 1 \text{ k}\Omega$, $C = 8 \text{ }\mu\text{F}$.

$$\left| \frac{E_0}{E_i} \right| = \frac{1}{\sqrt{1 + \left(\frac{1}{\omega RC}\right)^2}} = \frac{1}{\sqrt{1 + \left(\frac{40\pi}{\omega}\right)^2}}$$

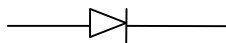
At 20 Hz,

$$\left| \frac{E_0}{E_i} \right| = \frac{1}{\sqrt{2}} = 0.707$$

At 20 kHz,

$$\left| \frac{E_0}{E_i} \right| = \frac{1}{\sqrt{1.000001}} = 1$$

2. A diode works as half-wave rectifier.



The average of voltage output from half-wave rectifier is determined from

$$\frac{\omega}{2\pi} \int_0^{\pi/\omega} V_m \sin(\omega t) dt = \frac{V_m}{\pi}$$

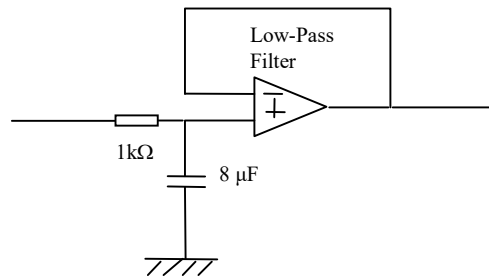
At 20 Hz,

$$\left| \frac{E_0}{E_i} \right| = \frac{0.707}{\pi} = 0.225$$

At 20 kHz,

$$\left| \frac{E_0}{E_i} \right| = \frac{1}{\pi} = 0.318$$

3. Low-pass filter is used to create DC signal.



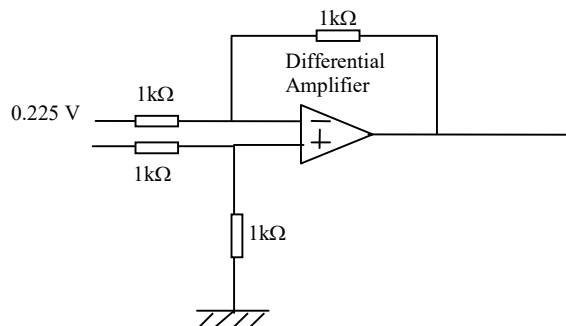
Cut-off frequency of the filter is selected also at 20 Hz.

$$f = \frac{1}{2\pi RC}$$

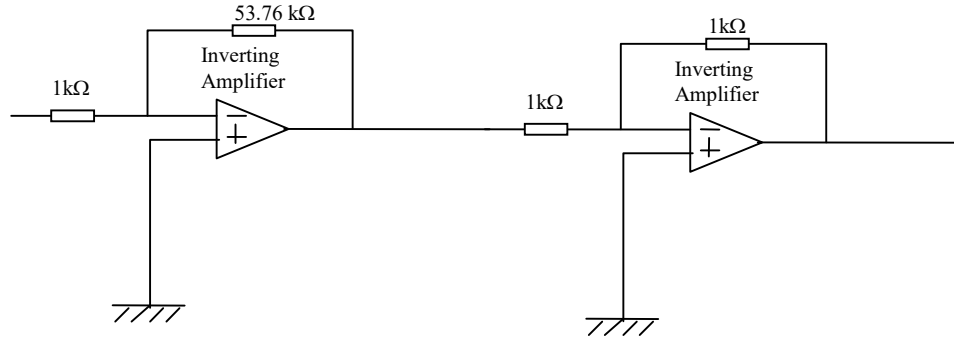
$$RC = \frac{1}{2\pi(20)} = 0.008$$

Select R = 1 kΩ, C = 8 μF.

4. Differential amplifier is used to change the voltage output range from 0.225 - 0.318 Volt to 0 - 0.093 Volt by connecting the inverting input with 0.225 Volt supply.



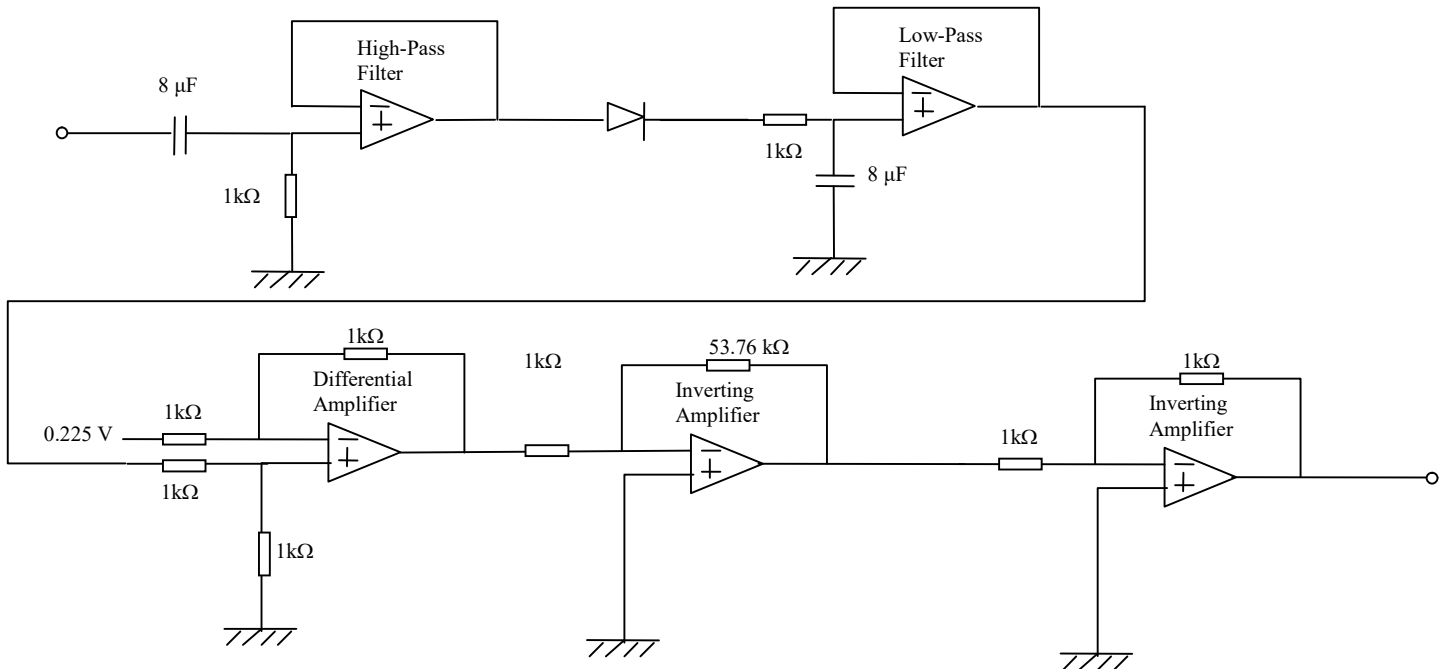
5. Two inverting amplifiers are used to convert the signal from 0 - 0.093 Volt to 0 - 5 Volts.



$$\frac{R_f}{R_1} = \frac{5}{0.093} = 53.76$$

Select $R_1 = 1 \text{ k}\Omega$, $R_f = 53.76 \text{ k}\Omega$.

6. Combine the circuits (1-5),



Q.2 A linear resistive potentiometer is used to measure a motion distance, x , in the range of 0-50 cm with the resistance varying from 0-10 kΩ. A voltmeter with input impedance of 100 MΩ is used to measure the voltage output in order to find the distance. When the distance is 20 cm, the voltage read at the voltmeter shows 4.8 Volts, determine the distance when the voltage read at the voltmeter shows 8 Volts. (25)

Solution

$$e_0 = \frac{1}{\frac{x_t}{x_i} + \frac{R_p}{R_M} \left(1 - \frac{x_i}{x_t}\right)} e_{ext}$$

When the distance is 20 cm, the voltage read at the voltmeter shows 4.8 Volts,

$$4.8 = \frac{1}{\frac{50}{20} + \frac{10}{100000} \left(1 - \frac{20}{50}\right)} e_{ext}$$

$$e_{ext} = 12$$

When the voltage read at the voltmeter shows 8 Volts,

$$8 = \frac{1}{\frac{50}{x_i} + \frac{10}{100000} \left(1 - \frac{x_i}{50}\right)} 12$$

$$x_i = 30 \text{ cm}$$

Q.3 A piezoelectric transducer having a transfer function expressed by $\frac{e_0}{x_i} = \frac{K\tau s}{\tau s + 1}$ is used to measure vibration from a system. When the vibration with amplitude 2 mm at 0.5 Hz is provided, the output reading from the transducer shows amplitude of 600 mV, and when the vibration with amplitude 5 mm at 2 Hz is provided, the output reading from the transducer shows amplitude of 2V. Determine the amplitude of vibration when the output reading shows 3 V at 3 Hz. (25)

Solution

$$\left| \frac{e_0}{x_i} \right| = \frac{K\tau\omega}{\sqrt{(\tau\omega)^2 + 1}}$$

When the vibration with amplitude 2 mm at 0.5 Hz,

$$\left| \frac{600}{2} \right| = 300 = \frac{K\tau(0.5 \times 2\pi)}{\sqrt{(\tau(0.5 \times 2\pi))^2 + 1}}$$

When the vibration with amplitude 5 mm at 2 Hz,

$$\left| \frac{2000}{5} \right| = 400 = \frac{K\tau(2 \times 2\pi)}{\sqrt{(\tau(2 \times 2\pi))^2 + 1}}$$

Thus,

$$(K\tau)^2 = \frac{1350000}{7\pi^2}$$

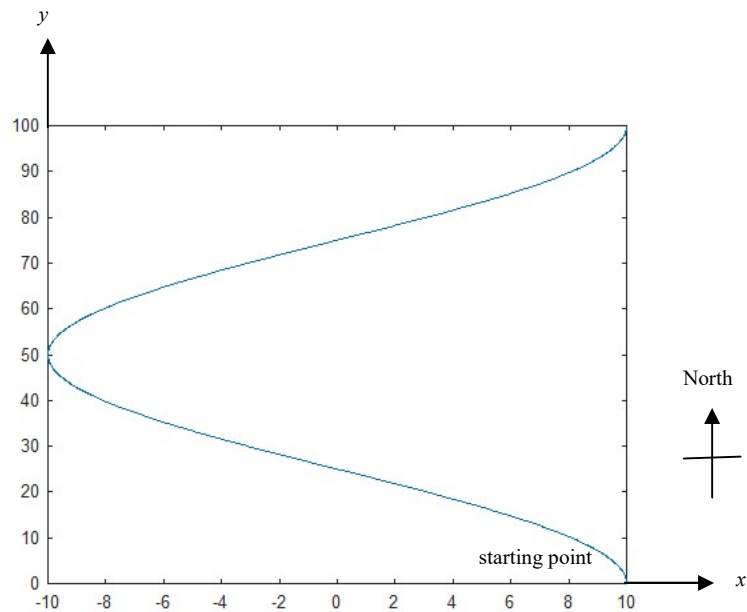
$$\tau^2 = \frac{8}{7\pi^2}$$

When the output reading shows 3 V at 3 Hz,

$$\left| \frac{3000}{x_i} \right| = \frac{K\tau(3 \times 2\pi)}{\sqrt{(\tau(3 \times 2\pi))^2 + 1}}$$

$$x_i = 7.39 \text{ mm}$$

Q.4 A rate-integrating gyro is used to determine heading direction of a vehicle running in a sinusoidal shape road as shown in the below figure. If the gyro has static sensitivity of 0.2 (positive in CW and negative in CCW), and the output deflection is reset to zero degree when the vehicle is at the starting point which heads to the north direction. Assume the gyro time constant is very short and negligible. Determine the output deflection as a function of y . The unit in both x and y axes in the figure is meter.



Solution

The road is expressed by

$$x = 10\cos\left(\frac{\pi y}{50}\right)$$

$$\frac{dx}{dy} = -\frac{\pi}{5}\sin\left(\frac{\pi y}{50}\right)$$

Heading

$$\text{atan}\left(\frac{dx}{dy}\right) = \text{atan}\left(-\frac{\pi}{5}\sin\left(\frac{\pi y}{50}\right)\right)$$

The output deflection in radian unit as a function of y ,

$$K \times \text{atan}\left(\frac{dx}{dy}\right) = 0.2\text{atan}\left(-\frac{\pi}{5}\sin\left(\frac{\pi y}{50}\right)\right)$$