

Time: 10:00-11:30 h.

Open Book

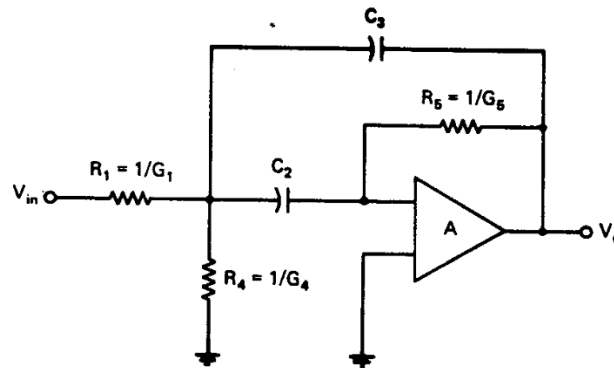
Marks: 100

Attempt all questions.

Q.1 Design a circuit that converts input signal expressed by $v_i(t) = 2 + 0.5\sin(50t) + 0.8\sin(200t)$ into output signal expressed by $v_o(t) = 4\sin(50t + \phi_1)$. The DC component is completely attenuated and the amplitude at 200 rad/s is less than 0.01. (25)

Solution

Since the signal at 50 rad/s is amplified while the signals at 0 rad/s and 200 rad/s are attenuated, band-pass filter with resonance frequency at 50 rad/s is selected.



Transfer function of this band-pass filter is expressed by

$$T = \frac{-sG_1C_2}{s^2C_2C_3 + G_5(G_1 + sC_2 + sC_3 + G_4)} = \frac{-sR_4R_5C_2}{s^2R_1R_4R_5C_2C_3 + sR_1R_4(C_2 + C_3) + R_1 + R_4}$$

The resonance frequency is at

$$\omega_r = 50 = \sqrt{\frac{G_5(G_1 + G_4)}{C_2C_3}} = \sqrt{\frac{R_1 + R_4}{R_1R_4R_5C_2C_3}}$$

The gain at the resonance frequency

$$T_r = \frac{4}{0.5} = 8 = \frac{(R_5/R_1)C_2}{C_2 + C_3}$$

The gain at 0 rad/s

$$T_0 = \frac{0}{2} = 0$$

The gain at 200 rad/s

$$T_{200} = \frac{0.01}{0.8} = 0.0125 > \frac{200R_4R_5C_2}{\sqrt{(R_1 + R_4 - 40000R_1R_4R_5C_2C_3)^2 + (200R_1R_4(C_2 + C_3))^2}}$$

There are 2 equations and 1 inequality with 5 unknown parameters. There are many solutions. Select one of these solutions. If select $C_2 = C_3 = 1 \mu\text{F}$, thus,

$$8 = \frac{R_5}{2R_1}$$

Then if select $R_1 = 1 \text{ M}\Omega$, thus

$$R_5 = 16 \text{ M}\Omega$$

From the resonance frequency equation,

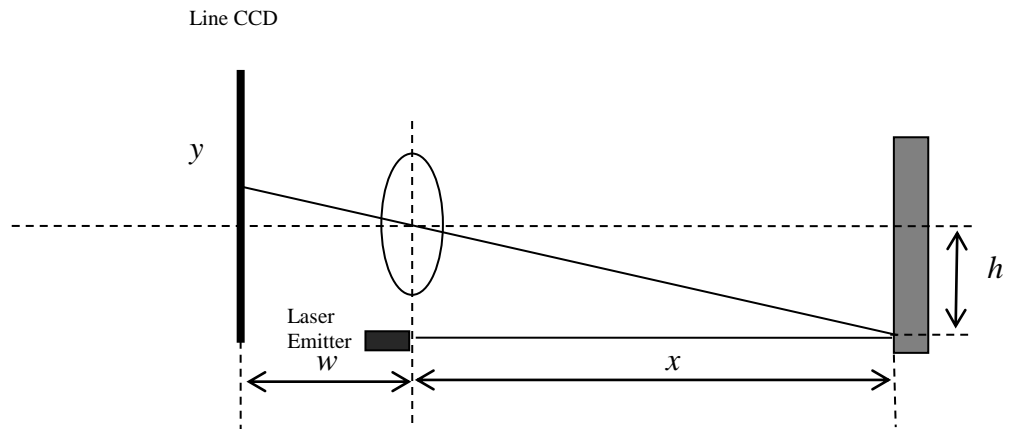
$$200 = \sqrt{\frac{1 \times 10^6 + R_4}{R_4}}$$

$$R_4 = 25 \Omega$$

Check for the gain at 200 rad/s.

$$0.0125 > \frac{200R_4R_5C_2}{\sqrt{(R_1 + R_4 - 40000R_1R_4R_5C_2C_3)^2 + (200R_1R_4(C_2 + C_3))^2}} = 0.0053$$

Q.2 Laser range finder is used to determine the range to an object as shown in the below figure. Determine the range to the object, x , as a function of position of high intensity point on the line CCD, y , and the resolution, $R = \frac{dx}{dy}$. (25)



Solution

From the similar triangles,

$$\frac{y}{h} = \frac{w}{x}$$

Thus,

$$x = \frac{wh}{y}$$

Resolution is rate of change of input respect to rate of change of output

$$R = \frac{dx}{dy} = -\frac{wh}{y^2}$$

Q.3 A rate-integrating gyro is used to determine roll angle of a satellite. Normally, when the satellite rolls at 5 degree, the deflection of the gyro is measured at 1 degree. If the spinning speed of the gyro drops to 75% of the normal speed, determine the roll angle of the satellite when the deflection of the gyro is measured at 2 degree. (25)

Solution

The sensitivity of rate-integrating gyro follows

$$K = \frac{\theta}{\phi} = \frac{H_s}{B}$$

At the normal case, when the satellite rolls at 5 degree, the deflection of the gyro is measured at 1 degree, thus

$$K_{normal} = \frac{1}{5} = \frac{H_s}{B}$$

When the spinning speed drops to 75%, thus

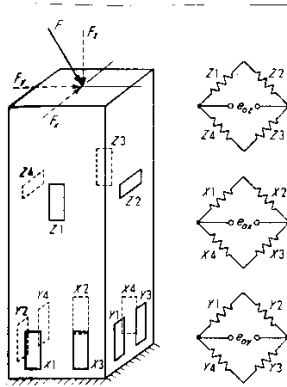
$$K_{75\%} = \frac{0.75H_s}{B} = \frac{3}{20}$$

And the deflection is measured at 2 degree, thus

$$K_{75\%} = \frac{3}{20} = \frac{2}{\phi}$$

$$\phi = \frac{40}{3} = 13.33 \text{ degree}$$

Q.4 12 identical strain gauges are used to determine magnitude and direction of unknown force as shown in the below figure.



When the force with $F_x = 20$ N, $F_y = 10$ N, $F_z = 30$ N, is applied, the voltage outputs from X, Y, and Z-Wheatstone bridge circuits are measured as 0.5 V, 1.2 V, and 2.4 V respectively. Determine the magnitude of an unknown force when the voltage outputs from X, Y, and Z-Wheatstone bridge circuits are measured as -1 V, 3 V, and -4 V respectively. (25)

Solution

The relation between force and voltage for each axis is linear relation. Sensitivity of each axis is determined.

$$S_x = \frac{V_x}{F_x} = \frac{0.5}{20} = 0.025$$

$$S_y = \frac{V_y}{F_y} = \frac{1.2}{10} = 0.12$$

$$S_z = \frac{V_z}{F_z} = \frac{2.4}{30} = 0.08$$

Determine the force along each axis,

$$\frac{-1}{F_x} = 0.025; F_x = -40$$

$$\frac{3}{F_y} = 0.12; F_y = 25$$

$$\frac{-4}{F_z} = 0.08; F_z = 50$$

Thus, the magnitude of the unknown force

$$F = \sqrt{(-40)^2 + 25^2 + 50^2} = 68.74 \text{ N}$$