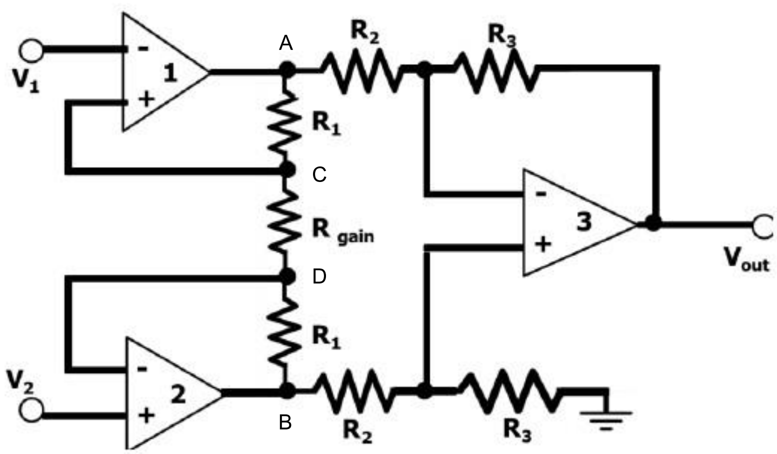
Midterm Examination Sensing and Actuation AT74.03 September 27, 2019

Time: 10:00-11:30 h. Open Book

Marks: 100

Attempt all questions.

Q.1 An instrumentation amplifier is used to amplify very low-level signals, rejecting noise and interference signals. A circuit of the instrumentation amplifier is shown below using 3 op-amps. (a) Determine the output voltage, V_{out}, as a function of the difference of the input voltage, V_2 -V_1. (b) Determine the amplifying gain when all the resistors are selected as 10 kΩ. (25)



Solution

(a)

The circuit of op-amp 3 is differential amplifier, thus.

$$V_{out} = \frac{R_3}{R_2}(V_B - V_A)$$

The voltage at both inputs of op-amp 1 and op-amp 2 must be the same, thus

$$V_C = V_1$$

$$V_D = V_2$$

The current, I, flowing between B through D through C and A is the same current, thus

$$V_D - V_C = V_2 - V_1 = IR_{gain}$$

$$I = \frac{V_2 - V_1}{R_{gain}}$$

$$V_B - V_A = I(2R_1 + R_{gain}) = \frac{V_2 - V_1}{R_{gain}}(2R_1 + R_{gain})$$

Substitute the last equation into the first equation,

$$V_{out} = \frac{R_3}{R_2}\left(\frac{V_2 - V_1}{R_{gain}}\left(2R_1 + R_{gain}\right)\right) = \frac{R_3\left(2R_1 + R_{gain}\right)}{R_2 R_{gain}}(V_2 - V_1)$$

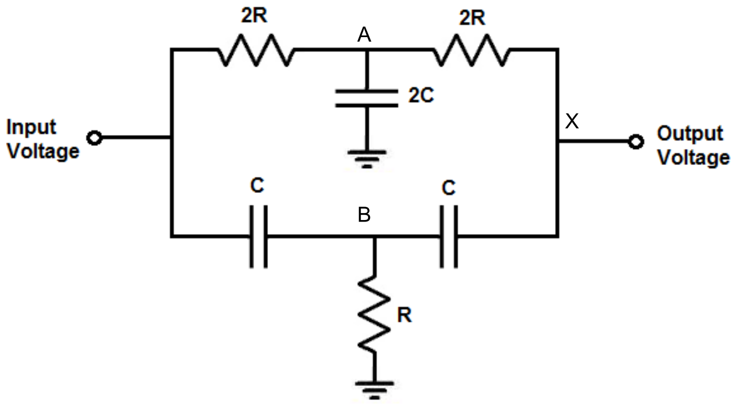
(b) When all the resistors are 10 kΩ, the amplifying gain becomes

$$\frac{V_{out}}{V_2 - V_1} = \frac{10(20 + 10)}{10(10)} = 3$$

Q.2 A passive notch filter has the circuit as shown below.

(a) Determine the transfer function from the input voltage, V_{in}, to the output voltage, V_{out}, and the notch frequency (in rad/s), ω_n, whose the output voltage becomes zero.

(b) If $R = 10$ kΩ is selected, determine value of C that eliminates the input signal at 50 Hz completely. (25)



<u>**Solution**</u>

(a)

At node A,

$$\frac{V_{in} - V_A}{2R} + \frac{V_{out} - V_A}{2R} - 2CsV_A = 0$$

$$\frac{V_{in}}{2R} + \frac{V_{out}}{2R} = \frac{(1 + 2RCs)}{R}V_A$$

$$\frac{V_{in}}{2(1 + 2RCs)} + \frac{V_{out}}{2(1 + 2RCs)} = V_A$$

At node B,

$$(V_{in} - V_B)Cs + (V_{out} - V_B)Cs - \frac{V_B}{R} = 0$$

$$V_{in}Cs + V_{out}Cs = \frac{(1+2RCs)}{R}V_B$$

$$\frac{RCsV_{in}}{(1+2RCs)} + \frac{RCsV_{out}}{(1+2RCs)} = V_B$$

At node X,

$$\frac{V_{out} - V_A}{2R} + (V_{out} - V_B)Cs = 0$$

$$\frac{V_{out}}{2R} + V_{out}Cs = \frac{V_A}{2R} + V_B Cs$$

$$\left(\frac{1+2RCs}{2RCs}\right)V_{out} - \frac{V_A}{2RCs} = V_B$$

Equate V_B,

$$\frac{RCsV_{in}}{(1+2RCs)} + \frac{RCsV_{out}}{(1+2RCs)} = \left(\frac{1+2RCs}{2RCs}\right)V_{out} - \frac{V_A}{2RCs}$$

$$-\left(\frac{2(RCs)^2}{(1+2RCs)}V_{in} + \frac{2(RCs)^2 - (1+2RCs)^2}{(1+2RCs)}V_{out}\right) = V_A$$

Equate V_A,

$$\frac{1}{2(1+2RCs)}V_{in} + \frac{1}{2(1+2RCs)}V_{out} = -\frac{2(RCs)^2}{(1+2RCs)}V_{in} - \frac{2(RCs)^2 - (1+2RCs)^2}{(1+2RCs)}V_{out}$$

$$\frac{1 + 4(RCs)^2 + 2(1+2RCs)^2}{2(1+2RCs)}V_{out} = -\frac{1+4(RCs)^2}{2(1+2RCs)}V_{in}$$

$$\frac{V_{out}}{V_{in}} = G(s) = -\frac{1+4(RCs)^2}{1+4(RCs)^2 + 2(1+2RCs)^2} = -\frac{1+4R^2C^2s^2}{3+8RCs+12R^2C^2s^2}$$

Substitute $s = \omega j$,

$$G(\omega j) = -\frac{1-4R^2C^2\omega^2}{3-12R^2C^2\omega^2 + 8RC\omega j}$$

Thus,

$$\omega_n = \frac{1}{2RC}$$

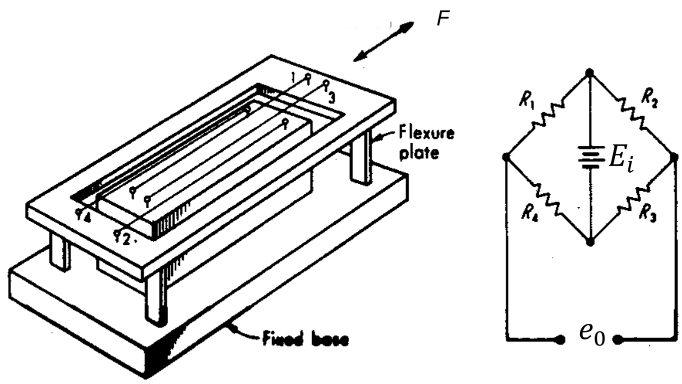
(b)

$$f_n = \frac{1}{4\pi RC}$$

$$50 = \frac{1}{40{,}000\pi C}$$

$$C = 159.2nF$$

Q.3 4 identical unbonded strain-gages; each with gage factor, S_g, of 2, nominal length, L, of 10 mm, are used to determine the distance, ΔL, resulting from the force, F, applied to the upper plate as shown below. Determine the distance when the output reading, e_0, from the DC Wheatstone bridge circuit indicates 20 mV when the supplied voltage, E_i, is 5 V. Assume that nonlinearity of the bridge circuit is negligible. (25)



Solution

The voltage output from DC Wheatstone bridge circuit is determined from

$$e_0 = \frac{r}{(1+r)^2}\left[\frac{\Delta R_1}{R_1} - \frac{\Delta R_2}{R_2} + \frac{\Delta R_3}{R_3} - \frac{\Delta R_4}{R_4}\right] E_i = \frac{\Delta R}{R} E_i = S_g \varepsilon E_i = S_g \frac{\Delta L}{L} E_i$$

$$20 = 2\frac{\Delta L}{10}5000$$

$$\Delta L = 0.02mm$$

Q.4 3-axis identical accelerometer is used to determine attitude (roll (θ), and pitch (α)) of a robot body by measuring the gravitational acceleration. When roll axis of the robot is aligned with y axis of the accelerometer and pointing to the front direction of the robot, pitch axis is aligned with x axis and pointing to the right, and yaw axis is aligned with z axis and pointing up. When the robot rolls 30 degrees then pitch -90 degrees, the output voltages from x, y, and z axes of the accelerometer become 1 V, 1.732 V, and 0 V respectively. Determine the roll angle then the pitch angle of the robot if the output voltages show -1.732 V, -0.7071 V, and -0.7071 V respectively. (25)

Solution

The gravitational acceleration detected from the accelerometer depends on the roll angle (θ), and pitch angle (α) of the robot body.

$$a_x = g sin(\theta)$$

$$a_y = -gcos(\theta)sin(\alpha)$$

$$a_z = -gcos(\theta)cos(\alpha)$$

When the accelerometer has the sensitivity of K, the output voltages become

$$V_x = Kgsin(\theta)$$

$$V_y = -Kgcos(\theta)sin(\alpha)$$

$$V_z = -Kgcos(\theta)cos(\alpha)$$

When the robot rolls 30 degrees then pitch -90 degrees,

$$1 = Kgsin(30°)$$

$$1.732 = -Kgcos(30°)sin(-90°)$$

$$0 = -Kgcos(30°)cos(-90°)$$

Thus,

$$Kg = 2$$

If the output voltages show -1.732 V, -0.7071 V, and -0.7071 V,

$$-1.732 = 2sin(\theta)$$

$$-0.7071 = -2cos(\theta)sin(\alpha)$$

$$-0.7071 = -2cos(\theta)cos(\alpha)$$

Thus,

$$\theta = -60°$$

$$\alpha = 45°$$